Minimum Wireless Charger Placement with Individual Energy Requirement

Xingjian Ding[1], Jianxiong Guo[2], Deying Li[1](✉), and Ding-Zhu Du[2]

[1] School of Information, Renmin University of China, Beijing 100872, China
{dxj,deyingli}@ruc.edu.cn

[2] Department of Computer Science, University of Texas at Dallas, Richardson, TX 75080, USA
{jianxiong.guo,dzdu}@utdallas.edu

Abstract. Supply energy to battery-powered sensor devices by deploying wireless chargers is a promising way to prolong the operation time of wireless sensor networks, and has attracted much attention recently. Existing works focus on maximizing the total received charging power of the network. However, this may face the unbalanced energy allocation problem, which is not beneficial to prolong the operation time of wireless sensor networks. In this paper, we consider the individual energy requirement of each sensor node, and study the problem of minimum charger placement. That is, we focus on finding a strategy for placing wireless chargers from a given candidate location set, such that each sensor node's energy requirement can be met, meanwhile the total number of used chargers can be minimized. We show that the problem to be solved is NP-hard, and present two approximation algorithms which are based on the greedy scheme and relax rounding scheme, respectively. We prove that both of the two algorithms have performance guarantees. Finally, we validate the performance of our algorithms by performing extensive numerical simulations. Simulation results show the effectiveness of our proposed algorithms.

Keywords: Wireless charger placement · Wireless sensor network · Individual energy requirement

1 Introduction

Over the past ten years, there is a growing interesting of using Wireless Sensor Networks (WSNs) to collect data from the real world. A WSN system mainly consists of lots of sensor nodes that are powered by on-board batteries. Due to the inherent constraints on the battery technology, these on-board batteries can only provide limited energy capacity, and thus it limits the operating time of the wireless sensor networks. To achieve perpetual operation of the network

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system, prolonging the operation time of these battery-powered sensor nodes has been an important task. The great progress in Wireless Power Transfer (WPT) based on magnetic resonant coupling [8] bring a novel way to replenish the batteries of wireless sensor networks. Prolonging the operation time of sensor nodes by using WPT has many advantages. Such as, WPT is insensitive to external environments and can provide relatively stable energy supply for each sensor node; the charging power to sensor nodes is controllable and thus can be flexibly adjusted according to the energy requirement of the wireless sensor network; WPT provides an efficient way to charge sensor nodes without any interconnecting conductors. As a promising way to deliver energy to wireless sensor networks, the study of deploying wireless chargers has attracted significant attention in a few years.

In previous studies, researchers mainly concentrate on charging utility maximization problem [3,4,15,18]. That is, their object is to get a maximum total charging power of a given sensor network under some certain constraints, such as the number of used chargers or the overall working power of deployed chargers. These studies may face the unbalanced energy allocation problem. For example, with the object of maximizing the total charging power of the network, there might be some sensor nodes receive lots of wireless power, but others receive rare or even no wireless power, which is not beneficial to prolong the lifetime of the wireless network. Moreover, there are some studies focus on studying how to efficient place wireless chargers so that a sensor device deployed anywhere in the network area always can receive enough energy [6]. However, these works didn't consider the individual energy requirement of sensor nodes. In a real wireless sensor network application, the energy consumption rates of different sensor nodes are significantly different [16]. On the one hand, sensor nodes may perform different sensing tasks that require different energy support. On the other hand, those sensor nodes that are around the base station need to forward data for remote nodes and thus have much higher energy consumption rates than others.

In contrast to existing works, we consider a more practice scenario that sensor nodes have different energy requirement. Our object is to get a charger placement strategy with minimum number of used chargers so that every sensor node in a network area can receive enough wireless power to meet its energy requirement. The main contributions of this paper are as follows.

– In this paper, we consider the individual energy requirement of each sensor node in a WSN, and study the problem of minimum charger placement with individual energy requirement (problems PIO). We prove the problem to be solved is NP-hard.
– We propose two approximation algorithms for the PIO problem, which are based on greedy and relax rounding, respectively. Moreover, we give detail theoretical performance analysis of the two algorithms.
– We validate the performance of the proposed algorithms by performing lots of numerical simulations. The results show the effectiveness of our designs.

The rest of this paper is organized as follows. Section 2 introduces the state-of-art work of this paper. Section 3 introduces the definition the problem to be solved. Sections 4 describes the two proposed algorithms for the PIO problem. Section 5 validates our algorithms through numerical simulations. Finally, Section 6 draws the conclusion of this paper.

