












Superconducting and antiferromagnetic properties of dual-phase V_3Ga

Cite as: Appl. Phys. Lett. **117**, 062401 (2020); <https://doi.org/10.1063/5.0015535>

Submitted: 29 May 2020 . Accepted: 27 July 2020 . Published Online: 11 August 2020

 Michelle E. Jamer, Brandon Wilfong,  Vasily D. Buchelnikov,  Vladimir V. Sokolovskiy,  Olga N. Miroshkina,  Mikhail A. Zagrebin,  Danil R. Baigutlin,  Jared Naphy, Badih A. Assaf,  Laura H. Lewis,  Aki Pulkkinen,  Bernardo Barbiellini, Arun Bansil, and  Don Heiman



View Online



Export Citation



CrossMark

ARTICLES YOU MAY BE INTERESTED IN

Hybrid anisotropic pentamode mechanical metamaterial produced by additive manufacturing technique

Applied Physics Letters **117**, 061901 (2020); <https://doi.org/10.1063/5.0014167>

Light-emitting diodes with AlN polarization-induced buried tunnel junctions: A second look

Applied Physics Letters **117**, 061104 (2020); <https://doi.org/10.1063/5.0015097>

Cherenkov-type three-dimensional breakdown behavior of the Bloch-point domain wall motion in the cylindrical nanowire

Applied Physics Letters **117**, 062402 (2020); <https://doi.org/10.1063/5.0013002>





Webinar
How to Characterize Magnetic
Materials Using Lock-in Amplifiers



Zurich
Instruments



[Register now](#)

Superconducting and antiferromagnetic properties of dual-phase V_3Ga

Cite as: Appl. Phys. Lett. **117**, 062401 (2020); doi: 10.1063/5.0015535

Submitted: 29 May 2020 · Accepted: 27 July 2020 ·

Published Online: 11 August 2020



View Online



Export Citation



CrossMark

Michelle E. Jamer,^{1,a)} Brandon Wilfong,¹ Vasily D. Buchelnikov,^{2,3} Vladimir V. Sokolovskiy,^{2,3} Olga N. Miroshkina,^{2,4} Mikhail A. Zagrebin,^{2,3,5} Danil R. Baigutlin,^{2,4} Jared Naphy,¹ Badih A. Assaf,⁶ Laura H. Lewis,⁷ Aki Pulkkinen,^{4,8} Bernardo Barbiellini,^{4,9} Arun Bansil,⁹ and Don Heiman⁹

AFFILIATIONS

¹Physics Department, United States Naval Academy, Annapolis, Maryland 20899, USA

²Faculty of Physics, Chelyabinsk State University, 454001 Chelyabinsk, Russia

³National University of Science and Technology "MISIS," 119049 Moscow, Russia

⁴Department of Physics, School of Engineering Science, LUT University, FI-53850 Lappeenranta, Finland

⁵National Research South Ural State University, 454080 Chelyabinsk, Russia

⁶Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556, USA

⁷Chemical Engineering Department, Northeastern University, Boston, Massachusetts 02115, USA

⁸Département de Physique and Fribourg Center for Nanomaterials, Université de Fribourg, CH-1700 Fribourg, Switzerland

⁹Physics Department, Northeastern University, Boston, Massachusetts 02115, USA

^{a)}Author to whom correspondence should be addressed: jamer@usna.edu

ABSTRACT

The binary compound V_3Ga can exhibit two near-equilibrium phases, the A15 structure that is superconducting and the Heusler $D0_3$ structure that is semiconducting and antiferromagnetic. Density functional theory calculations show that these two phases are nearly degenerate, being separated in energy by only ± 10 meV/atom. Our magnetization measurements on bulk-grown samples show antiferromagnetism and superconducting behavior below 14 K. These results indicate the possibility of using V_3Ga for quantum technology devices exploiting the co-existence of superconductivity and antiferromagnetism in a dual-phase material.

