Optimal Network Coding Policy for Joint Cellular and Intermittently Connected D2D Networking

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Abstract*—In this paper, we study the problem of transmitting a common content to a number of cellular users with the help of device-to-device (D2D) links via instantly decodable network coding (IDNC). In particular, a common content that is broadcast by a base station may be partially received by cellular users due to packet erasures over cellular links. D2D links among cellular users, which may be intermittently available due to mobility and link imperfections, are used to recover the missing content. In this setup, we design an IDNC mechanism to minimize *packet completion time*, defined as the amount of time it takes for the common content to be received by all cellular users. We develop an optimal packet completion time strategy by constructing a two-layer IDNC conflict graph where the higher layer represents all feasible network coded packets that can be transmitted over cellular links, while the lower layer consists of network coded packets (as well as corresponding cellular users that can generate and transmit these packets) that can be transmitted via D2D links. We show that finding the optimal IDNC packets to minimize the packet completion time problem is equivalent to finding the maximum independent set of the two-layer IDNC conflict graph. We also design a sub-optimal IDNC mechanism, `aIDNC`, that efficiently finds IDNC packets to be transmitted over cellular and D2D links. Finally, the efficiency of our proposed approach is verified through numerical experiments.

I. Introduction

5G and beyond wireless networks are foreseen to experience a huge increase in the number of connected devices [1] and the amount of mobile data traffic [2]. To avoid severe performance degradation of modern wireless systems, academic and industry researchers alike have prioritized designing appropriate technologies to efficiently tackle this challenge. Device-to-Device (D2D) communication stands out among emerging technologies thanks to its capability of alleviating cellular traffic, thereby circumventing the damaging effect of traffic growth. In fact, D2D technology which enables spatially close mobile users to establish short range communication without relaying through the base station (BS), presents a compelling potential for 5G systems [3].

Network coding (NC), which advocates the transmission of a smart combination of packets, emerges as a promising technique for enhancing the performance of wireless systems. NC is a significant breakthrough for wireless broadcast channels enabling efficient packet transmission and judicious resource utilization through packet coding. Network coding can be categorized roughly into two classes: random linear network coding (RLNC) [4] and opportunistic network coding (ONC) [5]. The latter class, which leverages the diversity of the information available at the receiver's end to achieve a specific network's optimization goal, has been widely investigated in the literature [5]–[7]. One example of the ONC category is the instantly decodable network coding (IDNC) [8]–[10] which relies on binary XOR ($\oplus$) operations [11] to encode and decode packets at the transmitter and receiver end. System designers favor the simplicity of the XOR, coupled with the fact that IDNC does not require extensive buffer storage.

D2D-enabled communication with IDNC has been investigated in the literature [6], [12]–[14]. The problem of mean video distortion minimization for an IDNC-based partially connected D2D network is investigated in [12]. The problem of reducing decoding delay of IDNC for a multi-hop D2D communication is addressed in [13]. The authors in [14] focus on deriving the encoded packets along with the set of transmitting users for a partially connected D2D network. They formulated the problem as a maximum independent set problem using a conflict graph model and designed a heuristic.

The aforementioned works consider only D2D links while neglecting the communications on cellular links. This may lead to performance loss as argued in [15], which demonstrated that concurrent operation on both D2D and cellular links improves throughput. Hence, it is important to assess the performance of a network coding-enabled cellular and D2D systems. Considering a fully connected D2D network, the authors in [16], [17] investigated the problem of packet completion time minimization - the number of transmission slots necessary to recover all missing packets - over joint cellular and D2D systems with IDNC. They designed several heuristics to find a sub-optimal solution to the problem.

In real-world scenarios, users are sparsely scattered over a large area, and utilize single-hop or multi-hop short range D2D links whenever possible. However, depending on their locations, some users may be isolated and unable to use D2D links and can receive their data only via cellular links. Isolated users are referred to as singleton users. To the best of our knowledge, existing research has neglected singleton users, consequently the schemes proposed in works such as [12]–[14], [16], [17] will fail in the presence of isolated users.

Motivated by this observation, we consider a general network topology that takes into account the heterogeneity of

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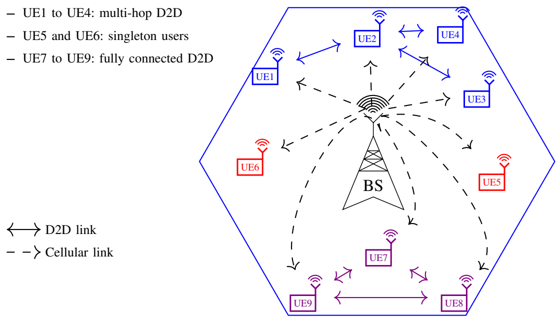

Fig. 1: Example of an intermittently connected D2D network.

D2D connections among users as depicted in Fig. 1. We refer this setup as *intermittently connected D2D network*. It includes singleton users and disjoint clusters of multi-hop D2D networks, and disjoint clusters of single-hop fully connected D2D networks. Moreover, multiple users can transmit simultaneously; e.g., UE1 can transmit to UE2, while UE9 transmits to both UE7 and UE8 in Fig. 1. The intermittently connected D2D topology considered in this paper is a general case of the fully connected D2D scenario studied in [16], [17]. To the best of our knowledge, existing literature has not studied the IDNC codes for joint cellular and intermittently connected D2D links, which is the focus of this paper.