2 Related Works

In the past few years, replenish energy to sensor networks by placing wireless chargers has been widely studied. There are lots of existing works that related to ours. Some works consider the charging utility maximization problem from different aspects. Dai et al. [4] focus on maximizing charging utility under the directional charging model, they aim to get a charger placement strategy for a certain number of directional chargers so that the overall charging utility of the network can be maximized. Yu et al. [18] consider that wireless chargers could communicate with each other, and they address the connected wireless charger placement problem, that is, they aim to place a certain number of wireless chargers into a WSN area to maximize the total charging utility under the constraint that the deployed wireless chargers are connected. Wang et al. [15] first deal with the problem of heterogeneous charger placement under directional charging model with obstacles. They aim to efficiently deploy a set of heterogeneous wireless chargers such that the total charging utility can be maximized while considering the effect of obstacles in the network area.

Different from the above works, some researchers focus on making sure that each sensor node could get sufficient power to achieve perpetual operation. Li et al. [9] investigate how to efficiently deploy wireless chargers to charge wearable devices, and they aim to place wireless chargers in a 2-D area with minimum cardinality to guarantee that the power non-outage probability of the wearable device is not smaller than a given threshold. The work [10] is most similar to ours, in which their object is to get a charger placement strategy with the minimum cardinality to make sure that each sensor node can receive enough energy. The key difference between this work and ours lies on we consider the individual energy requirement of sensor nodes, other than with the assumption that the same energy requirement of each sensor node is the same. Moreover, our proposed algorithms have approximation ratios which guarantee the performance in theory.

3 System Model and Problem Formulation

3.1 System Model and Assumptions

We consider a wireless sensor network that contains m rechargeable sensor nodes denoted by $\mathcal{S} = \{s_1, s_2, \ldots, s_m\}$. These sensor nodes are deployed in a limited 2-D area randomly, and their locations are fixed and known in advance. There are n candidate locations in the network area which are chosen for placing wireless chargers. The candidate locations are chosen by end-users and at most one wireless charger can be placed at each candidate location. The set of candidate

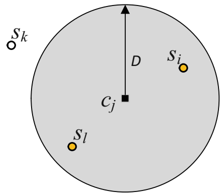


Fig. 1. Omni charging model

locations is denoted by by $\mathcal{C} = \{c_1, c_2, \ldots, c_n\}$. With a little abuse of notations, the wireless charger placed at the *i-th* location is also denoted by c_i.

As shown in Fig. 1, in this paper, we consider problem to be addressed under omnidirectional charging model. In the omnidirectional charging model, both rechargeable sensor nodes and wireless chargers are equipped with omni antennas. Each charger symmetrically radiates its wireless power and shape a disk charging area centering the charger. A sensor node can receive the wireless power from any direction, it can be charged by a wireless charger as long as it located within the charging area of the charger. In practice, the wireless power decays with distance increases, and thus each wireless charger has a bounded charging area. We consider the scenario that all the wireless chargers of end-users are homogeneous, and assume that each charger can only charge sensor nodes within the range of D. Next we describe the energy transfer model and explain the way to calculate the receiving power of a sensor node from wireless chargers.

Based on the Friis's free space equation [2], the receiving radio frequency (RF) power P_r of a receiver from a transmitter can be calculated as

$$P_r = G_t G_r \left(\frac{\lambda}{4\pi d} \right)^2 P_t, \tag{1}$$

where G_t and G_r are antenna gains of transmitter and receiver, respectively, d is the line-of-sight distance between transmitter and receiver, λ is the electromagnetic wavelength, and P_t is the transmitting RF power of the transmitter.

The Friis's free space function is used for far-field wireless power transmission such as satellite communication. For wireless rechargeable sensor networks, the polarization loss should be considered. Based on this, He et al. [6] use a more empirical model to formulate the wireless charging process in wireless rechargeable sensor systems:

$$P_r = \frac{G_t G_r \eta}{L_p} \left(\frac{\lambda}{4\pi (d + \beta)} \right)^2 P_t, \tag{2}$$

where η is the rectifier efficiency, L_p is the polarization loss, and β is a parameter to make Friis's free space equation suitable for short distance wireless power transmission.

As mentioned before, the wireless power decays with distance increases, it's difficult for a receiver which is very far from the transmitter to capture the wireless power as the RF signal is very weak. The symbol D is used to represent the bound distance, that is, if $d > D$, $P_r = 0$. Therefore, in our omnidirectional charging model, we use the following function to evaluate the RF power at sensor node s_i receiving from charger c_j:

$$P_{rx}(s_i, c_j) = \begin{cases} \dfrac{\alpha P_{tx}(c_j)}{(\|s_i - c_j\| + \beta)^2}, & \|s_i - c_j\| \leq D \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where $\alpha = \frac{G_t G_r \eta}{L_p} \left(\frac{\lambda}{4\pi}\right)^2$, $P_{tx}(c_j)$ is the antenna power of the charger c_j, $\|s_i - c_j\|$ is the line-of-sight distance between s_i and c_j, and β is an empirically-determined constant determined by the hardware parameters of chargers and the surroundings.

