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Cross Service Providers Workload Balancing for Data Centers in Deregulated Electricity Markets

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***Abstract*—The emerging Internet of things (IoT) and 5G applications boost a continuously increasing demand for data processing, which results in an enormous energy consumption of data centers (DCs). Considering that existing distributed geographical load balancing is approaching the limit in reducing the energy cost of DCs, cloud service providers (SPs) are motivated to pursue a higher level cooperation. In this context, cross-SP workload balancing among the DCs operated by different SPs represents a future trend of the DC industry. This paper investigates the optimal cross-SP workload balancing when it couples with the electricity markets. First we assume that there is a central operator (CO) coordinating the DCs owned by various SPs. A noncooperative game is formulated to model the interaction between utilities and CO which serves as a price maker. Under the centralized coordination of CO, an optimal solution is obtained with an iterative algorithm. Taking into account the computation and privacy issues, a decentralized algorithm is then proposed by utilizing techniques in state based potential game. Numerical results corroborate the effectiveness of the proposed algorithm. Simulations using Google workload trace show that the workload balancing among cross-SP DCs results in a lower DC operation cost than existing price-taker approach.**

***Index Terms*—cloud service provider, electricity market, marker power, decentralized coordination**

I. Introduction

THE growing Internet service as well as other data intensive industries in recent years has boosted the demand of data centers (DCs) for computing service. Along with the prosperity of DC industry, the huge electricity consumption of DCs has caught public attention during the past decade. The need to reduce energy cost has been driving the DC owners to improve the energy efficiency. Previous efforts have experienced three phases: single server, single DC (intra DC) and single cloud service provider (SP) (inter DCs). To be specific, the first phase focuses on hardware development to enhance the efficiency of individual servers [1]. The second phase involves DC infrastructure and computer network technology. DCs are carefully designed for the computing machines and even are built in remote areas with low temperature or plenty of water for cooling [2]. In addition, networking technologies such as, virtualization, live migration, and consolidation are developed for DC [3]. These technologies allow dynamic resource allocation and consolidation of the workload in a small number of servers and turn off inactive ones [4], [5]. The third phase devotes to workload balancing among geo-distributed DCs [6]. These DCs locating in different electricity markets are operated by a singe SP which coordinates the workload distribution taking advantage of electricity price variability. However, the space for further reducing energy cost of DCs by above mentioned methods is limited. The energy demand still remains overwhelming, which motivates SPs to pursue more powerful energy cost reduction schemes. This is even more urgent because of the upcoming Internet of things (IoT) and 5G era when the volume of data is expected to grow exponentially.

The limitations of existing technologies and the new data explosion naturally prompt SPs to cooperate with each other. As pointed out by [7], [8], transferring workload across SPs brings benefits in cost, performance and reliability, and thus it represents a future trend. To this end, SPs will embrace a more open DC market where they can share their resources by sharing the workload, which resembles the direct energy sharing [9]. Specifically, SPs connect their DCs with those of other SPs and their workload can be transferred across SPs for mutual benefits. In order to distinguish from geo-distributed workload balancing, we term it as cross-SP workload balancing. Instead of transferring workload only among self owned DCs, SPs are offered a lager space for optimization, which enables underutilized DCs to contribute more to cutting electricity bill. To the best of our knowledge, this paper is the first work in literature to investigate workload balancing across SPs from the perspective of energy cost.

Associated with this particular scenario are challenges concerning computation and privacy. In previous studies, an SP usually operates a relatively small number of DCs, and the SP possesses all the information and resources of its DCs. Therefore, the SP naturally plays the role of central operator (CO) to calculate the optimal workload transferring strategy. However, in our considered problem, the DCs are of large number and are owned by different SPs. Hence, centralized

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optimization can induce intensive computation. Moreover, a centralized optimizer requires to collect the DC and workload information from different SPs, which gives rise to privacy concern. It is concluded that centralized optimization is inappropriate to deal with cross-SP workload balancing problems because of the possible computation issue and privacy leakage. Therefore, a decentralized optimization method without any central coordinator is preferred.

Besides, since each data center usually has a large electricity consumption, its market power, the ability to change electricity prices, is nonnegligible [10]. SPecifically, the workload transfer depends primarily on the electricity prices of different locations. However, the flexibility of workload being transferred across DCs itself significantly changes the local electricity demand, and consequently has impact on electricity prices [11], [12]. Therefore, a model charactering the reciprocal dependency of electricity prices and cross-SP workload balancing is critical.

Among prior studies, the most relevant category to our work is geo-distributed workload balancing. It is reported that implementing geographical workload balancing can help DCs participate in wholesale electricity markets and offer ancillary services [13]. The workload transfer strategy of data centers in electricity market adopting locational marginal price (LMP) is investigated in [14]. A min-max integer programming problem is formulated in [15] to optimize the task scheduling on distributed DCs. A receding horizon control based online algorithm is proposed in [16] to coordinate the electricity generation and workload scheduling. Coalition game method is employed in [17] to coordinate multiple DCs to cope with workload and electricity price uncertainty. The spatial-temporal diversity of electricity prices is first exploited by [18], [19] to reduce the electricity bills of SPs. Furthermore, temporal and spatial flexibility of workload and multiple energy options of distributed DCs are explored in [20]. The market power of DC is taken into consideration in [21] that models the prices with supply function method. A matching game is constructed to investigate the temporal workload scheduling and utilities choice in deregulated electricity market [22]. The market power of DCs is also respected in [23], [24] where the prices are determined by the Stackelberg game or two-stage interaction between utilities and DC operators.

Although geo-distributed DC workload balancing problem has been studied extensively, previous schemes do not apply to our problem. All the works [13], [15]–[19], [21]–[24] focus on the DCs owned by a single SP who coordinates the workload distribution. [13], [15]–[20] employ price taker model, ignoring the the market power of DCs. Price maker model is adopted by [21]–[24] in different scenarios from ours. The DCs are assumed to run in an isolated manner in [22], i.e., DCs do not conduct workload balancing. In [23], the optimization is of central free on the utility side, but is centralized on DC side. The problem is studied in [24] under the assumption that all the DCs purchase electricity from a single utility company. Further, the authors assume that utilities have complete information of DCs, which deviates significantly from the setting of our problem.

The essential difference between existing work and the proposed work of this paper is that the cross-SP workload balancing problem is placed in deregulated electricity markets where the market power of DCs is exercised. Because DCs and utilities can be distributed across continents, DCs procure electricity from different utilities that release electricity prices independently. Thus, either the utilities or DCs possess a global knowledge of the whole system and only concern about their local interests.

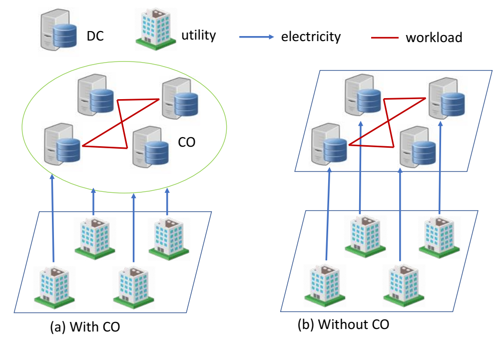

Fig. 1. The infrastructures of centralized/decentralized coordination

Our contributions are summarized as follows: 1) The impact of DCs on electricity markets is considered and a noncooperative game is established to depict the interaction between utilities and CO which coordinates all DCs. 2) We prove that there exists a unique Nash equilibrium that can be obtained by an iterative algorithm. 3) Considering the practical situation where the CO does not exist, we construct a state based potential game to develop a decentralized algorithm that yields the same result as the centralized one. Note that the centralized algorithm minimizes the total cost of the data centers. Thus if the decentralized algorithm yields the same solution, it results in the minimum overall cost. Because we assume that all the data centers are allied to reduce the overall cost, their ultimate goal is to obtain a global optimal solution as the centralized algorithm.

The remaining of the paper is organized as follows: Section II describes the electricity price model and DCs model and formulates a noncooperative game between utilities and DCs. A centralized algorithm assuming a CO coordinates all DCs is proposed to attain the unique Nash Equilibrium. Then we propose a distributed algorithm based on the framework of state based potential game in Section III, providing the same solution as the centralized counterpart. Section IV presents the simulation results which verify the effectiveness of the proposed method. Finally, in Section V we conclude this paper.

