Title: Symbolic Magnitude Understanding Predicts Preschoolers' Later Addition Skills

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Abstract

Young children's symbolic magnitude understanding, or knowledge of how written numerals and number words can be ordered and compared, is thought to play an important role in their mathematical development. There is consistent evidence that symbolic magnitude skills predict mathematical achievement in later childhood and adulthood. Yet less is known about symbolic magnitude understanding before the start of formal schooling, a time when children are rapidly developing knowledge of small whole numbers. In this study, preschoolers (N = 140, Mean age = 4 years, 5 months) were assessed using measures of numerical skills (cardinality, symbolic magnitude, addition) and executive functioning (working memory, inhibitory control, attention shifting) in the winter and spring of the school year. Symbolic magnitude predicted later addition skills, fully mediating the relation between children's cardinality and addition skills. Moreover, children's domain-general executive functioning skills and domain-specific numeracy skills explained a similar amount of variability in children's later addition skills. Results highlight the role of symbolic magnitude in the development of children's understanding of mathematics.

Keywords: preschool mathematics; symbolic magnitude; cardinality; addition; executive functioning

Symbolic Magnitude Understanding Predicts Preschoolers' Later Addition Skills

The path towards mathematics success starts early. Children's numeracy skills in preschool and the beginning of kindergarten predict their math achievement in later grades, controlling for early reading skills, general cognitive skills, and demographic variables (Duncan et al., 2007; Watts et al., 2014). Furthermore, variability in starting points matters: Children who enter school behind in math have a slower rate of improvement throughout early elementary school, effectively widening the achievement gap with their higher performing peers (Jordan et al., 2009).

Children's understanding of numerical magnitudes plays an important role in their mathematical development (De Smedt et al., 2013; Fazio et al., 2014; Mussolin et al., 2016; Siegler, 2016). Early on in childhood, children learn the numerical symbols of their culture, including number words (e.g., one, two, three) and numerals (e.g., 1, 2, 3). Symbolic magnitude understanding refers to how these number symbols can be compared to one another and ordered. This type of numerical magnitude knowledge can support children's broader mathematical achievement through grounding the higher order manipulations of more advanced math problems, helping students learn arithmetic, select appropriate strategies when solving math problems, and identify more plausible answers (Mussolin et al., 2016). In support of this perspective, meta-analyses show the relation between symbolic magnitude knowledge and broader mathematical competence is statistically significant and positive across samples of children and adults (*rs*= .20 – .44; Chen & Li, 2014; Fazio et al., 2014; Schneider et al., 2017; 2018).

However, the majority of research examining the relations between magnitude and mathematical performance has relied on studies of school-aged children and adults. This raises

the question of whether the relation exists for children prior to the start of kindergarten, a period in which children are rapidly developing magnitude knowledge for numbers 1 through 10 (Siegler, 2016). The findings are mixed across the few available studies of preschoolers.

Symbolic Magnitude Understanding in Early Childhood

VanMarle and colleagues (2014) found that preschoolers' symbolic magnitude knowledge was significantly related to their concurrent and later math achievement. However, when children's verbal counting (e.g., counting out loud as high as possible without error) and numeral knowledge (e.g., labeling written numerals from 1 to 15) skills were included in the model, their symbolic magnitude skills no longer predicted math achievement. Kolkman and colleagues (2013) found that preschoolers' symbolic magnitude understanding predicted their symbolic magnitude understanding at age 5, which was the only direct pathway to their math achievement at age 6. Similarly, Toll and Van Luit (2014) found that preschoolers' symbolic magnitude skills predicted both the intercept and slope of their mathematical development between ages 4 and 6. These longitudinal studies suggest that early symbolic magnitude skills predict later math achievement, but other related number skills may be more predictive of preschoolers' math achievement.

Experimental studies that trained preschoolers' symbolic magnitude skills by comparing symbolic numerals or playing numerical linear board games have shown significant improvements to children's arithmetic skills (Honoré & Noël, 2016; Siegler & Ramani, 2009), but only when the training successfully increased children's symbolic magnitude understanding relative to control conditions (Cankaya et al., 2014). These findings are similar to other symbolic magnitude interventions with kindergarten children (Maertens et al., 2016).