To replenish energy to the batteries, receivers need to convert the RF energy to electric energy. In practical applications, the conversion efficiency from RF to electricity is non-linear [11]. We denote the electric power got by s_i from c_j as $P_{in}(s_i, c_j)$, and use ξ to denote the conversion efficiency, where ξ is related to the receiving RF power $P_{rx}(s_i, c_j)$, and is calculated as $\xi = f(P_{rx}(s_i, c_j))$. Then $P_{in}(s_i, c_j) = \xi P_{rx}(s_i, c_j) = f(P_{rx}(s_i, c_j))P_{rx}(s_i, c_j)$. This function also can be expressed as $P_{in}(s_i, c_j) = g(P_{rx}(s_i, c_j))$, where $g(\cdot)$ is a non-linear function. In this work, we use the 2^{nd} order polynomial model proposed by [17], we have

$$P_{in}(s_i, c_j) = \mu_1 (P_{rx}(s_i, c_j))^2 + \mu_2 P_{rx}(s_i, c_j) + \mu_3, \tag{4}$$

where $\mu_1, \mu_2, \mu_3 \in \mathbb{R}$ are the empirically-determined parameters.

We assume that all of the used chargers are homogeneous, and the transmitting RF power of each charger is P_{tx}. According to the above charging model, the minimum RF power that a sensor node receives from a charger can be calculated by $P_{rx}^{min} = \frac{\alpha P_{tx}}{(D+\beta)^2}$. Correspondingly, the minimum electric energy that a sensor node got from a charger is estimated as $P_{in}^{min} = g(P_{rx}^{min})$. In energy-harvesting sensor systems, each sensor node needs to manage its electric energy for achieving a long-term operation [7]. In order to make decision efficiently, the sensor node use integers rather than reals to evaluate its energy. In this paper, therefore, we use the *charging levels* present in [5] to evaluate the actual charging power of a sensor node in a discretized way. The charging levels of a sensor node s_i received from a charger c_j can be calculated as follows.

$$L(s_i, c_j) = \left\lfloor \frac{P_{in}(s_i, c_j)}{P_{rx}^{min}} \right\rfloor. \tag{5}$$

According to [4,14], we can use multiple chargers to charge a sensor node simultaneously, and the charging power of the sensor node got from these chargers is accumulative. Thus we measure the charging levels of a sensor node from multiple chargers as the summation of the charging levels provided by each charger. Limited by the hardware of the sensor nodes, the charging levels of a sensor node is bounded. We use $\mathcal{L}_{th}$ to denote the bounded charging levels of a sensor node. Then the charging levels of a sensor node s_i obtained from a given charger set C is formulated as

$$\mathcal{L}(s_i, C) = \begin{cases} \sum\limits_{c_j \in C} L(s_i, c_j), & \text{if } \sum\limits_{c_j \in C} L(s_i, c_j) \leq \mathcal{L}_{th} \\ \mathcal{L}_{th}, & \text{otherwise.} \end{cases} \tag{6}$$

3.2 Problem Formulation

In real wireless rechargeable sensor networks, sensor nodes may have different energy requirement, as these sensor nodes may execute different tasks, besides, the consumed energy for forwarding data is also variant. Therefore, in this work, we study the strategy to place wireless chargers to meet the energy requirement of every sensor node while the total number of used chargers can be minimized. The problem to be addressed under omnidirectional charging model is formulated as follows.

Problem 1. minimum charger Placement with Individual energy requirement under Omnidirectional charging model (PIO). *Given m rechargeable sensor nodes $\mathcal{S} = \{s_1, s_2, \ldots, s_m\}$, and n pre-determined candidate locations $\mathcal{C} = \{c_1, c_2, \ldots, c_n\}$. The charging levels requirement for each sensor node $s_i \in \mathcal{S}$ is $\alpha_i \leq \mathcal{L}_{th}$. Our object is to find a subset $C \subseteq \mathcal{C}$ with minimum cardinality to place wireless chargers, such that every sensor node meets its charging levels requirement.*

Formally, the PIO problem can be present as

$$\begin{aligned} &\textbf{min} \quad |C| \\ &\boldsymbol{s.t.} \quad \mathcal{L}(s_i, C) \geq \alpha_i, \forall s_i \in \mathcal{S} \end{aligned}$$

In our study, we assume that the PIO problem always has feasible solutions, as end users will determine sufficient candidate locations to provide enough wireless power to wireless networks.