II. Centralized Coordination

A. System Model

Suppose that DCs of different SPs collaborate to reduce the total operational cost of all SPs. Since DCs have market power, the workload transfer will influence the local electricity prices. It is clear that the workload transfer and electricity prices depend on each other. Therefore, there is an intersection among the DCs and the utilities. For simplicity, we first assume that there is a virtual CO dictating workload transfer and

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representing DCs to interact with utilities, which is illustrated in Fig. 1(a). In this scenario, CO takes over all DCs and purchase electricity for them from different utilities. In this case, we focus on the interaction between DCs and utilities without the need to consider the competition among DCs. In next section we will show that even without this central operator as illustrated in Fig. 1(b), the DCs can still achieve the same result with a central-free algorithm.

1) Electricity Market Model: Suppose there is a set $\mathcal{I} = \{1, 2, \cdots, I\}$ of locations where utilities and DCs interact with each other. Assume local demand responds to the price following widely used linear demand function [25]:

$$d_i(p_i) = d_i^0 - \kappa_i p_i,\ 0 \leq p_i \leq \frac{d_i^0}{\kappa_i}, \tag{1}$$

where d_i^0, $k_i > 0$ are parameters featuring the largest allowed demand and elasticity of the demand. To focus our attention on the interaction between utilities and DCs, we assume that each utility i has possessed the local parameters d_i^0 and κ_i. With the demand of DC at location i denoted as z_i , the total demand at location i is $d_i + z_i$, which incurs a generation cost:

$$C_i(p_i) = \frac{1}{2}\alpha_i(d_i + z_i)^2 + \beta_i(d_i + z_i), \tag{2}$$

where α_i and β_i are the parameters of utility i regarding the electricity generation condition.

The profit of utility company i is

$$\begin{aligned} U_i(p_i) =& p_i(z_i + d_i) - C_i(p_i) \\ =& (1 + \alpha_i\kappa_i)[a_i p_i^2 + b_i p_i + z_i p_i] \\ &- \frac{1}{2}\alpha_i(d_i^0 + z_i)^2 - \beta_i(d_i^0 + z_i), \end{aligned} \tag{3}$$

where $a_i = -\frac{\kappa_i + \frac{1}{2}\alpha_i\kappa_i^2}{1+\alpha_i\kappa_i}$ and $b_i = (d_i^0 + \frac{\beta_i\kappa_i}{1+\alpha_i\kappa_i})$. Note that κ_i and α_i are positive, thus $U_i(p_i)$ is a strictly concave function of p_i. The goal of utility i is to maximize its profit, which is equivalent to the problem below because the last two terms in (3) are constants for utility i.

$$\max_{p_i \in \mathcal{P}_i}\ a_i p_i^2 + b_i p_i + z_i p_i, \tag{4}$$

where $\mathcal{P}_i$ is the feasible set of p_i as indicated in (1).

2) Data Center Model: Assume there is one DC i at location i, connected to a set of neighboring DCs $\mathcal{N}_i$. The workload transferred from DC i to its neighbor j is m_{ij}, $i \in \mathcal{I}$, $j \in \mathcal{N}_i$. Provided that the workload originally generated at DC i is z_i^0 and it receives from and sends to its neighbor a fraction of workload, the resultant workload to be processed at DC i is:

$$z_i = z_i^0 - \sum_{j\in\mathcal{N}_i} m_{ij} + \sum_{j\in\mathcal{N}_i} m_{ji}. \tag{5}$$

Naturally, DC i has its capacity:

$$0 \leq z_i \leq Z_i, \forall i \in \mathcal{I}. \tag{6}$$

For ease of expression in the rest of the paper, we also write constraint (6) as

$$l_i \triangleq -z_i^0 + \sum_{j\in\mathcal{N}_i} m_{ij} - \sum_{j\in\mathcal{N}_i} m_{ji} \leq 0, \forall i \in \mathcal{I}, \tag{7a}$$

$$u_i \triangleq z_i^0 - \sum_{j\in\mathcal{N}_i} m_{ij} + \sum_{j\in\mathcal{N}_i} m_{ji} - Z_i \leq 0, \forall i \in \mathcal{I}. \tag{7b}$$

Suppose that the workload transfer incurs cost including the bandwidth cost and the delay, which can be summarized by a convex function [26], [27]:

$$g_{ij}(m_{ij}) := g_{ij}^a m_{ij}^2 + g_{ij}^b m_{ij},$$

where $g_{ij}^a > 0$ and g_{ij}^b are constants determined by the distance and the communication condition between the two DCs. Obviously, there is bandwidth limit of each communication link:

$$0 \leq m_{ij} \leq M_{ij}, \forall i \in \mathcal{I}, j \in \mathcal{N}_i. \tag{8}$$

For brevity, let $m_i = \{m_{ij}\}_{j\in\mathcal{N}_i}$, $\forall i \in \mathcal{I}$ and $m = \{m_{ij}\}_{i\in\mathcal{I}, j\in\mathcal{N}_i}$ collect the workload transfer profile of DC i and the workload transfer across all DCs, respectively. The total cost of the DC i consists of the electricity bill and workload transfer cost:

$$f_i(m_i) := p_i z_i + \gamma \sum_{j\in\mathcal{N}_i} g_{ij}(m_{ij}).$$

B. Interaction Between CO and Utilities

In this part, we assume there exists a CO that takes charge of all DCs. The CO coordinates the workload transfer among DCs to minimize the total cost of all the DCs:

$$F_1(m; p) := \sum_{i\in\mathcal{I}} f_i(m_i).$$

In the electricity markets, the selfish and rational participants, the utilities and the data centers, take actions for their own interest— the utilities maximize their profits and the CO (or the data centers) minimizes the cost of the data centers, or equivalently maximizes the negative cost. With the demand reported from the data centers, the utilities release the electricity prices, and accordingly the data centers transfer workload and purchase electricity from the utilities. The best strategies of the utilities and the data centers rely on each other. The profit of the utilities comes from part of the cost of the data centers. This is naturally a noncooperative game; namely, the interaction between the CO and the utilities can be formulated as a noncooperative game. Following the convention, we use $p = (p_1, p_2, \cdots, p_I)$ to denote the strategy profile of the utilities, and p_{-i} to represent the strategy of all the utilities other than utility i. Denoted by $\mathcal{P}$ and $\mathcal{M}$ the feasible set of p and m, respectively. The interaction between CO and utilities is modeled as following game $\mathcal{G}_1$:

- **Players** CO and utility i, $i \in \mathcal{I}$.
- **Strategy** CO: workload transfer $m \in \mathcal{M}$; Utility i: electricity price $p_i \in \mathcal{P}_i$, $i \in \mathcal{I}$;
- **Payoff** CO: $-\sum_{i\in\mathcal{I}} f_i(m_i)$; Utility i: $U_i(p_i)$.

Remark 1. *Deregulated electricity markets feature the competitive environment where the electricity consumers are free to choose the electricity producers (or providers) and the producers attract consumers by offering low prices. In the considered model, the competition among utilities can be understood from the following two perspectives:*

I. The utilities at different locations attract the electricity consumption (i.e., the workload of the data centers) via tuning the prices, because the CSP can transfer the workload *from one site to another. In this regard, the CSP maintaining multiple data centers are free to choose the distributed utilities and the utilities compete indirectly with each other taking the data centers as bridge.*

II. Although we assume for simplicity that there is at most one utility at each location, this model can capture the scenario where there are multiple utilities at a location. This corresponds to the case in our model that there are multiple utilities and data centers with different indexes sharing the same geographical location and the workload transfer prices across these data centers (physically one singe data center) are accordingly 0. Therefore, at a single location, whether it is oligopolistic, monopolistic or competitive market depends on how many electricity providers are there in practice. Nevertheless, our model is able to encompass all these situations in theory.

We will show by the theorem below that $\mathcal{G}_1$ has a unique Nash equilibrium.

Theorem 1. *The Nash equilibrium of preceding game $\mathcal{G}_1$ exists and is unique.*

Before proving Theorem 1, we first present the following lemma, of which the proof is in the appendix.

Lemma 1. *Define $H(x,y)$ as*

$$H(x,y) = h_1(x) + h_2(y) + cxy, x \in \mathcal{X}, y \in \mathcal{Y},$$

where $h_1(x)$, $x \in \mathcal{X}$ is strictly concave and $h_2(y)$, $y \in \mathcal{Y}$ is strictly convex and c is a constant, then $H(x,y)$ has a unique saddle point $(\widetilde{x}, \widetilde{y})$, that is,

$$H(x,\widetilde{y}) \leq H(\widetilde{x},\widetilde{y}) \leq H(\widetilde{x},y), \forall x \in \mathcal{X}, y \in \mathcal{Y}.$$

Then we prove Theorem 1.