<https://doi.org/10.1063/5.0015535>

Superconductivity and magnetism were once thought to be mutually exclusive because magnetic fields are efficient at closing the superconducting gap. Nevertheless, it was found that superconductivity has been found in 3d materials with magnetic transition-metal atoms and magnetic lattices as well.¹ High- T_c cuprate superconductors, for example, were found to have exceedingly strong magnetic exchange,² while Fe-based superconductors were found to have large Fe moments of several Bohr magnetons.^{3,4} Also of interest here are the binary vanadium compounds, such as V_3Al , which belong to a class of simple superconductors with an A15 (β -W) crystal structure.^{5–8} Interestingly, V_3Al has also been synthesized in a non-superconducting $D0_3$ Heusler phase with antiferromagnetic (AFM) order.⁹ This $D0_3$ phase of V_3Al was predicted to be a gapless semiconductor^{10,11} and found experimentally⁹ to be a G-type antiferromagnet having a Néel temperature of $T_N = 600$ K. It is clear that V_3Z -type compounds represent a class of hybrid materials that could possess both superconducting and magnetic properties at the same temperature, which could provide potential

next-generation platforms for hosting Majorana modes¹² for applications in possible fault-tolerant quantum computer hosting and other quantum technology applications.

Another well-known binary compound in the vanadium family is V_3Ga , which has been used in superconducting applications for many years.¹³ The remarkable low-temperature elastic, electric, magnetic, and superconducting properties of this material have been investigated extensively both experimentally and theoretically (see, e.g., Refs. 14–20). The critical temperature of superconducting V_3Ga in the A15 phase is 15 K.

V_3Ga can exist in two near-equilibrium phases, the A15 superconducting phase and the AFM $D0_3$ phase—an interesting and potentially useful result of their similar formation energies. Since the arrangement of atoms in binary V_3Ga can accommodate both $D0_3$ and A15 structures (Fig. 1), one must study the stability of these two phases using density functional theory (DFT). DFT was used here to compute the formation energies for various crystalline and magnetic

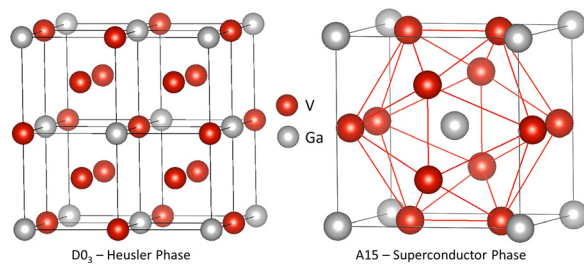


FIG. 1. $D0_3$ (space group: $Fm\bar{3}m$, no.: 225, and prototype, BiF_3) and β -W A15 (space group: $Pm\bar{3}n$, no.: 223, and prototype, Cr_3Si) crystal structures of V_3Ga .

structures. Previous calculations for the $D0_3$ structure of V_3Ga by Galanakis *et al.*¹¹ predict a Heusler G-type AFM phase with a Néel temperature well above room temperature, which makes the compound attractive for spintronic applications.^{9,21,22} A recent study reported on an AFM phase of V_3Ga in the β -W structure.²³ The present magnetization measurements on bulk samples show a clear AFM behavior, in addition to a strong Meissner effect, indicating the presence of a superconducting transition temperature of 15 K.

Bulk samples of V_3Ga were synthesized via arc-melting using an Edmund Buehler MAM-1. The ingots were subsequently annealed at 1050 °C for 48 h in an argon environment to promote homogeneity and quenched in an ice bath. The composition was confirmed using energy dispersive spectroscopy (EDS) to be within $\pm 2\%$ of the nominal composition. Magnetic measurements were performed on a Quantum Design MPMS XL-5 SQUID magnetometer with magnetic fields up to 5 T and at temperatures from $T = 2$ to 400 K. Synchrotron x-ray diffraction was measured at beamline 11-BM at the Advanced Photon Source (APS). The structure was refined using TOPAS.²⁴ Overall, the structural analysis indicates the presence of both A15 and $D0_3$ phases of V_3Ga . Rietveld refinement in which the oxide contribution was not taken into account found 81% A15 and 19% $D0_3$ phases (see the [supplementary material](#) for further discussion.)

