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## ABSTRACT

With the help of quantum entanglement, quantum dense metrology (QDM) is a technique that can make joint estimates of two conjugate quantities such as phase and amplitude modulations of an optical field, with an accuracy beating the standard quantum limit simultaneously. SU(1,1) interferometers (SUIs) can realize QDM with detection loss tolerance but is limited in absolute sensitivity. Here, we present a QDM scheme with a linear or SU(2) interferometer nested inside an SUI. By using a degenerate SUI and controlling the phase angle of the phase-sensitive amplifiers in the SUI, we can achieve the optimum quantum enhancement in the measurement precision of an arbitrary mixture of phase and amplitude modulation.



It is well-known[1] that the phase uncertainty of an interferometer with classical sources is bound by the standard quantum limit (SQL) that scales as $1/\sqrt{N}$, with $N$ being the photon number sensing the phase. This total number $N$ is viewed as classical resources. This is why strong power of light is employed inside the Laser Interferometer Gravitational-Wave Observatory (LIGO)[2] for the highest absolute sensitivity. However, high power causes various problems such as thermal effects. In the field of biological imaging, light toxicity of the probe field is a huge issue.[3] A possible way to reach a high absolute sensitivity of phase measurement, but at a relatively low light level, is to place the unused input port of the interferometer in a squeezed state of light in order to reduce the injected vacuum fluctuation.[1,4–8]

A quantity that is equally important at the fundamental level but relatively less in practical applications is the amplitude of a field, whose noise can also be reduced by the squeezed states.[9] An example is the radiation pressure noise in LIGO induced by light intensity fluctuations.[1] However, because of the Heisenberg uncertainty principle, it is impossible to beat SQL with a single-mode squeezed state in the measurement of the phase and amplitude simultaneously. In recent years, with the help of quantum entanglement, two-mode squeezed light has been demonstrated to be capable of embedding two or more noncommuting observables in information with their measurement precision beyond the SQL simultaneously. This is so-called quantum dense coding in quantum information science[10] or quantum dense metrology (QDM) in quantum metrology.[11–15]

An alternative way to achieve quantum-enhanced sensitivity is to amplify noiselessly the signal,[16–18] achieved in the so-called SU(1,1) interferometer (SUI),[19–24] which is first proposed by Yurke *et al.*[25] some thirty years ago. SUIs, while achieving a quantum-enhanced phase measurement, possess a number of advantages[26] over the conventional squeezed state interferometers especially the property of detection loss tolerance.[20,23,27,29] Recently, it was demonstrated that an SUI is also capable of splitting quantum resources for measuring multiple noncommuting parameters and beating the standard quantum limit simultaneously.[14,15] In spite of so many advantages over the conventional interferometers, SUIs suffer low phase sensing photon numbers ($I_{ps}$) due to saturation of parametric amplifiers. Thus, the absolute sensitivity of such a type of interferometer is still not comparable to that of classical interferometers. Moreover, resources can, in principle, be all devoted to the measurement of one parameter such as the phase or amplitude to achieve optimum sensitivity. However, so far, there was only report of phase measurement that achieves full use of the available resources for optimum sensitivity.[14,30,31] The reported scheme is not suitable for the amplitude measurement.[30]

In this paper, we consider a more practical scheme, embedding a linear interferometer [also known as an SU(2) interferometer[25]] inside an SUI. Although the linear interferometer has a strong injection for increasing the absolute sensitivity, it operates in the dark fringe mode so that the SUI basically works with no injection and, thus, a low photon number inside. We will demonstrate that the scheme is suitable for the simultaneous measurement of the phase and amplitude, thus realizing QDM, but without the limitation on the phase sensing photon number. Its degenerate version can be used to measure either the phase or amplitude modulation signal or any arbitrary mixture of the two, with an optimum sensitivity that is achieved by full use of both classical and quantum resources.

