Age of Information in Uncoordinated Unslotted Updating

Roy D. Yates
WINLAB, ECE Dept.,
Rutgers University
email: ryates@winlab.rutgers.edu

Sanjit K. Kaul
Wireless Systems Lab,
IIIT-Delhi
email: skkaul@iiitd.ac.in

***Abstract*—Sensor sources submit updates to a monitor through an unslotted, uncoordinated, unreliable multiple access collision channel. The channel is unreliable; a collision-free transmission is received successfully at the monitor with some transmission success probability. For an infinite-user model in which the sensors collectively generate updates as a Poisson process and each update has an independent exponential transmission time, a stochastic hybrid system (SHS) approach is used to derive the average age of information (AoI) as a function of the offered load and the transmission success probability. The analysis is then extended to evaluate the individual age of a selected source. When the number of sources and update transmission rate grow large in fixed proportion, the limiting asymptotic individual age is shown to provide an accurate individual age approximation, even for a small number of sources.**

I. Introduction

Consider a collection of sensors that transmit updates to a central monitor. In many applications, complexity and energy considerations dictate that the sensors be transmit-only devices that blindly send update measurements without regard to the activity of other sensors [1]–[3]. Because these transmit-only sources cannot coordinate, the transmissions are subject to collisions and the system operation is necessarily unslotted.

Since timeliness may be important, this work examines the age of information (AoI) of these sensor updates. When the newest received update has time stamp $u(t)$, the age process is $\Delta(t) = t - u(t)$ [4] and the average age is $\lim_{t\to\infty} \mathrm{E}[\Delta(t)]$.

We note there has been growing interest in the AoI of sources sharing a communication facility, starting with multiple sources submitting updates through queues [5]–[13]. In addition, AoI has been analyzed for multiple users sharing a slotted system with various levels of system coordination, including round-robin and Aloha-like contention [14], [15], scheduled access [16]–[22], CSMA [23], and random access with source-optimized contention policies [24]–[26]. However, age of information (AoI) in transmit-only sensor updates has not been studied. The graphical method of age analysis introduced in [4] and then employed in e.g. [27]–[35] has not enabled age analysis of the collision channel.

A. System Model

In the collision channel, a transmission is collision-free if all other transmitters are idle during that transmission. If an update suffers a collision, it is not received by the monitor. In addition, the communication channel is unreliable; a collision-free update will suffer an error and fail to be received by the monitor with probability P_e.

A key advantage of an unslotted system is that the transmission times can have arbitrary durations [36]. To avoid a combinatorial explosion of the state space, we assume the transmission times of the updates are modeled as independent exponential (μ) random variables. Furthermore, the collection of sensors in aggregate initiate update transmissions as a rate λ Poisson point process. This is consistent with the "infinite user" model of historical importance in the analysis of the maximum stable throughput of collision resolution protocols [36]–[40].

B. Paper Summary

For the collection of uncoordinated sensors, we consider two types of age metrics. The *system age* is defined as the age of the most recent update received from any sensor in the system. For the system age, a fresher update from any sensor reduces the age at the monitor. This is in contrast to the *individual age* of a selected sensor among N sensors. Poisson arrivals of transmitted updates and exponential update transmission times enable the method of stochastic hybrid systems (SHS) for age analysis. Section II-A, provides a short introduction to the SHS method and then uses SHS to analyze the system age in Section II-B.

Using the probability of correct detection $P_c = 1 - P_e$, the system age analysis is extended to evaluate the individual age in Section III. The individual age, in the limit of a large number of users and proportional system service rate, is shown to converge to simple function of the offered load, that approximates the individual age even for a small number of sources. The paper concludes with a discussion of open issues in Section IV.

II. Average System Age

A. SHS Background

A stochastic hybrid system (SHS) [41] has state $[q(t), \mathbf{x}(t)]$ such that $\mathbf{x}(t) \in \mathbb{R}^{1\times n}$ and $q(t) \in \mathcal{Q} = \{0, \ldots, M\}$ is a continuous-time Markov chain.

For AoI analysis, $q(t)$ describes the discrete state of a network while the age vector $\mathbf{x}(t)$ describes the continuous-time evolution of a collection of age-related processes. The

978-1-7281-6432-8/20/$31.00 ©2020 IEEE

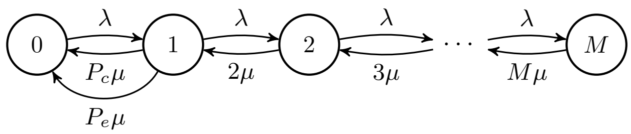

Fig. 1. SHS Markov chain for the system age over an unslotted collision channel.

SHS approach was introduced in [10], where it was shown that age tracking can be implemented as a simplified SHS with non-negative linear reset maps in which the continuous state is a piecewise linear process [42]–[44]. For finite-state systems, this led to a set of age balance equations and simple conditions [10, Theorem 4] under which $\mathrm{E}[\mathbf{x}(t)]$ converges to a fixed point.

A description of this simplified SHS for AoI analysis now follows. In the graph representation of the Markov chain $q(t)$, each state $q \in \mathcal{Q}$ is a node and each transition $l \in \mathcal{L}$ is a directed edge (q_l, q'_l) with transition rate $\lambda^{(l)}$ from state q_l to q'_l. Associated with each transition l is a transition reset mapping $\mathbf{A}_l \in \{0,1\}^{n\times n}$ that induces a jump $\mathbf{x}' = \mathbf{x}\mathbf{A}_l$ in the continuous state $\mathbf{x}(t)$.