The results of magnetic measurements are shown in Figs. 2 and 3. In particular, Fig. 2 shows a magnetic hysteresis from the Meissner effect at low temperature, which is characteristic of flux pinning in the A15 type-II superconducting phase. The superconducting transition temperature $T_c = 13.6$ K is deduced from the plot of the magnetization as a function of temperature (Fig. 2, inset). This value of the critical temperature is in line with previous studies, although it is slightly lower than the maximal value reported for optimal stoichiometric V_3Ga .^{25,26} It is likely that our A15 V_3Ga phase is slightly off-stoichiometry within the 2% error in our EDS measurements, thus giving rise to the slightly decreased critical temperature in our measurements. Magnetic susceptibility shown in the [supplementary material](#) indicates $\sim 90\%$ superconducting volume fraction, which is in line with the compositional analysis of the x-ray diffraction data through Rietveld refinement. The inset of Fig. 3 shows the magnetic moment as a function of applied field at $T = 300$ K. The moment is linear up to at least $\mu_0 H = 5$ T, where the moment is only $\mu = 0.007 \mu_B$ per formula unit (f.u.). This linear-in- H moment is similar to that we previously found in the AFM $D0_3$ phase of V_3Al .⁹ The H_{c2} of our A15 phase is about 3.5 T for isothermal magnetization at 10 K (see the [supplementary material](#)), which is lower than the currently accepted value of 15 T at 10 K for A15 V_3Ga .^{26,27} The decrease in H_{c2} is most likely due to the decreased T_c of

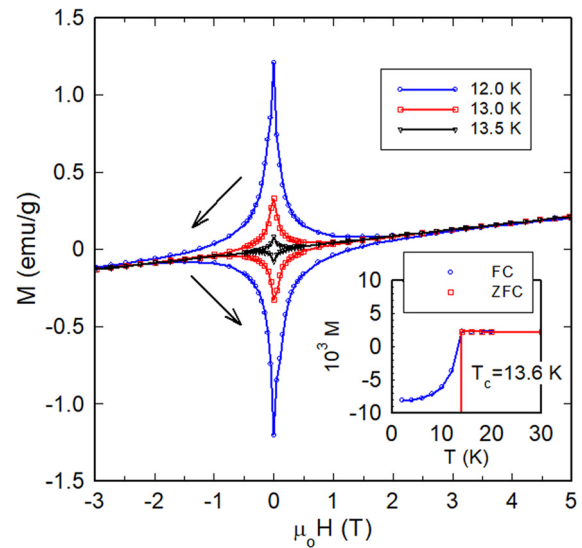


FIG. 2. Magnetization of V_3Ga vs magnetic field at low temperatures under zero field cooling (ZFC), showing a hysteretic peak around $\mu_0 H = 0$ characteristic of flux pinning in a type-II superconductor. The inset shows the temperature-dependent Meissner flux exclusion below $T_c = 13.6$ K taken in a field of $\mu_0 H = 0.05$ T.

the superconducting phase. The reduced T_c could result from the presence of the antiferromagnetic phase causing disorder in the compound, the possible off-stoichiometry, or grain size effects. Grain size effects on H_{c2} cannot be ruled out; however, since our synthesis method and those previously reported are similar, it is unlikely that grain size for the as-synthesized A15 V_3Ga phase would differ drastically from those previous reports.^{26,27} Notably, off-stoichiometry of A15 V_3Ga ²⁶ in previous studies was reduced T_c to a larger degree than what we observed;