Before discussing QDM, let us first define the phase (PM) and amplitude (AM) measurement. A phase modulation signal $\delta$ can be added to an incoming probe field $\hat{a}_{in}$ by a phase factor: $e^{i\delta}\hat{a}_{in}$ without adding extra vacuum noise. However, since amplitude modulation $\epsilon$ changes the intensity or energy of the probe field, which is equivalent to a loss, we need to model it with a beam splitter,

$$\hat{a} = t\hat{a}_{in} + r\hat{a}_{\nu}, \tag{1}$$

with $t = e^{-\epsilon}$ and $r = \sqrt{1-t^2}$. Here, $\hat{a}_{\nu}$ is the vacuum field coupled in through loss. However, if the modulation signal $\epsilon$ is very small, $\epsilon \ll 1$ so that $t \approx 1-\epsilon, r \approx \sqrt{2\epsilon}$, we can neglect the vacuum contribution because the noise $\langle \Delta^2 X\rangle = t^2\langle \Delta^2 X_{in}\rangle + r^2\langle \Delta^2 X_{\nu}\rangle \approx (1-2\epsilon)\langle \Delta^2 X_{in}\rangle + 2\epsilon\langle \Delta^2 X_{\nu}\rangle \approx \langle \Delta^2 X_{in}\rangle$. With this in mind, we will drop the vacuum term in Eq. (1) in all treatment of amplitude modulation later. Therefore, for small $\delta, \epsilon (\ll 1)$, we have the modulated field as $\hat{a}' \approx e^{i\delta}e^{-\epsilon}\hat{a}_{in} \approx (1 + i\delta - \epsilon)\hat{a}_{in}$.

We start by first considering how to make simultaneous AM and PM measurements with a classical coherent state. The simplest scheme is by the direct homodyne measurement, as shown in Fig. 1(a), and a simultaneous measurement is achieved by splitting the modulated field with a beam splitter. An alternative way is by a linear or SU(2) interferometer, as shown in Fig. 1(b) with similar signal splitting. Both

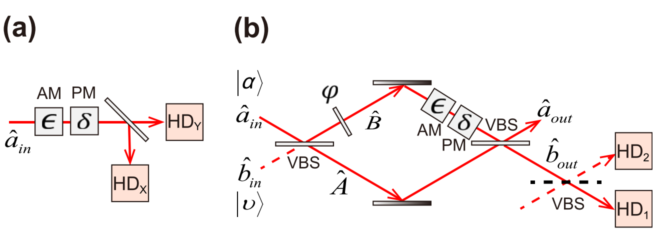


FIG. 1. Schematic diagram for (a) direct homodyne measurement; (b) interferometer with the homodyne measurement. VBS: variable beam splitter; HD: homodyne detection; AM: amplitude modulation; PM: phase modulation.

schemes have been shown[29] to achieve optimum sensitivity in the joint measurement by a classical field. However, the interferometric scheme is advantageous to the direct homodyne detection scheme because the former can operate at the dark port without a large coherent component and it is especially suitable for increasing the intensity of the probe field for higher sensitivity. To prepare the discussion on QDM with SUIs, we analyze in detail the interferometric scheme next.

For the interferometric scheme in Fig. 1(b), we can find the output field by using the following beam splitter relations [refer to Fig. 1(b) for notations]:

$$\begin{aligned} \hat{A} &= \sqrt{T_1}\hat{a}_{in} + \sqrt{R_1}\hat{b}_{in},\ \hat{B} = \sqrt{T_1}\hat{b}_{in} - \sqrt{R_1}\hat{a}_{in},\\ \hat{a}_{out} &= \sqrt{T_2}\hat{A} - \sqrt{R_2}\hat{B}e^{-\epsilon}e^{i\varphi},\\ \hat{b}_{out} &= \sqrt{T_2}\hat{B}e^{-\epsilon}e^{i\varphi} + \sqrt{R_2}\hat{A}. \end{aligned} \tag{2}$$