The Role of Cardinality

Recent research suggests preschoolers' understanding of the cardinal value of numbers, or the quantities represented by individual number words and numerals, is the most significant predictor of their later math achievement (Geary & vanMarle, 2018). A child who understands the cardinal value of the number three would be able to give their parent three forks to set the table and understand that counting "one, two, three" means they have three in total (Gelman & Gallistel, 1978; Wynn, 1992). The cardinal-knowledge proposal suggests that once children understand the cardinality principle, they undergo a significant shift in their numerical knowledge that launches their understanding of symbolic mathematics (Geary et al., 2017). In particular, this proposal states that only after young children understand cardinality can they begin to understand the relations among symbolic numbers, suggesting cardinality may be a necessary antecedent of symbolic magnitude understanding (Geary et al., 2017; Geary & vanMarle, 2018).

The Role of Executive Functioning

Beyond foundational numerical skills such as cardinality, domain-general executive functioning skills may influence children's symbolic magnitude understanding. Executive functioning (EF) is defined as a set of top-down cognitive processes consisting of three core skills: inhibition, working memory, and cognitive flexibility (Diamond, 2013). EF skills relate to children's skill development broadly, including their math achievement in early childhood (Blair & Razza, 2007; Bull & Lee, 2014; Duncan et al., 2007; Geary, 2011; Nguyen et al., 2019). EF skills may allow children to concentrate, pay attention to, and learn from symbolic magnitude information in their surroundings. Furthermore, EF skills may help children inhibit incorrect responses, keep relevant information in mind, and shift their attention to the relative features of

the tasks that assess symbolic magnitude understanding. Indeed, young children's EF skills positively correlate to their symbolic magnitude and predict improvements in symbolic magnitude understanding over time (Geary et al., 2008; Geary & vanMarle, 2016; Passolunghi et al., 2007; Toll & Van Luit, 2014).

The Present Study

The present study used a longitudinal design to investigate whether symbolic magnitude understanding relates to arithmetic skills in early childhood. Our study builds from a wealth of previous research in mathematics education that views children's counting, number knowledge, and addition skills as intertwined (e.g., Baroody & Purpura, 2017; Fuson, 1992; Frye et al., 2013; Steffe, von Glasersfeld, & Cobb, 1983; Sarama & Clements, 2009). By focusing on children's year-end addition skills, we provide a more nuanced look at children's mathematical conceptual development than predicting performance on a general mathematics achievement assessment assessing multiple numerical skills (e.g., Ginsburg & Baroody, 2003; Purpura et al., 2015). Specifically, we tested whether preschoolers' symbolic magnitude understanding in the winter of the school year predicted their spring symbolic addition skills, controlling for their EF skills. We further asked whether children's symbolic magnitude knowledge mediates the relation between their cardinality and addition skills. While previous research has identified cardinality knowledge as a more substantive predictor of their later mathematics achievement than symbolic magnitude knowledge (Geary & vanMarle, 2016, 2018), the present study included more comprehensive measures of children's symbolic magnitude knowledge that could account for additional variability in their broader math ability. Specifically, we chose measures of symbolic magnitude that allowed children to compare and order numbers based on their knowledge of symbolic number words, rather than requiring knowledge of symbolic written numerals. Our inclusion of

multiple measures of each skill is both an extension from previous work and a recommended practice for more thoroughly assessing children's conceptual knowledge (e.g., Bisanz & LeFevre, 1992; Bisanz et al., 2009; Rittle-Johnson et al., 2011). In the landscape of early numerical skills, symbolic magnitude knowledge is arguably the most complex – integrating knowledge of symbolic numbers (both number words and numerals), an understanding of what quantity each number refers to, and the consecutive and non-consecutive relations among numbers from across the count sequence. Thus, symbolic magnitude knowledge may represent the culmination of the foundational numerical skills and was therefore expected to fully mediate the relation between children's cardinality and later symbolic addition skills. Our study sheds light on the developmental trajectory of preschool children's numerical skills across the school year, identifying the early skills most relevant to math learning.

Method

Participants

Participants were $140\ 3-5$ year old children (M=4 years, 5 months; range = 3 years, 2 months – 5 years, 4 months; 49% female; 60% African American/Black, 19% Asian/Pacific Islander, 9% Caucasian/White, 1% Native American/Alaskan, 11% Biracial/Multiracial; 21% Hispanic/Latino). Children were recruited from four Head Start Centers in a mid-Atlantic state during the 2018-2019 school year. Four additional children were recruited but excluded from the study due to: extreme distraction that prevented the experimenter from completing assessments (n=1); limited English comprehension and production (n=2); repeated declinations to participate (n=1).

