Light Confinement in Coreless Twisted Photonic Crystal Fibers

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Abstract – **Recent work have shown light confinement can occur during propagation through a twisted coreless photonic crystal fiber (a chiral fiber). In the absence of a twist, the modal profile is assumed known from Bloch theory and assumed not to be confined. By use of asymptotic techniques applied to the field propagation equation, we provide a theoretical framework in support of observed confinement. While we do this for a particular periodic index profile, recent experiments suggest this to be a robust effect. In this work, we also explore the problem both in the linear and the nonlinear regime. We show that an increase in twist rate will result in more confined modes and indications that nonlinearity plays a secondary role on confinement.**

I. Introduction

Guiding light by ways of manipulating the transverse index of refraction profile is a classic well studied topic. Its relevance stems from applications to optical communications, laser systems, sensors and many more. In the early years, the traditional optical fiber used in most industrial applications utilized a guiding core that is surrounded by a cladding of material with a lower refractive index [1]. As in many similar research explorations, one encounters new propagation properties due to topological features and a better understanding of effects proposed in other fields such as quantum mechanics and new applications. Motivated by recent beautiful experiments by Russell's group, in this paper we theoretically examine light confinement in helical photonic crystals. In particular we extend the work to include nonlinear effects. The structure studied in this paper is that of a twisted *coreless* PCF with features similar to the one used in [2], shown to display a variety of interesting phenomea. In particular, we will closely examine light confinement in helical fibers in the absence of a guiding core.

Inspired by the work of Beravat et.al. [2], here we consider a photonic crystal structure with a refractive index profile similar to the one fabricated and used in experiments from [2] whose structure consists of hollow chambers with radii spaced around 5μm apart and grouped together in the shape of a hexagon. The fiber is twisted around its axis during the drawing process. In their experiments, light propagated through this fiber for around 20cm and was shown to have confinement proportional to the twist rate of the fiber. Here, we will verify and show by analytical and numerical means through field theory, how this type of confinement can occur due to the geometry of the fiber.

We suggest the dynamics of the rapid twisting is analogous to that of the rapid vibrating base in the classical Kapitza pendulum. Previous investigations into periodic modulations of the transverse refractive index have been connected to this "Kapitza effect" and can also provide confinement and guiding of light [3]. In this work, we assert the rapid motion of the pendulum's base is analogous to the rapid twisting of the fiber and thus creating a trapping region for light.

II. Field Theory - Analysis

As a first step in our study, we propose a smooth approximation of the refractive index profile of the fiber as

$$n(x,y,z) = 1 - \delta\cos[kx\cos(\alpha z) - ky\sin(\alpha z)]\cos[kx\sin(\alpha z) + ky\cos(\alpha z)], \quad (1)$$

where δ is a small positive constant, k is a parameter that determines the spacing between holes and α is a parameter that determines the rate of twist; an increase in α will result in a tighter twist. Note that $\frac{1}{k} \sim \mu$m and $\frac{1}{\alpha} \sim$ mm. In this work, we analyze light as described by Bloch waves. The result is the appearance of an envelope function around the wave that decays radially outward from the center. Starting with the linear equation for the propagation

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of an electric field, $\Delta E + (\frac{n\omega}{c})^2 E = 0$, we assume the electric field has the form $E = F(x, y, z)e^{i\beta z - i\omega t}$. Next, we introduce the non-dimensional variables $X = kx$, $Y = ky$, and $Z = \alpha z$ and the rotational frame variables $U = X\cos(Z) - Y\sin(Z)$, and $V = X\sin(Z) + Y\cos(Z)$. Since $n = 1 - \delta\cos(U)\cos(V)$, the propagation equation can be written as

$$2i\frac{\beta}{\alpha}(F_Z - VF_U + UF_V) + \epsilon^{-2}\Delta_{UV}F - \frac{\omega^2\delta}{c^2\alpha^2}(\mu + f(U,V))F = 0, \tag{2}$$