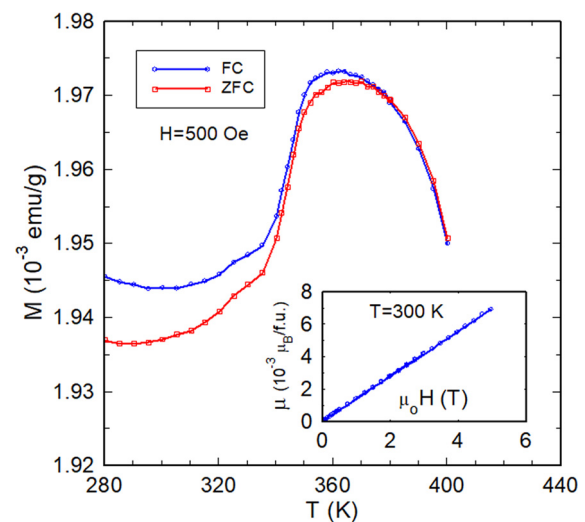


FIG. 3. Magnetic properties of AFM V_3Ga vs temperature and magnetic field. Magnetization vs temperature taken at $\mu_0 H = 0.05$ T shows a peak at 360 K, indicating a magnetic transition. Inset: Magnetic moment vs magnetic field taken at $T = 300$ K, showing a small moment of only $\mu = 0.007 \mu_B$ f.u. at $\mu_0 H = 5$ T.

TABLE I. Calculated equilibrium properties of various magnetic states (Mag. st.) of V_3Ga . Lattice constants (a_0 in Å), atom-resolved and total magnetic moments (μ_{V_i} and μ_{tot} in $\mu_B/f.u.$), total energy (E_0 in eV/atom), and the energy difference between the $D0_3$ and A15 structures (ΔE in meV/atom) are given. The results are based on various exchange-correlation approaches (LDA, GGA, GGA+U, and SCAN) as described in the text. For the A15 structure, the SCAN solutions for the FM and AFM-III states, which lie within 5 meV/atom, are nearly degenerate. See the [supplementary material](#) for details.

D0 ₃								A15								
	Mag. st.	a_0	μ_{V_1}	μ_{V_2}	μ_{V_3}	μ_{tot}	E_0		Mag. st.	a_0	μ_{V_1}	μ_{V_2}	μ_{V_3}	μ_{tot}	E_0	ΔE
LDA	AFM	5.902	0.429	−0.429	0.0	0.0	−8.487	FM	4.678	0.089	0.164	0.112	0.368	−8.572	85	
GGA	AFM	6.064	−1.314	1.314	0.0	0.0	−7.600	FM	4.788	0.222	0.390	0.303	0.916	−7.645	45	
GGA+ <i>U</i>	AFM	6.130	−1.916	1.917	0.0	0.0	−6.647	AFM-III	4.879	±1.351	±1.508	±1.502	0.0	−6.632	−15	
SCAN	AFM	6.035	−1.848	1.848	0.0	0.0	−17.486	FM	4.744	0.308	0.523	0.414	1.245	−17.442	−44	
								AFM-III	4.744	±0.268	±0.326	±0.330	0.0	−17.437	...	

a reduction of 1.7 K was shown to be caused by the stoichiometry being off by 10%, much larger than in our samples. It is reasonable to attribute some of the decrease in T_c to the AFM signal of another magnetic V_3Ga phase present in the as-synthesized compound mixture.

At higher temperatures, there is a notable peak in the temperature dependence of the low-field ($\mu_0 H = 0.05$ T) moment at $T = 360$ K, shown in Fig. 3. A similar feature was seen in the temperature-dependent resistivity of V_3Ga at 350 K;²³ however, the peak was not assigned to a magnetic transition. Similar temperature-dependent features in both the magnetization and resistivity could arise from either a magnetic or a magnetostructural transition. A small peak in the magnetization of an antiferromagnet is generally characteristic of a Néel temperature,⁹ so there is a reasonable case to assign the small peak observed here in the magnetization of V_3Ga to a magnetic transition.