Proof. Define $H(p,m)$ as

$$H(p,m) = \sum_{i\in\mathcal{I}}(a_i p_i^2 + b_i p_i + z_i p_i) + \gamma \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{N}_i} g_{ij}(m_{ij}).$$

It is straightforward that $H(p,m)$ is the sum of a strictly concave function in p, a strictly convex function in m and a bilinear term of p and m (the bilinear term comes from substituting (5) in to $p_i z_i$). According to Lemma 1, there exists a unique saddle point $(\widetilde{p}, \widetilde{m})$:

$$H(p,\widetilde{m}) \leq H(\widetilde{p},\widetilde{m}) \leq H(\widetilde{p},m), \forall p \in \mathcal{P}, m \in \mathcal{M}. \quad (9)$$

It can be observed from its definition that $H(p,m)$ is separable across the utilities. Thus, that $\tilde{p} = (\tilde{p}_1, \tilde{p}_2, \cdots, \tilde{p}_I)$ maximizes $H(p,\tilde{m})$ indicates $\tilde{p}_i$ individually maximizes $H(p_i, \tilde{p}_{-i}, \tilde{m})$ for all $i \in \mathcal{I}$. That is, the saddle point satisfies the following inequilities.

$$H(\tilde{p}_i, \tilde{p}_{-i}, \tilde{m}) \geq H(p_i, \tilde{p}_{-i}, \tilde{m}), \ \forall p_i \in \mathcal{P}_i$$
$$H(\tilde{p}_1, \tilde{p}_2, \cdots, \tilde{p}_I, \tilde{m}) \leq H(\tilde{p}_1, \tilde{p}_2, \cdots, \tilde{p}_I, m), \ \forall m \in \mathcal{M}$$

As for game $\mathcal{G}_1$, it is not difficult to verify that given m utility i adopts a strategy p_i optimizing the problem:

$$\max_{p_i \in \mathcal{P}_i} H(p_i, p_{-i}, m).$$

Algorithm 1 Centralized coordination of DCs

1: set $\epsilon_1 = \delta_1$ to be small enough positive constant, $\eta_1 = 0.5$, $k = 0$, and randomly initialize p^k.
2: **while** $(\delta_1 \geq \epsilon_1)$ **do**
3: update m^{k+1} according to (10) and submit z_i to utilities.
4: update p_i^{k+1} according to (11) and release prices to CO.
5: update $\delta_1 = \|p^{k+1} - p^k\|$ and set $k \leftarrow k+1$.
6: **end while**

Given p the CO adopts a strategy m optimizing the problem:

$$\min_{m \in \mathcal{M}} H(p,m).$$

Suppose that $(p_1^*, p_2^*, \cdots, p_I^*, m^*)$ is a Nash equilibrium, then it must satisfy:

$$H(p_i^*, p_{-i}^*, m^*) \geq H(p_i, p_{-i}^*, m^*), \ \forall p_i \in \mathcal{P}_i$$
$$H(p_1^*, p_2^*, \cdots, p_I^*, m^*) \leq H(p_1^*, \tilde{p}_2, \cdots, p_I^*, m), \ \forall m \in \mathcal{M}$$

Therefore, the Nash equilibrium of the game (p^*, m^*) and the saddle point of $H(p,m)$ are equivalent. We have obtained that there exists a unique saddle point $(\widetilde{p}, \widetilde{m})$. Thus, the Nash equilibrium of the game exists and is unique.

□

C. Centralized Iterative Algorithm for the Game

We propose an algorithm that describes the interaction between CO and utilities to approach the equilibrium of $\mathcal{G}_1$ by CO and utilities iteratively updating their strategies. At each iteration, CO acting as a central coordinator determines the workload transfer strategy according the electricity prices:

$$m^{k+1} = \arg\min_{m \in \mathcal{M}} H(p^k, m). \quad (10)$$

Then, CO submits the bid z_i to the local utility i, $i \in \mathcal{I}$. Next, the utilities update the prices following (11) and release them to CO.

$$p_i^{k+1} = \mathrm{Pr}_{\mathcal{P}_i}[p_i^k + \eta_1 \frac{\partial H(p_i^k, p_{-i}^k, m^{k+1})}{\partial p_i}], \forall i \in \mathcal{I}, \quad (11)$$

where $\mathrm{Pr}_{\mathcal{P}_i}[\cdot]$ denotes the projection to feasible set $\mathcal{P}_i$. This process repeats until the prices converge. We summarize this iterative process in Algorithm 1, of which the convergence is guaranteed [28].

In Algorithm 1, there is a virtual CO coordinating the workload transfer among DCs. We refer to this algorithm as *centralized* interaction between CO and utilities to distinguish it from the *decentralized* algorithm to be presented in the next section.

III. Decentralized Coordination of DCs

In this section, we focus on the realistic scenario without CO, and the problem becomes how DCs attain the workload transfer strategy in a decentralized fashion. To be specific, in previous centralized algorithm, at each iteration given the electricity prices, the CO minimizes the aggregated cost of all DCs, namely, solves the problem below:

$$\textbf{P1}: \quad \min_{m} \quad \sum_{i\in\mathcal{I}} f_i(m_i)$$
$$s.t. \quad (6), (8).$$

Now the DCs have to solve **P1** in a decentralized manner.

In essence, **P1** couples all DCs in both the objective function and the constraints. Therefore, information exchange is necessary for the decentralized algorithm. Here we have two assumptions for the communication: 1) the DCs are connected, and 2) the communication is bidirectional.

The goal of the decentralized algorithm is to obtain the same solution as the centralized algorithm. However, each DC only possesses local information, and can only conduct local optimization based on the estimates of global information. Communication with neighbors helps the DCs update the estimates and drives them to the real value. Therefore, to solve **P1** requires an iterative process where the DCs need to exchange information, update estimates and conduct optimization. The decentralized algorithm will be developed in the framework of state based potential game introduced in the ensuing subsection.

A. State Based Potential Game Preliminary

A state based game is typically characterized by the following components [29]:

(i) A set of players: $\mathcal{N} = \{1, 2, \cdots, n\}$;
(ii) A state space: $\mathcal{X}$;
(iii) An action set: $\mathcal{A}(x) = \prod_{i\in\mathcal{N}} \mathcal{A}_i(x)$, $x \in \mathcal{X}$, where $\mathcal{A}_i(x)$, $\forall i \in \mathcal{N}$, is the strategy set of player i;
(iv) A cost function: $J_i(x, a)$: $\mathcal{X} \times \mathcal{A} \to \mathbb{R}$ for each $i \in \mathcal{N}$;
(v) A state transition function $f(x, a)$: $\mathcal{X} \times \mathcal{A} \to \mathcal{X}$, that is, $x(t+1) = f(x(t), a(t))$.

A null action $\mathbf{0} \in \mathcal{A}(x)$ is the one that does not change the current state, namely, $x = f(x, \mathbf{0})$. State based game captures a dynamic process in which the players play the game repeatedly. The actions adopted by the game drive the system to different states, according to which the cost for each player is generated.

Definition 1. *Stationary state Nash equilibrium: A state action pair (x^*, a^*) is called stationary state Nash equilibrium if*

c-1 a_i^* minimizes J_i given x^* and a_{-i}^*:
$J_i(x^*, a_i^*, a_{-i}^*) \le J_i(x^*, a_i, a_{-i}^*), \forall i \in \mathcal{N}, \forall a_i \in \mathcal{A}_i$;
c-2 x^* is a fixed point of f: $x^* = f(x^*, a^*)$.

Definition 2. *State based potential game: A state based game with a null action is a state based potential game if there exists a potential function Φ: $\mathcal{X} \times \mathcal{A} \to \mathbb{R}$ such that the following two conditions hold for any $x \in \mathcal{X}$:*

c-1 For any player $i \in \mathcal{N}$, action $a \in \mathcal{A}(x)$ and action $a_i' \in \mathcal{A}_i(x)$, the equality below holds:

$$J_i(x, a_i', a_{-i}) - J_i(x, a) = \Phi(x, a_i', a_{-i}) - \Phi(x, a).$$

c-2 The potential function satisfies the following equality for any action $a \in \mathcal{A}(x)$ and the consequent state $x' = f(x, a)$:

$$\Phi(x, a) = \Phi(x', \mathbf{0})$$

The reason we resort to potential game is that the existence of a Nash equilibrium is guaranteed in a potential game [30].

B. Potential game based decentralized Coordination

In this part, we propose a distributed algorithm based on the framework of state based potential game. First, we construct a game according to our problem and show that this game is a potential game. Then, we prove that the equilibrium of the game is the optimal solution to **P1**. Last, we specify the action of the game and demonstrate that the action can lead the game to its equilibrium.