where $\epsilon = \alpha/k$, $\mu = \frac{\beta^2c^2-\omega^2}{\delta\omega^2}$ and $f(U,V) = 2\cos U\cos V - \delta\cos^2 U\cos^2 V$. We assume $\frac{\omega^2\delta}{c^2\alpha^2}$ is on the order of ϵ^{-2}, and can therefore analyze the last two terms on the same order. Continuing with a perturbative analysis, F can be expanded as $F = F_0 + \epsilon F_1 + \epsilon^2 F_2 + \ldots$ and we will also assume F_0 has a Bloch wave form of

$$F_0 = A(Z,b)F_B(U,V)e^{i\psi(Z,b)}, \tag{3}$$

where $b = \epsilon(U^2 + V^2)$. The envelope and phase of the Bloch wave is then slowly dependent on U and V while radially symmetric in the cross section. From the leading order terms at $\mathcal{O}(\epsilon^{-2})$, $L(F_B) = 0$ where the operator $L = \Delta_{UV} - (\mu + 2\cos U\cos V)$. At $\mathcal{O}(\epsilon^{-1})$, $L(F_1) = R$ where R involves complex terms of partial derivatives of F_0. In order for this equation to have a solution, we look at the solvability condition on R. That is, R must be orthogonal to the null space of the L operator. Since F_B is in the null space of L, we multiply R by F_B and integrate over one period. This integral must be equal to zero. Separating the resulting equation into its imaginary and real parts we arrive at

$$\frac{\partial A}{\partial Z}C_1 - AC_2 + A\frac{\partial\psi}{\partial b}C_3 = 0, \qquad A\frac{\partial\psi}{\partial Z}C_1 + \frac{\partial A}{\partial b}C_3 = 0, \tag{4}$$

where the C_i's are constants given by $C_1 = \int_p \bar{\beta}F_B^2 dUdV$, $C_2 = \int_p \bar{\beta}F_B(V\frac{\partial F_B}{\partial U} - U\frac{\partial F_B}{\partial V})dUdV$, and $C_3 = \int_p 2F_B(F_B + U\frac{\partial F_B}{\partial U} + V\frac{\partial F_B}{\partial V})dUdV$ and $\bar{\beta} = \frac{\beta}{k}$. The partial derivatives with respect to b have been pulled out of the integrals as an approximation since b depends slowly on U and V. Notice the integrals are over the cross section of the PCF and are all positive. We find the solution to (4) where A does not depend on Z is

$$A = A_0 e^{-\epsilon(U^2+V^2)}, \qquad \psi = \frac{C_2}{C_3}\epsilon(U^2 + V^2) + \frac{C_3}{C_1}Z + \psi_0. \tag{5}$$

As in the experiments in [2], the Bloch wave has a radially symmetry envelope gaussian profile. This comes with a radially symmetric chirp as an effective correction of the stationary phase. The last one can be explained in terms of effective longer paths of the rays as they propagate in the twisted configuration. This is intuitively consistent with a classical result that prechirping a phase in free space propagation may induce an initial confinement of the beam. It is the case here that this is induced by the twist rather than by an initial shaping of the phase. Furthermore, if the twist rate is increased, the confinement is also increased. While this is only a qualitative picture, our study clearly supports the experimental observation.

III. Field Theory (Nonlinear)- Numerical

In this section, we investigate the effects of the twisted configuration by numerically integrating the nonlinear Schrödinger equation (NLS) (6).

$$i\frac{\partial E}{\partial z} = \Delta_{xy}E + n^2E + \gamma|E|^2E \tag{6}$$

We utilize a numerical method proposed by Kirkpatrick and Zhang [4] for solving the fractional NLS. The right hand side of (6) is split into two parts, one containing the transverse Laplacian and one containing the index of refraction and nonlinear components. Each are numerically solved independently and then coupled using the second-order Strang method. The Fourier pseudospectral method is used to discretize the Riesz fractional Laplacian. For simplicity, the numerical domain is given periodic boundaries and we submit an initial field profile of $E = \bar{A}\exp\{-(x^2 + y^2)/s\}$. Due to the approximations we have made and the periodic domain, it is important to note that these simulations are not representative of the physical experiments given in [2] but rather an additional exploratory analysis into the role of twist in photonic crystals.