Next, we will prove that problem PIO is NP-hard through a theorem. The following introduced problems are helpful for our proof.

The Set Cover Problem (SC): Given a set $\mathcal{S}'$ and a collection $\mathcal{C}'$ of the subset of $\mathcal{S}'$, assume that $\cup_{c'_j \in \mathcal{C}'} c'_j = \mathcal{S}'$, the SC problem is to find a sub-collection $C' \subseteq \mathcal{C}'$ with minimum cardinality such that $\cup_{c'_j \in C'} c'_j = \mathcal{S}'$.

The Decision Version of the SC Problem (d-SC): For a given integer k, whether there is a sub-collection $C' \subseteq \mathcal{C}'$ so that $\cup_{c'_j \in C'} c'_j = \mathcal{S}'$ and $|C'| \leq k$?

The Decision Version of the PIO Problem (d-PIO): For a given integer l, whether there is a location subset $C \subseteq \mathcal{C}$, such that every sensor's charging levels requirement can be met and $|C| \leq l$ if we place wireless chargers on C?

Theorem 1. *The PIO problem is NP-hard.*

Proof. We prove the theorem by reduction, where we reduce the well-known SC problem to PIO. Consider such an instance of d-SC: given an integer k, a set $\mathcal{S}' = \{s'_1, s'_2, \ldots, s'_m\}$, and a collection $\mathcal{C}' = \{c'_1, c'_2, \ldots, c'_n\}$, where $c'_j \subseteq \mathcal{S}'$ for any $c'_j \in \mathcal{C}'$. Next, we construct an instance d-PIO as follows. For each $s'_i \in \mathcal{S}'$, we generate a rechargeable sensor node s_i. We also generate a virtual candidate location c_j for each $c'_j \in \mathcal{C}'$, where the distance between c_j and s_i is less than the charging range D only when $s'_i \in c'_j$. Besides, we set the charging levels requirement α_i for each sensor s_i to be 1, and let $l = k$.

Obviously, this reduction will terminated in polynomial time, and we can get a "yes" answer from the generated instance of problem d-PIO if and only if the given instance of problem d-SC has a "yes" answer. As the SC problem is a well-known NP-complete problem [13], we know that the PIO problem is at least NP-hard.

4 Algorithms for the PIO Problem

In this section, we design two algorithms with performance guarantees for problem PIO: one is a greedy algorithm, named gPIO; another one is based on relax and rounding, named rPIO. In the following, we will describe our algorithms in detail, and given theoretical performance analysis of the two algorithms, respectively.

4.1 The Greedy Based Algorithm

Algorithm Description. We first introduce some useful concepts for making the description of algorithm gPIO more clearly. As each sensor node only needs to meet its charging levels requirement, given a set of placed wireless chargers C, we define the *useful charging levels* of a sensor s_i as $\mathcal{L}^U(s_i, C) = \min\{\mathcal{L}(s_i, C), \alpha_i\}$. The overall useful charging levels provided by charger set C for the whole network is calculated as $\mathcal{L}^U(C) = \sum_{s_i \in \mathcal{S}} \mathcal{L}^U(s_i, C)$. Clearly, $\mathcal{L}^U(\emptyset) = 0$. Consider a location set C which has been deployed with wireless chargers, the *marginal increment* about total useful charging levels is the difference between $\mathcal{L}^U(C \cup \{c_i\})$ and $\mathcal{L}^U(C)$, when a candidate location c_i is selected for placing a wireless charger.

The basic idea of algorithm gPIO is as follows. In each step, the candidate location which brings maximum marginal increment of overall useful charging levels will be selected to place wireless charger. After a candidate location is selected to be placed a charger, algorithm gPIO will update the overall useful charging levels. gPIO terminates after every sensor's charging levels requirement is achieved. Algorithm 1 shows the details of algorithm gPIO.