Procedure

Children completed three 15–20-minute sessions across the school year (Figure 1). The first two sessions occurred in the winter and assessed children's cardinality, symbolic magnitude, and EF skills. The third session occurred 3-4 months later (M = 3.7 months, SD = 0.23 months) and assessed children's symbolic magnitude and arithmetic skills. Each type of children's skills was assessed with two or measures and combined to form composite scores (see Preliminary Analyses section below).

Measures

Cardinality. Children's cardinality knowledge was assessed with three tasks. In the Give-N task (Wynn, 1990), children are given 15 plastic tokens and asked to give the experimenter one. If the child provided a correct response, the experimenter next asked them for N + 1 tokens. If the child provided an incorrect response, the experimenter next asked for N - 1 tokens. The task ended when the child reached six tokens correctly or gave at least two correct responses for N and two incorrect responses for N + 1. The dependent measure was the highest number of tokens that children provided correctly.

In the Point-to-X task, children were shown two sets of objects on a piece of paper, separated by a vertical line (adapted from Levine et al., 2010). On each of 15 trials, children were asked to point to X (e.g., "two candies"). Quantities ranged from 1 to 6. The dependent variable was the percentage of trials in which the child correctly identified the target set.

In the How-Many task (adapted from Wynn, 1992), children were shown drawings of stars and asked to count them. After the child counted, the experimenter turned over the picture to "hide" the stars and asked the child how many stars were hiding. Children were shown sets of

2, 3, 4, 5, and 6 stars presented in random order. The dependent variable was the percentage of trials in which the child correctly stated the number of hidden stars.¹

Symbolic magnitude. Children's symbolic numerical magnitude knowledge was assessed with two tasks. In the symbolic magnitude comparison task, children were asked to compare 24 pairs of symbolic numerals ranging from 1-9 presented on a paper flipbook (Ramani & Siegler, 2008). After two practice trials with experimenter feedback, participants were shown 22 test pairs of numbers and asked to indicate the larger number. The test pairs were read aloud by the experimenter without accuracy feedback. Each number was counterbalanced for side of presentation (i.e., 3|8, 8|3). The dependent measure was the percentage of correct comparisons.

In the number line estimation task, children were shown 20 cm lines on a tablet computer, with 0 labeled at the left end and 10 labeled at the right end, and asked to make a mark on the line where a target number would go (Ramani & Siegler, 2008). After practice making marks on an example trial, children were administered 18 trials with numbers ranging from 1 to 9 presented in random order. The dependent variable was the accuracy of children's estimates compared to the target quantity, measured by percentage of absolute error (PAE = (|estimate - estimated quantity| / scale of estimates) x 100). PAE scores were reversed prior to analyses to aid in interpretability.

Symbolic addition. Children's symbolic addition skills were assessed with two tasks. In the forced-choice addition task (adapted from Daubert, 2018; Prather & Alibali, 2011), children were asked to make judgments about which of two imaginary children answered addition

¹ Our scoring procedure required children to correctly state the number of hidden stars, however, the original scoring procedure from Wynn (1992) only required that children's verbal count and set label matched, allowing for inaccurate counts to be scored as correct if the label followed the last-count-word principle. To account for this discrepancy, we also scored children's responses to this task using the Wynn (1992) criteria and found a comparable pattern of results to our analyses presented below.

problems correctly. Children were shown two complete problems (e.g., 2 + 1 = 3, 2 + 1 = 1). All equations were shown in the form of symbolic numerals and their corresponding non-symbolic quantities (i.e., drawings of cookies). The experimenter read both of the problems aloud and the child was asked to indicate which option "is right". Children completed seven trials and the dependent variable was the percentage of items the child answered correctly.

In the story problem task, the experimenter described someone who had some number of tokens (e.g., 3) then received additional tokens (e.g., 2), and asked how many tokens they had altogether. On each trial, the experimenter provided the number of tokens referenced for the participant to use in problem solving. Each set of tokens was placed in front of the child separately in a discrete group. Each child completed 10 trials, and the dependent variable was the percentage of trials that the child answered correctly.