The outputs are related to the input by

$$\begin{aligned} \hat{a}_{out} &= \left(\sqrt{T_1T_2} + \sqrt{R_1R_2}e^{-\epsilon}e^{i(\varphi+\delta)}\right)\hat{a}_{in}\\ &\quad + \left(\sqrt{R_1T_2} - \sqrt{T_1R_2}e^{-\epsilon}e^{i(\varphi+\delta)}\right)\hat{b}_{in}\\ \hat{b}_{out} &= \left(\sqrt{T_1R_2} - \sqrt{R_1T_2}e^{-\epsilon}e^{i(\varphi+\delta)}\right)\hat{a}_{in}\\ &\quad + \left(\sqrt{R_1R_2} + \sqrt{T_1T_2}e^{-\epsilon}e^{i(\varphi+\delta)}\right)\hat{b}_{in}, \end{aligned} \tag{3}$$

where $\varphi$ is the phase added to field $B$ to account for the overall phase difference in the interferometer.

For simplicity without loss of generality, we assume identical beam splitters: $T_1 = T_2 \equiv T, R_1 = R_2 \equiv R$. Note that when $\varphi = 0$, the interferometer without modulations ($\delta = 0 = \epsilon$) acts as if nothing is there: $\hat{a}_{out} = \hat{a}_{in}$ and $\hat{b}_{out} = \hat{b}_{in}$. This is the so-called dark fringe operation. We will work at this point throughout this paper. Now, let input field $\hat{a}_{in}$ be in a coherent state $|\alpha\rangle$ and field $\hat{b}_{in}$ be in vacuum. Using small modulation assumption ($\delta, \epsilon \ll 1$), we obtain for the dark port output,

$$\hat{b}_{out} \approx \hat{b}_{in} + \hat{a}_{in}\sqrt{TR}(\epsilon - i\delta). \tag{4}$$

Here, we only keep the first non-zero order terms. Since the coherent state has the same noise as a vacuum state and $\delta, \epsilon \ll 1$, $\hat{b}_{out}$ has fluctuations dominated by that of $\hat{b}_{in}$ and also contains information about modulations $\delta, \epsilon$.

Without loss of generality, we assume $\alpha =$ real for the input coherent state $|\alpha\rangle$ at $\hat{a}_{in}$. Then, the homodyne measurement of quadrature phase amplitudes $\hat{X}_{b_{out}} \equiv \hat{b}_{out} + \hat{b}^{\dagger}_{out}, \hat{Y}_{b_{out}} \equiv (\hat{b}_{out} - \hat{b}^{\dagger}_{out})/i$ gives signals and noise, respectively, as

$$\begin{aligned} \langle \hat{X}_{b_{out}}\rangle = 2\alpha\epsilon\sqrt{TR},\ \langle \hat{Y}_{b_{out}}\rangle = 2\alpha\delta\sqrt{TR},\\ \langle \Delta^2\hat{X}_{b_{out}}\rangle = 1 = \langle \Delta^2\hat{Y}_{b_{out}}\rangle, \end{aligned} \tag{5}$$

leading to the signal-to-noise ratio (SNR) for the measurement of phase and amplitude modulations,

$$\begin{aligned} SNR_{\delta} &\equiv \langle \hat{Y}_{b_{out}}\rangle^2/\langle \Delta^2\hat{Y}_{b_{out}}\rangle = 4TI_{ps}\delta^2\\ SNR_{\epsilon} &\equiv \langle \hat{X}_{b_{out}}\rangle^2/\langle \Delta^2\hat{X}_{b_{out}}\rangle = 4TI_{ps}\epsilon^2, \end{aligned} \tag{6}$$

where $I_{ps} \equiv R\alpha^2$ is the photon number for the probe field to the modulations, which can be considered as the overall classical resource for the measurement. Optimum SNRs[26] of $4I_{ps}\delta^2, 4I_{ps}\epsilon^2$ are achieved for a very unbalanced interferometer with $T \approx 1, R = 1 - T \ll 1$.