Unlike an ordinary continuous-time Markov chain, the SHS Markov chain may include self-transitions in which the discrete state is unchanged because a reset occurs in the continuous state. Furthermore, for a given pair of states $q, q' \in \mathcal{Q}$, there may be multiple transitions l and $\hat{l}$ in which $q(t)$ jumps from q to q' but the transition maps $\mathbf{A}_l$ and $\mathbf{A}_{\hat{l}}$ are different.

For each state $\bar{q}$, we denote the respective sets of incoming and outgoing transitions by

$$\mathcal{L}'_{\bar{q}} = \{l \in \mathcal{L} : q'_l = \bar{q}\}, \quad \mathcal{L}_{\bar{q}} = \{l \in \mathcal{L} : q_l = \bar{q}\}. \tag{1}$$

Assuming the discrete state Markov chain is ergodic, $q(t)$ has unique stationary probabilities $\bar{\boldsymbol{\pi}} = [\bar{\pi}_0 \ \cdots \ \bar{\pi}_M]$ satisfying

$$\bar{\pi}_{\bar{q}} \sum_{l\in\mathcal{L}_{\bar{q}}} \lambda^{(l)} = \sum_{l\in\mathcal{L}'_{\bar{q}}} \lambda^{(l)} \bar{\pi}_{q_l}, \quad \bar{q} \in \mathcal{Q}, \quad \text{and} \sum_{\bar{q}\in\mathcal{Q}} \bar{\pi}_{\bar{q}} = 1. \tag{2}$$

The next theorem provides a way to derive the limiting average age vector $\mathrm{E}[\mathbf{x}] = \lim_{t\to\infty} \mathrm{E}[\mathbf{x}(t)]$.

Theorem 1: [10, Theorem 4] If the discrete-state Markov chain $q(t)$ is ergodic with stationary distribution $\bar{\boldsymbol{\pi}} > 0$ and there exists a non-negative vector $\bar{\mathbf{v}} = [\bar{\mathbf{v}}_0 \cdots \bar{\mathbf{v}}_M]$ such that

$$\bar{\mathbf{v}}_{\bar{q}} \sum_{l\in\mathcal{L}_{\bar{q}}} \lambda^{(l)} = \mathbf{1}\bar{\pi}_{\bar{q}} + \sum_{l\in\mathcal{L}'_{\bar{q}}} \lambda^{(l)} \bar{\mathbf{v}}_{q_l}\mathbf{A}_l, \quad \bar{q} \in \mathcal{Q}, \tag{3}$$

then the average age vector is $\mathrm{E}[\mathbf{x}] = \sum_{\bar{q}\in\mathcal{Q}} \bar{\mathbf{v}}_{\bar{q}}$.

In the next section, Theorem 1 is employed to find the average age for uncoordinated unslotted updating.

B. SHS Modeling of the System Age

For an SHS age model of the unslotted collision channel, the discrete state Markov chain for $q(t)$ is shown in Figure 1 and the corresponding set of SHS transitions is given in Table I. The discrete state $q(t) \in \{0, 1, 2, \ldots\}$ is the number of active transmitters. In the idle state 0, the start of a transmission causes the system to jump to state 1. This update is successfully delivered if it completes service before another update begins transmission. Otherwise, a jump to state 2 begins a collision period in which transmitted updates suffer collisions and are unsuccessful. In states $k \geq 2$, there are k updates being transmitted in a k-way collision. A collision period ends when the system returns to the idle state.

TABLE I
SHS TRANSITIONS FOR TRACKING THE OVERALL AGE IN THE MARKOV CHAIN OF FIG. 1.

l	$q_l \to q'_l$	$\lambda^{(l)}$	$\mathbf{x}\mathbf{A}_l$	$\mathbf{A}_l$	$\mathbf{v}_{q_l}\mathbf{A}_l$
1	$0 \to 1$	λ	$[0 \ x_2]$	$\left[\begin{smallmatrix}0&0\\0&1\end{smallmatrix}\right]$	$[0 \ v_{02}]$
2	$1 \to 0$	$P_c\mu$	$[x_1 \ x_1]$	$\left[\begin{smallmatrix}1&1\\0&0\end{smallmatrix}\right]$	$[v_{11} \ v_{11}]$
3	$1 \to 0$	$P_e\mu$	$[x_2 \ x_2]$	$\left[\begin{smallmatrix}0&0\\1&1\end{smallmatrix}\right]$	$[v_{12} \ v_{12}]$
4	$1 \to 2$	λ	$[x_2 \ x_2]$	$\left[\begin{smallmatrix}0&0\\1&1\end{smallmatrix}\right]$	$[v_{12} \ v_{12}]$
5	$2 \to 1$	2μ	$[x_1 \ x_2]$	$\mathbf{I}$	$[v_{21} \ v_{22}]$
6	$2 \to 3$	λ	$[x_1 \ x_2]$	$\mathbf{I}$	$[v_{21} \ v_{22}]$
7	$3 \to 2$	3μ	$[x_1 \ x_2]$	$\mathbf{I}$	$[v_{31} \ v_{32}]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$M \to M-1$	$M\mu$	$[x_1 \ x_2]$	$\mathbf{I}$	$[v_{M1} \ v_{M2}]$

The age state is $\mathbf{x}(t) = [x_1(t) \ x_2(t)]$ where $x_2(t)$ is the age at the monitor and $x_1(t)$ is what the age at the monitor would become if an update in service were to complete transmission at time t. In each state $q(t) = \bar{q}$, the continuous state evolves according to $\dot{\mathbf{x}}(t) = \mathbf{1} = [1 \ 1]$. Our objective is to calculate the average age at the monitor $\Delta = \lim_{t\to\infty} \mathrm{E}[x_2(t)]$.

