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Polarized jet fragmentation functions

Zhong-Bo Kang^{a,b,c}, Kyle Lee^{d,e,*}, Fanyi Zhao^{a,b}

^a Department of Physics and Astronomy, University of California, Los Angeles, CA 90095, USA

^b Mani L. Bhaumik Institute for Theoretical Physics, University of California, Los Angeles, CA 90095, USA

^c Center for Frontiers in Nuclear Science, Stony Brook University, Stony Brook, NY 11794, USA

^d C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, NY 11794, USA

^e Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794, USA

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ABSTRACT

We develop the theoretical framework needed to study the distribution of hadrons with general polarization inside jets, with and without transverse momentum measured with respect to the standard jet axis. The key development in this paper, referred to as "polarized jet fragmentation functions", opens up new opportunities to study both collinear and transverse momentum dependent (TMD) fragmentation functions. As two examples of the developed framework, we study longitudinally polarized collinear Λ and transversely polarized TMD Λ production inside jets in both *pp* and *ep* collisions. We find that both observables have high potential in constraining spin-dependent fragmentation functions with sizeable asymmetries predicted, in particular, at the future Electron-Ion Collider.

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1. Introduction

Over the last few years, the study of hadron distributions inside jets has received increasing attention as an effective tool to understand the fragmentation process, describing how the color carrying partons transform into color-neutral particles such as hadrons. Understanding such a fragmentation process is important as it will provide us with a deep insight into the elusive mechanism of hadronization. Theoretical objects which describe the momentum distribution of hadrons inside a fully reconstructed jet are called jet fragmentation functions (JFFs). The usefulness of studying the longitudinal momentum distribution of the hadron in the jet rather than the hadron production itself stems from the former process being differential in the momentum fraction $z_h \equiv p_{hT}/p_{JT}$, where p_{hT} and p_{JT} are the transverse momenta of the hadron and the jet with respect to the beam axis, respectively. Collinear JFFs in the first process can be matched onto the standard collinear fragmentation functions (FFs), enabling us to extract the usual universal FFs more directly by "scanning" the differential z_h dependence. The theoretical developments on the JFFs were first studied in the context of exclusive jet production [1–4] and was later extended to the inclusive jet production case [5–9].

At the same time, the transverse momentum distribution of the hadrons within jets can be sensitive to the transverse momentum dependent fragmentation, described by transverse momentum dependent jet fragmentation functions (TMDJFFs). In [10], it was demonstrated that such TMDJFFs are closely connected to the standard transverse momentum dependent FFs (TMDFFs) [11–13] when the transverse momentum of the hadron is measured with respect to the standard jet axis. For the TMD study of the hadron with respect to the Winner-Take-All jet axis, see [14,15]. As for the TMD study inside the groomed jet, see [16–18]. For the recent works on resummation of $\ln z_h$ and $\ln(1 - z_h)$, see [19,20].

Because of its phenomenological relevance and effectiveness, study of the JFFs has become a very important topic over recent years at the LHC and RHIC, producing measurements for a wide range of identified particles within the jet. Calculations for the JFFs have been performed for single inclusive jet production in unpolarized proton-proton collisions in the context of light charged hadrons [4,6,7], heavy-flavor mesons [4,21,22], heavy quarkonium [23,24], and photons [25]. For the relevant experimental results for the LHC and RHIC, see [26–38] and [39,40], respectively. The study of JFFs is not only important at the LHC and RHIC, as has already been proven, but also provides novel insights at the future Electron-Ion Collider (EIC) [41–44] as we will show below.

* Corresponding author.

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E-mail addresses: zkang@physics.ucla.edu (Z.-B. Kang), kunsu.lee@stonybrook.edu (K. Lee), fanyizhao@physics.ucla.edu (F. Zhao).

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Fig. 1. Illustration for the distribution of hadrons inside a jet, which is initiated by a quark.

In this paper, we provide the general theoretical framework for studying the distribution of hadrons inside a jet by taking full advantage of the polarization effects. We introduce polarized jet fragmentation functions, where the parton that initiates the jet and the hadron that is inside the jet can both be polarized, as illustrated in Fig. 1. We do this in the context of both *pp* colliders like LHC and RHIC, and *ep* colliders like the future EIC. Analogous to the standard FFs, we find a slew of different JFFs that have close connection with the corresponding standard FFs.