In order to interpret our experimental results, first-principles electronic structure calculations were carried out within the framework of the DFT as implemented in the VASP software package.^{28,29} Various approximations for the exchange-correlation energy employed to check the robustness of our results include the local-density approximation (LDA),³⁰ the generalized-gradient approximation (GGA),^{31–33} GGA+U with Hubbard U correction, and the strongly constrained and appropriately normed (SCAN) meta-GGA.^{34–36} Lattice and spin relaxations were included in the computations. Total energies and residual forces were converged to an accuracy of 10^{-8} eV and 10^{-7} eV/Å, respectively. Following He *et al.*,²³ $U = 2.0$ eV and $J = 0.67$ eV were used in GGA+U calculations.

The equilibrium properties of various magnetic states of V_3Ga are summarized in Table I (see the [supplementary material](#) for details). Electron correlation effects beyond the GGA as seen from the energy differences (ΔE) for the GGA+U and SCAN results tend to stabilize the $D0_3$ with respect to the A15 structure. Both LDA and GGA yield a ferromagnetic (FM) ground state for the A15 phase. With an improved treatment of electron correlations using the GGA+U and SCAN, however, the ground state becomes the AFM G-type $D0_3$ solution. Regarding the A15 solution, the effect of correlations is to stabilize the AFM-III state²³ (see the [supplementary material](#)) with respect to the FM solution. Within SCAN, the FM and AFM-III solutions in the A15 structure are almost degenerate and lie within 5 meV/atom (FM is marginally more stable). GGA+U, however, fully stabilizes the AFM-III solution. The AFM-III solution with a net zero magnetic moment is representative of the superconducting properties of the A15 structure.

Given the observation of the dual phase in the present V_3Ga samples, SCAN seems to exaggerate the stabilization of the $D0_3$ solution, while GGA+U gives almost degenerate A15 and $D0_3$ solutions within 15 meV/atom, which is in better accord with the experimental results and in agreement with previous DFT studies.²³ We note that SCAN is known to exaggerate magnetic moments in some materials.^{37–40}

Figure 4 compares the calculated electronic density of states (DOS) for the AFM $D0_3$ solution using various schemes. LDA and GGA essentially yield a gapless semiconductor, while the correlation effects beyond the GGA within GGA+U and SCAN lead to the opening of a bandgap of about 0.2 eV. Similar gap openings have been observed by Buchelnikov *et al.*⁴¹ in other Heusler alloys.

We have estimated the magnetic transition temperature for the AFM G-type $D0_3$ phase based on our GGA solution. Using Monte Carlo simulations with *ab initio* exchange integrals and a Heisenberg model (see the [supplementary material](#)) we obtain the Néel temperature $T_N = 590$ K, which is higher than the peak position in the experimental $M(T)$ data at 360 K in Fig. 3. In order to account for disorder effects, we also calculated the Néel temperature using the mean field approximation within the SPR-KKR scheme (see the [supplementary material](#) for details). Figure 5 shows how T_N collapses rapidly with increasing disorder (x) when the Ga atoms are exchanged randomly with the nonmagnetic V atoms that lie between the two antiferromagnetically coupled magnetic V atoms. T_N is observed to go to zero at $x = 0.2$ or at the 20% exchange level. In particular, a reduction of the Néel temperature to $T_N = 360$ K would require only 6% of the Ga atoms exchanging with nonmagnetic V atoms. Note, however, that there is a sizeable energy barrier for V-Ga exchange. Nevertheless, the results of Fig. 5 emphasize the sensitivity of the Néel temperature to disorder effects.

In summary, we discuss the magnetism and superconductivity of V_3Ga . We show that this compound can crystallize in two distinct crystal structures: A15 that is superconducting and $D0_3$ that is an antiferromagnetic gapless semiconductor. At low temperatures, these two crystal structures are found to co-exist. Our experimental findings are supported by our in-depth first-principles computations using several different exchange-correlation functionals, which allow us to conclude that the ground states of A15 and $D0_3$ phases of V_3Ga are nearly degenerate. The Néel temperature of the ideal AFM $D0_3$ phase of V_3Ga is theoretically estimated to be 590 K, which is higher than the experimental peak at 360 K in the magnetization. We show, however, that the Néel temperature is quite sensitive to disorder effects.