An age reduction in $x_2(t)$ occurs only when a collision-free update is delivered successfully. This event must be preceded by a transition $l = 1$ in which the system goes from idle to having a single update in service. In this transition, the mapping $\mathbf{x}' = \mathbf{x}\mathbf{A}_1 = [0 \ x_2]$ resets x_1 to $x_1 = 0$, the age of the fresh update that just began transmission. On the other hand, $x'_2 = x_2$ is unchanged because it tracks the age at the monitor. In state 1, the transition $l = 2$ corresponds to the update being transmitted collision-free and also being successfully received. In this transition, $\mathbf{x}' = \mathbf{x}\mathbf{A}_2 = [x_1 \ x_1]$ resets x_2 to $x'_2 = x_1$, the age of the update that was just successfully received.

By contrast, transition $l = 3$ corresponds to the update being transmitted collision-free but it fails to be received. In this transition, $\mathbf{x}' = \mathbf{x}\mathbf{A}_3 = [x_2 \ x_2]$ leaves the age x_2 at the monitor unchanged. This transition also resets x_1 to $x'_1 = x_2$ to indicate that there is no update in transmission whose delivery can yield an age reduction. Similarly, $\mathbf{A}_4 = \mathbf{A}_3$ because in transition $l = 4$, a second update collides with an update in a transmission. Since this collision guarantees that neither update in transmission is successfully received, this transition also sets $x'_1 = x_2$,

While state 1 has exactly one update being in service, this update may or may not be collision-free. This information is encoded in the continuous state $\mathbf{x}(t)$. If $x_1(t) < x_2(t)$, then there is single update with age $x_1(t)$ in the middle of a

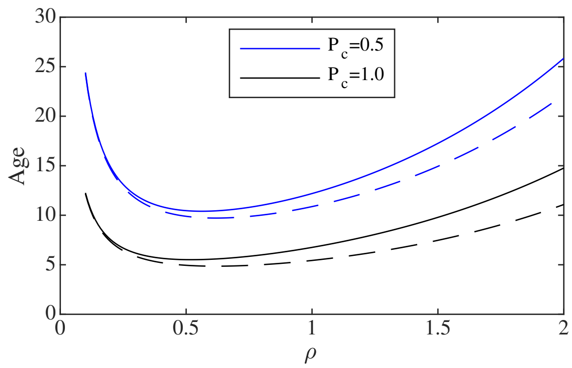

Fig. 2. Average system age Δ in Theorem 2 as a function of offered load ρ; time is normalized so that $\mu = 1$. Dashed lines show the lower bound $\Delta_1(\rho)/(\mu P_c)$.

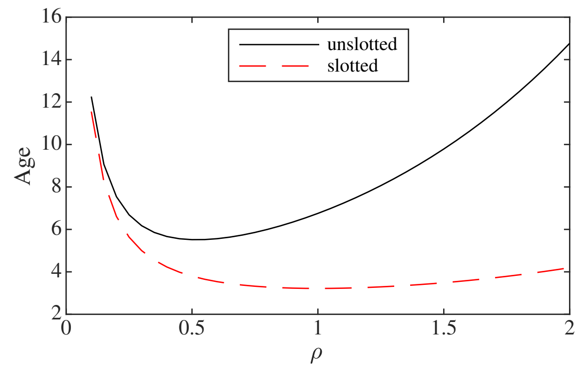

Fig. 3. Average system age Δ in Theorem 2 of the unslotted system vs. the average system age Δ_{slotted} of the corresponding slotted Aloha system as a function of offered load ρ.

collision-free transmission at time t; otherwise $x_1(t) = x_2(t)$. In particular, the transition $l = 4$ into state 2 initiates a collision period in which $x_1(t) = x_2(t)$. This condition is preserved throughout the collision period, including when the system transitions through state 1 and back to the idle state 0.

C. SHS Analysis of the System Age

A consequence of the Poisson update arrival process is that the number of updates being simultaneously transmitted can be arbitrarily large. However, in order to apply Theorem 1, the state space is truncated so that the largest collision has M updates. We start by finding Δ_M, the average age in this truncated system.

To employ Theorem 1, observe first that (2) implies $\lambda\bar{\pi}_0 = \mu\bar{\pi}_1$ and for $k = 1, \ldots, M-1$,

$$(\lambda + k\mu)\bar{\pi}_k = \lambda\bar{\pi}_{k-1} + (k+1)\mu\bar{\pi}_{k+1}. \tag{4}$$

Solving for $\bar{\pi}_k$, $k = 1, \ldots, M$, in terms of $\rho = \lambda/\mu$ and enforcing the normalization constraint yields

$$\bar{\pi}_0 = \left(\sum_{j=0}^{M} \rho^j/j!\right)^{-1}, \quad \bar{\pi}_k = \frac{\rho^k}{k!}\bar{\pi}_0. \tag{5}$$