We tackle the problem of broadcasting a common content, which may be partially received by multiple cellular users. Intermittently available D2D links among users are used to recover the missing content. We design an IDNC mechanism to minimize the *packet completion time*. Specifically, we derive an optimal packet completion time strategy by constructing a two-layer IDNC conflict graph where the higher layer represents all feasible IDNC codes that can be transmitted over cellular links, while the lower layer consists of network coded packets (as well as corresponding cellular users that can generate and transmit these packets) that can be transmitted via D2D links. We demonstrate the equivalence between finding the optimal IDNC packets to minimize the packet completion time problem and the maximum independent set problem of the two-layer IDNC conflict graph. To circumvent the high computational complexity to find the maximum independent set, we invoke the principle of alternating optimization, and develop a computationally efficient algorithm, which we name `aIDNC`. The efficiency of `aIDNC` as compared to the global optimum is verified through numerical experiments.

II. System Setup and Problem Formulation

We consider an intermittently connected device-to-device-enabled cellular network composing of one base station that seeks to communicate with a set of $\mathcal{N} = \{1, 2, \cdots, N\}$ cellular users equipped with D2D interface as exemplified in Fig. 1. The D2D network topology is represented by the connection matrix $\mathbf{C} \in \{0,1\}^{|\mathcal{N}|\times|\mathcal{N}|}$ with entries $c_{jk} \in \{0,1\}$ and is set to 1 if users j and k (UEj and UEk) are in the transmission range of each other, hence directly connected. We define the coverage area of each user as follows:

Definition 1 *[12] The coverage area $\mathcal{Y}_j$ of user j, is defined as the set of neighboring users that are directly connected to it, i.e., $\mathcal{Y}_j \triangleq \{k \in \mathcal{N} | c_{jk} = 1\}$.*

We assume that all users are interested in receiving a popular content and are requesting a common file from a BS. A content (e.g., a video or music file) is divided into a set of $\mathcal{M} = \{1, 2, \cdots, M\}$ packets. Furthermore, it is assumed that the system operates in two stages. During the first stage, the BS uses cellular link to broadcast $M = |\mathcal{M}|$ packets to all $N = |\mathcal{N}|$ users. Due to packet erasures over cellular, some packets may not be received by some users. Thus, after the first stage transmission, all users inform the BS their packet reception status via an error-free link. The packet reception status is determined by a feedback matrix $\mathbf{F} \in \{0,1\}^{M\times N}$ whose entries f_{nm} are defined as follows:

$$f_{mn} \triangleq \begin{cases} 1 & \text{if packet } m \in \mathcal{H}_n \\ 0 & \text{if packet } m \in \mathcal{W}_n \end{cases} \tag{1}$$

where $\mathcal{H}_n$, which is referred to as *Has* set, denote the set of packets that have been successfully received by user $n \in \mathcal{N}$, and $\mathcal{W}_n$ known as the *Wants* set is the set of packets missing at the nth user after the first stage.

In the second stage, a packet retransmission scheme is employed to ensure delivery/recovery of all missing packets. Packets in the *Wants* sets of the devices are recovered by simultaneously broadcasting IDNC packets on both cellular and D2D links. We note that cellular and D2D links operate on different frequencies, so simultaneous transmissions over these links are possible [17], [18]. During the retransmission process, we assume that the communication between either the BS to the cellular users or users to users is established via a lossless channel, yet it is straightforward to relax this assumption for the lossy-channel scenario [17].

Definition 2 *A packet received by a user is instantly decodable if it contains exactly and at most one packet from the Wants set of the user.*

Example 1: *Benefit of IDNC when cellular links are used.* Consider the following example where three users; u_1, u_2, and u_3 are requesting four packets from the BS. After the first stage of transmission, the *Has* sets of the users are given by $\mathcal{H}_1 = \{p_1, p_4\}$, $\mathcal{H}_2 = \{p_1, p_2, p_3\}$ and $\mathcal{H}_3 = \{p_2, p_3\}$ and their *Wants* sets are $\mathcal{W}_1 = \{p_2, p_3\}$, $\mathcal{W}_2 = \{p_4\}$ and $\mathcal{W}_3 = \{p_1, p_4\}$. Without network coding, the BS needs four time slots to recover all missing packets. By using network coding, only two transmissions are required. More precisely, the BS may broadcast coded packet $p_3 \oplus p_4$ in the first time slot and $p_1 \oplus p_2$ for the second time slot or inversely. The transmission time reduces to two from four thanks to IDNC.

On the other hand, if both cellular and D2D links are used simultaneously (they can operate simultaneously thanks to using different parts of the spectrum [17], [18]), the benefit of network coding is amplified [16], [17]. Let us consider Example 1 again. Two transmission slots are needed for the BS to retransmit all missing packets. By assuming a fully connected D2D network, u_2 can broadcast $p_1 \oplus p_3$ to u_1 and u_3 via D2D link, while the BS conveys $p_2 \oplus p_4$ to all users on the cellular link. In this case, all packets can be recovered in a single transmission. This shows the benefit of IDNC when both cellular and D2D links are used. Yet, the performance benefit of IDNC is still not studied when D2D connections are intermittent, which is the focus of this paper.

Problem 1 Given an intermittently connected D2D network topology and the *Has* and *Wants* sets of each cellular user, find the optimal IDNC codes to minimize the packet completion time simultaneously leveraging cellular and D2D links during the second stage.