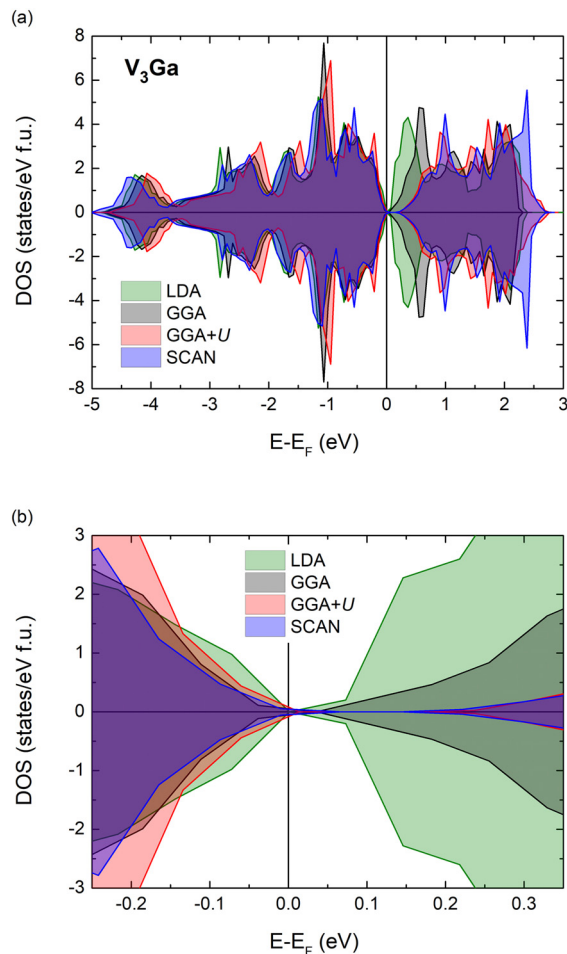


FIG. 4. (a) Comparison of the total DOS for the AFM D0₃ structure obtained using the LDA, GGA, GGA+U, and SCAN. (b) The gapless region near the Fermi level in (a) is shown on an expanded energy scale.

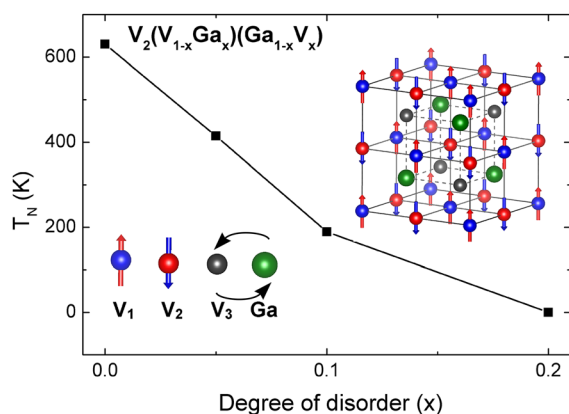


FIG. 5. Effect of disorder on the Néel temperature T_N of V₃Ga based on computations using the SPR-KKR scheme (see the [supplementary material](#) for details). x denotes the fraction of Ga atoms that are randomly exchanged with the nonmagnetic V atoms.

involving random exchange of Ga and non-magnetic V atoms and that the smaller experimental value of the Néel temperature could be achieved through as small as 6% Ga/V substitution. Our study demonstrates the dual-phase nature of V₃Ga and suggests that this material could provide a unique platform for exploiting co-existing superconducting and antiferromagnetic properties for quantum technology applications.

See the [supplementary material](#) for the additional data. The theoretical part includes details and subsequent results of VASP, SPR-KKR, and Monte Carlo calculations for ordered and disordered structures. The experimental part contains the discussion concerning the Rietveld analysis and superconductivity, and x-ray absorption spectroscopy measurements.