1) Construction of state based game: The state based game $\mathcal{G}_2$ is formed as below:

(i) Players: The DCs $\mathcal{I} = \{1, 2, \cdots, I\}$.

(ii) State: The state x_i, $i \in \mathcal{I}$ includes inner state $\{m_{ij}\}_{j\in\mathcal{N}_i}$ and communication state $e_i = \{r_{ij}^i, r_{ji}^i, l_k^i, u_k^i\}_{k\in\mathcal{I}, j\in\mathcal{N}_i}$, where r_{ij}^i and r_{ji}^i are the DC i's estimates of m_{ij} and m_{ji}, respectively; l_k^i and u_k^i are DC i's estimates of l_k and u_k, which are defined in (7). All states are collected as $x = \{x_i\}_{i\in\mathcal{I}}$.

(iii) Action: a_i includes inner action $\{\hat{m}_{ij}\}_{j\in\mathcal{N}_i}$ and communication action $\hat{e}_i = \{\hat{r}_{ij}^i, \hat{r}_{ji}^i, \hat{l}_k^i, \hat{u}_k^i\}_{k\in\mathcal{I}, j\in\mathcal{N}_i}$. All actions are collected as $a = \{a_i\}_{i\in\mathcal{I}}$.

(iv) State transition functions for any $i, k \in \mathcal{N}$ and $j \in \mathcal{N}_i$:

$$m_{ij}(t+1) = m_{ij}(t) + \hat{m}_{ij}(t); \tag{12a}$$
$$r_{ij}^i(t+1) = r_{ij}^i(t) + 2\hat{m}_{ij}(t) - \hat{r}_{ij}^i(t) + \hat{r}_{ij}^j(t); \tag{12b}$$
$$r_{ji}^i(t+1) = r_{ji}^i(t) - \hat{r}_{ji}^i(t) + \hat{r}_{ji}^j(t); \tag{12c}$$
$$l_k^i(t+1) = l_k^i(t) + \mathbb{1}_i^a(k) \sum_{j\in\mathcal{N}_i} \hat{m}_{ij}(t) - \mathbb{1}_i^b(k)\hat{m}_{ik} \tag{12d}$$
$$- \sum_{j\in\mathcal{N}_i} \hat{l}_k^i + \sum_{j\in\mathcal{N}_i} \hat{l}_k^j; \tag{12e}$$
$$u_k^i(t+1) = u_k^i(t) - \mathbb{1}_i^a(k) \sum_{j\in\mathcal{N}_i} \hat{m}_{ij}(t) + \mathbb{1}_i^b(k)\hat{m}_{ik} \tag{12f}$$
$$- \sum_{j\in\mathcal{N}_i} \hat{u}_k^i + \sum_{j\in\mathcal{N}_i} \hat{u}_k^j; \tag{12g}$$

where $\mathbb{1}_i^a(k)$ and $\mathbb{1}_i^b(k)$ are two indicator functions: $\mathbb{1}_i^a(k) = 1$, if $k = i$, otherwise, $\mathbb{1}_i^a(k) = 0$; $\mathbb{1}_i^b(k) = 1$, if $k \in \mathcal{N}_i$, otherwise, $\mathbb{1}_i^b(k) = 0$.

The decentralized algorithm is built upon the communications among the neighboring DCs. The information to be exchanged is the communication state e_i and the communication action $\hat{e}_i$. Each DC updates its own state according to above state transition function (12).

From the transition function, the game can be compared to a bargaining process between each DC and its neighbors. Take m_{ij}, r_{ij}^i and r_{ij}^j as an example where m_{ij} is the amount of workload that DC i is going to transfer to DC j, r_{ij}^i can be regarded as the amount DC i applies to DC j that it wants to transfer, and r_{ij}^j is the amount that DC j is willing to accept. At beginning, DC i tends to ask for a large r_{ij}^i, but DC j wants a small r_{ij}^j. Then, they exchange their opinion, $\hat{r}_{ij}^i$ and $\hat{r}_{ij}^j$, and update their estimates by compromising with their neighbors, i.e, attaching more weight to the neighbors' opinion and cutting the interests of their own. Through the repeated bargaining, the DCs hope to reach an agreement with their neighbors.

During the bargaining, $m_{ij}(t)$ should capture the willingness of DC i and DC j, that is,

$$2m_{ij}(t) = r^i_{ij}(t) + r^j_{ij}(t) \tag{13}$$

At the end when agreement is reached ($[\cdot]^*$ denotes the variable at the equilibrium),

$$r^{i*}_{ik} = r^{k*}_{ik} = m^*_{ik}, \ \forall i \in \mathcal{I}, k \in \mathcal{N}_i \tag{14}$$

By carefully elaborating the initial value of the state, it can be guaranteed that (13) is satisfied, and it will be shown later by lemma 2 that by properly designing local cost function for DCs, (14) hold at the equilibrium.

Suppose that the DCs can manage their workload without workload transfer; thus the initial value can be set as $m_{ij}(0) = 0$, $i \in \mathcal{I}, j \in \mathcal{N}_i$. It can be verified that the following initial values can guarantee (13).

$$r^i_{ij}(0) = 2m_{ij}(0), r^i_{ji}(0) = 0, i \in \mathcal{I}, j \in \mathcal{N}_i \tag{15}$$

Similarly, if the initial state $l^i_k(0)$, $i, k \in \mathcal{I}$ satisfies $\sum_{i\in\mathcal{I}} l^i_k(0) = l_k(0) = -z^0_k + \sum\limits_{j\in\mathcal{N}_k} m_{kj}(0) - m_{jk}(0)$, then during the evolution induced by above state transition function, $l^i_k(t)$, $i, k \in \mathcal{I}$ always satisfies:

$$\sum_{i\in\mathcal{I}} l^i_k(t) = l_k(t) \tag{16}$$

It is straightforward that the following initialization of $l^i_k(t)$, $i, k \in \mathcal{I}$, satisfies above condition:

$$l^i_k(0) = \begin{cases} l_k(0), & i = k \\ 0, & i \neq k \end{cases} \tag{17}$$

In this way, constraint (7a) is equivalent to

$$\sum_{i\in\mathcal{I}} l^i_k(t) \leq 0, \forall k \in \mathcal{I}$$

Similarly, when $u^i_k(t)$ is initialized as

$$u^i_k(0) = \begin{cases} u_k(0), & i = k \\ 0, & i \neq k \end{cases} \tag{18}$$

$u^i_k(t)$ always satisfies:

$$\sum_{i\in\mathcal{I}} u^i_k(t) = u_k(t), \forall k \in \mathcal{I} \tag{19}$$

Remark 2. *Particularly in this paper, by privacy preserving we mean that the private parameters of each data center are not released to other data centers. The private information includes the workload processing capacity Z_i, the original workload z^0_i, the processed workload z_i, and the workload transfer capacity between data centers M_{ij}, ($i \in \mathbb{I}, j \in \mathcal{N}_i$). In this sense, the initialization,* (15), (17) *and* (18) *respects the privacy of all DCs, because each DC i only requires the workload originating in DC i, z^0_i and the DC capacity Z_i.*

In addition, the state transition together with the privacy-preserving initialization ensures (13), (16) and (19) hold, which, as will be shown later, is essential to guarantee that the equilibrium of the potential game is the optimal solution of **P1**.

(v) Cost function: For ease of exposition, we drop index t when no confusion occurs, and define f^j_i and z^j_i for any $i, j \in \mathcal{I}$ respectively as:

$$f^j_i = p_i z^j_i + \sum_{k\in\mathcal{N}_i} g_{ik}(r^j_{ik})$$

and

$$z^j_i = z^0_i - \sum_{k\in\mathcal{N}_i} r^j_{ik} + \sum_{k\in\mathcal{N}_i} r^j_{ki}.$$

The cost of each DC consists of three components:

$$J_i(x, a) = J^0_i(x, a) + \mu_1 J^1_i(x, a) + \mu_2 J^2_i(x, a),$$

where

$$\begin{aligned}
&J^0_i(x(t), a(t)) = J^0_i(x(t+1), \mathbf{0}) \\
&= f^i_i(t+1) + \sum_{j\in\mathcal{N}_i} (f^j_i(t+1) + f^i_j(t+1) + f^j_j(t+1)); \\
&J^1_i(x(t), a(t)) = J^1_i(x(t+1), \mathbf{0}) \\
&= \sum_{j\in\mathcal{N}_i} (r^i_{ij}(t+1) - r^j_{ij}(t+1))^2 + (r^i_{ji}(t+1) - r^j_{ji}(t+1))^2; \\
&J^2_i(x(t), a(t)) = J^2_i(x(t+1), \mathbf{0}) \\
&= \sum_{k\in\mathcal{N}} max\{0, l^i_k(t+1)\}^2 + max\{0, u^i_k(t+1)\}^2 \\
&+ \sum_{j\in\mathcal{N}_i} \sum_{k\in\mathcal{N}} max\{0, l^j_k(t+1)\}^2 + max\{0, u^j_k(t+1)\}^2
\end{aligned}$$

Notice that J^0_i centers the cost of DC i and its neighbors, J^1_i represents a penalty to the disagreement on the estimates of the same variable by different DCs and J^2_i charges the violation of constraint (6). For DC i, the first part of cost J^0_i includes its own cost and the cost of its neighbors. These cost are based on the estimation of itself and its neighbors. This shows that DCs respect the interest and opinion of their neighbors.