When a proton with a general polarization collides with an unpolarized proton or lepton, different JFFs appear with different parton distribution functions (PDFs) and characteristic modulations in the azimuthal angles measured with respect to the scattering plane. Therefore, these observables are not only useful in exploring the spin-dependent FFs, but also in understanding the polarized PDFs. For instance, with the extra handle in z_h , we would be able to reduce uncertainties coming from the final state fragmentation functions by restricting to a well-determined z_h region. Alternatively, with well-determined polarized PDFs at hand, we can directly probe spin-dependent FFs through a study of different JFFs. Some applications of spin-dependent JFFs relevant for the RHIC were considered in [39,45–49], but other applications are far and wide. To demonstrate this, we consider two phenomenological applications in detail. We demonstrate how one can use spin-dependent JFFs to study the collinear helicity FFs and so-called TMD polarizing fragmentation functions (TMD PFFs). There are, of course, many more possible applications of studying other polarized JFFs which we also list in this paper and will present the details in a forthcoming long paper. Other potential applications include probing the polarization of heavy quarkonium inside the jet [23], which is very promising at the LHC and RHIC.

The rest of the paper is organized as follows. In section 2, we lay out the theoretical framework for the leading power JFFs with various correlations between the polarization of the hadrons and the fragmenting partons. We provide their physical meaning and connections to the standard fragmentation functions. In section 3, we first write down the general structure functions that appear in the cross section with different characteristic modulation in the azimuthal angles. We then use the developed framework to study collinear JFFs describing the longitudinally polarized Λ -hyperons production at the RHIC and EIC kinematics. We further study transversely polarized Λ -hyperons production from unpolarized scatterings at the LHC, RHIC, and EIC kinematics. We conclude our paper in section 4.

2. Theoretical framework

In this section, we introduce the concept of polarized jet fragmentation functions. As shown in Fig. 1, we consider a fully reconstructed jet which is initiated by a parton, either a quark or a gluon. One further observes a hadron inside the jet, which carries a longitudinal momentum fraction z_h of the jet and a transverse momentum \mathbf{j}_{\perp} with respect to the jet axis direction. We consider the general situation where both the parton and the hadron can be polarized. We first provide the definition and parametrization of these novel polarized JFFs, and we then study their connection to the standard FFs.

2.1. Polarized jet fragmenting functions

To properly define the momentum and the spin vectors, we define a light-cone vector $n^{\mu} = (1, 0, 0, 1)$ and its conjugate vector $\bar{n}^{\mu} = (1, 0, 0, -1)$, such that $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$. We can then decompose any four-vector p^{μ} as $p^{\mu} = (p^+, p^-, p_{\perp})$ with $p^+ = n \cdot p$, $p^- = \bar{n} \cdot p$. That is,

$$p^{\mu} = p^{-} \frac{n^{\mu}}{2} + p^{+} \frac{\bar{n}^{\mu}}{2} + p^{\mu}_{\perp}.$$
(1)

Let us specify the kinematics of the hadron inside the jet. If the hadron is in a reference frame in which it moves along the +z-direction and has no transverse momentum, then the p_h^- component of its momentum would be very large while the p_h^+ component is small, $p_h^+ \ll p_h^-$. We can parameterize the momentum p_h and the spin vector S_h of the hadron, respectively, as

$$p_h = \left(\frac{M_h^2}{p_h^-}, p_h^-, 0\right), \qquad S_h = \left(-\Lambda_h \frac{M_h}{p_h^-}, \Lambda_h \frac{p_h^-}{M_h}, \boldsymbol{S}_{h\perp}\right),$$
(2)

where M_h is the mass of the hadron, and Λ_h and $S_{h\perp}$ describe the longitudinal and transverse polarization of the hadron inside the jet, respectively. It is evident that they satisfy the relation $p_h \cdot S_h = 0$ as required.

The general correlators that define jet fragmentation functions in such a hadron frame are given by

$$\Delta^{h/q}(z, z_h, \boldsymbol{j}_\perp, S_h) = \frac{z}{2N_c} \delta\left(z_h - \frac{p_h^-}{p_J^-}\right) \langle 0|\delta\left(p^- - \bar{n} \cdot \mathcal{P}\right) \delta^2(\mathcal{P}_\perp/z_h + \boldsymbol{j}_\perp) \chi_n(0)|(Jh)X\rangle \langle (Jh)X|\bar{\chi}_n(0)|0\rangle, \tag{3a}$$

$$\Delta^{h/g,\ \mu\nu}(z,z_h,\boldsymbol{j}_{\perp},S_h) = \frac{z\,p^-}{(d-2)(N_c^2-1)}\delta\left(z_h - \frac{p_h^-}{p_J^-}\right)\langle 0|\delta\left(p^- - \bar{n}\cdot\mathcal{P}\right)\delta^2(\mathcal{P}_{\perp}/z_h + \boldsymbol{j}_{\perp})\mathcal{B}_{n\perp}^{\mu}(0)|(Jh)X\rangle\langle(Jh)X|\mathcal{B}_{n\perp}^{\nu}(0)|0\rangle, \quad (3b)$$