From (3), we have for $\bar{q} \in \{0, 1, 2, M\}$ that

$$\lambda\bar{\mathbf{v}}_0 = \mathbf{1}\bar{\pi}_0 + P_c\mu\bar{\mathbf{v}}_1\mathbf{A}_2 + P_e\mu\bar{\mathbf{v}}_1\mathbf{A}_3, \tag{6a}$$

$$(\lambda + \mu)\bar{\mathbf{v}}_1 = \mathbf{1}\bar{\pi}_1 + \lambda\bar{\mathbf{v}}_0\mathbf{A}_1 + 2\mu\bar{\mathbf{v}}_2, \tag{6b}$$

$$(\lambda + 2\mu)\bar{\mathbf{v}}_2 = \mathbf{1}\bar{\pi}_2 + \lambda\bar{\mathbf{v}}_1\mathbf{A}_4 + 3\mu\bar{\mathbf{v}}_3, \tag{6c}$$

$$M\mu\bar{\mathbf{v}}_M = \mathbf{1}\bar{\pi}_M + \lambda\bar{\mathbf{v}}_{M-1}, \tag{6d}$$

and for $\bar{q} = k \in \{3, \ldots, M-1\}$,

$$(\lambda + k\mu)\bar{\mathbf{v}}_k = \mathbf{1}\bar{\pi}_k + \lambda\bar{\mathbf{v}}_{k-1} + (k+1)\mu\bar{\mathbf{v}}_{k+1}. \tag{6e}$$

Solving (6) for $\bar{\mathbf{v}}_0, \ldots, \bar{\mathbf{v}}_M$, the average system age in the truncated system is

$$\Delta_M = \mathrm{E}[x_2] = \sum_{k=0}^{M} \bar{v}_{k2}. \tag{7}$$

The average system age with an infinite user population is then $\Delta = \lim_{M\to\infty} \Delta_M$. These steps can be found in the Appendix. For $j = 1, 2, \ldots$, we adopt the shorthand notation

$$\beta_j \equiv \sum_{i=j}^{\infty} \frac{\rho^i}{i!} e^{-\rho}, \qquad \gamma_j \equiv \sum_{k=0}^{\infty} \frac{j!}{(j+k)!}\rho^k, \tag{8}$$

in order to state the following claim.[1]

Theorem 2: Poisson updates through a collision channel achieve the average system age

$$\Delta = \frac{(1+\rho)e^\rho}{\mu P_c \rho} + \frac{\beta_1}{\mu} + \frac{(3+\rho)\beta_2}{2\mu} + \frac{\rho(1+\rho)\beta_2\gamma_3}{6\mu} + \sum_{j=3}^{\infty} \frac{\beta_j\gamma_j}{j\mu}.$$

D. System Age: Numerical Results

With time normalized so that $\mu = 1$, Figure 2 depicts the system age in Theorem 2 as a function of the offered load ρ for probability of correct reception $P_c \in \{0.5, 0.8, 1\}$. For all P_c, the age becomes high when ρ approaches zero or when ρ becomes large and the system has too many collisions. For $P_c = 1$, the average age happens to be minimized at $\rho = \rho^* = 0.5195$, achieving the minimum age of $\Delta^* = 5.513$. As P_c decreases, the optimal offered load increases slightly. For example, when $P_c = 0.5$, the optimal load is $\rho^* = 0.5625$; this achieves an average age of $\Delta^* = 10.40$. We see from Figure 2 that the average age is not particularly sensitive to variations in ρ near ρ^*.

We further observe that all terms in Δ are non-negative. With the definition

$$\Delta_1(\rho) \equiv \left(1 + \frac{1}{\rho}\right)e^\rho, \tag{9}$$

the average system age satisfies the lower bound

$$\Delta \geq \frac{\Delta_1(\rho)}{\mu P_c} \tag{10}$$

This simple lower bound, depicted in Figure 2 with dashed lines, is tight for small ρ and nontrivial for large ρ.

It is also instructive to compare the system age of the unslotted and slotted systems. Consider the corresponding

[1] Note that $\beta_j = \mathrm{P}[K \geq j]$ and $\gamma_j = \beta_j/\mathrm{P}[K = j]$ for a Poisson (ρ) random variable K, While it is possible to state Theorem 2 with γ_j replaced by $\beta_j/\mathrm{P}[K = j]$, this ratio of quantities that both go to zero as $j \to \infty$ can induce numerical stability issues in the calculation of Δ.

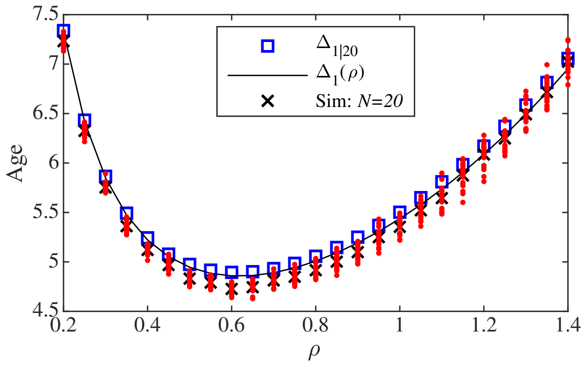

Fig. 4. Average individual age for $N = 20$ sources. Time is normalized so that $\mu = 20$. The figure compares $\Delta_{1|20}$ and $\Delta_1(\rho)$ against a simulation of $N = 20$ on/off sources with aggregate update arrival rate 20ρ. At each ρ, $\bullet$ marks the time-average individual ages of each of the 20 sources while $\times$ marks the average age averaged over the 20 sources.