The potential function Φ is defined to be:

$$\Phi = \sum_{i\in\mathcal{I}} J^0_i + \mu_1 \sum_{i\in\mathcal{I}} J^1_i + \mu_2 \sum_{i\in\mathcal{I}} J^2_i$$

It can be verified that game $\mathcal{G}_2$ is a state based potential game. Therefore, it is guaranteed that there exists a Nash equilibrium of this game.

2) Equilibrium of Game $\mathcal{G}_2$: Suppose that (x^*, a^*) is the equilibrium of the state based potential game, namely, (x^*, a^*) satisfies conditions c-1 and c-2 in Definition 1. First, we will show that at the equilibrium, the states of the game exhibit the properties stated in the lemma below.

Lemma 2. *The local estimates of different DCs at the equilibrium satisfy for any $i, j, q \in \mathcal{I}$ and $k \in \mathcal{N}_i$:*
(a) $r^{i*}_{ik} = r^{k*}_{ik} = m^*_{ik}$;
(b) $max\{0, l^{i*}_q\} = max\{0, l^{j*}_q\} = \frac{1}{I} max\{0, -z^*_q\}$;
(c) $max\{0, u^{i*}_q\} = max\{0, u^{j*}_q\} = \frac{1}{I} max\{0, z^*_q - Z_q\}$.

All the proofs for here and the rest of the paper can be found in the appendix. This lemma implies that at the equilibrium, the DCs will reach a consensus about the estimates: any

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Algorithm 2 Decentralized Coordination of DCs

set $\epsilon_1 = \delta_1$, $\epsilon_2 = \delta_2$ to be small enough positive constants, $\eta_1 = 0.5$, $\eta_2 = 0.01$, $k = 0$, and randomly initialize p^k.
while $(\delta_1 \geq \epsilon_1)$ **do**
  set $t = 0$, initial $m_{ij}(0) = 0$ and initialize e_i according to (15), (17) and (18).
  while $(\delta_2 \geq \epsilon_2)$ **do**
    calculate action $a_i(t)$ following (20) and broadcast $a_i(t)$ to its neighbors.
    update state $x_i(t)$ according to (12) and broadcast $x_i(t)$ to its neighbors.
    update $\delta_2 = ||a(t)||$ and set $t \leftarrow t + 1$.
  end while
  update $m^{k+1} = m(t)$ and submit z_i $(i \in \mathbb{I})$ to utilities.
  update p_i^{k+1} $(i \in \mathbb{I})$ according to (11) and release prices to corresponding data cneters.
  update $\delta_1 = ||p^{k+1} - p^k||$ and set $k \leftarrow k + 1$.
end while

two connected DCs agree on the workload transfer between them, and all the DCs have the same opinion on whether any constraint in (6) is violated.

Then by the theorem below it is established that the constructed game is effective in obtaining the optimal solution of problem **P1**.

Theorem 2. *With μ_2 selected such that $\mu_2 > \frac{\Gamma B I}{2d}$, the equilibrium of game $\mathcal{G}_2$ corresponds to the optimal solution of problem **P1** (Γ, B and d are finite constants that are specified in the appendix).*

Due to space limitations, the complete proof for the lower bound of μ_2 is included in the supplementary file, i.e., the one-column version. At this point, only the action a leading game $\mathcal{G}_2$ to its equilibrium remains to be determined.

3) Action and Convergence: A simple projected gradient action is adopted for each DC i, $i \in \mathcal{I}$:

$$a_i(t) = \mathrm{Pr}_{\mathcal{A}_i(x_i(t))}[-\eta_2 \frac{\partial J_i(x(t), 0)}{\partial a_i}] \tag{20}$$

where η_2 is a positive small enough step size and $\mathrm{Pr}_{\mathcal{A}_i(x_i(t))}[\cdot]$ means projection to the feasible set $\mathcal{A}_i(x_i(t))$. Specifically, $\hat{m}_{ij}$ should satisfy $0 \leq m_{ij}(t) + \hat{m}_{ij}(t) \leq M_{ij}$, thus

$$\hat{m}_{ij}(t) = \begin{cases} -m_{ij}(t), & m_{ij}(t) + \eta \frac{\partial J_i(x(t),0)}{\partial m_{ij}} < 0; \\ M_{ij} - m_{ij}(t), & m_{ij}(t) + \eta_2 \frac{\partial J_i(x(t),0)}{\partial m_{ij}} > M_{ij}; \\ \eta_2 \frac{\partial J_i(x(t),0)}{\partial m_{ij}}, & otherwise. \end{cases}$$

All the other actions are unconstrained, thus the projection imposes no additional operation on the gradient.

Due to limited space, we omit the calculation of the gradient. It is worth mentioning that for any DC the calculation of the gradient only requires the electricity prices and state from its neighbors. That is, the workload information and the parameters of other DCs are not necessary. Therefore, the privacy of the DCs is protected.

Now, we demonstrate by the following theorem that the gradient based action guarantees to lead game $\mathcal{G}_2$ to its equilibrium.

Theorem 3. *Let $\nabla\Phi(x, a)$ denote the derivative of $\Phi(x, a)$ with respect to a, i.e., $\nabla\Phi(x, a) = [\frac{\partial\Phi(x,a)}{\partial a_1}, \cdots, \frac{\partial\Phi(x,a)}{\partial a_I}]^T$. Assume $\Phi(x, a)$ is L-smooth in a, or equivalently, there exists a constant L such that*

$$\Phi(x, a) - \Phi(x, \mathbf{0}) - a^T \nabla\Phi(x, \mathbf{0}) \leq \frac{L}{2}||a||_2^2. \tag{21}$$

Let the stepsize $\eta = \min\{\frac{1}{L}, \frac{\|x(1)-x^*\|}{L\sqrt{\xi t}}\}$, then $\Phi(x(t), 0)$ enjoys $\frac{1}{\sqrt{t}}$-sublinear convergence, and the state-action will converge to equilibrium $(x^*, \mathbf{0})$. To be specific, $\Phi(x(t), \mathbf{0}) - \Phi(x^*, \mathbf{0})$ satisfies

$$\Phi(x(t), \mathbf{0}) - \Phi(x^*, \mathbf{0}) \leq \frac{\xi L \|x(1) - x^*\|_2}{\sqrt{t}} \tag{22}$$

where ξ is a constant such that

$$\frac{\partial\Phi(x, a)}{\partial x} \leq \xi \nabla\Phi(x, a) \tag{23}$$

Note that ξ exists since the state transition function $f(x, a)$ is a linear function of a (Please see the appendix for detailed explanation).

Here the decentralized algorithm design following the framework of state based potential game is finished and it is summarized in Algorithm 2. It is a two-loop algorithm, where the inner loop yields the optimal workload transfer strategy given the prices in a decentralized manner and the outer loop generates the electricity prices.

Remark 3. *In the proposed decentralized algorithm, the data centers only use their own local parameters and the communication information from their neighbors. This concept of privacy protection is also adopted in some distributed optimization and federated learning literature [31], [32], where the raw data or parameters are maintained in the local agents and the communicated information is derived from the local information. Indeed there are a bunch of works that deal with privacy in a different way. For example, in the literature of differential privacy, when the private data have to be shared, noise is injected to them and the privacy is quantified to capture how much of the original information can be recovered from the perturbed data [33], [34].*

We have to admit that in the present algorithm, the communication information is calculated from the private information which thus might be partially inferred by reverse engineering. We agree on that this paper does not aim to provide a fully private solution under this consideration. Henceforth, to develop a differentially private algorithm is in our future agenda.

IV. Simulations

1) Synthetic Simulation: In this part, we use workload randomly generated to validate the effectiveness of the proposed algorithm. We emulate 6 distributed DCs in our simulation.