for quark and gluon jets, respectively, in *d* space-time dimensions. Here the momenta of the parton that initiates the jet, the jet, and the hadron are given by p, p_j and p_h , respectively. The energy fractions z and z_h are defined as

$$z = \frac{p_J^-}{p^-}, \qquad z_h = \frac{p_h^-}{p_J^-},$$
 (4)

and thus *z* is the momentum fraction of the parton carried by the jet, while z_h is the momentum fraction of the jet carried by the hadron. Here, we have used the gauge invariant quark and gluon fields, χ_n and $\mathcal{B}_{n\perp}^{\mu}$, in the Soft Collinear Effective Theory [50–53], with *n* denoting the direction of the jet in this case. Note that the state $|(Jh)X\rangle$ represents the final-state unobserved particles *X* and the observed jet *J* with an identified hadron *h* inside, denoted collectively by (Jh). Because of this, the equations above also contain the kinematics of the jet, such as the jet radius *R*. We will suppress them here for simplicity, but express them out explicitly when we discuss their evolution equation in the next section.

Taking into account the spins of both the parton that initiates the jet and the final hadron inside the jet, we can parametrize the above correlators at the leading power as

$$\Delta^{h/q}(z, z_h, \boldsymbol{j}_\perp, S_h) = \Delta^{h/q} [\boldsymbol{j}] \, \frac{\boldsymbol{\vec{n}}}{2} - \Delta^{h/q} [\boldsymbol{j}\gamma_5] \, \frac{\boldsymbol{\vec{n}}\gamma_5}{2} + \Delta^{h/q} [\boldsymbol{i}n_\nu \, \sigma^{i\nu} \gamma_5] \, \frac{\boldsymbol{i}\bar{n}_\mu \, \sigma^{i\mu} \, \gamma_5}{2} \,, \tag{5a}$$

$$\Delta^{h/g,\ ij}(z,z_h,\boldsymbol{j}_\perp,\boldsymbol{S}_h) = \frac{1}{2}\,\delta_T^{ij}\left(\delta_T^{kl}\Delta^{h/g,\ kl}\right) - \frac{i}{2}\,\epsilon_T^{ij}\left(i\epsilon_T^{kl}\Delta^{h/g,\ kl}\right) + \hat{\boldsymbol{S}}\,\Delta^{h/g,\ ij}\,,\tag{5b}$$

where we have $\Delta^{h/q[\Gamma]} \equiv \frac{1}{4} \text{Tr}[\Delta^{h/q}\Gamma]$. The three terms on the r.h.s. of eq. (5a) correspond, in order, to unpolarized, longitudinally polarized, and transversely polarized quarks. On the other hand, the three terms on the r.h.s. of eq. (5b) correspond to unpolarized, circularly polarized, and linearly polarized gluons. Note that we have used $\delta_T^{ij} = -g_T^{ij}$ with $g_T^{\mu\nu} = g^{\mu\nu} - n^{\mu}\bar{n}^{\nu}/2 - n^{\nu}\bar{n}^{\mu}/2$, $\epsilon_T^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}\bar{n}_{\alpha}n_{\beta}/2$, and the symmetrization operator \hat{S} which is defined by [54–56],

$$\hat{S}O^{ij} \equiv \frac{1}{2} \left(O^{ij} + O^{ji} - \delta_T^{ij} O^{kk} \right).$$
(6)

Using the symmetry arguments [57,58], the leading TMDJFFs are parametrized for quarks as

$$\Delta^{h/q[\sharp]} = \mathcal{D}_1^{h/q}(z, z_h, \boldsymbol{j}_\perp) - \frac{\epsilon_T^{ij} j_\perp^i S_{h\perp}^j}{z_h M_h} \mathcal{D}_{1T}^{\perp h/q}(z, z_h, \boldsymbol{j}_\perp), \qquad (7a)$$

$$\Delta^{h/q[\not\!\!\!|p\gamma_5]} = \Lambda_h \, \mathcal{G}_{1L}^{h/q}(z, z_h, j_\perp) - \frac{\boldsymbol{j}_\perp \cdot \boldsymbol{S}_{h\perp}}{z_h \, M_h} \mathcal{G}_{1T}^{h/q}(z, z_h, \boldsymbol{j}_\perp) \,, \tag{7b}$$