The above is for individual $\delta$ or $\epsilon$ measurements alone. For simultaneous measurements of $\delta$ and $\epsilon$, an extra BS ($T_3$, $R_3$) can be used to split the output but with a split SNR as well: $SNR_\delta^{(s)} = 4T_3 I_{ps}\delta^2$, $SNR_\epsilon^{(s)} = 4R_3 I_{ps}\epsilon^2$, similar to Fig. 1(a). Note that simultaneous measurements of $\delta$ and $\epsilon$ need to share the overall resource of $I_{ps}$ ($T_3 I_{ps} + R_3 I_{ps} = I_{ps}$).[29]

When a single-mode squeezed state is used to replace the vacuum state at the unused input port of $\hat{b}_{in}$, the vacuum quantum noise can be reduced and the SNR in either the phase[1,4,5] or amplitude measurement[9] is enhanced, but not in both because only one quadrature is squeezed at a time. However, with the employment of two entangled fields, the technique of quantum dense metrology (QDM) is capable of measuring two non-orthogonal quadratures simultaneously.[13] SUIs, which possess the property of detection loss tolerance, also have the ability to measure simultaneously multiple quadrature-phase amplitudes at arbitrary angles with quantum enhanced precision.[14,15] However, despite their advantages over traditional interferometers, SUIs have limited absolute sensitivity due to practical issues. Here, to circumvent these problems, we study a variation of the SUI, which combines a traditional linear or SU(2) interferometer with an SUI for QDM and inherits the advantages of both interferometers.

Consider the scheme shown in Fig. 2. It is a Mach–Zehnder interferometer (MZI) nested in an SUI, which was shown recently[32] to possess all the advantages of the SUI for the phase measurement but without the limit on the intensity of the probe field. To achieve QDM, we perform the homodyne measurement on both outputs of the second parametric amplifier (PA2), one for the phase ($d_1$) and the other for amplitude ($d_2$). If the MZI works at the dark fringe point, we have the input-output relation in Eq. (4). Since $\hat{b}_{out} \approx \hat{b}_{in}$ as if MZI were not there, the overall performance of the SUI is not affected by MZI and it does not have any injection at its inputs ($\hat{a}_0, \hat{b}_0$), but the modulation signals $\delta, \epsilon$ are contained in $\hat{b}_{out}$ for the SUI to measure.

Using the input–output relation for PA1 and PA2,

$$\begin{aligned} \hat{b}_{in} &= G_1\hat{b}_0 + g_1\hat{a}_0^\dagger, \quad \hat{C} = G_1\hat{a}_0 + g_1\hat{b}_0^\dagger, \\ \hat{d}_1 &= G_2\hat{C}e^{i\phi} + g_2\hat{b}_{out}^\dagger, \; \hat{d}_2 = G_2\hat{b}_{out} + g_2\hat{C}^\dagger e^{-i\phi}, \end{aligned} \tag{7}$$

we obtain the outputs of the SUI for QDM as

$$\begin{aligned} \hat{d}_1 &= (G_1G_2e^{i\phi} + g_1g_2)\hat{a}_0 + (g_1G_2e^{-i\phi} + G_1g_2)\hat{b}_0^\dagger + g_2\sqrt{R}(\epsilon + i\delta)\hat{a}_{in}^\dagger \\ \hat{d}_2 &= (G_1G_2 + g_1g_2e^{-i\phi})\hat{b}_0 + (g_1G_2 + G_1g_2e^{-i\phi})\hat{a}_0^\dagger + G_2\sqrt{R}(\epsilon - i\delta)\hat{a}_{in}, \end{aligned} \tag{8}$$

where $\phi$ is a phase to account for the overall phase of the SUI. Here, we used Eq. (4) with $T \approx 1$ for optimum performance of the MZI.