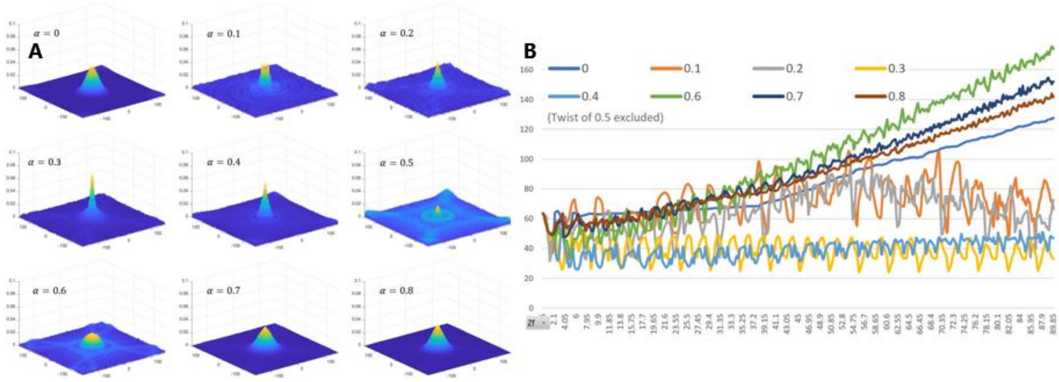

Fig. 1: (A) Mode profiles for increasing rates of twist after propagating a length of 12 rotations for $\alpha = 0.8$. (B) Full width at half maximum measurement as the fields propagate along the z-axis.

Fig.1 (A) shows the mode profiles as the degree of twist is increased. The fields were integrated along the z-axis to a length that allowed the highest degree of twist, 0.8, to propagate through 12 rotations of the fiber. Fig. 1 (B) shows the evolution of the full width at half maximum measurement for each degree of twist. Twist rates of $0.1 - 0.4$ have small amounts of radiation spreading from the center however they have a tighter beam and higher peak values than in the case of no twist. When $\alpha = 0.5$, the field quickly spreads and this is possibly the result of a resonance of a parametric process. Twist rates of $0.6 - 0.8$ show modes that are less confined than the case of no twist but regain confinement as the twist is increased. The results above are for the nonlinear focusing case with $\gamma = 1$. The initial condition parameters were set to $\bar{A} = 0.8$ and $s = 160$. In the linear case where $\gamma = 0$, the same behavior of confinement was noticed with slightly smaller peaks. The nonlinear defocusing case, $\gamma = -1$, also showed the same behavior with smaller peaks than that of the linear case.

From these results, we notice that certain degrees of twist result in better confinement of light. However there is some degree of twist in which the mode quickly loses its shape but then regains confinement if the twist is increased further. This also suggests similarities to the Kapitza pendulum and Paul trap where specific frequencies are required in order to obtain a confining potential; frequencies that are too high or two low may be unstable.

IV. Conclusions

Extended Bloch modes propagating through a helical periodic medium can be shown confined to where the envelope and phase of the mode adapt to follow the twist for the helical structure. Through a change of variables, we see that the allowed modes will have a radially symmetric chirp and the envelope should decay away from the axis of rotation. The energy is thus confined and will propagate along the axis with little energy loss. It is also shown that an increase in twist will increase confinement as the model predicts in (5). By numerically integrating the NLS with such a refractive index it is also seen that enhanced confinement may exist at certain rates of twist. The numerical results also show that larger twist rates may cause the mode to quickly spread, after which even higher twist rates can allow confinement similar to the non-twisted case. We suggest that the confinement effect shown here due to the rapid spiralling of hollow channels is analogous to the creation of a stable region in the classical Kapitza pendulum. Outside of the traditional optical fibers that contain one or multiple cores, these results show that a twisted PCF structure provides an additional method of guiding light in the absence of a core. This work was supported by the US National Science Foundation, grant number 1909559.

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