An Improved Ptychographic Algorithm for Multi-Pulse Phase Retrieval

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Abstract: We present a ptychographic phase retrieval algorithm which solves the square root problem in second order pulse measurement techniques and reconstructs the fields of multiple incoherent pulses simultaneously from a single dispersion scan trace.

1. Basic Ptychographic Phase Retrieval Algorithm

The extremely short duration of ultrafast pulses makes them difficult to measure and has spawned many techniques, such as FROG and SPIDER, to measure their phase and amplitude structure [1]. The dispersion scan (d-scan) was recently developed as a simple method for characterizing ultrafast pulses [2]. In d-scan, the pulse compressor itself is used to iteratively vary the dispersion of the pulse while recording the spectrum of the second harmonic generation (SHG). This generates a two dimensional trace that can be inverted using an iterative algorithm to retrieve the phase and amplitude of the fundamental pulse.

We have developed our own ptychographic phase retrieval algorithm, inspired by similar algorithms used for SHG FROG and d-scan [3, 4], to retrieve the field from a d-scan trace. Each column of a d-scan trace represents a SH spectrum acquired with different amounts of applied dispersion, $\phi_k^{App}(\omega)$. The standard serial version of our algorithm schematically works by making a guess for the fundamental field, $\tilde{E}_k(\omega)$, and then propagating the guess through the compressor at the k^{th} dispersion setting: $\tilde{X}_k(\omega) = \tilde{E}_k(\omega)\exp(i\phi_k^{App}(\omega))$. The guess field is then propagated through the doubling crystal assuming an instantaneous and lossless nonlinearity to obtain the SHG field: $\psi_k(t) = \left(\mathscr{F}^{-1}\{\tilde{X}_k(\omega)\}\right)^2$. A modulus constraint is then applied to the SHG guess by replacing the modulus of the guess with the amplitude of the measured SHG spectrum, $T_k(\omega)$, at the k^{th} dispersion setting while preserving the guess phase $\tilde{\psi}'_k(\omega) = \sqrt{T_k(\omega)}\frac{\tilde{\psi}_k(\omega)}{|\tilde{\psi}_k(\omega)|}$. The constrained SHG field is then propagated back through the doubling crystal with $X'_k(t) = X_k(t) + \alpha \frac{X_k(t)^*}{|X_k(t)|^2}_{max}(\psi'_k(t) - \psi_k(t))$ to obtain an updated guess for the fundamental field after the compressor. Here, α is a weighting parameter that controls the strength of the update, and *max* refers to the peak value of $|X_k(t)|^2$. The *max* operation is used to numerically stabilize against creating singularities in the field. Lastly, the field is propagated back through the compressor by removing the current applied dispersion $\tilde{E}_k(\omega) = \tilde{X}_k(\omega)\exp(-i\phi_k^{App}(\omega))$ to obtain the true fundamental field before the compressor. This process repeats through all K applied dispersion settings (columns) in the trace until the algorithm converges on a stable solution for the field $\tilde{E}(\omega)$.

2. Extensions to the Basic Algorithm

In SHG pulse measurement techniques, a fundamental problem exists where the SHG field must be back-propagated through the doubling crystal to arrive at a new guess for the fundamental. This amounts to taking the square root of the SHG field and gives rise to an ambiguity in the overall sign of the fundamental field (i.e. $\sqrt{z} = \pm\sqrt{r}e^{i\theta/2}$). Typically the principal root is chosen to solve the ambiguity. However, the principal root may not always be the correct choice, and could lead to errors in the reconstruction of the pulse. The connection between ptychographic algorithms and Newton's method as an overall amplitude minimization has previously been made [5]. However, it has not previously been shown that in the special case of $\chi^{(2)}$ nonlinearities, the fundamental field update ($X'(t)$) can be shown to be exactly calculating the square root of the SHG field with the selection of $\alpha = 1/2$. The fundamental field update then becomes: $X'_k(t) = \frac{1}{2}\left[2X_k(t) + \frac{X_k(t)^*}{|X_k(t)|^2}_{max}(\psi'_k(t) - \psi_k(t)\right]$.

This simple alteration of the field update is a direct analogy of Newton's second order method for calculating the square root of a number [6]. In the ptychographic algorithm, this field update allows the algorithm to progress naturally without enforcing a principal root. Additionally, we have shown that this n^{th} root field update can be extended to other perturbative nonlinearities such as third harmonic generation (THG).