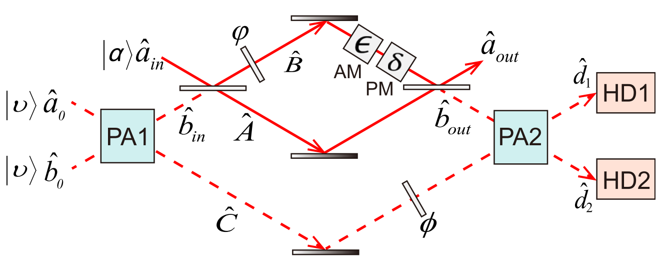


**FIG. 2.** QDM with non-degenerate SU(1,1) interferometry. PA1 and PA2: parametric amplifier; HD: homodyne detection.

With input fields $\hat{a}_0$ and $\hat{b}_0$ in the vacuum state, $R \to 0$, and small modulations ($\delta, \epsilon \ll 1$), it is straightforward to calculate the noise of the outputs of the SUI as

$$\begin{aligned} \langle \Delta^2 \hat{Y}_{d_1} \rangle &= \langle \Delta^2 \hat{X}_{d_2} \rangle \\ &= |G_1G_2 + g_1g_2e^{-i\phi}|^2 + |g_1G_2 + G_1g_2e^{-i\phi}|^2 \\ &= (G_1^2 + g_1^2)(G_2^2 + g_2^2) + 4G_1G_2g_1g_2\cos\phi, \end{aligned} \tag{9}$$

where $\hat{Y}_{d_1} = (d_1 - d_1^\dagger)/i, \hat{X}_{d_2} = d_2 + d_2^\dagger$. The noise is minimum when $\phi = \pi$, corresponding to dark fringes. The signals are calculated as

$$\langle \hat{Y}_{d_1} \rangle = 2g_2\delta\sqrt{I_{ps}}, \quad \langle \hat{X}_{d_2} \rangle = 2G_2\epsilon\sqrt{I_{ps}}, \tag{10}$$

with $I_{ps} = R\alpha^2$. So the SNRs for simultaneous measurements of $\delta$ and $\epsilon$ are

$$\begin{aligned} SNR_{SUI}(\hat{Y}_{d_1}) &= \frac{4g_2^2 I_{ps}\delta^2}{(G_2G_1 - g_1g_2)^2 + (G_1g_2 - G_2g_1)^2} \\ SNR_{SUI}(\hat{X}_{d_2}) &= \frac{4G_2^2 I_{ps}\epsilon^2}{(G_2G_1 - g_1g_2)^2 + (G_1g_2 - G_2g_1)^2}. \end{aligned}$$

Both reach optimum values when $G_2 \approx g_2 \gg 1$,

$$\begin{aligned} SNR_{SUI}^{(op)}(\hat{Y}_{d_1}) &= 2I_{ps}\delta^2(G_1 + g_1)^2 \\ SNR_{SUI}^{(op)}(\hat{X}_{d_2}) &= 2I_{ps}\epsilon^2(G_1 + g_1)^2. \end{aligned} \tag{11}$$

When $(G_1 + g_1)^2/2 > 1$, the measurement of both $\delta$ and $\epsilon$ beats simultaneously the classical sensitivity expressed in Eq. (6) by the same factor of $(G_1 + g_1)^2/2$, thus achieving QDM. Note that the total resource of $I_{ps}(G_1 + g_1)^2$ is shared equally, satisfying the quantum resource sharing law.[29,31] The resource now consists of photon number $I_{ps}$ and quantum entanglement characterized by $(G_1 + g_1)^2$.

Since the resource is shared equally between phase and amplitude measurements, when SUIs are used for the phase or amplitude measurement alone, only half the optimum sensitivity is realized. Recently, a variation of SUI[30] has employed a dual beam to sense the modulations for taking all the resource for the phase measurement and achieving optimum sensitivity. However, the scheme is not suitable for amplitude. In practice, the information embedded may not be purely in the phase but could be in amplitude or something of a mixture of phase and amplitude. To achieve optimum sensitivity for these, we consider a variation to the SUI. We use a degenerate parametric amplifier (DPA) instead of the regular non-degenerate PA, as shown in Fig. 3. Benefiting from the degenerate PA, which can be regarded as a phase-sensitive amplifier, we can selectively distribute all resource to any mixture of phase and amplitude for optimum measurement sensitivity, as shown next.