III. Two-Layer IDNC Conflict Graph

We tackle Problem 1 by leveraging concepts from graph theory. Specifically, we construct a two-layer IDNC conflict graph to determine the IDNC packets that can be transmitted over the cellular links, users that can create and transmit packets via D2D links, and IDNC packets that can be transmitted over D2D links. A similar approach of constructing IDNC graphs has been considered in [12], [14], but only for partially connected D2D networks without considering (i) cellular links, and (ii) isolated users. Next, we describe the construction of our two-layer IDNC conflict graph.

1) Higher-layer IDNC graph: which is built from the perspective of the BS, aims at constructing feasible IDNC packets that can be transmitted over the cellular link. We denote the higher-layer IDNC graph as $\mathcal{G}_1(\mathcal{V}_1, \mathcal{E}_1)$. It is an undirected graph constructed with the set of packets from $\mathcal{M}$ yet to be recovered by the users. The set of vertices $\mathcal{V}_1$ corresponds to the set of missing packets of all users, i.e., $\cup_{n \in \mathcal{N}} \mathcal{W}_n$. For each packet $p_l \in \cup_{n \in \mathcal{N}} \mathcal{W}_n$, there is a vertex $v_l^{(\mathrm{BS})} \in \mathcal{V}_1$. Two vertices are connected by an edge if the coded combination of the two packets associated with the vertices is not instantly decodable for a least one user. That is,

$$\forall (v_n^{(\mathrm{BS})}, v_l^{(\mathrm{BS})}) \in \mathcal{V}_1, (v_n^{(\mathrm{BS})}, v_l^{(\mathrm{BS})}) \in \mathcal{E}_1 \text{ if } \exists\, k \in \mathcal{N} | (p_n, p_l) \in \mathcal{W}_k$$

Example 2: Consider that four users; $\{u_1, \dots u_4\}$ want to receive three packets; p_1, p_2, p_3. Assume that both u_1 and u_3 are connected to u_2, and u_4 is a singleton. The *Has* and *Wants* sets of the users are given respectively by $\mathcal{H}_1 = \{p_2\}$, $\mathcal{H}_2 = \{p_1, p_3\}$, $\mathcal{H}_3 = \{p_2\}$, $\mathcal{H}_4 = \{p_2, p_3\}$ and $\mathcal{W}_1 = \{p_1, p_3\}$, $\mathcal{W}_2 = \{p_2\}$, $\mathcal{W}_3 = \{p_1, p_3\}$, $\mathcal{W}_4 = \{p_1\}$. Given that $\cup_{n=1,\cdots,4} \mathcal{W}_n = \{p_1, p_2, p_3\}$, then $\mathcal{V}_1 = \{v_1^{\mathrm{BS}}, v_2^{\mathrm{BS}}, v_3^{\mathrm{BS}}\}$. For this case, there is only one edge connecting v_1^{BS} to v_3^{BS} because $\mathcal{W}_3 = \{p_1, p_3\}$.

2) Lower-layer IDNC graph: denoted as $\mathcal{G}_2(\mathcal{V}_2, \mathcal{E}_2)$ is an undirected graph constructed from the perspective of all cellular users. Its vertices and edges are derived as follow: Let $v_l^{(i)}$ be associated with ith user ($i \in \mathcal{N}$) and the lth packet ($\forall p_l \in \mathcal{M} \cap \mathcal{H}_i$). $v_l^{(i)}$ is a vertex of the graph $\mathcal{G}_2(\mathcal{V}_2, \mathcal{E}_2)$ if there exists a user k in the coverage area of user i that wants packet p_l. That is, $\forall i \in \mathcal{N}, v_l^{(i)} \in \mathcal{V}_2 \rightarrow \exists (k,l) | \{k \in \mathcal{Y}_i\} \cap \{p_l \in \mathcal{H}_i \cap \mathcal{W}_k\}$. For example, we have $\mathcal{V}_2 = \{v_1^2, v_3^2, v_2^1, v_2^3\}$ in *Example 2*. In fact, $\{v_1^2, v_3^2\} \in \mathcal{V}_2$, because u_2 is connected to u_1 and $\mathcal{W}_1 = \mathcal{H}_2$. $v_2^1 \in \mathcal{V}_2$, because both u_1 and u_2 are connected, and $p_2 \in \mathcal{H}_1 \cap \mathcal{W}_2$.

Two vertices of the graph $\mathcal{G}_2(\mathcal{V}_2, \mathcal{E}_2)$ are connected by an edge if at least one of the following conditions is satisfied.

- (C1) The vertices correspond to different packets that are in the *Has* set of one user and in the *Wants* set of a second user, and the two users are connected. For example, v_1^2 and v_3^2 are connected in *Example 2*, because $\{p_1, p_3\} \in \mathcal{H}_2 \cap \mathcal{W}_1$ and u_1 and u_2 are connected.
- (C2) The vertices are generated by two different users that are directly connected to each other; e.g., v_1^2 is connected to v_2^1 because u_1 and u_2 are connected in *Example 2*.
- (C3) The vertices correspond to two different users that are connected to a third user; e.g., v_2^1 is connected to v_2^3 in *Example 2*, because u_1 and u_3 are connected to u_2.