Executive functioning. Three tasks were administered corresponding to the core components of EF processes: inhibition, working memory, and cognitive flexibility (Diamond, 2013). Each task was administered on a tablet computer using the National Institutes of Health (NIH) Toolbox Cognition Battery (Weintraub et al., 2013).

The Flanker task was used to assess children's inhibitory control skills (Eriksen & Eriksen, 1974). Children were shown displays with five fish and asked to touch a button indicating the direction that the middle fish was facing, which was either in the same or opposite direction as the surrounding fish. Children completed four practice trials with accuracy feedback and twenty test trials. The dependent variable was children's uncorrected standardized scores, calculated as the sum of the accuracy and reaction time scores and converted into a normative score.

Children's working memory skills were assessed with the List Sorting task (Tulsky et al., 2013). Children were shown a series of stimuli (e.g., pictures of animals) while simultaneously hearing verbal labels (e.g., dog, horse) and were asked to repeat the stimuli back to the experimenter from smallest to largest. Trials began with a list of two stimuli; if a child correctly ordered the stimuli by size, the next trial increased the list length by one. If a child incorrectly ordered a list by size, the next trial repeated the same list length. The task ended if a child incorrectly responded to two trials of the same list length. The dependent measure is the uncorrected standardized score, calculated as the normalized sum of scores across all lists presented.

The Dimensional Change Card Sort task was used to assess children's attention shifting or cognitive flexibility (Zelazo, 2006). Children were asked to sort objects by either color or shape. Children were shown four practice trials to sort by color and four to sort by shape. If a child responded correctly to three or more practice trials, they completed additional test trials where they were asked to sort by color or by shape. The dependent variable is children's uncorrected standardized scores, calculated as the sum of the accuracy and reaction time scores that are then converted into a normative score.

Results

Preliminary analyses. Descriptive statistics and bivariate correlations for all tasks are presented in Table 1. Composites were created for each construct (cardinality, symbolic magnitude, addition, EF) using principal components analysis (PCA) in SPSS Statistics Version 25 (Table 2). Unlike using a summed score or averaging standardized task scores, PCA explains the total variation in the observed variables. Eleven children were missing scores on at least one task, representing less than 3% of scores on individual tasks. To address this issue, the treatment

of missing data was handled with full information maximum likelihood (FIML) conditioned on the model covariates.

Path analysis models. We used measured variable path analysis (MVPA) conducted in Mplus8 Demo Editor, Version 1.6 to address the study aims. MVPA allows for the estimation of the complete model comprising multiple direct and indirect effects simultaneously, as opposed to conducting a multi-step mediation test with linear regression (e.g., Baron & Kenny, 1986). Our model included direct paths from cardinality to symbolic magnitude, symbolic magnitude to symbolic addition, and controlled for child age, gender, and EF skills (Figure 2). The chi-square for the model was significant $\chi 2(1) = 6.17$, p = .01, and the fit indices met the benchmark criteria for a good fit to the data. The Comparative Fit Index (CFI) was 0.98, greater than the benchmark value of 0.95 (Hu & Bentler, 1999), suggesting the model explains a good amount of variance in the data compared to a null model. The SRMR is 0.02, less than the benchmark of 0.08 (Hu & Bentler, 1999), suggesting there is little variance left unexplained after accounting for the model. These results suggest that there is adequate data-model fit and the proposed model could be retained.

All of the hypothesized structural pathways of interest were statistically significant and positive (Table 3). As hypothesized, the MVPA model indicated that children's cardinality skills significantly predicted their symbolic magnitude skills, $\beta = 0.53$, S.E. = 0.09, p < .001, and children's symbolic magnitude skills significantly predicted their addition skills, $\beta = 0.23$, S.E. = 0.09, p = .011. Because our model included single time point measures we ran additional linear regressions that demonstrated these predictive associations held when controlling for prior performance on each task (Appendix). In addition, all of the direct paths between children's EF skills and numerical skills were statistically significant. The total standardized direct and indirect

effects of children's cardinality, symbolic magnitude, and EF skills on addition are shown in Table 3. The total effects of both numerical skills and EF skills were positive, statistically significant, and comparable in magnitude.