In the following, we give the analysis of the time complexity of gPIO. The calculation of $\mathcal{L}^U(C \cup \{c_i\})$ costs $\mathcal{O}(mn)$ time, where m and n are the number of sensor nodes and candidate locations, respectively. In each iteration of the while loop, every candidate location in $\mathcal{C} \setminus C$ needs to be checked to find the "best" one. Therefore, it costs $\mathcal{O}(mn^2)$ time for each iteration of the while loop. It's easy

Algorithm 1. The greedy algorithm for PIO (gPIO)

Input: $\mathcal{S}$,$\mathcal{C}$, and α_i for each sensor $s_i \in \mathcal{S}$
Output: a subset of candidate locations C
1: $C \leftarrow \emptyset$
2: **while** $\mathcal{L}^U(s_i, C) < \alpha_i$, $\exists s_i \in \mathcal{S}$ **do**
3: choose $c_i \in \mathcal{C} \backslash C$ that maximizes $\mathcal{L}^U(C \cup \{c_i\}) - \mathcal{L}^U(C)$, and break tie arbitrarily;
4: $C \leftarrow C \cup \{c_i\}$;
5: **end while**
6: **return** C

to know that if the feasible solution exists, the gPIO algorithm must terminate within n iterations after it scanned all the candidate locations. In summary, the time complexity of algorithm gPIO is $\mathcal{O}(mn^3)$.

Performance Analysis. We analyze the approximation ratio of algorithm gPIO through the following theorem.

Theorem 2. gPIO *is a* $(1+\ln\gamma)$*-approximation algorithm for the PIO problem, where* $\gamma = \max_{c_i \in \mathcal{C}} L^U(\{c_i\})$.

Proof. We assume the solution found by algorithm gPIO contains g candidate locations, and we renumber the locations in the order of their selection into the solution, i.e., $C = \{c_1, c_2, \ldots, c_g\}$. We use C_i to denote the location set get by gPIO after the *i-th* iteration, where $i = 0, 1, \ldots, g$, i.e., $C_i = \{c_1, c_2, \ldots, c_i\}$ and $C_0 = \emptyset$. Denote the optimal solution by C^*, and assume there are t number of candidate locations in C^*. The PIO problem is to get a minimum set of candidate locations to place wireless chargers such that every sensor node meets its charging levels requirement, according to the definition of *useful charging levels*, the PIO problem also can be described as to get a minimum set of candidate locations so that the overall useful charging levels equals to $\sum_{s_i \in \mathcal{S}} \alpha_i$. Obviously, we have $\mathcal{L}^U(C) = \mathcal{L}^U(C^*) = \sum_{s_i \in \mathcal{S}} \alpha_i$.

We use $\mathcal{L}^{-U}(C_i)$ to represent the difference of useful charging levels between $\mathcal{L}^U(C_i)$ and $\mathcal{L}^U(C^*)$, i.e., $\mathcal{L}^{-U}(C_i) = \mathcal{L}^U(C^*) - \mathcal{L}^U(C_i)$. In other words, after the *i-th* iteration of gPIO, there still need $\mathcal{L}^{-U}(C_i)$ useful charging levels to meet the charging levels requirement of every sensor node.

As the optimal solution contains t candidate locations, it's easy to know that given a location set C_i, there exists a set with no more than t locations in $\mathcal{C} \setminus C_i$ that can provide $\mathcal{L}^{-U}(C_i)$ useful charging levels for the network. By the pigeonhole principle, there must exist a location $c_j \in \mathcal{C} \setminus C_i$ that provides at least $\frac{\mathcal{L}^{-U}(C_i)}{t}$ marginal increment of useful charging levels. According to the greedy criterion of algorithm gPIO, in each step, we select the location with maximum marginal increase of overall useful charging levels. Therefore, we have

$$\mathcal{L}^U(C_{i+1}) - \mathcal{L}^U(C_i) \geq \frac{\mathcal{L}^{-U}(C_i)}{t} = \frac{\mathcal{L}^U(C^*) - \mathcal{L}^U(C_i)}{t}. \tag{7}$$

Equivalently, we get

$$\mathcal{L}^U(C^*) - \mathcal{L}^U(C_{i+1}) \leq \left(\mathcal{L}^U(C^*) - \mathcal{L}^U(C_i)\right) \cdot \left(1 - \frac{1}{t}\right). \tag{8}$$

By induction, we have

$$\begin{aligned}\mathcal{L}^{-U}(C_i) &= \mathcal{L}^U(C^*) - \mathcal{L}^U(C_i) \\ &\leq \mathcal{L}^U(C^*) \cdot \left(1 - \frac{1}{t}\right)^i \leq \mathcal{L}^U(C^*) \cdot e^{-\frac{i}{t}}.\end{aligned} \tag{9}$$

In each iteration, $\mathcal{L}^{-U}(C_i)$ decreases from $\mathcal{L}^U(C^*)$ to 0, so we can always find an positive integer $i \leq g$ such that $\mathcal{L}^{-U}(C_{i+1}) < t \leq \mathcal{L}^{-U}(C_i)$. Each location selected by algorithm gPIO provides at least 1 useful charging levels. Thus we can conclude that after $(i+1)$-th iterations, the gPIO algorithm will terminate after at most $t-1$ more iterations (i.e., selects at most $t-1$ more candidate locations). Therefore, we get $g \leq i+t$. As $t \leq \mathcal{L}^{-U}(C_i) \leq \mathcal{L}^U(C^*) \cdot e^{-\frac{i}{t}}$, we have $i \leq t \cdot \ln\left(\frac{\mathcal{L}^U(C^*)}{t}\right) \leq t \cdot \ln \gamma$. Thus we have

$$g \leq i + t \leq t(1 + \ln \gamma). \tag{10}$$

Thus the theorem holds.