3) Two-layer IDNC conflict graph: combines higher and lower layer IDNC graphs. The two-layer IDNC conflict graph is an undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ where the vertices and edges are constructed using the set of vertices and edges of both the higher and lower layer IDNC graphs $\mathcal{G}_1(\mathcal{V}_1, \mathcal{E}_1)$ and $\mathcal{G}_2(\mathcal{V}_2, \mathcal{E}_2)$, i.e., $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$. In the combined graph, new edges are created between higher and lower layers to reduce redundancy. In particular, a vertex from the higher layer IDNC graph is connected to a vertex from lower layer IDNC graph if both vertices are induced by the same packet; e.g., v_2^{BS} is connected to both $v_2^1$and v_2^3 in *Example 2*, because the three vertices are generated by packet p_2.

A. Characterization of the Optimal Solution

We now proceed to characterize the optimal solution of Problem 1. We first state the following theorem.

Theorem 1 *Finding users that will create and transmit IDNC packets over D2D links as well as IDNC packets themselves (both the ones that are transmitted from BS over cellular or from one user to another through D2D) is equivalent to finding an independent set of the two-layer IDNC conflict graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$.*

Proof: The proof of the theorem is built upon the following two lemmas.

Lemma 1 *Finding an IDNC packet that can be transmitted over the cellular link is equivalent to finding an independent set of the higher-layer IDNC graph $\mathcal{G}_1(\mathcal{V}_1, \mathcal{E}_1)$.*

Lemma 2 *Finding the set of users that generate and transmit IDNC packets as well as their corresponding IDNC packets is equivalent to finding an independent set of the lower-layer IDNC graph $\mathcal{G}_2(\mathcal{V}_2, \mathcal{E}_2)$.*

The proofs of Lemma 1 and Lemma 2 can be found in the extended version of this paper [19].

To prove Theorem 1, we start by showing the sufficient condition. Suppose that $\mathcal{N}_2$ is the set of users scheduled to broadcast on the D2D links and $\mathcal{M}_{\text{BS, D2D}}$ is the set of codes that can be transmitted by the BS or any user in $\mathcal{N}_2$, and are instantly decodable. $\mathcal{M}_{\text{BS, D2D}}$ is the union of $\mathcal{M}_{\text{D2D}}$ and $\mathcal{M}_{\text{BS}}$, where $\mathcal{M}_{\text{BS}}$ is the set of feasible IDNC packets that can be transmitted from BS via cellular links while $\mathcal{M}_{\text{D2D}}$ is the set of feasible IDNC packets that can be transmitted from one user to another via D2D links.

With the indices of packets in $\mathcal{M}_{\text{BS}}$, we can construct a set of vertices $\mathcal{V}_{\text{BS}} \subseteq \mathcal{V}_1 \subseteq \mathcal{V}$. Similarly, the indices of the users in $\mathcal{N}_2$ together with the indices of their associated feasible IDNC packets enable to construct a set of vertices $\mathcal{V}_{\text{D2D}} \subseteq \mathcal{V}_2 \subseteq \mathcal{V}$. Thus, $\mathcal{V}_{\text{BS, D2D}} \triangleq \mathcal{V}_{\text{BS}} \cup \mathcal{V}_{\text{D2D}}$ is the set of vertices associated with feasible codes in $\mathcal{M}_{\text{BS, D2D}}$. It holds true that $\mathcal{V}_{\text{BS, D2D}} \subseteq \mathcal{V}$. We proceed to show that no two vertices in $\mathcal{V}_{\text{BS, D2D}}$ are connected. Suppose that at least two vertices are connected. Given that $\mathcal{M}_{\text{BS}}$ is the set of feasible codes that BS can transmit via cellular links, we know by Lemma 1 that no two vertices in $\mathcal{V}_{\text{BS}}$ can be linked by an edge. Connections between at least two vertices in $\mathcal{V}_{\text{BS, D2D}}$ cannot happen in $\mathcal{V}_{\text{BS}}$. Similarly, using the result of Lemma 2, we can state that the connection will not happen in $\mathcal{V}_{\text{D2D}}$. This means that there exists at least one vertex in $\mathcal{V}_{\text{BS}}$ that is connected to at least one vertex in $\mathcal{V}_{\text{D2D}}$. Consequently, the vertices correspond to at least one common packet. This leads to a contradiction. Therefore, $\mathcal{V}_{\text{BS, D2D}}$ is an independent set.

Suppose the existence of an independent set $\widetilde{\mathcal{V}} \subseteq \mathcal{V}$ of the two-layer IDNC conflict graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$. Let $\widetilde{\mathcal{M}}$ be the set of all IDNC packets associated with the vertices in $\widetilde{\mathcal{V}}$. We need to demonstrate that the codes in $\widetilde{\mathcal{M}}$ are *instantly decodable and efficient*, which requires the following conditions to be satisfied: (i) instant decodability, (ii) collision free D2D transmission (i.e., there should not be transmissions to the same user at the same time), and (iii) no redundant transmissions (i.e., network coded packets over cellular and D2D should not carry the same/redundant packets). In particular:

1) Suppose $\widetilde{\mathcal{V}} \subseteq \mathcal{V}_1$, i.e., all vertices of $\widetilde{\mathcal{V}}$ belong to the higher-layer IDNC graph $\mathcal{G}_1(\mathcal{V}_1, \mathcal{E}_1)$. Given that $\widetilde{\mathcal{V}} \subseteq \mathcal{V}_1$, it is known from Lemma 1 that $\widetilde{\mathcal{M}}$ is a feasible IDNC packet for the cellular link. Given that no transmission occurs via D2D links, we therefore conclude that coded packets in $\widetilde{\mathcal{M}}$ are instantly decodable.

