EXPLORING PRESERVICE TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE OF TEACHING FRACTIONS

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Teachers' pedagogical content knowledge (PCK) influences their instruction and, by consequence, their children's opportunities to learn better. Within the domain of fractions, items assessing PCK are nested within larger assessments, with little explicit focus on the PCK domain. We report on the development and initial validity argument for a PCK for Fractions assessment that assesses preservice teachers' (PSTs') knowledge of students' fractional reasoning. Results suggest the assessment can differentiate between PSTs of different levels in their teacher education program, and that items appear to assess the intended construct. Implications for future study, and for how PCK may develop among PSTs is discussed.

Keywords: Teacher Knowledge; Number Concepts and Operations

Teachers' professional knowledge influences their instruction and, as a result, their students' mathematical learning (Hill et al., 2008a; Hill et al., 2008b). Within mathematics education, such professional knowledge is described by Ball et al. (2008) as Mathematical Knowledge for Teaching (MKT), with a pragmatic distinction between content knowledge (CK) and pedagogical content knowledge (PCK). Although both domains are considered essential, there are relatively few measures of teachers' PCK (Copur-Gencturk et al., 2019; Hill et al., 2008a). Copur-Gencturk et al. (2019) note that many items designed to assess PCK may be better described as CK items both at a theoretical and statistical level. We conjecture that one potential reason for such misalignment of PCK items is due to the nature of the overarching assessments themselves. Rather, by designing an MKT assessment that measures both CK and PCK, there is a risk of unintentionally overemphasizing elements of CK and underrepresenting elements of PCK (Copur-Gencturk et al., 2019).

In the current paper, we report on efforts to design and pilot an assessment of preservice teachers' (PSTs) PCK for teaching fractions in grades 3-5. Current measures of PCK for fractions tend to situate such items as part of an overarching MKT assessment (Depaepe et al., 2015; Kazemi & Rafiepour, 2018). Such assessments tend to distinguish CK and PCK items and may align certain items with key elements of PCK. However, many designed PCK items are open responses, which are time-consuming to code. Our own efforts focus on designing closed-response items to allow for less time-consuming coding for practitioners who may use the assessment. Additionally, we chose to initially focus on one element of PCK, knowledge of students' conceptions, and reasonings about fractions. Our intentional narrowed focus allows for a more concerted effort to examine the nature of this subconstruct, as well as aspects of item design that may inform development of measures for additional aspects of PCK. Thus, the purpose of this study is to construct an initial validity argument for a measure of PSTs' PCK for elementary children's reasoning about fractions.

Theoretical Framework

Developing professional knowledge is an essential component of teacher education programs (AMTE, 2017). Such knowledge is distinct from content knowledge and general pedagogical knowledge and it is called pedagogical content knowledge (PCK) (Shulman, 1986). PCK represents essential knowledge for teaching to facilitate student learning of the subject matter. Ball, Hill, and colleagues extended Shulman's work to articulate a framework for Mathematical Knowledge for

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Teaching (MKT) (Hill et al., 2008b). Extending Ball et al.'s (2008) framework, different scholars have examined teachers CK and PCK on various grounds. Herbst and Kosko (2014) developed an instrument to assess high school teachers' mathematical knowledge for teaching geometry (MKT-G). Khakasa & Berger (2016) applied six domains of MKT to categorize secondary school teachers' mathematical knowledge based on their interpretations of open-ended tasks. They found both the amount of experience and quality of experiences affect teachers' MKT (Herbst & Kosko, 2014; Khakasa & Berger, 2016).

Of particular interest in the present study are those analyses of teachers' MKT for fractions. Depaepe and colleagues (2015) compared CK and PCK on rational numbers among secondary and elementary preservice teachers. They found that CK items are generally easier for PSTs to answer correctly than PCK items. Additionally, PSTs' CK scores were considered low, despite taking a course related to teaching rational numbers. Although secondary PSTs significantly outperformed elementary PSTs on CK for rational numbers, there was no observable difference in how both groups scored on PCK for rational numbers. This is an interesting finding, considering that PSTs' CK and PCK scores are positively correlated (Depaepe et al., 2015; Kazemi & Raflepour, 2018). A common assumption in the literature on teacher knowledge is that strong CK is required to have strong PCK (Izsák et al., 2019; Shulman, 1986). However, in comparing professional development approaches for elementary PSTs, Trobst et al. (2018) found that focusing on enhancing CK of PSTs was less effective than focusing specifically on PCK. Collectively, these findings suggest that teachers' PCK for fractions is related to CK for fractions, but growth in PCK stems from a focus on contexts related to the teaching and learning of fractions.

Method

Sample & Measure

Participants included 58 preservice teachers enrolled in a teacher education program in a Midwestern U.S. university. Participants included 47 early childhood education majors (grades Preschool to 3rd, with optional 4^{th} & 5^{th} grade endorsement) and 11 middle childhood education majors (grades 4-9). Each licensure program includes two mathematics methods course and participants were solicited from each course across both programs (31 juniors; 27 seniors).

An initial version of the PCK assessment included 20 questions, which were subjected to cognitive interviews with two experienced elementary math coaches (Karabenick et al., 2007). In the cognitive interviews, we asked two expert teachers to interpret each item and explain their rationale for their responses. Following cognitive interviews, we analyzed responses related to each item. Some items were interpreted as intended and remained unchanged, while others were revised or removed based on participants' responses. The revised PCK assessment included 15 questions; 9 multiple-choice, 5 multiple-response. Figure 1 illustrates an example question including four items (where an item is counted as a singular response, a question may group one or more items). The question was inspired by literature describing children's fraction learning progressions (Battista, 2012; Hackenberg et al., 2016). Each image represents a specific task of teaching that illustrates a scenario in the form of student work (assessing the reasoning of students' shading 2/3 of 12). Items in the assessment followed a similar structure of assessing children's reasoning based on descriptions from the research literature.