$$\Delta^{h/q[in_{\nu}\sigma^{i\nu}\gamma_{5}]} = S_{h\perp}^{i}\mathcal{H}_{1}^{h/q}(z,z_{h},\boldsymbol{j}_{\perp}) + \frac{\epsilon_{T}^{ij}j_{\perp}^{j}}{z_{h}M_{h}}\mathcal{H}_{1}^{\perp h/q}(z,z_{h},\boldsymbol{j}_{\perp}) - \frac{j_{\perp}^{i}}{z_{h}M_{h}}\Lambda_{h}\mathcal{H}_{1L}^{\perp h/q}(z,z_{h},\boldsymbol{j}_{\perp}) + \frac{j_{\perp}^{i}\boldsymbol{j}_{\perp}\cdot\boldsymbol{S}_{h\perp} - \frac{1}{2}\boldsymbol{j}_{\perp}^{2}S_{h\perp}^{i}}{z_{h}^{2}M_{h}^{2}}\mathcal{H}_{1T}^{\perp h/q}(z,z_{h},\boldsymbol{j}_{\perp}),$$
(7c)

and for gluons as

$$\delta_T^{ij} \Delta^{h/g,ij} = \mathcal{D}_1^{h/g}(z, z_h, \boldsymbol{j}_\perp) - \frac{\epsilon_T^{ij} \, \boldsymbol{j}_\perp^i \, \boldsymbol{S}_{h\perp}^j}{z_h \, M_h} \, \mathcal{D}_{1T}^{\perp h/g}(z, z_h, \boldsymbol{j}_\perp) \,, \tag{8a}$$

$$i\epsilon_T^{ij}\Delta^{h/g,ij} = \Lambda_h \mathcal{G}_{1L}^{h/g}(z, z_h, \boldsymbol{j}_\perp) - \frac{\boldsymbol{j}_\perp \cdot \boldsymbol{S}_{h\perp}}{z_h M_h} \mathcal{G}_{1T}^{h/g}(z, z_h, \boldsymbol{j}_\perp),$$
(8b)

$$\hat{S} \Delta^{h/g,ij} = \hat{S} \left[-\frac{j_{\perp}^{i} \epsilon_{T}^{jk} S_{h\perp}^{k}}{2z_{h} M_{h}} \mathcal{H}_{1}^{h/g}(z, z_{h}, \boldsymbol{j}_{\perp}) + \frac{j_{\perp}^{i} j_{\perp}^{j}}{2z_{h}^{2} M_{h}^{2}} \mathcal{H}_{1}^{\perp h/g}(z, z_{h}, \boldsymbol{j}_{\perp}) \right. \\ \left. + \frac{j_{\perp}^{i} \epsilon_{T}^{jk}}{2z_{h}^{2} M_{h}^{2}} \left(\Lambda_{h} j_{\perp}^{k} \mathcal{H}_{1L}^{\perp h/g}(z, z_{h}, \boldsymbol{j}_{\perp}) - \left(\frac{\boldsymbol{j}_{\perp} \cdot \boldsymbol{S}_{h\perp} j_{\perp}^{k} - \frac{1}{2} \boldsymbol{j}_{\perp}^{2} S_{h\perp}^{k}}{z_{h} M_{h}} \right) \mathcal{H}_{1T}^{\perp h/g}(z, z_{h}, \boldsymbol{j}_{\perp}) \right] \right].$$

$$(8c)$$

Summary of the results with physical interpretation of the quark and gluon TMDJFFs at leading power are given in Table 1. Except that they represent the hadron fragmentation inside a fully reconstructed jet, their physical meaning is similar to that of standard TMDFFs as summarized in [13]. For this reason, we choose to employ the scripted version of the letters used for the usual TMDFFs to label TMDJFFs. In this table, U, L, and T represent unpolarized, longitudinally, and transversely polarized states, while Circ and Lin represent circularly and linearly polarized states. Thus $\mathcal{D}_1^{h/q,g}$ describe the distribution of unpolarized hadrons inside a jet which is initiated by an unpolarized quark or gluon. It is also instructive to mention several TMDJFFs that lead to important spin asymmetries in experiments. For example, $\mathcal{D}_{1T}^{\perp h/q,g}$ describes the transverse polarization of hadrons inside the jet that is initiated by an unpolarized quark/gluon, and thus can be used to describe the transverse polarization of Hyperons inside a jet. $\mathcal{H}_1^{\perp h/q}$ is a transversely polarized quark that initiates a jet in which unpolarized hadrons are observed. This function leads to the so-called Collins azimuthal asymmetry for hadrons inside a jet [48]. On the other hand, $\mathcal{H}_1^{\perp h/g}$ represents the situation where a linearly polarized gluon generates a jet in which unpolarized hadrons are observed.