2) Suppose that $\widetilde{\mathcal{V}} \subseteq \mathcal{V}_2$, i.e., $\widetilde{\mathcal{V}}$ is in the the lower-layer IDNC graph $\mathcal{G}_2(\mathcal{V}_2, \mathcal{E}_2)$. Given that $\widetilde{\mathcal{V}} \subseteq \mathcal{V}_2$, the result of Lemma 2 implies that the set of users that can create network coded packets can be found. Furthermore, $\widetilde{\mathcal{M}}$ determines the corresponding feasible IDNC packets that can be transmitted over D2D links. Since the BS is not transmitting any packets, we conclude that the codes in $\widetilde{\mathcal{M}}$ are instantly decodable and collision free thanks to the construction of $\mathcal{G}_2(\mathcal{V}_2, \mathcal{E}_2)$.

3) Suppose $\widetilde{\mathcal{V}}$ consists of vertices that are both in $\mathcal{V}_1$ and $\mathcal{V}_2$. We denote $\widetilde{\mathcal{V}}_{\text{BS}}$ as the vertices of $\widetilde{\mathcal{V}}$ that are in $\mathcal{V}_1$, and $\widetilde{\mathcal{V}}_{\text{D2D}}$ as the vertices of $\widetilde{\mathcal{V}}$ that are in $\mathcal{V}_2$. This means that $\widetilde{\mathcal{V}} = \widetilde{\mathcal{V}}_{\text{BS}} \cup \widetilde{\mathcal{V}}_{\text{D2D}}$, and that both sets $\widetilde{\mathcal{V}}_{\text{BS}}$ and $\widetilde{\mathcal{V}}_{\text{D2D}}$ are independent sets. Following the decomposition of the set $\widetilde{\mathcal{V}}$, we can also decompose the set $\widetilde{\mathcal{M}}$ of IDNC packets associated with the vertices in $\widetilde{\mathcal{V}}$ as $\widetilde{\mathcal{M}}_{\text{BS}}$ and $\widetilde{\mathcal{M}}_{\text{D2D}}$ which are respectively, associated with the vertices in $\widetilde{\mathcal{V}}_{\text{BS}}$ and $\widetilde{\mathcal{V}}_{\text{D2D}}$. From Lemma 1, we know that $\widetilde{\mathcal{M}}_{\text{BS}}$ is the set of feasible IDNC packets over cellular links. Also, using the result of Lemma 2, we can say that $\widetilde{\mathcal{M}}_{\text{D2D}}$ is the set of users that transmit network coded packets and the associated feasible IDNC packets over D2D links. To complete the proof, it remains to show that the set $\widetilde{\mathcal{M}}_{\text{BS}} \cup \widetilde{\mathcal{M}}_{\text{D2D}}$ of codes are instantly decodable and efficient. Suppose this does not hold; given that the previous two items are satisfied (i.e., the instant decodability of packets in $\widetilde{\mathcal{M}}$ and their collision free nature over D2D), the infeasibility of the codes in $\widetilde{\mathcal{M}}_{\text{BS}} \cup \widetilde{\mathcal{M}}_{\text{D2D}}$ violates the redundancy condition. In particular, there is at least one common packet in both sets $\widetilde{\mathcal{M}}_{\text{BS}}$ and $\widetilde{\mathcal{M}}_{\text{D2D}}$, which means that there exists at least one edge that connects at least one vertex in $\mathcal{V}_1$ with one vertex in $\mathcal{V}_2$. This leads to a contradiction since $\widetilde{\mathcal{V}}$ is an independent set. Therefore, the coded packets in $\widetilde{\mathcal{M}} = \widetilde{\mathcal{M}}_{\text{BS}} \cup \widetilde{\mathcal{M}}_{\text{D2D}}$ are instantly decodable and efficient. ■

Theorem 1 creates a one-to-one mapping between an independent set of the two-layer IDNC graph and IDNC packets that are instantly decodable and efficient.

Corollary 1 *Finding an optimal strategy at each transmission for Problem 1 is equivalent to finding the maximum independent set of the two-layer IDNC conflict graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$.*

Proof: The proof is built on the following proposition, whose proof is provided in the extended version [19].

Proposition 1 *Let $\overline{\mathcal{V}} \subseteq \mathcal{V}$ be an independent set of the two-layer IDNC conflict graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$. Suppose $\overline{\mathcal{N}}$ is the number of users scheduled to broadcast over the D2D links and $\overline{\mathcal{M}}_{BS,\ D2D}$ is the set of IDNC packet associated with $\overline{\mathcal{V}}$. Let $\breve{v}_i^{(j)} \in \{\mathcal{V} \backslash \overline{\mathcal{V}}\}$ with $j \in (BS, 1, \cdots, N)$, $i \in (1, \cdots M)$ and let p_i be the packet associated with vertex $\breve{v}_i^{(j)}$. $\overline{\mathcal{V}} \cup \{\breve{v}_i^{(j)}\}$ is an independent set of the two-layer IDNC conflict graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ if and only if $\overline{\mathcal{M}}_{BS,\ D2D} \oplus p_i$ is an instantly decodable and efficient IDNC packet.*