4.2 The Relax Rounding Algorithm

Algorithm Description. We first rewrite the PIO problem as an integer linear program problem. We use n variables $x_1, x_2, \ldots, x_n$ to be indicators to denote whether the candidate locations are selected to be placed with wireless chargers. If a location is selected, then $x_j = 1$ and $x_j = 0$ otherwise, for $1 \leq j \leq n$. Then problem PIO can be rewritten as

$$\begin{aligned}\mathbf{min} \quad & \sum_{c_j \in \mathcal{C}} x_j \\ \boldsymbol{s.t.} \quad & \sum_{c_j \in \mathcal{C}} x_j \cdot L(s_i, c_j) \geq \alpha_i, \quad \forall s_i \in \mathcal{S} \\ & x_j \in \{0, 1\}, \quad 1 \leq j \leq n.\end{aligned} \tag{11}$$

By relaxing the constraints of $x_j \in \{0,1\}$ to the constraints of $0 \leq x_j \leq 1$, for $1 \leq j \leq n$, the integer linear program is transformed into a linear program:

$$\begin{aligned}\mathbf{min} \quad & \sum_{c_j \in \mathcal{C}} x_j \\ \boldsymbol{s.t.} \quad & \sum_{c_j \in \mathcal{C}} x_j \cdot L(s_i, c_j) \geq \alpha_i, \quad \forall s_i \in \mathcal{S} \\ & 0 \leq x_j \leq 1, \quad 1 \leq j \leq n.\end{aligned} \tag{12}$$

Solving the linear program problem (12) and we get its optimal solution. To get a a feasible solution of (11), i.e., a feasible solution of PIO, we need to rounding the optimal solution of (12) to integers. Next, we will show the details of the rounding process.

Denote the optimal solution of problem (12) by $X^* = \{x_1^*, x_2^*, \dots, x_n^*\}$, then we sort the elements in X^* in descending order of the value of x_j^* and renumber them, that is, let $x_1^* \geq x_2^* \geq \cdots \geq x_n^*$. Note that we also renumber the candidate location set according to X^* such that x_j^* indicates whether location c_j is selected to be placed with a charger. We denote a solution of problem (11) as $X_A = \{x_1^A, x_2^A, \dots, x_n^A\}$. In the beginning, we let $x_j^A = 0$ for $1 \leq j \leq n$, and then we make X_A feasible for problem (11) through iterative operations. In the *j-th* operation, we let $x_j^A = 1$. The iteration terminates until X_A be a feasible solution for the integer linear program problem (11), that is, every sensor node meets its charging levels requirement. We show the details of rPIO in Algorithm 2.

Algorithm 2. The relax rounding algorithm for PIO (rPIO)

Input: $\mathcal{S}, \mathcal{C}$, and α_i for each sensor $s_i \in \mathcal{S}$
Output: a feasible solution X_A for problem (11)
1: Calculate $L(s_i, c_j)$ for each $s_i \in \mathcal{S}$ and $c_j \in \mathcal{C}$, and then convert the PIO problem to an integer linear program as shown in (11);
2: Relax problem (11) and construct a corresponding linear program (12);
3: Solve the linear program (12) and get an optimal solution X^*;
4: Sort the elements in X^* in descending order of the value of x_j^*, and renumber them such that $x_1^* \geq x_2^* \geq \cdots \geq x_n^*$;
5: Renumber candidate locations such that x_j^* indicates whether location c_j is selected;
6: $X_A = \{x_1^A, x_2^A, \dots, x_n^A\} = \{0, 0, \dots, 0\}$;
7: $k = 1$;
8: **while** $\sum_{c_j \in \mathcal{C}} x_j \cdot L(s_i, c_j) < \alpha_i, \exists s_i \in \mathcal{S}$ **do**
9: $\quad x_k^A = 1$;
10: $\quad k = k + 1$;
11: **end while**
12: **return** X_A