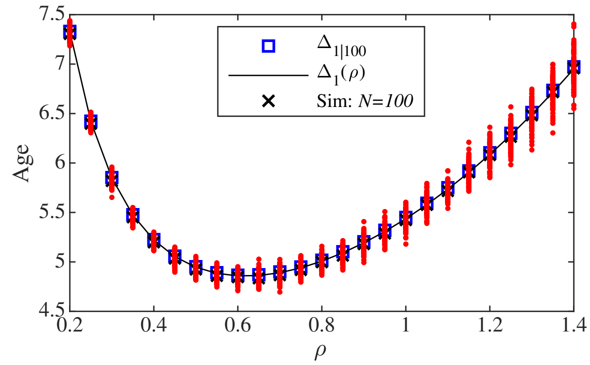

Fig. 5. Average individual age for $N = 100$ sources. Time is normalized so that $\mu = 100$. The figure compares $\Delta_{1|100}$ and $\Delta_1(\rho)$ against a simulation of $N = 100$ on/off sources with aggregate update arrival rate 100ρ. At each ρ, $\bullet$ marks the time-average individual ages of each of the 100 sources while $\times$ marks the average age averaged over the 100 sources.

infinite-user slotted system. In each unit time slot, the number of fresh transmitted updates is a Poisson random variable K with $\mathrm{E}[K] = \rho$. A fresh update is successfully transmitted in each time slot with probability $P_s = \mathrm{P}[K = 1] = \rho e^{-\rho}$. The average system age is [14, Equation (23)]

$$\Delta_{\text{slotted}} = \frac{1}{2} + \frac{1}{P_s} = \frac{1}{2} + \frac{e^\rho}{\rho}. \tag{11}$$

Figure 3 compares average system age in the slotted and unslotted systems. We see that the age penalty for unslotted operation is negligible when the offered load ρ is small. However, when the offered load is large, the age penalty becomes large because of the long collision periods induced by unslotted operation. This highlights how slotting is is able to destroy the memory of the collision process.

III. Individual Age Analysis

In practice, the number of sources N will be finite and it is desirable to characterize the age process of an individual source. Fortunately, the infinite user model of Theorem 2 can be employed to evaluate the individual age for one of N sources by reinterpreting P_c, the probability of correct detection of a collision-free update, as the probability that the collision-free update reduces the age of a selected user. Specifically, suppose the aggregate updating rate λ in the infinite user model is from N independent sources, each offering updates as a Poisson process of rate λ/N. In this case, a transmitted update belongs to a source i with probability $1/N$. Hence, Theorem 2 can be employed with an update that is transmitted collision-free as belonging to source i (and thus offering an age reduction for source i) with probability $P_c = 1/N$. This yields the individual age

$$\Delta_{1|N} = \frac{N(1+\rho)e^\rho}{\mu\rho} + \frac{\beta_1}{\mu} + \frac{(3+\rho)\beta_2}{2\mu} + \frac{\rho(1+\rho)\beta_2\gamma_3}{6\mu} + \sum_{j=3}^{\infty} \frac{\beta_j\gamma_j}{j\mu}. \tag{12}$$

For fixed service rate μ and fixed offered load ρ, (12) implies that the individual age grows linearly with the number of users N. This is not surprising since the system bandwidth, as embodied in the fixed service rate μ, is shared among N sources. However, to provide good age performance as N becomes large, the system needs bandwidth to grow in proportion to N. In this case, we assume the system has N sources, each offering updates at rate λ_0 but the system bandwidth grows with N so that the service rate of a transmission is $\mu = N\mu_0$. The normalized offered load remains fixed at $\rho = (N\lambda_0)/(N\mu_0) = \lambda_0/\mu_0$. This is essentially the same scaling previously employed in [17]. A transmitted update belongs to the selected source with probability $P_c = 1/N$. We also assume time is normalized so that $\mu_0 = 1$. Under these conditions, we observe as $N \to \infty$ that

$$\Delta_{1|N} \to \Delta_1(\rho). \tag{13}$$

Here we can interpret $\Delta_1(\rho)$ as the individual age on a collision channel in the limit of the number of sources becoming large and the transmission time of an update approaching zero. In this asymptotic limit, the individual average age is minimized at $\rho = (\sqrt{5} - 1)/2 = 0.618$.[2]

We will see this individual age model is somewhat pessimistic. The Poisson update process of source i can generate self-colliding updates that are time-overlapping with prior source i updates. In practice, each source transmits one update at a time and never has a self-collision. In this sense, $\Delta_{1|N}$ and $\Delta_1(\rho)$ are approximations for the individual average age in a practical system.

To evaluate these approximations, we simulate a system with N independent on/off sources. Each source is either transmitting an update of exponential duration with expected value $1/\mu$, or being silent for an exponential period with expected length $1/\lambda_0 - 1/\mu$. By this construction, the two-state update process of each source offers updates at the longterm rate of $\lambda_0 = \lambda/N$ updates per unit time. As N becomes large, we expect the aggregate update process to be reasonably

[2]It can be shown that $\rho = 0.618$ also maximizes the probability the system is transmitting a collision-free update.

approximated by a rate $N\lambda_0 = \lambda$ Poisson process. We also expect each source to obtain average individual age that is approximated by $\Delta_{1|N}$.

Under these conditions, Figures 4 and 5 compare $\Delta_{1|N}$ and $\Delta_1(\rho)$ against the simulated time-average ages experienced by each of the N on/off sources, each generating $50,000$ updates. Time is normalized so that $\mu = N$ and the average update transmission time is $1/\mu = 1/N$. The aggregate offered load is $\rho = (N\lambda_0)/N = \lambda_0$.