The use of the Advanced Photon Source at the Argonne National Laboratory was supported by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences, under Contract No. DE-AC02-06CH11357. The work at Northeastern University was supported by the U.S. Department of Energy (DOE), Office of Science, Basic Energy Sciences, under Grant No. DE-SC0019275 (materials discovery for QIS applications), and benefited from Northeastern University's Advanced Scientific Computing Center and the National Energy Research Scientific Computing Center through DOE Grant No. DE-AC02-05CH11231. The theoretical study with VASP was supported by the Russian Science Foundation project No. 17-72-20022. Monte Carlo simulations and SPR-KKR calculations were performed with the support of the Ministry of Science and Higher Education of the Russian Federation within the framework of the Russian State Assignment under Contract No. 075-00250-20-03. This work was supported by NSF under Grant Nos. DMR-1904446 (M.E.J.) and DMR-1905662 (D.H.). V.D.B. and M.A.Z. gratefully acknowledge the financial support of the Ministry of Science and Higher Education of the Russian Federation in the framework of Increase Competitiveness Program of NUST "MISIS" (No. K2-2020-018), implemented by a governmental decree, N 211. B.B. acknowledges support from the COST Action CA16218 and from the CSC IT Center for Science, Finland, for computational resources. A.P. acknowledges the Osk. Huttunen Foundation for financial support. M.E. Jamer thanks D.A. Arena and G.E. Sterbinsky for their help in acquiring the XAS measurements.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

- ¹P. C. Canfield, P. L. Gammel, and D. J. Bishop, *Phys. Today* **51**(10), 40 (1998).
- ²P. A. Lee, N. Nagaosa, and X.-G. Wen, *Rev. Mod. Phys.* **78**, 17 (2006).
- ³Y. Mizuguchi and Y. Takano, *J. Phys. Soc. Jpn.* **79**, 102001 (2010).
- ⁴M. D. Lumsden and A. D. Christianson, *J. Phys.: Condens. Matter* **22**, 203203 (2010).
- ⁵Y. Du, G. Z. Xu, X. M. Zhang, Z. Y. Liu, S. Y. Yu, E. K. Liu, W. H. Wang, and G. H. Wu, *Europhys. Lett.* **103**, 37011 (2013).
- ⁶A. Bansil, S. Kaprzyk, P. Mijnares, and J. Toboła, *Phys. Rev. B* **60**, 13396 (1999).
- ⁷L. R. Testardi, T. Wakiyama, and W. A. Royer, *J. Appl. Phys.* **48**, 2055 (1977).