Our MVPA model assumes that symbolic magnitude fully mediates the relation between children's cardinality and addition skills. To determine whether a partial mediation model with a direct path between cardinality and addition skills better fit the data, we compared it to the full mediation model. The partial mediation model was just-identified, therefore absolute model fit could only be assessed with a comparative measure of fit. We used the Akaike Information Criteria with a second-order bias correction (AIC_c ; Anderson, 2008). The absolute value of the difference in AIC_c s between the two models was equal to the benchmark value of 4 (AIC_c partial model = 1303.7, AIC_c full model = 1299.5), indicating some support for the full mediation model (Burnham & Anderson, 2004). This suggested the more parsimonious full mediation model may be a better fit for the data than the saturated partial mediation model.

Discussion

The present study is one of the first to investigate the relations between children's symbolic magnitude understanding and math achievement during preschool, a period when numerical understanding undergoes rapid improvement and yields wide variability in children's skills. The results provide evidence that symbolic magnitude understanding in preschool relates to children's later addition skills above and beyond cardinality and executive functioning skills.

Cardinality Predicts Symbolic Magnitude

Our findings indicate that preschoolers' cardinality knowledge predicts their symbolic magnitude understanding. Emerging research suggests that cardinality is a gateway skill to children's broader mathematical achievement (e.g., Geary et al., 2017; Geary & vanMarle, 2018;

Spaepen et al., 2018). We hypothesized that an understanding of cardinality, or what individual numbers mean, could serve as the foundation for symbolic magnitude understanding, or the relative meanings of numbers.

Our results are consistent with previous studies of young children's numerical skills, which found the age at which children understood the cardinal principle significantly predicted their symbolic magnitude skills (Geary et al., 2017; Geary & vanMarle, 2018). We extend previous studies by testing the association with a more rigorous assessment of cardinality and symbolic magnitude understanding, including multiple measures of each skill with many opportunities for children to demonstrate their knowledge. Importantly, children's understanding of cardinality as defined by the research literature can be thought of as a combination of several areas of knowledge that build on and relate to one another, including verbal-based understanding of small numbers assessed by the Point-to-X and How-Many tasks with subitizable sets, the understanding of the count-cardinal concept assessed by the How-Many task with larger sets, and finally the understanding of the cardinal-count concept assessed by the larger sets required in the Give-N task (Fuson, 1988; 1992). By including measures of each type, our results provide a broader picture of children's cardinality understanding than approaches that use only a single measure. Moreover, the selected symbolic magnitude measures allow for children to use their knowledge of symbolic magnitudes based on number words, unlike previous studies that required children to make comparisons based on their knowledge of written numerals.

Symbolic Magnitude Predicts Addition

As hypothesized, we found that children's symbolic magnitude understanding predicted their later addition skills. The Integrated Theory of Numerical Development suggests magnitude understanding provides the foundation for later math achievement in part by helping students

learn arithmetic (Siegler, 2016). One potential developmental mechanism between children's symbolic magnitude understanding and addition skills may relate to their understanding of part-whole relations. Specifically, it is possible that understanding the magnitudes of numbers and how they relate to one another could help children to recognize set combinations and the part-whole relations that form the basis of simple addition problems. This in turn could facilitate children's understanding of basic arithmetic facts, particularly in problems with non-symbolic representations of quantities like those in the present study.

Our results contrast with previous longitudinal work that found there was no significant relation between preschoolers' symbolic magnitude understanding and later math achievement when controlling for cardinality and EF skills (e.g., vanMarle et al., 2014; Geary & vanMarle, 2016). This discrepancy in findings may be due in part to our operationalization of math achievement. While previous work used children's standardized scores on the Test of Early Mathematical Ability-3 (TEMA-3; Ginsburg & Baroody, 2003), the present study used addition tasks. The TEMA-3 assesses multiple types of numerical knowledge, but our focus on simple addition problems informs a more nuanced skill development model. Furthermore, our measures of symbolic magnitude understanding included substantially more trials and presented each trial using both number words and written numerals.