Next, we give the analysis of the time complexity of algorithm rPIO. It costs $\mathcal{O}(mn)$ time to get $L(s_i, c_j)$ for each $s_i \in \mathcal{S}$ and $c_j \in \mathcal{C}$ in the first line in algorithm rPIO. Convert problem PIO to an integer linear program and then relax it to a corresponding linear program takes $\mathcal{O}(1)$ time. Solving the linear program costs $\mathcal{O}(n^{2.5}L)$ according to [12], where L is the number of bits in the input. Sort the elements in X^* takes $\mathcal{O}(n \log n)$ time by using the Quicksort method. $L(s_i, c_j)$ has been calculated for each $s_i \in \mathcal{S}$ and $c_j \in \mathcal{C}$ in the first line, so the judgement of the while loop costs $\mathcal{O}(m)$ time. The while loop contains at most n iterations, and thus the while loop costs $\mathcal{O}(mn)$ time. To sum up, the time complexity of algorithm rPIO is $\mathcal{O}(mn) + \mathcal{O}(n^{2.5}L) + \mathcal{O}(n \log n) + \mathcal{O}(mn)$, that is $\mathcal{O}(mn + n^{2.5}L)$ time.

Performance Analysis. We use N_i to represent the summation of charging levels of sensor s_i provided by every candidate location, that is, $N_i = \sum\limits_{c_j \in \mathcal{C}} L(s_i, c_j)$. Then we have the following theorem.

Theorem 3. rPIO *is a δ-approximation algorithm for the PIO problem, where $\delta = \max\limits_{s_i \in \mathcal{S}} \{N_i - \alpha_i + 1\}$.*

Proof. To prove the theorem, we need first prove that for any $x_j^A \in X_A$, if $x_j^A = 1$, then $x_j^* \geq \frac{1}{\delta}$. We prove this condition by contradiction. We first divide the candidate location set into two parts according to the values of elements in X^*. For any $c_j \in \mathcal{C}$, we put location c_j into C^+ if $x_j^* \geq \frac{1}{\delta}$, otherwise, we put location c_j into C^-, that is, $C^+ = \{c_j | x_j^* \geq \frac{1}{\delta}\}$ and $C^- = \{c_j | x_j^* < \frac{1}{\delta}\}$.

Assume that there exists an indicator $x_k^A \in X_A$ where $x_k^A = 1$ but $x_k^* < \frac{1}{\delta}$. According to condition of the while loop in algorithm rPIO, there must exist a sensor $s_i \in \mathcal{S}$ such that $\sum_{j=1}^{k-1} x_j \cdot L(s_i, c_j) < \alpha_i$, otherwise, algorithm rPIO will terminates in $k-1$ iterations, and then $x_k^A = 0$. We can easy know that $|C^+| \leq k - 1$, as $x_k^* < \frac{1}{\delta}$ and the elements in X^* have been sorted in the descending order of the value of each element. In other words, the candidate location set C^+ cannot provide enough charging levels for sensor s_i. We use L_i^+ to denote the summation of the charging levels of s_i provided by each location in C^+, i.e., $L_i^+ = \sum_{c_j \in C^+} L(s_i, c_j) < \alpha_i$. Then the summation of the charging levels of s_i provided by each location in C^- can be calculated by $N_i - L_i^+$.

$$\begin{aligned}
\sum_{c_j \in \mathcal{C}} x_j^* \cdot L(s_i, c_j) &= \sum_{c_j \in C^+} x_j^* \cdot L(s_i, c_j) + \sum_{c_j \in C^-} x_j^* \cdot L(s_i, c_j) \\
&< \sum_{c_j \in C^+} 1 \cdot L(s_i, c_j) + \sum_{c_j \in C^-} \frac{1}{\delta} \cdot L(s_i, c_j) \\
&= L_i^+ + (N_i - L_i^+) \cdot \frac{1}{\delta}
\end{aligned} \tag{13}$$

As L_i^+ is an positive integer, and $L_i^+ < \alpha_i$, so we know that $L_i^+ \leq \alpha_i - 1$. We only consider the case that problem PIO has feasible solutions, it's easy to know that $\delta \geq 1$, and then $L_i^+ + (N_i - L_i^+) \cdot \frac{1}{\delta}$ hits its maximum value when $L_i^+ = \alpha_i - 1$. Therefore, we have

$$\begin{aligned}
\sum_{c_j \in \mathcal{C}} x_j^* \cdot L(s_i, c_j) &< L_i^+ + (N_i - L_i^+) \cdot \frac{1}{\delta} \\
&\leq \alpha_i - 1 + (N_i - \alpha_i + 1) \cdot \frac{1}{\delta} \leq \alpha_i.
\end{aligned} \tag{14}$$