First, we show the converging process of the inner loop, that is, given price, the distributed coordination of the DCs. Fig. 2(a) exhibits the bargaining procedure between DC 1 and DC 2 on how much workload to transfer between them. Fig. 2(b) demonstrates the real and estimated constraint functions. Fig. 2(a) and Fig. 2(b) verify Lemma 2 and equalities

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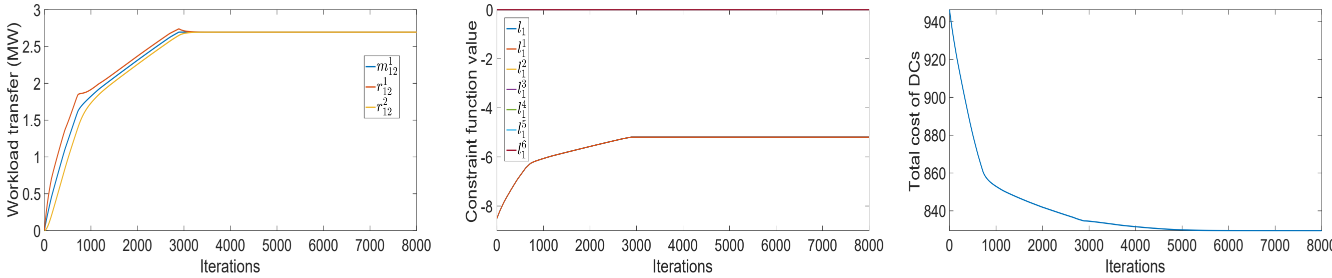

(a) Workload transfer between DC 1 and DC 2

(b) Local constraint estimates

(c) The total cost of the DCs

Fig. 2. Convergence of the decentralized coordination of the DCs

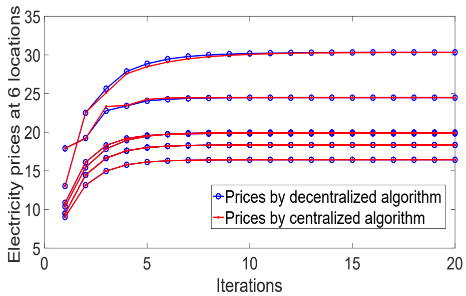

Fig. 3. The electricity prices at different locations

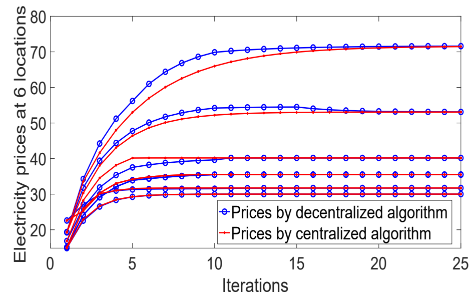

Fig. 5. Convergence with inexact optimal inner solution

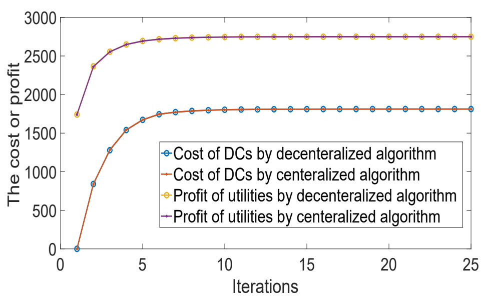

Fig. 4. The cost of DCs and profit of utilities

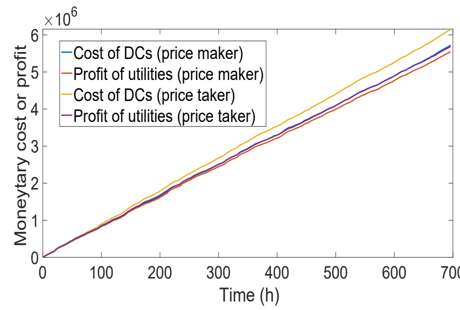

Fig. 6. Accumulate cost of DC and profit of utilities

(13) and (16). Fig 2(c) illustrates during the interaction between DCs, the total cost of DCs keeps diminishing. Then Fig. 3 and Fig 4 show the convergence of the interaction between DCs, or CO and utility companies. The prices obtained by the decentralized algorithm coincide with the prices by the centralized algorithm. Fig. 4 demonstrates that the cost of DCs and profit of utilities completely match despite of slight price mismatch.

Fig. 2 shows that it takes thousands of iterations for the inner loop to converge to the optimal point. Although that each inner iteration only involves simple calculation of the gradients and state update, large number of inner loop iterations translates to intensive communications. Therefore, we propose to reduce the communication by cut the number of inner loop iterations. In other words, we set a limit on the number of inner iteration and stop it before it reaches the optimal. Fig. 5 shows the convergence process by implementing this idea. At first 20 outer loop iterations, we set the limit as 1000, and it is demonstrated that at the end when 25 outer iterations are finished the result is the same as that in Fig. 3. In this way, the communication expenditure is significantly reduced.

The algorithms are tested using a MacBook Pro with 2.3 GHz Intel Core i5. The running times for the centralized algorithm, the decentralized algorithm, and the decentralized algorithm with reduced iterations are 30.21 seconds, 854 seconds and 201 seconds, respectively. Note that because the decentralized algorithms is actually simulated in a single laptop, the communication time is not taken into account.

To explore the impact of the coordination of the data centers,

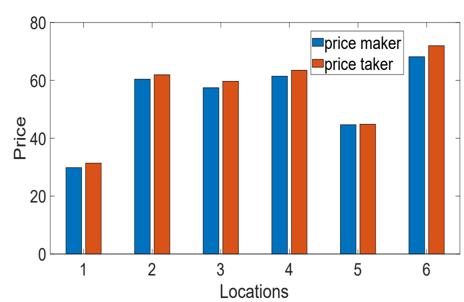

Fig. 7. Prices at different locations

TABLE I
IMPACT OF WORKLOAD TRANSFER ON THE COST OR PROFIT OF THE MARKET PARTICIPANTS (THE COST AND PROFIT IS IN TERMS OF 10^3 DOLLARS. ABBREVIATIONS USED IN THE TALBLE: POU—PROFIT OF UTILITIES, CODC—COST OF DATA CENTERS, COOU—COST OF OTHER USERS)

	PoU	CoDC	CoOU
With coordination	1.82	2.75	3.96
No coordination	2.09	3.03	4.27

we compare the profit of the utilities, the cost the data centers and the cost of other users when there is workload transfer among data centers and when there is none. As demonstrated in Table I, the costs of the data centers and other users are cut down, while the overall profit of the utilities is reduced. Thus, it is concluded that the workload transfer is beneficial for the electricity users. This is because the workload transfer results in lower electricity prices.

2) Simulation based on Google Cluster Workload Trace: For the realistic data analysis, we use the workload trace from Google cluster [35]. The workload trace represents 29 days' workload on a cluster of about 12.5k servers in May 2011. We scale and translate the trace to produce the workload of 6 DCs. We compare the results with that of the price taker model in [20] where the DCs are not aware that the workload transfer can change the electricity prices. Hence, the historical average prices are used to guide the workload transfer, and there is no interaction between DCs and utilities. The proposed scheme in this paper acknowledges the market power of DCs, and adopts an iteration between DCs and utilities. The simulation in this part verifies that the price maker model is superior to price taker model in that it results in better electricity generation. As illustrated in Fig. 6 that workload transfer aware of market power reduces the aggregated cost of DCs by about 8% in one month. The total profit of utilities also declines compared with that of the price taker model. This is because that the price maker model reduces the electricity prices, as shown in Fig. 7. The reductions of electricity prices are the results of the negotiation between DCs and utilities. It should be noted that in the deregulated electricity market, some more competent utilities will gain more profit. The competitiveness of the utilities is determined by the generation, the elasticity of local demand, and the connection of DCs, which is implicit and is not the scope of this paper. It is seen in Table. II that the profit of utility 1 and 4 increases, meaning that they are more competent in free market. (In Table. II, the profits are in terms of million dollars, and profit(m) and profit(t) denote the profit of price maker model and price taker model, respectively).