Addition skills are a hallmark of early elementary mathematics instruction (Ginsburg et al., 2008). Our results suggest that magnitude understanding is also a key school readiness skill. Although intervention studies have demonstrated that children's symbolic magnitude skills are malleable with brief trainings (e.g., Maertens et al., 2016; Ramani & Siegler, 2008; Scalise et al., 2017, 2019; Whyte & Bull, 2008), observational research suggests many parents and educators refer to symbolic magnitude less frequently than other basic numerical concepts in their

conversations with young children (Klibanoff et al., 2006; Ramani et al., 2015). Practice with symbolic magnitude skills may be especially important for children from low-income households who show significant gaps on their symbolic magnitude skills compared to children from middle-income households (Scalise et al., 2017; Siegler & Ramani, 2008). More generally, our findings suggest that children's semantic proficiency with symbolic numbers predict their later computational fluency in arithmetic. Looking at the field of cognitive development broadly, future research should investigate whether this pattern of semantic understanding as a predictor of computational fluency is specific to the domains of number, mathematics, or a general pattern of cognition across domains.

Symbolic Magnitude Mediates the Association Between Cardinality and Addition

The results of the present study suggest that children's symbolic magnitude skills mediate the relation between their cardinality and addition skills. However, children's cardinality skills were a significant predictor of their symbolic magnitude skills, suggesting that both cardinality and magnitude skills are important to supporting mathematical development. Specifically, this finding suggests that children may need to master cardinality and symbolic magnitude before they are equipped to understand simple arithmetic. Our results provide further support to the theorized learning trajectories of children's counting, magnitude, and addition skills proposed by researchers in mathematics education (e.g., Baroody & Purpura, 2017; Frye et al., 2013).

Magnitude screeners can be used by researchers and educators to identify children who are in the most need of additional support (Nosworthy et al., 2013). Screening measures may also identify children who struggle with cardinality or have mastered cardinality and symbolic magnitude and are ready for symbolic arithmetic problems. The use of broad, adaptive interventions that target multiple skills may help parents and educators support children with

varying needs more effectively. For example, existing symbolic magnitude interventions with numerical card games (e.g., Scalise et al., 2017, 2019) could be modified to include more explicit cardinality scaffolding, such as strategically pairing verbal counting and set labeling to underscore the cardinal principle (Mix et al., 2012).

Executive Functioning and Addition

Children's EF skills had a significant total effect on their addition skills. EF skills also predicted children's cardinality and symbolic magnitude skills. Interestingly, children's numerical and EF skills had comparable effects on their addition skills. This suggests that EF skills in early childhood may be equivalently important to explaining variability in later addition skills as other numerical skills, particularly with samples of children from low-income households. Previous research has similarly highlighted the importance of EF skills for later math achievement for children from low-income households (e.g., Blair & Razza, 2007; Nayfeld et al., 2013), with some researchers hypothesizing that income-related gaps in children's EF skills may partially explain the observed gaps in children's math achievement (Ramani et al., 2017). Given that EF and numerical skills explained a similar amount of variability in children's addition scores, researchers should incorporate domain-general skills into broadened models of numerical development rather than as control variables.

Limitations

There are several limitations of the present study. The conceptual model and path analysis presume a causal influence between the numerical and EF skills. Although the longitudinal nature of the assessments provides some evidence in favor of the causal hypotheses, an experimental design would be more conclusive. Similarly, although the present study established support for the proposed conceptual model of symbolic magnitude understanding, it

remains an open question whether children's general ability (i.e., intelligence or processing speed) would help to explain variability in children's numerical skills over time. Furthermore, it is likely that children's other core symbolic number skills relate to their development of symbolic magnitude understanding and addition performance, including verbal counting, ordinality, and numeral identification skills (Carey, 2009; Purpura et al., 2013; Reynvoet & Sasanguie, 2016). Additional longitudinal and microgenetic work is needed to better capture incremental changes in children's symbolic number knowledge, bolstering the evidence from landmark cross-sectional studies of children's early math skills (e.g., Gelman & Gallistel, 1978; Schaeffer et al., 1974; Wynn, 1992). In keeping with previous studies (e.g., Levine et al., 2010; Prather & Alibali, 2011; Ramani & Siegler, 2008), we analyzed children's responses to measures with two options as interval data, however future research should consider adapting scoring methods that capture children's performance as significantly below-chance, at-chance, or significantly above-chance. Finally, our measures of addition skills may have overestimated children's abilities to solve addition word problems by including non-symbolic representations of quantities. It is possible that children who did not understand addition word problems interpreted the goal of these tasks as counting objects, which could in turn lead to correct responses without an underlying understanding of the concept. However, this seems unlikely given that there was wide variability in children's performance and the average was not at ceiling (M = 57.2% of trials correct, SD = 30.3%). Moreover, the intuition to see two sets and recognize to count them together as a single set could signal an early form of children's addition understanding – recognizing that separate parts can form a whole.