Table 1

Transverse momentum dependent JFFs for quarks and gluons. Here U, L, and T represent unpolarized, longitudinally, and transversely polarized states, while Circ and Lin represent circularly and linearly polarized states.

h\ q	U	L	Т	h∖ g	U	Circ	Lin
U	${\cal D}_1^{h/q}$		$\mathcal{H}_1^{\perp \ h/q}$	U	$\mathcal{D}_1^{h/g}$		$\mathcal{H}_1^{\perp \ h/g}$
L		${\cal G}^{h/q}_{1L}$	$\mathcal{H}_{1L}^{h/q}$	L		$\mathcal{G}_{1L}^{h/g}$	$\mathcal{H}_{1L}^{h/g}$
Т	$\mathcal{D}_{1T}^{\perp \ h/q}$	${\cal G}_{1T}^{h/q}$	$\mathcal{H}_1^{h/q}$, $\mathcal{H}_{1T}^{\perp \ h/q}$	Т	$\mathcal{D}_{1T}^{\perp h/g}$	$\mathcal{G}_{1T}^{h/g}$	$\mathcal{H}_1^{h/g}$, $\mathcal{H}_{1T}^{\perp \ h/g}$

Integration over the transverse momentum j_{\perp} dependence formally leads to the collinear JFFs,

$$\mathcal{D}_1^{h/c}(z, z_h) = \int d^2 \boldsymbol{j}_\perp \, \mathcal{D}_1^{h/c}(z, z_h, \, \boldsymbol{j}_\perp) \,, \tag{9a}$$

$$\mathcal{G}_{1}^{h/c}(z,z_{h}) = \int d^{2} \boldsymbol{j}_{\perp} \mathcal{G}_{1L}^{h/c}(z,z_{h},\boldsymbol{j}_{\perp}), \qquad (9b)$$

$$\mathcal{H}_1^{h/c}(z, z_h) = \int d^2 \boldsymbol{j}_\perp \, \mathcal{H}_1^{h/c}(z, z_h, \, \boldsymbol{j}_\perp) \,, \tag{9c}$$

where c = q, g. We assume that the hadron h has a spin- $\frac{1}{2}$, for which there is no integrated linearly polarized gluons as a consequence of conservation of angular momentum, i.e. $\mathcal{H}_1^{h/g}(z, z_h) = 0$. This is like for PDFs where no collinear gluon transversity exists for a spin- $\frac{1}{2}$ target. Lastly, we note that the correlators in eq. (3) and parametrizations from eqs. (5) to (8) generalize simply to the exclusive jet production case as well, where now the *z* dependence drops out. See [1–4,9] for examples of unpolarized case in exclusive processes. In this paper, we focus on the inclusive jet processes for phenomenological applications.

2.2. Connection to the standard fragmentation functions

Both collinear and transverse momentum dependent JFFs have close relations to the conventional collinear and transverse momentum dependent FFs. For the unpolarized case, the relations between collinear (TMD) JFFs and the standard collinear (TMD) FFs were derived in [7] and [10], respectively. For example, the collinear unpolarized JFFs $\mathcal{D}_1^{h/i}(z, z_h, p_{JT}R, \mu)$ is related to the collinear unpolarized FFs $\mathcal{D}_1^{h/i}(z_h, \mu)$ as follows

$$\mathcal{D}_{1}^{h/i}(z, z_{h}, p_{JT}R, \mu) = \sum_{j} \int_{z_{h}}^{1} \frac{dz'_{h}}{z'_{h}} \mathcal{J}_{ij}(z, z'_{h}, p_{JT}R, \mu) D_{1}^{h/j}\left(\frac{z_{h}}{z'_{h}}, \mu\right),$$
(10)

where the coefficient functions \mathcal{J}_{ij} can be found in [7]. Note that we have explicitly included the dependence on the jet kinematics, the jet transverse momentum p_{JT} and the jet radius R, as well as the renormalization scale μ . By studying the perturbative behavior of these JFFs, one can derive their renormalization group (RG) equations, which are the same as the usual time-like DGLAP evolution equations,

$$\mu \frac{d}{d\mu} \mathcal{D}_{1}^{h/i}(z, z_{h}, p_{JT} R, \mu) = \frac{\alpha_{s}(\mu)}{\pi} \sum_{j} \int_{z}^{1} \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}, \mu\right) \mathcal{D}_{1}^{h/j}(z', z_{h}, p_{JT} R, \mu),$$
(11)

where P_{ji} are the splitting functions for unpolarized fragmentation functions [59,60]. We will derive all the other relations between TMDJFFs and TMDFFs, as well as their corresponding collinear version in the future publication [61]. Here we only list the coefficients that are relevant to the key spin observables in the next section.