From Theorem 1 and Proposition 1, it is known that for each IDNC strategy with a certain number of packets there exists an independent set of the same size and vice versa. Moreover, it was proven that an IDNC packet that is formed by the highest number of uncoded packets coincides with the optimal solution [20]. Finally, the IDNC packet composed of largest uncoded packets yields the largest independent set, in terms of number of vertices. This concludes the proof. ■

B. Sub-optimal IDNC Mechanism: `aIDNC`

The optimal solution to the maximum independent set problem can be found by invoking the Bron-Kerbosch algorithm [21]. The computational complexity of the Bron-Kerbosch method to find the maximum independent set of the graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is on the order of $\mathcal{O}\left(3^{\frac{|\mathcal{V}|}{3}}\right)$. Hence, the worst case complexity to solve Problem 1 is $\mathcal{O}\left(3^{\frac{|\mathcal{V}|}{3}} \max_{n \in \mathcal{N}} \frac{\mathcal{W}_n}{2}\right)$. To circumvent the computational burden, we propose an efficient sub-optimal mechanism, `aIDNC`, which consists of alternating the search of the maximum independent set between the higher layer IDNC graph $\mathcal{G}_1(\mathcal{V}_1, \mathcal{E}_1)$ and the lower layer IDNC graph $\mathcal{G}_2(\mathcal{V}_2, \mathcal{E}_2)$. `aIDNC` is summarized as follows.

Algorithm 1 `aIDNC`: Alternating-based algorithm

1: **Input**: $\mathbf{F}, \mathbf{C}, \mathcal{W}_n \, \forall n, \, \mathcal{H}_n, \, \forall n$.
2: Initialize $T^{\text{Opt}} = 0$;
3: **while** $\mathbf{1}_{N\times 1}^{\top}\mathbf{F}\mathbf{1}_{N\times 1} \neq N \times M$ **do**
4: Construct the higher-layer IDNC graph $\mathcal{G}_1(\mathcal{V}_1, \mathcal{E}_1)$;
5: Run the Bron-Kerbosch algorithm to find the maximum independent set $\mathcal{V}_1^\star$ of the undirected graph $\mathcal{G}_1(\mathcal{V}_1, \mathcal{E}_1)$;
6: Use $\mathcal{V}_1^\star$ to find IDNC packets for the BS;
7: Update $\mathbf{F}$ and $\mathcal{W}_n$, $\forall n$, $\mathcal{H}_n$, $\forall n$;
8: Construct the lower-layer IDNC graph $\mathcal{G}_2(\mathcal{V}_2, \mathcal{E}_2)$;
9: Run the Bron-Kerbosch algorithm to find the maximum independent set $\mathcal{V}_2^\star$ of the undirected graph $\mathcal{G}_2(\mathcal{V}_2, \mathcal{E}_2)$;
10: Find the set of D2D users and their corresponding IDNC packets for transmission over the D2D link;
11: Update $\mathbf{F}$ and $\mathcal{W}_n$, $\forall n$, $\mathcal{H}_n$, $\forall n$;
12: Update $T^{\text{Opt}} \leftarrow T^{\text{Opt}} + 1$;
13: **end while**
14: **Output**: T^{Opt}.

Algorithm 1 is implemented in a centralized fashion and requires information about the network connection and feedback matrices. It is worth pointing out that in Step 6 and Step 9 of Algorithm 1, there might be more than one set with same size as the solution of the maximum independent set. Whenever this scenario occurs, we choose the set with the maximum number of receivers. The worst case computational complexity of Algorithm 1 to find a solution to Problem 1 is on the order of $\mathcal{O}\left(3^{\frac{\max(|\mathcal{V}_1|, |\mathcal{V}_2|)}{3}} \max_{n\in\mathcal{N}} \frac{\mathcal{W}_n}{2}\right)$.

C. Illustration of `aIDNC`

We now illustrate how `aIDNC` in Algorithm 1 is executed using a toy example. We consider an intermittently connected D2D network topology with six users in Fig. 2.

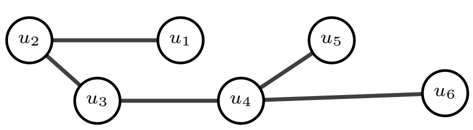

Fig. 2: An example network topology.

The network topology and the reception status are established through the connection matrix $\mathbf{C}$ and the feedback matrix $\mathbf{F}$, respectively.

$$\mathbf{C} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \mathbf{F} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

The *Has* and the *Wants* sets of the users are given by: $\mathcal{H}_1 = \{p_2, p_3\}$, $\mathcal{H}_2 = \{p_1, p_3, p_4\}$, $\mathcal{H}_3 = \{p_1, p_2\}$, $\mathcal{H}_4 = \{p_1, p_4\}$, $\mathcal{H}_5 = \{p_1, p_2, p_3\}$, $\mathcal{H}_6 = \{p_1, p_3, p_4\}$, $\mathcal{W}_1 = \{p_1, p_4\}$, $\mathcal{W}_2 = \{p_2\}$, $\mathcal{W}_3 = \{p_3, p_4\}$, $\mathcal{W}_4 = \{p_2, p_3\}$, $\mathcal{W}_5 = \{p_4\}$, $\mathcal{W}_6 = \{p_2\}$.