In Figure 4 with $N = 20$ sources, $\Delta_{1|N}$, which is derived from the infinite user model of Theorem 2, is pessimistic in slightly (by 2-3%) overestimating the average age received by a source. The asymptotic approximation, $\Delta_1(\rho)$, which discards terms of $\Delta_{1|N}$ that become negligible as $\mu = N$ becomes large, is observed to be an even better age approximation in the finite user system. In Figure 5 with $N = 100$ sources, we see that that with more sources, the approximation $\Delta_1(\rho)$ becomes an increasingly accurate approximation to the average individual age.

IV. Conclusion

For uncoordinated transmit-only sensors, this work provides an exact analysis for the system age. The uncoordinated transmit-only system works well as long as the normalized offered load is near $\rho^* = 0.6$. When these networks have a nontrivial number of sources, $\Delta_1(\rho)$ is a useful approximation for the individual age in a system with offered load ρ.

From $\Delta_1(\rho)$, we see that the individual age penalty is substantial (on the order of $10\times$) if the offered load is, say, $\rho^*/10$ or $10\rho^*$. Moreover, we saw in the comparison with the slotted system that age in the unslotted system is particularly sensitive to overloading the system. Configuring the network of transmit-only sources for the proper offered load would be important at time of deployment. On the other hand, adaptive configuration may also be possible if the sources have access to some minimal feedback.

In addition, there remain a number of open questions about how additional coordination mechanisms, such as collision detection and/or avoidance or state-dependent updating policies, can contribute to reducing AoI.

Acknowledgment This work was supported by the National Science Foundation award CNS-1717041.

Appendix

Proof of Theorem 2

The mappings $\mathbf{A}_l$ induce $x_1(t) = x_2(t)$ in all states $k \neq 1$. This implies (6) has a solution such that $\bar{\mathbf{v}}_k = [\bar{v}_k \ \bar{v}_k] = \bar{v}_k\mathbf{1}$ for all $k \neq 1$. Only $\bar{\mathbf{v}}_1 = [\bar{v}_{11} \ \bar{v}_{12}]$ has distinct non-identical components. In terms of $\bar{v}_{11}, \bar{v}_{12}$ and $\bar{v}_k$, $k \neq 1$, (6) becomes

$$\rho\bar{v}_0 = \mu^{-1}\bar{\pi}_0 + P_c\bar{v}_{11} + P_e\bar{v}_{12}, \tag{14a}$$
$$(1+\rho)\bar{v}_{11} = \mu^{-1}\bar{\pi}_1 + 2\bar{v}_2, \tag{14b}$$
$$(1+\rho)\bar{v}_{12} = \mu^{-1}\bar{\pi}_1 + \rho\bar{v}_0 + 2\bar{v}_2, \tag{14c}$$
$$(2+\rho)\bar{v}_2 = \mu^{-1}\bar{\pi}_2 + \rho\bar{v}_{12} + 3\bar{v}_3, \tag{14d}$$
$$M\bar{v}_M = \mu^{-1}\bar{\pi}_M + \rho\bar{v}_{M-1}, \tag{14e}$$

and for $3 \leq k \leq M-1$,

$$\rho_k\bar{v}_k = \mu^{-1}\bar{\pi}_k + \rho\bar{v}_{k-1} + (k+1)\bar{v}_{k+1}. \tag{14f}$$

The average age in Theorem 1 becomes

$$\Delta_M = \bar{v}_0 + \bar{v}_{12} + \bar{v}_2 + \sum_{j=3}^{M}\bar{v}_j. \tag{15}$$

In the limit of large M, we obtain the limiting average age $\Delta = \lim_{M\to\infty}\Delta_M$. Equations (14e) and (14f) admit the solution

$$\bar{v}_k = \frac{\rho}{k}\bar{v}_{k-1} + \frac{\beta_{k|M}}{k\mu}, \qquad 3 \leq k \leq M, \tag{16}$$

where

$$\beta_{k|M} = \sum_{i=k}^{M}\bar{\pi}_i = \bar{\pi}_0\sum_{i=k}^{M}\rho^i/i! \tag{17}$$

is the stationary probability of the system being in a collision of k or more updates. Now we observe that it follows from (16) that for $3 \leq l \leq M$,

$$\bar{v}_l = \sum_{j=3}^{l}\frac{(j-1)!}{\mu l!}\rho^{l-j}\beta_{j|M} + \frac{2\rho^{l-2}}{l!}\bar{v}_2. \tag{18}$$

Defining $V_{3:M} = \sum_{l=3}^{M}\bar{v}_l$, it then follows from (18) and reordering of the sums over l and j that

$$V_{3:M} = \sum_{j=3}^{M}\frac{\beta_{j|M}}{j\mu}\sum_{l=j}^{M}\frac{j!}{l!}\rho^{l-j} + \frac{\rho\bar{v}_2}{3}\sum_{m=0}^{M-3}\frac{\rho^m}{(m+3)!}. \tag{19}$$

Defining $\gamma_{j|M} = \sum_{k=0}^{M-j}\frac{j!}{(k+j)!}\rho^k$, the index shift $k = l - j$ in (19) yields

$$V_{3:M} = \sum_{j=3}^{M}\frac{\beta_{j|M}}{j\mu}\gamma_{j|M} + \frac{\rho}{3}\gamma_{3|M}\bar{v}_2. \tag{20}$$