In order to study the longitudinal polarization of a Λ particle inside a jet produced in single longitudinally polarized proton-proton and/or proton-lepton collisions, $\vec{p} + (p/e) \rightarrow (\text{jet}\vec{\Lambda}) + X$, one would need the polarized collinear JFFs $\mathcal{G}_1^{h/i}(z, z_h, p_{JT}R, \mu)$. The RG equations are given by

$$\mu \frac{d}{d\mu} \mathcal{G}_{1}^{h/i}(z, z_{h}, p_{JT} R, \mu) = \frac{\alpha_{s}(\mu)}{\pi} \sum_{j} \int_{z}^{1} \frac{dz'}{z'} \Delta_{L} P_{ji}\left(\frac{z}{z'}, \mu\right) \mathcal{G}_{1}^{h/j}(z', z_{h}, p_{JT} R, \mu),$$
(12)

where $\Delta_L P_{ji}(z_h)$ are the time-like splitting functions for helicity fragmentation functions [59,60]. This is expected since $\mathcal{G}_1^{h/i}$ describes longitudinally polarized Λ production inside a jet, which is initiated by a longitudinally polarized parton. Likewise, $\mathcal{G}_1^{h/i}$ can be matched onto helicity FFs $\mathcal{G}_1^{h/i}(z_h,\mu)$ as follows

$$\mathcal{G}_{1}^{h/i}(z, z_{h}, p_{JT}R, \mu) = \sum_{j} \int_{z_{h}}^{1} \frac{dz_{h}'}{z_{h}'} \mathcal{J}_{ij}^{L}(z, z_{h}', p_{JT}R, \mu) G_{1}^{h/j}\left(\frac{z_{h}}{z_{h}'}, \mu\right),$$
(13)

where the matching coefficients \mathcal{J}_{ij}^L at the next-to-leading order (NLO) for jets reconstructed via the anti- k_T algorithm are given by

$$\begin{aligned} \mathcal{J}_{qq}^{L}(z,z_{h},p_{JT}R,\mu) &= \delta(1-z)\delta(1-z_{h}) + \frac{\alpha_{s}}{2\pi} \left\{ \ln\left(\frac{\mu^{2}}{p_{JT}^{2}R^{2}}\right) \left[\Delta_{L}P_{qq}(z)\delta(1-z_{h}) - \Delta_{L}P_{qq}(z_{h})\delta(1-z) \right] \\ &+ \delta(1-z) \left[2C_{F}(1+z_{h}^{2}) \left(\frac{\ln(1-z_{h})}{1-z_{h}}\right)_{+} + C_{F}(1-z_{h}) + \Delta_{L}\mathcal{I}_{qq}^{anti-k_{T}}(z_{h}) \right] \\ &- \delta(1-z_{h}) \left[2C_{F}(1+z^{2}) \left(\frac{\ln(1-z)}{1-z}\right)_{+} + C_{F}(1-z) \right] \right\}, \end{aligned}$$
(14a)
$$\mathcal{J}_{qg}^{L}(z,z_{h},p_{JT}R,\mu) &= \frac{\alpha_{s}}{2\pi} \left\{ \ln\left(\frac{\mu^{2}}{p_{JT}^{2}R^{2}}\right) \left[\Delta_{L}P_{gq}(z)\delta(1-z_{h}) - \Delta_{L}P_{gq}(z_{h})\delta(1-z) \right] \\ &+ \delta(1-z) \left[2\Delta_{L}P_{gq}(z_{h}) \ln(1-z_{h}) - 2C_{F}(1-z_{h}) + \Delta_{L}\mathcal{I}_{gq}^{anti-k_{T}}(z_{h}) \right] \\ &- \delta(1-z_{h}) \left[2\Delta_{L}P_{gq}(z) \ln(1-z) - 2C_{F}(1-z) \right] \right\}, \end{aligned}$$
(14b)
$$\mathcal{J}_{gq}^{L}(z,z_{h},p_{JT}R,\mu) &= \frac{\alpha_{s}}{2\pi} \left\{ \ln\left(\frac{\mu^{2}}{p_{JT}^{2}R^{2}}\right) \left[\Delta_{L}P_{qg}(z)\delta(1-z_{h}) - \Delta_{L}P_{qg}(z_{h})\delta(1-z) \right] \\ &+ \delta(1-z) \left[2\Delta_{L}P_{gq}(z_{h}) \ln(1-z_{h}) + 2T_{F}(1-z_{h}) + \Delta_{L}\mathcal{I}_{qg}^{anti-k_{T}}(z_{h}) \right] \\ &- \delta(1-z_{h}) \left[2\Delta_{L}P_{qg}(z) \ln(1-z) + 2T_{F}(1-z_{h}) + \Delta_{L}\mathcal{I}_{qg}^{anti-k_{T}}(z_{h}) \right] \\ &- \delta(1-z_{h}) \left[2\Delta_{L}P_{qg}(z) \ln(1-z) + 2T_{F}(1-z_{h}) + \Delta_{L}\mathcal{I}_{qg}^{anti-k_{T}}(z_{h}) \right] \end{aligned}$$