TABLE II
PROFIT OF THE UTILITIES

	α_i	β_i	k_i	profit(m)	profit(t)
utility 1	0.16	0.08	1.6	0.820	0.806
utility 2	0.16	0.08	0.8	1.268	1.274
utility 3	0.8	0.8	0.8	0.952	1.015
utility 4	0.08	0.04	0.8	1.516	1.512
utility 5	0.16	0.08	0.8	1.427	1.429
utility 6	0.16	0.08	0.6	1.518	1.681

V. CONCLUSIONS

In this paper, we investigate the cross-SP DC workload transfer in deregulated electricity markets. The market power of DCs is taken into considertaion, which results in a noncooperative game capturing the interaction between utilities and DCs. It is shown that there exists a unique Nash equilibrium, which can be obtained by a centralized algorithm that admits a CO coordinating all DCs. Then, considering the practical computation and privacy issues induced by the centralized algorithm, we propose a decentralized algorithm following the framework of sate based potential game. The decentralized algorithm requires only communications among neighboring DCs and enables DCs to obtain the optimal strategies without central coordination. Simulations building upon Google workload trace show that the proposed method outperforms existing schemes in reducing the operational cost of DCs.

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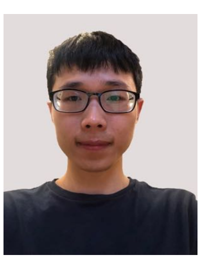

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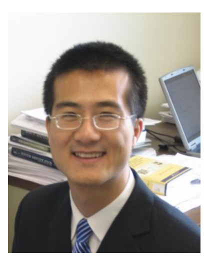

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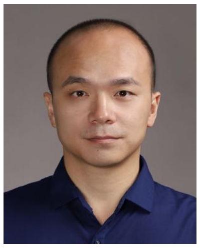

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Appendix A
Proof of Lemma 1

Proof. Since $h_1(x)$ is strictly concave,

$$h_1(x_1) < h_1(x_0) + \nabla h_1(x_0)(x_1 - x_0), \forall x_1, x_0 \in \mathcal{X}, x_1 \neq x_0$$

Therefore,

$$h_1(x_1) - h_1(x_0) < \nabla h_1(x_0)(x_1 - x_0)$$
$$h_1(x_0) - h_1(x_1) < \nabla h_1(x_1)(x_0 - x_1)$$

Adding above two inequalities yields: for any x_1, $x_0 \in \mathcal{X}, x_1 \neq x_0$,

$$(x_1 - x_0)(\nabla h_1(x_1) - \nabla h_1(x_0)) < 0 \tag{24}$$

Similarly, since $h_2(y)$ is strictly convex, we have for any y_1, $y_0 \in \mathcal{Y}, y_1 \neq y_0$,

$$(y_1 - y_0)(\nabla h_2(y_1) - \nabla h_2(y_0)) > 0 \tag{25}$$

Let $\mathcal{K} = \mathcal{X} \times \mathcal{Y}$, and $L(x,y) = [-\nabla_x H(x,y)\ \nabla_y H(x,y)]^T = [-\nabla h_1(x) - cy\ \nabla h_2(y) + cx]^T$. That z solves variational inequality $VI(\mathcal{K}, L)$, namely, $z \in SOL(\mathcal{K}, L)$, is equivalent to (please see the definitions of VI and SOL in Appendix F in the one-column version):

$$(z' - z)^T L(z) \geq 0, \forall z' \in \mathcal{K} \tag{26}$$

Suppose $(\widetilde{x}, \widetilde{y})$ is a saddle point of $H(x, y)$.

$$\begin{aligned} &H(\widetilde{x}, \widetilde{y}) \geq H(x, \widetilde{y}), \forall x \in \mathcal{X} \Leftrightarrow \\ &(x - \widetilde{x})^T[-\nabla_x H(\widetilde{x}, \widetilde{y})] \geq 0, \forall x \in \mathcal{X} \end{aligned} \tag{27}$$

$$\begin{aligned} &H(\widetilde{x}, \widetilde{y}) \leq H(\widetilde{x}, y), \forall y \in \mathcal{Y} \Leftrightarrow \\ &(y - \widetilde{y})^T \nabla_y H(\widetilde{x}, \widetilde{y}) \geq 0, \forall y \in \mathcal{Y} \end{aligned} \tag{28}$$

(27) and (28) can be written as

$$[(x - \widetilde{x})\ (y - \widetilde{y})]^T L(x, y) \geq 0, \forall x \in \mathcal{X}, y \in \mathcal{Y} \tag{29}$$

which means $(\widetilde{x},\ \widetilde{y})$ and $SOL(\mathcal{K}, L)$ are equivalent.

Next, we will show that $SOL(\mathcal{K}, L)$ exists and is unique by showing that $VI(\mathcal{K}, L)$ is strictly monotone.

Let $z_1 = [x_1\ y_1]^T$, $z_0 = [x_0\ y_0]^T \in \mathcal{K}$ and $z_1 \neq z_0$, then

$$\begin{aligned} &(z_1 - z_0)^T(L(z_1) - L(z_0)) \\ =& \begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \end{bmatrix}^T \begin{bmatrix} -(\nabla h_1(x_1) - \nabla h_1(x_0)) - c(y_1 - y_0) \\ \nabla h_2(y_1) - \nabla h_2(y_0) + c(x_1 - x_0) \end{bmatrix} \\ =& -(x_1 - x_0)(\nabla h_1(x_1) - \nabla h_1(x_0)) \\ &+ (y_1 - y_0)(\nabla h_2(y_1) - \nabla h_2(y_0)) > 0 \end{aligned} \tag{30}$$

where the inequality follows from (24) and (25). (30) indicates that $VI(\mathcal{K}, L)$ is strictly monotone. Therefore, $SOL(\mathcal{K}, L)$, i,e, the saddle point $(\widetilde{x},\ \widetilde{y})$ of $H(x, y)$ exists and is unique. □

Appendix B
Proof of Lemma 2

Proof. Directly from c-2 of Definition 1 we can obtain the following equalities for any $i, p \in \mathbb{I}, j \in \mathbb{N}_i$:

$$\frac{\partial J_i(x^*, a^*)}{\partial \hat{r}^i_{ij}} = -\gamma g'_{ij}(r^{i*}_{ij}) - r^{i*}_{ij} + \gamma g'_{ij}(r^{j*}_{ij}) + r^{j*}_{ij} = 0$$

$$\frac{\partial J_i(x^*, a^*)}{\partial \hat{l}^i_p} = -\sum_{j \in \mathbb{N}_i} 2max\{0, l^{i*}_p\} + 2max\{0, l^{j*}_p\} = 0$$

where the first equation indicates $r^{i*}_{ij} = r^{i*}_{ij}$, because function $\gamma g'(r) + r$ is strictly increasing. Note that (13) holds, thus $r^{i*}_{ij} = r^{i*}_{ij} = m^*_{ij}$, namely, (a) is proved.

The second equation, if written in a matrix form for any $p \in \mathbb{I}$, is as below

$$\mathbf{A}[max\{0, l^{1*}_p\} \quad max\{0, l^{2*}_p\} \cdots max\{0, l^{2*}_p\}]^T = \mathbf{0} \tag{31}$$

where the (i, j) entry of $\mathbf{A}$, $\mathbf{A}_{ij} = 1$, if $j \in \mathbb{N}_i$, and $\mathbf{A}_{ii} = -|\mathbb{N}_i|$, if $j = i$, otherwise $\mathbf{A}_{ij} = 0$. We have assumed that the communication graph is undirected and is connected. Therefore, it is straightforward that $\mathbf{A} \cdot \mathbf{1} = 0$, that is, $\mathbf{A}$ is irreducible and $rank(\mathbf{A}) = I - 1$. Thus, the solution of (31) must satisfies $max\{0, l^{i*}_p\} = max\{0, l^{j*}_p\}$, which together with (16) results in (b). Similarly, we can prove (c) holds. □

Appendix C
Proof of Theorem 2

Proof. First we show that the equilibrium is the optimal solution of the following problem:

$$\begin{aligned} \mathbf{P2} : \min_{m} \quad & F_1 m) \\ s.t. \quad & (8). \end{aligned}$$

where $F_2(m) = \sum_{i \in \mathbb{I}} f_i(m_i) + \frac{\mu_2}{2I} \sum_{i \in \mathbb{I}}[(max\{0, -z_i\})^2 + (max\{0, z_i - Z_i\})^2]$.