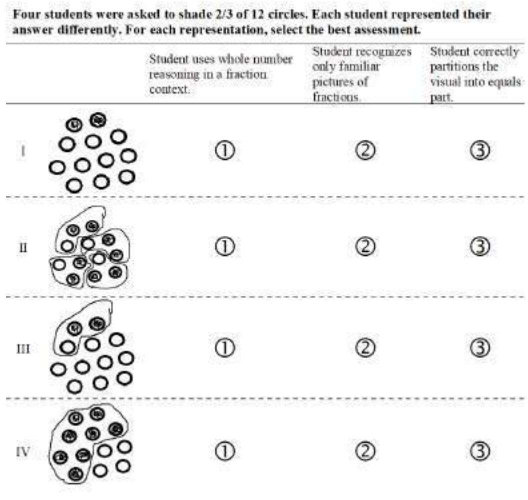

Figure 1: The Example of Item for Measuring PCK-Fractions

Analysis and Results

Analysis of data followed recommendations from the *Standards for Educational and Psychological Testing* (AERA et al., 2014), which states that survey/assessment development should integrate various sources of evidence across multiple studies to construct a validity argument. We collected validity evidence for response processes and test content. Validity evidence of *response processes* focuses on whether participants' responses to our items aligns with the intended theoretical design of the item. In this paper, we conducted a classical item analysis to examine the internal reliability of both the assessment and the items. We also used cognitive interview data to inform decisions on whether certain items should be retained or removed when conducting the item analysis. Evidence for *test content* focuses on how an assessment represents the content and whether scores can be interpreted as intended (AERA et al., 2014). Since our PCK assessment seeks to measure the effect of teacher education initiatives or interventions, we used an independent t-test to compare PCK scores of juniors and seniors as one example of such evidence (considering progress in a teacher education program as the intervention).

The initial item analysis of 30 items resulted in a Cronbach's alpha coefficient of .284. Although the customary threshold for Cronbach's alpha for piloted assessments is typically at or near .70 (Nunnally & Bernstein, 1994), many pilots of successfully validated PCK assessments have tended to report initial Cronbach's alpha coefficients above .60, but somewhat below .70 (e.g., Depaepe et al., 2015; Herbst & Kosko, 2012). Nevertheless, the initial model's reported alpha coefficient was below accepted norms. For each item, we examined point-biserial correlations as an indicator for potential removal. Point-biserial coefficients correlate an item's score (0 or 1) with the total score of the assessment, providing an index of potential fit for the assessment (Crocker & Algina, 2006). It is customary to identify items with point-biserial coefficients below the accepted norm of .30, and consider any interview data, aspects of face validity, etc. before determining whether an item should be removed. Items are removed one-at-a-time and the Cronbach's alpha and point-biserial coefficients are recalculated again.

Our final item analysis model yielded a Cronbach's alpha coefficient of .640. While the majority of remaining items had point-biserial coefficients at or near .30, we chose to retain a subset of items that had lower coefficients (~.20). We retained these items for several key reasons. First, initial pilots of assessments often include smaller samples and may not represent the variance in responses of participants from a larger, more representative sample. As our assessment included PSTs from a single university and no inservice teachers, we believe our sample is unlikely to be representative. Second, although point-biserial and Cronbach's alpha coefficients are useful psychometric indicators, they are but one part of the validity argument for developed measures (Wolf & Smith, 2007). Cognitive interview data for these particular items indicated that they were both interpreted in the manner they were intended and that both responding math coaches considered such content as normative (i.e., these were demonstrated student actions they had seen or seen something similar). Thus, the resulting PCK assessment includes 17 items nested in 7 questions (M=9.48, SD=3.04; *Range* = 4 to 15). Item difficulty ranged from .28 (28% of items answered correctly) to .93 (93% answered correctly), suggesting a wide range in difficulty.

Next, we used an independent samples t-test to compare PCK scores of juniors and seniors in our sample. Results were statistically significant (t = 2.23, df = 56, p = .03), suggesting that PSTs in the sample who were enrolled in their second mathematics methods course had higher scores (M=10.34) than their counterparts enrolled in the first mathematics methods course (M=8.62). This result was still statistically significant for early childhood majors when middle childhood majors were removed from the sample (t = 1.97, df = 45, p = .05), with a similar difference in scores between juniors (M=8.31) and seniors (M=10.00). These results suggest that the PCK assessment distinguishes between PSTs who are earlier and later in their teacher education program.

Discussion

Prior scholars' efforts to construct items assessing PCK are typically open-response and part of MKT measures covering both CK and PCK domains (Depaepe et al., 2015; Kazemi & Rafiepour, 2018; Trobst et al., 2018). Contrasting prior approaches, we sought to develop items exclusively focused on the PCK domain, with particular attention to the knowledge of students' conceptions and reasoning about fractions. The purpose of this study was to construct an initial validity argument for an assessment of PSTs' PCK. Thus, we reported results of a pilot on our assessment that included items on knowledge of students' conceptions and reasonings about fractions, with validity evidence supporting test content and response processes. Results from psychometric analysis, as well as evidence from previously conducted cognitive interviews, supports the claim that our assessment measures teachers' PCK for fractions. Findings from the independent t-test support the claim that our assessment can measure growth due to teacher education initiatives. Although preliminary, the evidence presented in this paper provides a useful baseline for an initial validity argument. Future study is needed to both verify these preliminary findings and to examine other features of validity for such assessments.

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