Applying (16) with $k = 3$ to (14d) yields

$$\bar{v}_2 = \frac{\bar{\pi}_2 + \beta_{3|M}}{2\mu} + \frac{\rho}{2}\bar{v}_{12} = \frac{\beta_{2|M}}{2\mu} + \frac{\rho}{2}\bar{v}_{12}. \tag{21}$$

From (21) and the identity $\bar{\pi}_1 + \beta_{2|M} = \beta_{1|M}$, it follows from (14b) and (14c) that

$$\bar{v}_{11} = \frac{\rho^2\bar{v}_0}{1+\rho} + \frac{\beta_{1|M}}{\mu}, \quad \bar{v}_{12} = \rho\bar{v}_0 + \frac{\beta_{1|M}}{\mu}. \tag{22}$$

From (22) and the identity $P_c + P_e = 1$, it follows from (14a) that

$$\bar{v}_0 = \frac{1+\rho}{\mu\rho P_c}. \tag{23}$$

It then follows from (21) and (22) that

$$\bar{v}_2 = \frac{\rho^2}{2}\bar{v}_0 + \frac{\beta_{2|M} + \rho\beta_{1|M}}{2\mu}. \tag{24}$$

Applying (20), (22), and (24) to (15) and observing that $\lim_{M\to\infty}\beta_{j|M} = \beta_j$ and $\lim_{M\to\infty}\gamma_{j|M} = \gamma_j$, the claim follows.

References

[1] B. Blaszczyszyn and B. Radunovic. Using transmit-only sensors to reduce deployment cost of wireless sensor networks. In *IEEE INFOCOM 2008 - The 27th Conference on Computer Communications*, pages 1202–1210, April 2008.

[2] Y. Zhang, B. Firner, R. Howard, R. Martin, N. Mandayam, J. Fukuyama, and C. Xu. Transmit only: An ultra low overhead MAC protocol for dense wireless systems. In *2017 IEEE International Conference on Smart Computing (SMARTCOMP)*, pages 1–8, May 2017.

[3] J. Zhao, C. Qiao, R. S. Sudhaakar, and S. Yoon. Improve efficiency and reliability in single-hop wsns with transmit-only nodes. *IEEE Transactions on Parallel and Distributed Systems*, 24(3):520–534, 2013.

[4] S. Kaul, R. Yates, and M. Gruteser. Real-time status: How often should one update? In *Proc. IEEE INFOCOM*, pages 2731–2735, March 2012.

[5] L. Huang and E. Modiano. Optimizing age-of-information in a multi-class queueing system. In *Proc. IEEE Int'l. Symp. Info. Theory (ISIT)*, June 2015.

[6] I. Kadota, E. Uysal-Biyikoglu, R. Singh, and E. Modiano. Minimizing the age of information in broadcast wireless networks. In *54th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pages 844–851, Sept 2016.

[7] Nikolaos Pappas, Johan Gunnarsson, Ludvig Kratz, Marios Kountouris, and Vangelis Angelakis. Age of information of multiple sources with queue management. In *2015 IEEE International Conference on Communications (ICC)*, pages 5935–5940. IEEE, 2015.

[8] I. Kadota, A. Sinha, E. Uysal-Biyikoglu, R. Singh, and E. Modiano. Scheduling policies for minimizing age of information in broadcast wireless networks. *IEEE/ACM Transactions on Networking*, 26(6):2637–2650, 2018.

[9] S.K. Kaul and R.D. Yates. Age of information: Updates with priority. In *Proc. IEEE Int'l. Symp. Info. Theory (ISIT)*, pages 2644–2648, June 2018.

[10] R. D. Yates and S. K. Kaul. The age of information: Real-time status updating by multiple sources. *IEEE Transactions on Information Theory*, 65(3):1807–1827, 2018.

[11] E. Najm and E. Telatar. Status updates in a multi-stream M/G/1/1 preemptive queue. In *IEEE Conference on Computer Communications (INFOCOM) Workshops*, pages 124–129, April 2018.

[12] R. Devassy, G. Durisi, G. C. Ferrante, O. Simeone, and E. Uysal-Biyikoglu. Delay and peak-age violation probability in short-packet transmissions. In *2018 IEEE International Symposium on Information Theory (ISIT)*, pages 2471–2475, 2018.

[13] Ali Maatouk, Mohamad Assaad, and Anthony Ephremides. Age of information with prioritized streams: When to buffer preempted packets? *arXiv preprint arXiv:1901.05871*, 2019.

[14] S. K. Kaul and R.D. Yates. Status updates over unreliable multiaccess channels. In *Proc. IEEE Int'l. Symp. Info. Theory (ISIT)*, pages 331–335, June 2017.

[15] Andrea Munari and Alexey Frolov. Average age of information of irregular repetition slotted aloha. arXiv preprint 2004.01998, 2020.

[16] Z. Jiang, B. Krishnamachari, X. Zheng, S. Zhou, and Z. Miu. Decentralized status update for age-of-information optimization in wireless multiaccess channels. In *Proc. IEEE Int'l. Symp. Info. Theory (ISIT)*, pages 2276–2280, June 2018.

[17] Zhiyuan Jiang, Bhaskar Krishnamachari, Xi Zheng, Sheng Zhou, and Zhisheng Niu. Timely status update in massive iot systems: Decentralized scheduling for wireless uplinks. *CoRR*, abs/1801.03975, 2018.

[18] Zhiyuan Jiang, Bhaskar Krishnamachari, Sheng Zhou, and Zhisheng Niu. Can decentralized status update achieve universally near-optimal age-of-information in wireless multiaccess channels? In *International Teletraffic Congress ITC 30*, September 2018.