In addition to the above serial algorithm, we have developed a parallel algorithm. In it, the guess for the field is applied to the K columns of the trace simultaneously, and the K updated fundamental guess fields are averaged together to form a guess for the next iteration. This enforces the idea that the trace is generated from a set of identical pulses that only vary in dispersion, and improves the stability and robustness of the algorithm in the presence of noise.

We can also extend the parallel algorithm to allow it to solve for multiple incoherent fields, or modes, simultaneously that may be present in a d-scan trace. Multi-mode traces are formed when multiple pulses generate an SHG signal during the integration time of the spectrometer, but do not produce interference. In such cases, the total measured trace, $T^{(T)}$, is the incoherent sum of the M individual traces from each mode: $T^{(T)} = \sum_m^M T^{(m)}$. An experimental example of this is where two pulses from successive round trips in a regenerative amplifier, who's spectral interference fringes cannot be resolved, are present in the trace. To accommodate multiple modes, multiple temporal and spectral axes can be used in the retrieval, one for each mode in the trace. The multi-mode version of the algorithm is then the same as the parallel algorithm where a guess is made for each of the M modes and is applied to the K columns in the trace simultaneously. The modulus constraint, however, is modified so that it represents the fact that the trace is generated from the incoherent sum of multiple traces: $\tilde{\psi}^{(m)\prime}(\omega) = \sqrt{T^{(T)}} \frac{\tilde{\psi}^{(m)}(\omega)}{\sqrt{\sum_m^M |\tilde{\psi}^{(m)}(\omega)|^2}}$. After this step, the algorithm proceeds just as in the single mode parallel case with all modes being calculated independently of one another. In Figure 1, we can see the result of a multi-mode retrieval of a simulated two-mode trace using grating compressor d-scan. The two modes originated as the same fifth-order limited pulse and one of the pulses was numerically propagated through an extra 30mm of BK7 glass.

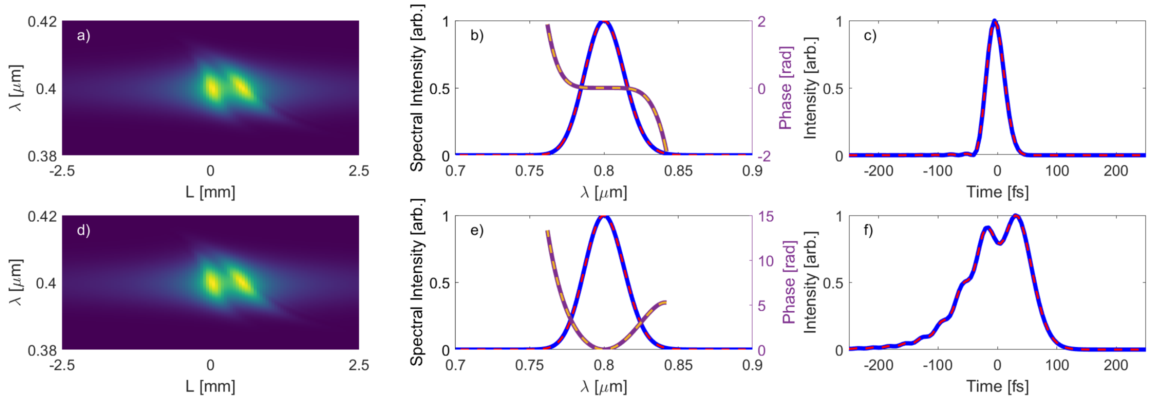

Fig. 1: Simulated multi-mode phase retrieval of a two mode trace. a) and d) are the simulated and retrieved traces respectively. b) and e) are the simulated (blue) and retrieved (red-dashed) spectra of the two modes as well as the simulated (purple) and retrieved (yellow-dashed) spectral phases of the two modes. c) and f) are the simulated (blue) and retrieved (red-dashed) temporal intensities of the two modes. The RMS error between the simulated and retrieved traces when the algorithm was terminated was approximately machine precision (10^{-16}).

The multi-modal algorithm shows promise as a way for diagnosing multi-pulse techniques such as pump-probe experiments without having to measure each pulse independently. Additionally, it can be used to help diagnose defects in a laser system when satellite pulses are separated far enough in time that other techniques such as FROG would not see them. We plan to experimentally implement the multi-modal algorithm using grating compressor d-scan in the near future.

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