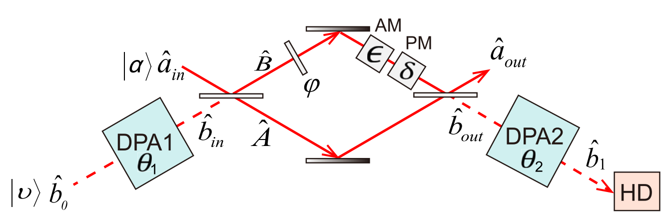


**FIG. 3.** QDM with degenerate SU(1,1) interferometry. DPA: Degenerate parametric amplifier.

Referring to the notations in Fig. 3, we write the input–output relations of DPA as

$$\hat{b}_{in} = G_1\hat{b}_0 + g_1 e^{i\theta_1}\hat{b}_0^\dagger,\\ \hat{b}_1 = G_2\hat{b}_{out} + g_2 e^{i\theta_2}\hat{b}_{out}^\dagger, \tag{12}$$

where phase $\theta_{1(2)}$ is for DPA1(2). Using Eq. (4) with $T \approx 1$ for $\hat{b}_{out}$, we find the output of DPA2 as

$$\begin{aligned}\hat{b}_1 &= G_2\Big[G_1\hat{b}_0 + g_1 e^{i\theta_1}\hat{b}_0^\dagger + \hat{a}_{in}\sqrt{R}(\epsilon - i\delta)\Big]\\ &\quad + g_2 e^{i\theta_2}\Big[G_1\hat{b}_0^\dagger + g_1 e^{-i\theta_1}\hat{b}_0 + \hat{a}_{in}^\dagger\sqrt{R}(\epsilon + i\delta)\Big]\\ &= G_T\hat{b}_0 + g_T\hat{b}_0^\dagger + \sqrt{R}G_2(\epsilon - i\delta)\hat{a}_{in} + \sqrt{R}g_2 e^{i\theta_2}(\epsilon + i\delta)\hat{a}_{in}^\dagger],\end{aligned} \tag{13}$$

where $G_T \equiv G_1G_2 + g_1g_2e^{i(\theta_2-\theta_1)}, g_T \equiv g_1G_2e^{i\theta_1} + G_1g_2e^{i\theta_2}$ are the overall gains of the degenerate SUI. Since $\hat{b}_0$ is in vacuum, the signal part of the output is then

$$\begin{aligned}\langle\hat{b}_1\rangle &= \alpha\sqrt{R}\big[G_2(\epsilon - i\delta) + g_2e^{i\theta_2}(\epsilon + i\delta)\big]\\ &= \alpha\sqrt{R}e^{i\theta_2/2}\Big[\epsilon(g_2e^{i\theta_2/2} + G_2e^{-i\theta_2/2}) + i\delta(g_2e^{i\theta_2/2} - G_2e^{-i\theta_2/2})\Big].\end{aligned} \tag{14}$$

If we measure $\hat{\mathcal{X}} \equiv \hat{b}_1e^{-i\theta_2/2} + \hat{b}_1^\dagger e^{i\theta_2/2}, \hat{\mathcal{Y}} \equiv (\hat{b}_1e^{-i\theta_2/2} - \hat{b}_1^\dagger e^{i\theta_2/2})/i$, we find that the signal becomes

$$\langle\hat{\mathcal{X}}\rangle = -2(G_2 + g_2)\sqrt{I_{ps}}\gamma_-\\ \langle\hat{\mathcal{Y}}\rangle = -2(G_2 - g_2)\sqrt{I_{ps}}\gamma_+, \tag{15}$$

where $\gamma_- \equiv -\epsilon\cos\frac{\theta_2}{2} + \delta\sin\frac{\theta_2}{2}, \gamma_+ \equiv \epsilon\sin\frac{\theta_2}{2} + \delta\cos\frac{\theta_2}{2}$ are two orthogonal modulation signals. Therefore, the measurement of $\hat{\mathcal{X}}, \hat{\mathcal{Y}}$ gives orthogonal quantities $\gamma_\mp$ with $\gamma_-$ amplified by $G_2 + g_2$ but $\gamma_+$ de-amplified by $G_2 - g_2$.