Assume that $\bar{m}$ is optimal solution of $\mathbf{P2}$, then the sufficient and necessary condition for $\bar{m}_{ij}$ to be the optimal solution is:

$$\frac{\partial F_2(\bar{m})}{\partial m_{ij}}(m'_{ij} - \bar{m}_{ij}) \geq 0, \forall i \in \mathbb{I}, j \in \mathbb{N}_i, m'_{ij} \in [0, M_{ij}].$$

where

$$\begin{aligned} \frac{\partial F_2(\bar{m})}{\partial m_{ij}} &= -p_i + p_j + \gamma g'(m_{ij}) + \frac{\mu_2}{I}(max\{0, -z_i\} \\ &- max\{0, -z_j\} - max\{0, z_i - Z_i\} + max\{0, z_j - Z_j\}) \end{aligned}$$

The first order condition for (x^*, a^*) minimizing J_i includes the inequality holds for any $i \in \mathbb{I}, j \in \mathbb{N}_i, m'_{ij} \in \mathcal{A}(x^*)$.

$$\frac{\partial J_i(x^*, a^*)}{\partial \hat{m}_{ij}}(\hat{m}'_{ij} - \hat{m}^*_{ij}) \geq 0 \tag{32}$$

which is equivalent to

$$\frac{\partial J_i(x^*, a^*)}{\partial \hat{m}_{ij}}(m'_{ij} - m^*_{ij}) \geq 0,\ \forall m'_{ij} \in [0, M_{ij}]. \tag{33}$$

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Applying chain rule, we can obtain

$$\frac{\partial J_i(x^*,a^*)}{\partial \hat{m}_{ij}} = 2[-p_i + p_j + \gamma g'(r_{ij}^{i*})] + 4\mu_1(r_{ij}^{i*} - r_{ij}^{j*}) + 2\mu_2 (max\{0,l_i^{i*}\} - max\{0,l_j^{i*}\} - max\{0,u_i^{i*}\} + max\{0,u_j^{i*}\}) \quad (34)$$

Substituting the equalities in Lemma 2 into (34) yields:

$$\frac{\partial J_i(x^*,a^*)}{\partial \hat{m}_{ij}} = 2[-p_i + p_j + \gamma g'(m_{ij}^*)] + 2\mu_2(max\{0,-z_i^*\} - max\{0,-z_j^*\} - max\{0,z_i^* - Z_i\} + max\{0,z_i^* - Z_i\}) = 2\frac{\partial F_2(m^*)}{\partial m_{ij}}. \quad (35)$$

Substituting (35) into (32) attains

$$\frac{\partial F_2 m^*)}{\partial m_{ij}}(m'_{ij} - m^*_{ij}) \geq 0, \forall i \in \mathbb{I}, j \in \mathbb{N}_i, m'_{ij} \in [0, M_{ij}].$$

which means that m^* is the optimal solution of $\mathbf{P2}$. When μ_2 is set to be large enough, m^* is the optimal solution of $\mathbf{P1}$.

Due to space limitations, the complete proof for the lower bound of μ_2 is included in the complete version.

□

Appendix D
Proof of Theorem 3

Proof. From the smoothness of $\nabla\Phi(x,a)$ we have that

$$\Phi(x(t+1),\mathbf{0}) - \Phi(x(t),\mathbf{0}) = \Phi(x(t),a(t)) - \Phi(x(t),\mathbf{0}) \leq a^T\nabla\Phi(x(t),\mathbf{0}) + \frac{L}{2}||a(t)||_2^2 \quad (36)$$

It can be verified that $\frac{\partial\Phi(x,a)}{\partial a_i} = \frac{\partial J_i(x,a)}{\partial a_i}$ holds for any $i \in \mathbb{I}$. Then, from the property of projection and the fact that $0 \in \mathcal{A}_i(x_i(t))$, we obtain

$$(\eta\nabla\Phi(x(t),\mathbf{0}) + a(t))^T(\mathbf{0} + a(t)) \leq 0$$

that is,

$$\eta\nabla\Phi(x(t),\mathbf{0})^T a(t) \leq -||a(t)||_2^2. \quad (37)$$

Substituting (37) into (36) results in

$$\Phi(x(t+1),\mathbf{0}) - \Phi(x(t),\mathbf{0}) \leq (\frac{L}{2} - \frac{1}{\eta})||a(t)||_2^2 \leq 0 \quad (38)$$

$\Phi(x(t),\mathbf{0})$ is lower bounded and decreasing along $x(t)$, therefore it will arrive at a fixed point $(x,\mathbf{0})$, the equilibrium.

The projected gradient update can be decomposed into two steps:

$$y(t+1) = x(t) - \eta\nabla\Phi(x(t),a(t)); \quad (39)$$
$$x(t+1) = \mathrm{Pr}_{\mathcal{X}}[y(t+1)]. \quad (40)$$

From the definition of $\Phi(x(t),a(t))$ we have that $\Phi(x(t),a(t))$ is convex in $x(t)$, which indicates

$$\Phi(x(t),\mathbf{0}) - \Phi(x^*,\mathbf{0}) \leq \left\langle \frac{\partial\Phi(x(t),\mathbf{0})}{\partial x(t)}, x(t) - x^* \right\rangle \quad (41)$$

Recalling the definition of $\nabla\Phi(x,a)$, it can be written as follows using chain rule.

$$\nabla\Phi(x,a) = \frac{\partial\Phi(x,a)}{\partial a} = \frac{\partial\Phi(x,a)}{\partial x}\frac{\partial x}{\partial a} \quad (42)$$

Since the the state transition function $f(x(t),a(t))$ in (12) is linear in $a(t)$, $\frac{\partial x}{\partial a}$ is a constant matrix. Consequently, from (42) we can obtain

$$\frac{\partial\Phi(x,a)}{\partial x} \leq \xi\nabla\Phi(x,a) \quad (43)$$

where ξ is a constant determined by the transition function (12).

Plugging (43) into (41) gives

$$\begin{aligned} &\Phi(x(t),\mathbf{0}) - \Phi(x^*,\mathbf{0}) \\ &\leq \xi\left\langle \nabla\Phi(x(t),\mathbf{0}), x(t) - x^*\right\rangle \\ &= \frac{\xi}{\eta}\left\langle x(t) - y(t+1), x(t) - x^*\right\rangle \\ &= \frac{\xi}{2\eta}\left[\|x(t) - y(t+1)\|_2^2 + \|x(t) - x^*\| - \|y(t+1) - x^*\|_2^2\right] \end{aligned} \quad (44)$$

Due to the fact that x^* is in $\mathcal{X}$ and $x(t)$ is the projection of $y(t)$ in $\mathcal{X}$, it holds that

$$\|y(t+1) - x^*\|_2^2 \geq \|x(t+1) - x^*\|_2^2. \quad (45)$$

With L-smoothness of Φ implies, we have that

$$\|x(t) - y(t+1)\|_2^2 = \eta^2\|\nabla\Phi(x,a)\|^2 \leq \eta^2L^2. \quad (46)$$

Combining (44), (45) and (46) yields

$$\begin{aligned} &\Phi(x(t),\mathbf{0}) - \Phi(x^*,\mathbf{0}) \\ &\leq \frac{\xi}{2\eta}\left[\eta^2L^2 + \|x(t) - x^*\| - \|x(t+1) - x^*\|_2^2\right]. \end{aligned} \quad (47)$$

Summing (47) over 1 to t and dividing the result gives

$$\begin{aligned} &\frac{1}{t}\sum_{\tau=1}^{t}\Phi(x(\tau),\mathbf{0}) - \Phi(x^*,\mathbf{0}) \\ &\leq \frac{\xi}{2\eta}\frac{1}{t}\sum_{\tau=1}^{t}[\eta^2L^2 + \|x(\tau) - x^*\|_2^2 - \|x(\tau+1) - x^*\|_2^2] \\ &\leq \frac{\xi\eta L^2}{2} + \frac{\xi}{2\eta t}\|x(1) - x^*\|_2^2 \end{aligned} \quad (48)$$

Recalling the obtained result (38) indicates that $\Phi(x(t),\mathbf{0}) - \Phi(x^*,\mathbf{0})$ is decreasing along $x(t)$, thus we obtain the following

$$\begin{aligned} &\Phi(x(t),\mathbf{0}) - \Phi(x^*,\mathbf{0}) \\ &\leq \frac{1}{t}\sum_{\tau=1}^{t}\Phi(x(\tau),\mathbf{0}) - \Phi(x^*,\mathbf{0}) \\ &\leq \frac{\xi\eta L^2}{2} + \frac{\xi}{2\eta t}\|x(1) - x^*\|_2^2 \end{aligned} \quad (49)$$

Let $\eta = \min\{\frac{2}{L}, \frac{\|x(1)-x^*\|}{L\sqrt{\xi t}}\}$, then we have

$$\Phi(x(t),\mathbf{0}) - \Phi(x^*,\mathbf{0}) \leq \frac{\xi L\|x(1) - x^*\|_2}{\sqrt{t}} \quad (50)$$

which means that $\Phi(x(t),\mathbf{0})$ enjoys $\mathcal{O}(\frac{1}{\sqrt{t}})$-sublinear convergence. Here we complete the proof.

□