Inequation (14) contradicts the fact that X^* is a feasible solution for linear program (12), i.e., it violates the condition that $\sum\limits_{c_j \in \mathcal{C}} x_j^* \cdot L(s_i, c_j) \geq \alpha_i$, $\forall s_i \in \mathcal{S}$. Hence, we now have prove that for any $x_j^A \in X_A$, if $x_j^A = 1$, then $x_j^* \geq \frac{1}{\delta}$. Then we have the following inequation,

$$\sum_{c_j \in \mathcal{C}} x_j^A \leq \delta \cdot \sum_{c_j \in \mathcal{C}} x_j^* \tag{15}$$

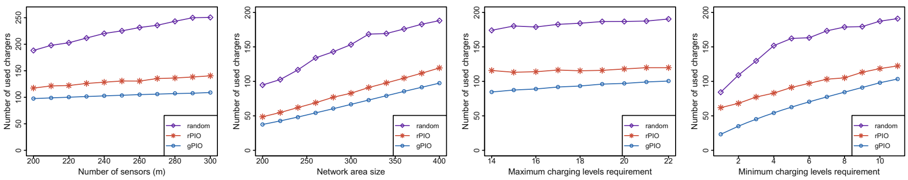
We complete the proof here.

5 Performance Evaluation

We assume that there is a wireless sensor network involves 200 rechargeable sensor nodes that are randomly deployed in a 400 m × 400 m square area, and each site of a sensor node is selected as a candidate location. We set the working RF power P_{tx} of each wireless charger to be 10^6 µW. The parameters α and β are set to be 2.5 and 15, respectively. The charging distance D is set to be 70 m. For the non-linear energy conversion, according to the data measured in [1], we set $\mu_1 = -0.00001$, $\mu_2 = 0.57$ and $\mu_3 = 10$. The required charging levels of each sensor node is randomly selected in $[10, 20]$. The data points plotted in this section under different settings are the average of 100 runs.

5.1 Performance Comparison

We implement a random algorithm, named random, as the baseline for problem PIO. Specifically, algorithm random repeatedly selects a candidate location in a random way to place an omnidirectional wireless charger until every sensor node's charging levels requirement is met. Next, we compare our algorithms with the baseline with different parameters.



(a) (b) (c) (d)

Fig. 2. Performance comparisons between our algorithms (gPIO and rPIO) and random in omnidirectional charging.

(1) Effect of the number of sensor nodes (m): Figure 2(a) shows the effect of the number of sensor nodes on the performance of our algorithms and the baseline. We can see that with the number of sensor nodes increases, all of the three algorithms will require more wireless chargers. However, our algorithms gPIO and gPIO always outperforms the random algorithm. More specifically, the growth rates of our algorithms are lower than that of the baseline algorithm. We can also see that algorithm gPIO is better than algorithm rPIO, which implies that the greedy algorithm is a simple but effective method to deal with the PIO problem.

(2) Effect of the number of the network area size: Figure 2(b) shows the effect of the network area size on the number of used wireless chargers. We keep the number of sensor nodes to be 200, and set the side length of the square area from 200 m to 400 m. We can see that, with the network area becomes larger, all of the algorithms will need more wireless chargers to meet the sensor nodes' charging levels requirements, and our algorithms always outperform the baselines.

(3) Effect of the charging levels requirements of sensor nodes: To evaluate the effect of the charging levels requirements of sensor nodes on the number of used wireless chargers, we design two different experiments. One is set the lower bound of the charging levels requirements to be 10, and range the upper bound of the charging levels requirements from 14 to 22, as shown in Fig. 2(c). It can be seen that with the upper bound of the charging levels requirements increases, the number of used wireless chargers slightly increases for all algorithms. In another set of experiments, we keep the upper bound of the charging levels requirements always be two times of the lower bound, and range the lower bound from 1 to 11, as shown in Fig. 2(d). We can see that the performance of our algorithms is always batter than the baseline, especially when the lower bound of charging levels requirement is small, for example, when the lower bound is et to be 1, the number of wireless chargers required by algorithm gPIO is only 27.6% of algorithm random, and 37.6% of algorithm rPIO.

6 Conclusions

In this study, we investigate the minimum wireless charger placement problem by considering individual energy requirement. We consider the problem under the omnidirectional charging model. We present two algorithms with performance guarantees for problem PIO. In addition, we give detail theoretical performance analysis of the two proposed algorithms. We perform lots of numerical simulations to validate the performance of our algorithms, simulation results show that our designs perform better than the baseline. The study of this problem under directional charging model will be our future work.

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