For the noise of the degenerate SUI, since $R \ll 1$, it is governed by $G_T$ and $g_T$ with $\hat{b}_0$ in vacuum, and we have

$$\begin{aligned}\langle\Delta^2\hat{\mathcal{X}}\rangle &= |G_Te^{-i\theta_2/2} + g_T^*e^{i\theta_2/2}|^2\\ &= (G_2 + g_2)^2|G_1 + g_1e^{-i\Delta}|^2\\ \langle\Delta^2\hat{\mathcal{Y}}\rangle &= |G_Te^{i\theta_2/2} - g_T^*e^{i\theta_2/2}|^2\\ &= (G_2 - g_2)^2|G_1 - g_1e^{-i\Delta}|^2,\end{aligned} \tag{16}$$

where $\Delta \equiv \theta_1 - \theta_2$ is the total phase of the SUI. With $\Delta = \pi$ for dark fringes (destructive interference), we have

$$\langle\Delta^2\hat{\mathcal{X}}\rangle = (G_2 + g_2)^2(G_1 - g_1)^2\\ \langle\Delta^2\hat{\mathcal{Y}}\rangle = (G_2 - g_2)^2(G_1 + g_1)^2. \tag{17}$$

So the SNRs for the measurement of $\hat{\mathcal{X}}, \hat{\mathcal{Y}}$ are

$$SNR_{\mathcal{X}} = 4I_{ps}\gamma_-^2(G_1 + g_1)^2\\ SNR_{\mathcal{Y}} = 4I_{ps}\gamma_+^2(G_1 - g_1)^2. \tag{18}$$

Because there is only one output for the degenerate SUI, we cannot make simultaneous measurements of the orthogonal quantities $\gamma_\pm$, and so the degenerate SUI is not suitable for QDM. However, a homodyne detection of $\mathcal{X}$ at the output of the degenerate SUI will give the measurement of $\gamma_- = -\epsilon\cos\theta_2/2 + \delta\sin\theta_2/2$, an arbitrary mixture of AM ($\epsilon$) and PM ($\delta$) signals, depending on $\theta_2$ with a quantum enhancement of $(G_1 + g_1)^2$. Setting $\theta_2 = 0$ gives the full amplitude modulation ($\epsilon$), whereas $\theta_2 = \pi$ gives the full phase modulation ($\delta$). The measurement reaches the optimum measurement sensitivity, making full use of the quantum resource of $I_{ps}(G_1 + g_1)^2$ of the probe field.

The physical meaning of the output signal in Eq. (15) and noise in Eq. (17) for the degenerate SUI is illustrated in the phase space representation of the state evolution in Fig. 4 at each stage of the SUI: (a) shows the initial input vacuum state (circle); In (b), DPA1 squeezes the vacuum state and amplifies the vacuum noise in the selected direction ($\theta_1/2$) but de-amplifies in the orthogonal direction; In (c), the SU(2) linear interferometer encodes weak signals of phase $\alpha\delta$ and amplitude $\alpha(-\epsilon)$ to the probe beam; In (d), DPA2 un-squeezes in the selected direction $\theta_2/2$ and noiselessly amplifies the mixed modulation signal $\alpha\gamma_-$. The actions of the two degenerate PAs are represented by