Conclusion

Symbolic magnitude understanding is thought to play an important role in mathematical achievement across the lifespan. However, this is one of few studies to have directly examined the role of symbolic magnitude understanding during the preschool period. The findings provide evidence that symbolic magnitude understanding predicts concurrent and later math skills, above and beyond the effects of cardinality, EF, child age, and child gender. The current study helps to explain the developmental trajectory of preschool children's numerical skills across the academic year, providing key insights for those seeking to support early mathematical development.

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Conflict of Interest Statement

We have no conflicts of interest to declare.

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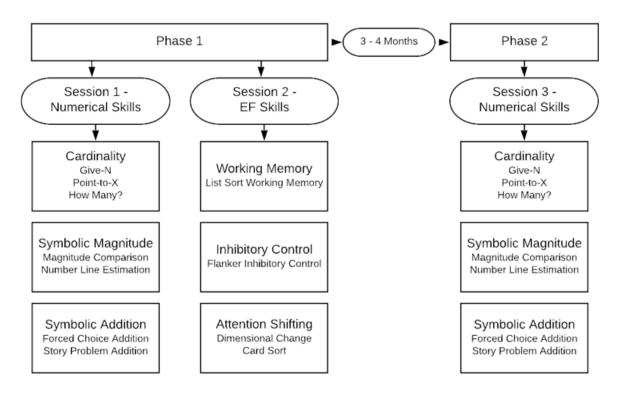


Figure 1. Study Procedure

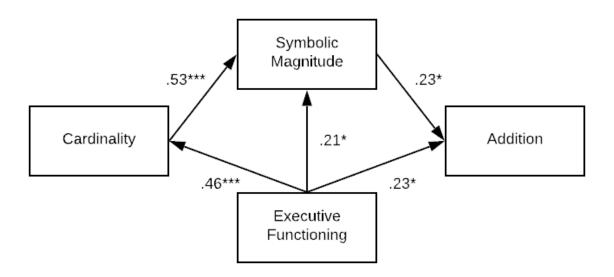


Figure 2. Measured variable path analysis of full mediation model

Table 1. Descriptive statistics and bivariate correlations

| | N | M | SD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------------------------|-----|------|------|--------|--------|--------|--------|--------|--------|--------|--------|------|----|
| 1. Give-N | 139 | 3.74 | 2.08 | | | | | | | | | | |
| 2. Point-to-X | 139 | 81.8 | 16.0 | .74*** | | | | | | | | | |
| 3. How many | 138 | 55.2 | 41.4 | .62*** | .61*** | | | | | | | | |
| 4. Magnitude comparison | 138 | 67.0 | 20.9 | .68*** | .63*** | .65*** | | | | | | | |
| 5. Number line estimation | 138 | 70.3 | 11.3 | .34*** | .33*** | .36*** | .41*** | | | | | | |
| 6. Working memory | 136 | 22.8 | 26.3 | .43*** | .44*** | .49*** | .53*** | .16 | | | | | |
| 7. Inhibitory control | 136 | 37.1 | 17.6 | .46*** | .35*** | .46*** | .45*** | .28** | .36*** | | | | |
| 8. Attention shifting | 136 | 40.7 | 19.2 | .41*** | .41*** | .44*** | .48*** | .31*** | .38*** | .50*** | | | |
| 9. Forced choice addition | 134 | 72.3 | 21.7 | .14 | .17 | .15 | .26* | .08 | .14 | .21* | .14 | | |
| 10. Story problem addition | 132 | 57.2 | 30.3 | .64*** | .58*** | .60*** | .53*** | .32*** | .43*** | .52*** | .39*** | .22* | |

Note: ***p < .001. ** p < .01. *p < .05.

Table 2. Factor Loadings by Composite Score

| Measure | Factor Loading |
|-----------------------------------|----------------|
| Cardinality $(N = 138)$ | |
| Give-N | .899 |
| Point to X | .895 |
| How Many? | .839 |
| Magnitude ($N = 138$) | |
| Magnitude comparison | .841 |
| Number line estimation | .841 |
| Executive functioning $(N = 136)$ | |
| Working memory | .723 |
| Inhibitory control | .804 |
| Attention shifting | .812 |
| Addition $(N = 132)$ | |
| Forced choice | .779 |
| Story problem | .779 |

Note: Composites were created from Time 1 scores for cardinality, magnitude, and executive functioning measures, and Time 2 scores for addition measures.