1) Higher-layer operation: `aIDNC` starts by constructing the higher-layer IDNC graph. Given that $\cup_n \mathcal{W}_n = \{p_1, \cdots, p_4\}$, there are 4 vertices associated with $\{p_1, \cdots, p_4\}$. Two vertices are connected if their associated packets are in the *Wants* set of a user. For example, the *Wants* set of u_1 is $\mathcal{W}_1 = \{p_1, p_4\}$. Therefore, there exists an edge between v_1^{BS} and v_4^{BS}. The higher-layer IDNC graph is depicted in Fig. 3.

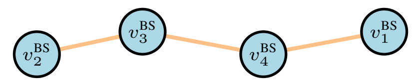

Fig. 3: Higher-layer IDNC graph

For the undirected graph given in Fig. 3, there are two maximum independent sets: $(v_1^{\text{BS}}, v_3^{\text{BS}})$ and $(v_2^{\text{BS}}, v_4^{\text{BS}})$. The associated IDNC packets are given by $p_1 \oplus p_3$ and $p_2 \oplus p_4$. To choose between these two IDNC packets, `aIDNC` determines the number of receivers for each packet. The IDNC packet $p_1 \oplus p_3$ is beneficial to three users, while $p_2 \oplus p_4$ is beneficial to six users. Therefore, $p_2 \oplus p_4$ will be transmitted by the BS over the cellular link. `aIDNC` updates the feedback matrix as

$$\mathbf{F} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \tag{2}$$

Then, `aIDNC` switches to the lower-layer operation.

2) Lower-layer operation: From the updated $\mathbf{F}$ in (2), p_1 and p_3 are yet to be transmitted. `aIDNC` gathers the users that have these two packets in their *Has* sets together with their corresponding coverage area in order to determine the vertices. For instance, consider u_2 whose coverage area encompasses u_1 and u_3. Moreover, the *Has* set of u_2 is $\mathcal{H}_2 = \{p_1, p_2, p_3, p_4\}$ and the *Wants* sets of u_1 and u_3 are $\mathcal{W}_1 = \{p_1\}$ and $\mathcal{W}_3 = \{p_3\}$, respectively. Given that $u_1 \in \mathcal{Y}_2$ and $p_1 \in \mathcal{H}_2 \cap \mathcal{W}_1$, then we have vertex v_1^2 meaning that u_2 can transmit p_1 since it will be beneficial to u_1. Also, there will be vertex v_3^2, because $u_3 \in \mathcal{Y}_2$ and $p_3 \in \mathcal{H}_2 \cap \mathcal{W}_3$. The remaining vertices are given by (v_3^5, v_3^6). Due to condition C3, there is an edge between vertices v_3^5 and v_3^6, because both u_5 and u_6 are transmitting to u_4. The lower-layer IDNC graph is shown in Fig. 4.

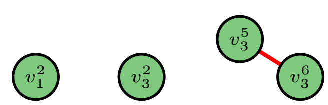

Fig. 4: Lower-layer conflict IDNC graph

The two maximum independent sets associated with the graph depicted in Fig. 4 are (v_1^2, v_3^2, v_3^5) and (v_1^2, v_3^2, v_3^6) associated respectively with the following two instances of transmission for the D2D link: (i) u_2 sends IDNC packets $p_1 \oplus p_3$ while u_5 sends packet p_3, (ii) u_2 broadcasts IDNC

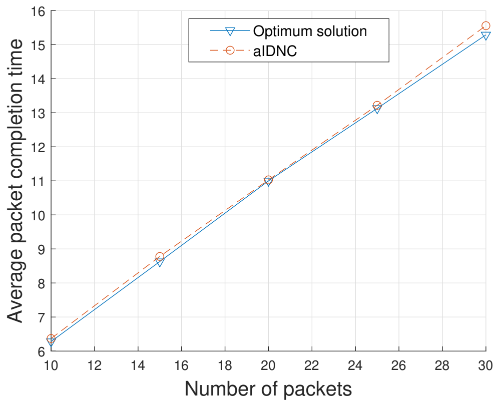

Fig. 5: Average packet completion time for $N = 10$

packets $p_1 \oplus p_3$ whereas u_6 conveys packet p_3. Given that the number of beneficial receivers in both independent sets is three, then aIDNC chooses randomly between the two. For this example, the packet completion time is 1.

IV. Simulation Results

In this section, we present numerical results to evaluate the performance of aIDNC. All results are obtained using Monte Carlo by averaging over 500 trials on both the connection matrix and the feedback matrix. The connection matrix $\mathbf{C}$ is generated randomly according to a uniform distribution. As a benchmark, we provide the performance of optimum solution obtained by running the Bron-Kerbosch approach to find the maximum independent set of the two-layer IDNC conflict graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ at each iteration.

Fig. 5 depicts the performance of the optimum solution and the performance of aIDNC in terms of average packet completion time versus the number of packets. We consider a network topology that consists of 10 users. As can be seen from Fig. 5, aIDNC achieves near-optimal solution given that the gap between the performance of the optimum solution and the one of the proposed aIDNC is negligible. There is a gain varying from 0.4% to 1.67% between the performance of both approaches. This leads us to conclude that our proposed aIDNC in Algorithm 1 yields near-optimal solution at least for low-scale intermittently connected D2D network. More simulation results are provided in the extended version [19].