- ⁸S. Ohshima, H. Ishida, T. Wakiyama, and K. Okuyama, *Jpn. Soc. Appl. Phys.* **28**, 1362 (1989).
- ⁹M. E. Jamer, B. A. Assaf, G. E. Sterbinsky, D. Arena, L. H. Lewis, A. A. Saúl, G. Radtke, and D. Heiman, *Phys. Rev. B* **91**, 094409 (2015).
- ¹⁰G. Gao and K. Yao, *Appl. Phys. Lett.* **103**, 232409 (2013).
- ¹¹I. Galanakis, S. Tirpanci, K. Özdoğan, and E. Şaşıoğlu, *Phys. Rev. B* **94**, 064401 (2016).
- ¹²L. Toikka, *New J. Phys.* **21**, 113033 (2019).
- ¹³W. Markiewicz, E. Mains, R. Vankeuren, R. Wilcox, C. Rosner, H. Inoue, C. Hayashi, and K. Tachikawa, *IEEE Trans. Magn.* **13**, 35 (1977).
- ¹⁴B. T. Matthias, E. A. Wood, E. Corenzwit, and V. B. Bala, *J. Phys. Chem. Solids* **1**, 188 (1956).
- ¹⁵M. Weger, *Rev. Mod. Phys.* **36**, 175 (1964).
- ¹⁶Y. A. Izyumov and Z. Kurmaev, *Sov. Phys. Usp.* **17**, 356 (1974).
- ¹⁷L. R. Testardi, *Rev. Mod. Phys.* **47**, 637 (1975).
- ¹⁸B. M. Klein, L. L. Boyer, D. A. Papaconstantopoulos, and L. F. Mattheiss, *Phys. Rev. B* **18**, 6411 (1978).
- ¹⁹T. Jarlborg and P. O. Nilsson, *J. Phys. C* **12**, 265 (1979).
- ²⁰A. Jain, S. Ong, G. Hautier, W. Chen, W. Richards, S. Dacek, S. Cholia, D. Gunter, D. Skinnerand, G. Ceder, and K. Persson, *APL Mater.* **1**, 011002 (2013).
- ²¹M. E. Jamer, G. E. Sterbinsky, G. M. Stephen, M. C. DeCapua, G. Player, and D. Heiman, *Appl. Phys. Lett.* **109**, 182402 (2016).
- ²²M. E. Jamer, Y. J. Wang, G. M. Stephen, I. J. McDonald, A. J. Grutter, G. E. Sterbinsky, D. A. Arena, J. A. Borchers, B. J. Kirby, L. H. Lewis, B. Barbiellini, A. Bansil, and D. Heiman, *Phys. Rev. Appl.* **7**, 064036 (2017).
- ²³B. He, Z. Bao, K. Zhu, W. Feng, H. Sun, N. Pang, N. Tsogbadrakh, and D. Odkhuu, *Phys. Status Solidi RRL* **13**, 1900483 (2019).
- ²⁴A. A. Coelho, *J. Appl. Crystallogr.* **51**, 210 (2018).
- ²⁵S. Abe, T. Kadokura, T. Yamaguchi, T. Shinoda, and Y. Oya-Seimiya, *J. Low Temp. Phys.* **130**, 477 (2003).
- ²⁶S. Foner, E. McNiff, Jr., S. Moehlecke, and A. Sweedler, *Solid State Commun.* **39**, 773 (1981).
- ²⁷D. Decker and H. Laquer, *J. Appl. Phys.* **40**, 2817 (1969).
- ²⁸G. Kresse and J. Furthmüller, *Phys. Rev. B* **54**, 11169 (1996).
- ²⁹G. Kresse and D. Joubert, *Phys. Rev. B* **59**, 1758 (1999).
- ³⁰J. P. Perdew and A. Zunger, *Phys. Rev. B* **23**, 5048 (1981).
- ³¹J. Perdew, *Physica B* **172**, 1 (1991).
- ³²K. Burke, J. Perdew, and M. Ernzerhof, *Int. J. Quantum Chem.* **61**, 287 (1997).
- ³³J. Perdew, K. Burke, and M. Ernzerhof, *Phys. Rev. Lett.* **77**, 3865 (1996).
- ³⁴J. P. Perdew, S. Kurth, A. C. V. Zupan, and P. Blaha, *Phys. Rev. Lett.* **82**, 2544 (1999).
- ³⁵J. Tao, J. P. Perdew, V. N. Staroverov, and G. E. Scuseria, *Phys. Rev. Lett.* **91**, 146401 (2003).
- ³⁶J. Sun, A. Ruzsinszky, and J. P. Perdew, *Phys. Rev. Lett.* **115**, 036402 (2015).
- ³⁷E. B. Isaacs and C. Wolverton, *Phys. Rev. Mater.* **2**, 063801 (2018).
- ³⁸Y. Fu and D. J. Singh, *Phys. Rev. Lett.* **121**, 207201 (2018).
- ³⁹M. Ekholm, D. Gambino, H. J. M. Jönsson, F. Tasnádi, B. Alling, and I. A. Abrikosov, *Phys. Rev. B* **98**, 094413 (2018).
- ⁴⁰D. Mejia-Rodriguez and S. B. Trickey, *Phys. Rev. B* **98**, 115161 (2018).
- ⁴¹V. D. Buchelnikov, V. V. Sokolovskiy, O. N. Miroshkina, M. A. Zagrebina, J. Nokelainen, A. Pulkkinen, B. Barbiellini, and E. Lähderanta, *Phys. Rev. B* **99**, 014426 (2019).