[19] Yu-Pin Hsu. Age of information: Whittle index for scheduling stochastic arrivals. In *Proc. IEEE Int'l. Symp. Info. Theory (ISIT)*, pages 2634–2638, June 2018.

[20] Shahab Farazi, Andrew G Klein, and D Richard Brown III. Fundamental bounds on the age of information in multi-hop global status update networks. *Journal of Communications and Networks*, 21(3):268–279, 2019.

[21] Antzela Kosta, Nikolaos Pappas, Anthony Ephremides, and Vangelis Angelakis. Age of information performance of multiaccess strategies with packet management. *Journal of Communications and Networks*, 21(3):244–255, 2019.

[22] Ali Maatouk, Saad Kriouile, Mohamad Assaad, and Anthony Ephremides. On the optimality of the Whittle's index policy for minimizing the age of information. *arXiv preprint arXiv:2001.03096*, 2020.

[23] Ali Maatouk, Mohamad Assaad, and Anthony Ephremides. Minimizing the age of information in a CSMA environment. arXiv:1901.00481, 2019.

[24] J. Sun, Z. Jiang, B. Krishnamachari, S. Zhou, and Z. Niu. Closed-form Whittle's index-enabled random access for timely status update. *IEEE Transactions on Communications*, 68(3):1538–1551, 2020.

[25] Xingran Chen, Konstantinos Gatsis, Hamed Hassani, and Shirin Saeedi Bidokhti. Age of information in random access channels. arXiv preprint arXiv:1912.01473, 2019.

[26] He Chen, Yifan Gu, and Soung-Chang Liew. Age-of-information dependent random access for massive IoT networks. arXiv preprint 2001.04780, 2020.

[27] C. Kam, S. Kompella, and A. Ephremides. Age of information under random updates. In *Proc. IEEE Int'l. Symp. Info. Theory (ISIT)*, pages 66–70, 2013.

[28] C. Kam, S. Kompella, and A. Ephremides. Effect of message transmission diversity on status age. In *Proc. IEEE Int'l. Symp. Info. Theory (ISIT)*, pages 2411–2415, June 2014.

[29] C. Kam, S. Kompella, G. D. Nguyen, and A. Ephremides. Effect of message transmission path diversity on status age. *IEEE Trans. Info. Theory*, 62(3):1360–1374, March 2016.

[30] M. Costa, M. Codreanu, and A. Ephremides. On the age of information in status update systems with packet management. *IEEE Trans. Info. Theory*, 62(4):1897–1910, April 2016.

[31] M. Costa, M. Codreanu, and A. Ephremides. Age of information with packet management. In *Proc. IEEE Int'l. Symp. Info. Theory (ISIT)*, pages 1583–1587, June 2014.

[32] J. P. Champati, H. Al-Zubaidy, and J. Gross. Statistical guarantee optimization for age of information for the D/G/1 queue. In *IEEE Conference on Computer Communications (INFOCOM) Workshops*, pages 130–135, April 2018.

[33] Yoshiaki Inoue, Hiroyuki Masuyama, Tetsuya Takine, and Toshiyuki Tanaka. A general formula for the stationary distribution of the age of information and its application to single-server queues. *CoRR*, abs/1804.06139, 2018.

[34] C. Kam, S. Kompella, G. D. Nguyen, J.E. Wieselthier, and A. Ephremides. Age of information with a packet deadline. In *Proc. IEEE Int'l. Symp. Info. Theory (ISIT)*, pages 2564–2568, 2016.

[35] S. Feng and J. Yang. Optimal status updating for an energy harvesting sensor with a noisy channel. In *IEEE Conference on Computer Communications (INFOCOM) Workshops*, pages 348–353, April 2018.

[36] R. Gallager. A perspective on multiaccess channels. *IEEE Transactions on Information Theory*, 31(2):124–142, March 1985.

[37] Norman Abramson. THE ALOHA SYSTEM: another alternative for computer communications. In *Proceedings of the November 17-19, 1970, fall joint computer conference*, pages 281–285. ACM, 1970.

[38] Lawrence G Roberts. Aloha packet system with and without slots and capture. *ACM SIGCOMM Computer Communication Review*, 5(2):28–42, 1975.

[39] Robert G Gallager. Conflict resolution in random access broadcast networks. In *Proc. AFOSR Workshop Commun. Theory Appl.(Provincetown, MA)*, pages 74–76, 1978.

[40] B. S. Tsybakov and V. A. Mikhailov. Random multiple packet access: part-and-try algorithm. *Problemy Peredachi Informatsii*, 16(4):65–79, 1980.

[41] J.P. Hespanha. Modelling and analysis of stochastic hybrid systems. *IEE Proceedings-Control Theory and Applications*, 153(5):520–535, 2006.

[42] D. Vermes. Optimal dynamic control of a useful class of randomly jumping processes. Technical Report PP-80-015, International Institute for Applied Systems Analysis, 1980.

[43] M. H. A. Davis. Piecewise-deterministic Markov processes: a general class of nondiffusion stochastic models. *J. Roy. Statist. Soc.*, 46:353–388, 1984.

[44] Lee DeVille, Sairaj Dhople, Alejandro D Domínguez-García, and Jiangmeng Zhang. Moment closure and finite-time blowup for piecewise deterministic Markov processes. *SIAM Journal on Applied Dynamical Systems*, 15(1):526–556, 2016.