$$\mathcal{J}_{gg}^{L}(z, z_{h}, p_{JT}R, \mu) = \delta(1-z)\delta(1-z_{h}) + \frac{\alpha_{s}}{2\pi} \left\{ \ln\left(\frac{\mu^{2}}{p_{JT}^{2}R^{2}}\right) \left[\Delta_{L}P_{gg}(z)\delta(1-z_{h}) - \Delta_{L}P_{gg}(z_{h})\delta(1-z)\right] + \delta(1-z) \left[4N_{c}\left(2(1-z_{h})^{2}+z_{h}\right) \left(\frac{\ln(1-z_{h})}{1-z_{h}}\right)_{+} - 4N_{c}(1-z_{h}) + \Delta_{L}\mathcal{I}_{gg}^{anti-k_{T}}(z_{h}) \right] - \delta(1-z_{h}) \left[4N_{c}\left(2(1-z)^{2}+z\right) \left(\frac{\ln(1-z)}{1-z}\right)_{+} - 4N_{c}(1-z)\right] \right\},$$
(14d)

with the anti- k_T algorithm term

$$\Delta_L \mathcal{I}_{ji}^{\text{anti}-k_T}(z_h) = 2\Delta_L P_{ji}(z_h) \ln z_h \,. \tag{15}$$

Now if one wants to explore the transverse momentum \mathbf{j}_{\perp} distribution of hadrons inside the jet, along with any spin-dependent correlations, one would need the TMDJFFs. For example, if one wants to measure both the unpolarized and transversely polarized Λ production inside the jet in unpolarized proton-proton and/or proton-lepton collisions, $p + (p/e) \rightarrow (\text{jet}\Lambda^{\uparrow}) + X$, one would need TMDJFFs $\mathcal{D}_1^{h/i}(z, z_h, \mathbf{j}_{\perp}, p_{JT}R, \mu)$ and $\mathcal{D}_{1T}^{\perp h/i}(z, z_h, \mathbf{j}_{\perp}, p_{JT}R, \mu)$, respectively. It has been demonstrated in [10] that unpolarized TMDJFF $\mathcal{D}_1^{h/i}$ is related to the standard TMDFF $\mathcal{D}_1^{h/i}$. Following the same procedure, we show only the final results here and leave the details in the future publication. First of all, since $\mathcal{D}_{1T}^{\perp h/i}$ involves an unpolarized parton that initiates the jet, its evolution follows the same RG equations as the unploarized case

$$\mu \frac{d}{d\mu} \mathcal{D}_{1T}^{\perp h/i}(z, z_h, \boldsymbol{j}_\perp, p_{JT} R, \mu) = \frac{\alpha_s(\mu)}{\pi} \sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}, \mu\right) \mathcal{D}_{1T}^{\perp h/j}(z', z_h, \boldsymbol{j}_\perp, p_{JT} R, \mu),$$
(16)

which allow us to evolve $\mathcal{D}_{1T}^{\perp h/i}$ from the typical jet scale $\mu_J \sim p_{JT}R$ to the hard scale $\mu \sim p_{JT}$ and thus resums the logarithm of the jet radius *R*. On the other hand, at the scale μ_J , we have

$$\mathcal{D}_{T}^{\perp h/i}(z, z_{h}, \boldsymbol{j}_{\perp}, p_{JT}R, \mu_{J}) = \mathcal{C}_{i \to j}(z, p_{JT}R, \mu_{J}) D_{1T}^{\perp h/j}(z_{h}, \boldsymbol{j}_{\perp}; \mu_{J}), \qquad (17)$$

where $D_{1T}^{\perp h/j}$ is the TMD PFFs that describes fragmentation of an unpolarized parton into a transversely polarized hadron. On the other hand, the coefficient functions $C_{i \rightarrow j}$ are the same as those in the unpolarized case and can be found in [10].

3. Application of polarized jet fragmentation functions

In this section, we demonstrate the application of the polarized jet fragmentation functions defined above and show how they can bring novel insights into the study of spin and TMD effects. We first set up the general framework, and we then choose two specific observables to show the numerical signatures at RHIC, LHC, and the future EIC.



Fig. 2. Illustration for the distribution of hadrons inside jets in the collisions of a polarized proton and an unpolarized proton or lepton.

3.1. General structure of the observables

We consider hadron distribution inside jets in either lepton-proton or proton-proton collisions, as illustrated in Fig. 2,

$$p(p_A, S_A) + \left(p(p_B)/e(p_\ell) \right) \to \left(\text{jet}(\eta_J, p_{JT}, R) h(z_h, \boldsymbol{j}_\perp, S_h) \right) + X,$$
(18)

where a polarized proton with the spin S_A and momentum p_A (moving along "+z" direction) scatters on an unpolarized proton (or lepton) with momentum p_B (p_ℓ) (moving along "-z" direction) and produces a jet reconstructed in the usual anti- k_T algorithm [62] with the jet radius parameter R, rapidity η_J , and transverse momentum p_{JT} . One further observes a hadron inside the jet with its spin S_h , which carries a longitudinal momentum fraction z_h of the jet and a transverse momentum \mathbf{j}_\perp with respect to the standard jet axis direction. The spin vector of the incoming proton in the center-of-mass frame is parametrized as

$$S_A = \left(-\lambda \frac{M_A}{p_A^-}, \lambda \frac{p_A^-}{M_A}, \boldsymbol{S}_T\right),\tag{19}$$

where λ is the helicity of the polarized proton with mass M_A , while S_T is its transverse spin vector. Needless to say, the observables in proton-proton collisions can be studied in both RHIC and LHC facilities, while we have EIC in mind for those in lepton-proton collisions.