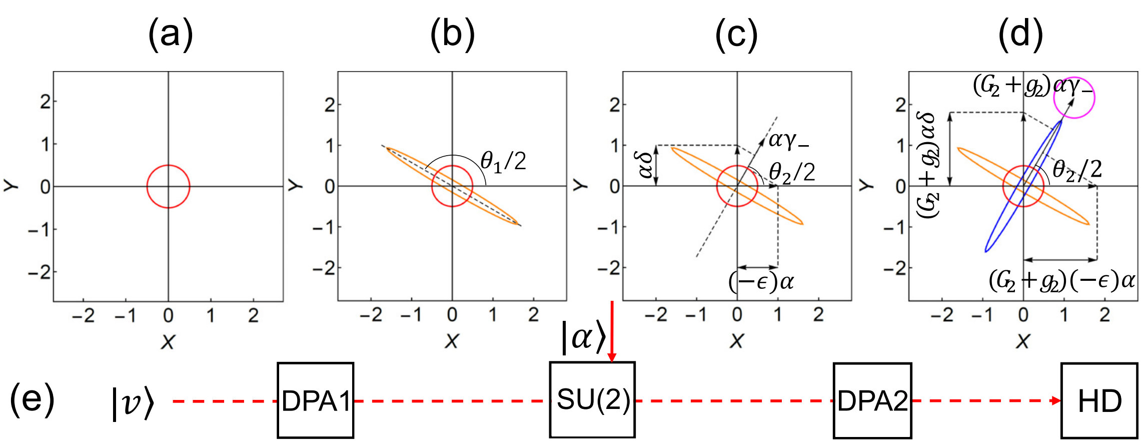


**FIG. 4.** State evolution of degenerate SU(1,1) interferometry. (a)–(d) show the phase-space (X-Y) representation of the evolved state at the corresponding position of the interferometer shown in (e), where $|v\rangle$ represents the vacuum state and $|\alpha\rangle$ represents the coherent state.

two ellipses. Note that $\theta_{1,2}/2$ gives the direction of maximum amplification $(G_{1,2}+g_{1,2})^2$ of the phase-sensitive DPA1,2. The orthogonal orientations of the two ellipses are due to the dark fringe operation condition: $(\theta_1-\theta_2)/2=\pi/2$ for the SUI. When $G_1=G_2$, the noise in Eq. (17) becomes vacuum noise of one (circle) at the output because the actions of DPA1,2 are opposite but equal to each other (squeezing and then un-squeezing), leading to vacuum noise output, but at the same time, DPA2 amplifies the modulation signal $\alpha\gamma_-$, leading to sensitivity enhancement of $(G_1+g_1)^2$.

Different from the non-degenerate SUI whose optimum sensitivity is achieved when $G_2\approx g_2\gg 1$, the degenerate SUI reaches optimum sensitivity at any gain $G_2$ for DPA2. On the other hand, to fully utilize the detection loss-tolerant property of the SUI,[20,27,28] we need to have $G_2\gg G_1$ so that Eq. (17) gives output noise $\langle\Delta^2\hat{\mathcal{X}}\rangle=(G_2+g_2)^2/(G_1+g_1)^2\gg 1$. In this way, any vacuum noise (size of 1) added through detection losses will be negligible compared to $\langle\Delta^2\hat{\mathcal{X}}\rangle$.

In summary, we have demonstrated that an SUI with a linear interferometer nested inside can make phase and amplitude measurements simultaneously with precision beating SQL, thus achieving quantum dense metrology. The phase and amplitude measurements share equally the overall resource of the photon number and quantum entanglement. The degenerate version of the SUI can devote all resources to one measurement of an arbitrary mixture of phase and amplitude with optimum measurement sensitivity. This should be particularly useful if the signal is encoded in both phase and amplitude such as the LIGO interferometer where both phase and intensity fluctuations play important roles. In fact, we noticed a recent work from LIGO Collaboration[33] in which they experimentally demonstrated quantum enhancement in a joint measurement of the phase of laser beams and the position of mirrors by choosing a proper squeezed angle of the initial squeezer (similar to $\theta_1$ in our work).

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The authors declare that there are no conflicts of interest related to this article.

## DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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