Table 3. Measured Variable Path Analysis Model of Numerical and Executive Functioning Skills With Full Information Maximum Likelihood

| | | Dependent Variable | | | | | | | | | | |
|------------------|-----|--------------------|------|-------------|------------|-------------|-------------|-----------|---------|-----|----------|------|
| | (| Cardinalit | y | Symb | olic Mag | nitude | Execu | tive Func | tioning | | Addition | 1 |
| Source | В | SE | p | В | SE | p | В | SE | p | В | SE | p |
| Gender | .04 | .01 | .001 | .01 | .01 | .563 | .07 | .01 | .001 | .03 | .01 | .019 |
| Age | .27 | .12 | .032 | 11 | .12 | .391 | .20 | .15 | .159 | .30 | .15 | .039 |
| EF | .46 | .07 | .001 | .21 | .08 | .013 | | | | .23 | .10 | .017 |
| Cardinality | | | | .53 | .09 | .001 | | | | | | |
| Magnitude | | | | | | | | | | .23 | .09 | .011 |
| | | | Tot | tal indirec | t and dire | ect effects | on addition | on | | | | |
| Direct effects | | | | .23 | .09 | .011 | .23 | .10 | .017 | | | |
| Indirect effects | .12 | .05 | .018 | | | | .10 | .05 | .021 | | | |
| Total effects | .12 | .05 | .018 | .23 | .09 | .011 | .33 | .09 | .001 | | | |

Note: N = 140 with Full Information Maximum Likelihood estimation.

Appendix

Multiple linear regression analysis was used to test if children's Time 1 cardinality skills significantly predicted their Time 2 symbolic magnitude skills, controlling for Time 1 symbolic magnitude skills, child age, and child gender. The results indicated that the regression was statistically significant and accounted for 58% of the variance in Time 2 symbolic magnitude skills ($R^2 = .58$, F(4, 127) = 43.42, p < .001). Specifically, Time 1 cardinality skills significantly predicted Time 2 symbolic magnitude skills ($\beta = .50$, t(127) = 5.85, p < .001), meaning a one standard deviation change in Time 1 cardinality skills was associated with a 0.50 standard deviation change in Time 2 symbolic magnitude skills, holding Time 1 symbolic magnitude skills, age, and gender constant (Table A1).

Multiple linear regression analysis was also used to test if children's Time 1 symbolic magnitude skills significantly predicted their Time 2 addition skills, controlling for Time 1 addition skills, child age, and child gender. The results indicated that the regression was statistically significant and explained 37% of the variance in children's Time 2 addition skills ($R^2 = .37$, F(4, 126) = 18.14, p < .001). Specifically, Time 1 symbolic magnitude skills significantly predicted Time 2 addition skills ($\beta = .23$, t(126) = 2.71, p = .008), meaning a one standard deviation change in Time 1 symbolic magnitude skills was associated with a 0.23 standard deviation change in Time 2 addition skills, holding Time 1 addition skills, age, and gender constant (Table A2).

Table A1 $Summary\ of\ Multiple\ Regression\ Analysis\ Predicting\ Time\ 2\ Symbolic\ Magnitude\ (N=132)$

| | В | SE | β |
|---------------------------|-----|----------|--------|
| Age | .02 | .01 | .12 |
| Gender | 06 | .12 | 03 |
| Time 1 symbolic magnitude | .25 | .08 | .25** |
| Time 1 cardinality | .51 | .09 | .50*** |
| R^2 | | .58 | |
| F | | 43.42*** | |

Note: ** p < .01; *** p < .001.

Table A2

Summary of Multiple Regression Analysis Predicting Time 2 Addition (N = 131)

| | В | SE | β |
|---------------------------|-----|----------|--------|
| Age | .03 | .01 | .18* |
| Gender | .27 | .14 | .14 |
| Time 1 addition | .31 | .08 | .32*** |
| Time 1 symbolic magnitude | .23 | .08 | .23* |
| R^2 | | .37 | |
| F | | 18.14*** | |

Note: * p < .05; ** p < .01; *** p < .001.