V. Conclusion

In this work, we investigated the problem of packet completion time minimization for joint cellular and intermittently connected D2D networking. We developed an optimal packet completion time strategy by constructing a two-layer IDNC conflict graph. We rigorously demonstrated that finding the optimal IDNC packets to minimize the packet completion time problem is equivalent to finding the maximum independent set of the two-layer IDNC conflict graph. Moreover, we designed an efficient sub-optimal IDNC mechanism, aIDNC , which was proved through numerical experiments to achieve a near-optimal solution.

References

[1] V. Cisco, "Cisco visual networking index: Forecast and trends, 2017–2022," *White Paper*, vol. 1, 2018.

[2] Ericsson, "Mobile data traffic growth outlook," June 2019.

[3] F. Boccardi, R. W. Heath, A. Lozano, T. L. Marzetta, and P. Popovski, "Five disruptive technology directions for 5G," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 74–80, February 2014.

[4] T. Ho, M. Medard, R. Koetter, D. R. Karger, M. Effros, J. Shi, and B. Leong, "A Random Linear Network Coding Approach to Multicast," *IEEE Trans. Inf. Theory*, vol. 52, no. 10, pp. 4413–4430, Oct 2006.

[5] S. Sorour and S. Valaee, "An Adaptive Network Coded Retransmission Scheme for Single-Hop Wireless Multicast Broadcast Services," *IEEE/ACM Trans. Netw.*, vol. 19, no. 3, pp. 869–878, June 2011.

[6] A. Douik, S. Sorour, T. Y. Al-Naffouri, and M. Alouini, "Instantly Decodable Network Coding: From Centralized to Device-to-Device Communications," *IEEE Commun. Surveys Tuts.*, vol. 19, no. 2, pp. 1201–1224, Secondquarter 2017.

[7] S. Katti, D. Katabi, W. Hu, H. Rahul, and M. Médard, "The importance of being opportunistic: Practical network coding for wireless environments," in *2005 43rd Annual Proc. Allerton Conference on Communication Control and Computing*, Sep. 2005, pp. 756–765.

[8] A. Le, A. S. Tehrani, A. G. Dimakis, and A. Markopoulou, "Instantly decodable network codes for real-time applications," in *2013 International Symposium on Network Coding (NetCod)*, June 2013, pp. 1–6.

[9] S. Sorour and S. Valaee, "Completion Delay Minimization for Instantly Decodable Network Codes," *IEEE/ACM Trans. Netw.*, vol. 23, no. 5, pp. 1553–1567, Oct 2015.

[10] ——, "Completion Delay Minimization for Instantly Decodable Network Codes," *IEEE/ACM Trans. Netw.*, vol. 23, no. 5, pp. 1553–1567, Oct 2015.

[11] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft, "XORs in the Air: Practical Wireless Network Coding," *IEEE/ACM Trans. Netw.*, vol. 16, no. 3, pp. 497–510, June 2008.

[12] M. S. Karim, S. Sorour, and P. Sadeghi, "Network Coding for Video Distortion Reduction in Device-to-Device Communications," *IEEE Trans. Veh. Technol.*, vol. 66, no. 6, pp. 4898–4913, June 2017.

[13] A. Douik, S. Sorour, T. Y. Al-Naffouri, H. Yang, and M. Alouini, "Delay Reduction in Multi-Hop Device-to-Device Communication Using Network Coding," *IEEE Trans. Wireless Commun.*, vol. 17, no. 10, pp. 7040–7053, Oct 2018.

[14] C. Zhan and K. Gao, "Conflict-Free Scheduling for Partially Connected D2D Networks With Network Coding," *IEEE Wireless Commun. Lett.*, vol. 5, no. 5, pp. 456–459, Oct 2016.

[15] H. Seferoglu, L. Keller, B. Cici, A. Le, and A. Markopoulou, "Cooperative video streaming on smartphones," in *2011 49th Annual Allerton Conference on Communication, Control, and Computing*, Sep. 2011, pp. 220–227.

[16] C. Zhan, Z. Wen, and L. Zhu, "Minimum Number of Transmission Slots in D2D-Assisted Wireless Coded Broadcast," in *2017 IEEE International Conference on Smart Computing (SMARTCOMP)*, May 2017, pp. 1–6.

[17] Y. Keshtkarjahromi, H. Seferoglu, R. Ansari, and A. Khokhar, "Device-to-Device Networking Meets Cellular via Network Coding," *IEEE/ACM Trans. Netw.*, vol. 26, no. 1, pp. 370–383, Feb 2018.

[18] L. Keller, A. Le, B. Cici, H. Seferoglu, C. Fragouli, and A. Markopoulou, "Microcast: Cooperative video streaming on smartphones," in *Proceedings of the 10th international conference on Mobile systems, applications, and services*, 2012, pp. 57–70.

[19] J. Denis and H. Seferoglu, "Packet Completion Time Minimization via Joint D2D and Cellular Communication: A Unified Network Coding Approach," 2020. [Online]. Available: https://arxiv.org/abs/2006.11606v1

[20] A. Le, A. S. Tehrani, A. Dimakis, and A. Markopoulou, "Recovery of Packet Losses in Wireless Broadcast for Real-Time Applications," *IEEE/ACM Trans. Netw.*, vol. 25, no. 2, pp. 676–689, 2017.

[21] C. Bron and J. Kerbosch, "Algorithm 457: finding all cliques of an undirected graphs," *Commun. ACM*, vol. 16, no. 9, pp. 575–577, 1973.