In this paper, we consider the case where the jet fragmentation function measurement is performed for an inclusive jet sample in proton-proton (lepton) collisions, where the sum over all particles in the final state *X* besides the observed jet is performed. In this case, it is the so-called semi-inclusive JFFs defined in the previous section that are probed. On the other hand, if one performs the measurements in an exclusive jet production, such as the back-to-back lepton-jet in lepton-proton collisions, or *Z*-jet correlations in proton-proton collisions, one should instead use the exclusive version of the JFFs. For details, see e.g. [9,43,63].

For the scattering illustrated in Fig. 2, and for single inclusive jet production, the most general azimuthal dependence for the hadron distribution inside the jet differential in both z_h and j_{\perp} can be written in the following form

$$\frac{d\sigma^{p(S_{A})+p/e \to (jeth(S_{h}))X}}{dp_{JT}d\eta_{J}dz_{h}d^{2}\boldsymbol{j}_{\perp}} = F_{UU,U} + |\boldsymbol{S}_{T}|\sin(\phi_{S_{A}} - \hat{\phi}_{h})F_{TU,U}^{\sin(\phi_{S_{A}} - \hat{\phi}_{h})} + \Lambda_{h} \left[\lambda F_{LU,L} + |\boldsymbol{S}_{T}|\cos(\phi_{S_{A}} - \hat{\phi}_{h})F_{TU,L}^{\cos(\phi_{S_{A}} - \hat{\phi}_{h})}\right]
+ |\boldsymbol{S}_{h\perp}| \left\{\sin(\hat{\phi}_{h} - \hat{\phi}_{S_{h}})F_{UU,T}^{\sin(\hat{\phi}_{h} - \hat{\phi}_{S_{h}})} + \lambda\cos(\hat{\phi}_{h} - \hat{\phi}_{S_{h}})F_{LU,T}^{\cos(\hat{\phi}_{h} - \hat{\phi}_{S_{h}})}
+ |\boldsymbol{S}_{T}| \left(\cos(\phi_{S_{A}} - \hat{\phi}_{S_{h}})F_{TU,T}^{\cos(\phi_{S_{A}} - \hat{\phi}_{S_{h}})} + \cos(2\hat{\phi}_{h} - \hat{\phi}_{S_{h}} - \phi_{S_{h}})F_{TU,T}^{\cos(2\hat{\phi}_{h} - \hat{\phi}_{S_{h}} - \phi_{S_{h}})}\right\},$$
(20)

where $F_{AB,C}$ are the spin-dependent structure functions, with *A*, *B*, and *C* indicating the polarization of the polarized incoming proton, the unpolarized incoming proton or lepton, and the outgoing hadron inside the jet, respectively. Note that the situation where both incoming protons (or proton-lepton) are polarized has been extensively studied in the literature [64], and can be used to explore the polarized PDFs. Our focus here is to understand how the polarization in the fragmentation functions inside the jet can couple with the polarization in the incoming proton and thus to figure out the novel insights that come with these observables. Explicitly, we note that

$$F_{UU,C} \sim f_{a/A}(x_a,\mu), \qquad F_{LU,C} \sim g_{a/A}(x_a,\mu), \qquad F_{TU,C} \sim h_{a/A}(x_a,\mu), \tag{21}$$

where $f_{a/A}$, $g_{a/A}$, and $h_{a/A}$ stand for unpolarized, helicity, and transversity collinear PDFs in the incoming proton, respectively. As the transverse momentum is measured with respect to the jet axis, its dependence on the inherent transverse momentum of the partons from the incoming hadrons is power suppressed [7]. Therefore, incoming distributions remain collinear whether we have the transverse momentum measured or not for the final hadron inside the jet. Then each hadron in A = U, L, T state create unpolarized, longitudinally (circularly), and transversely (linearly) polarized quarks (or gluons) respectively that fragments into hadron of state *C*. Comparing this fact with the Table 1 show which JFFs each $F_{AB,C}$ are sensitive to. That is,



Fig. 3. The longitudinally polarized fragmentation functions plotted at $\mu = 1$ GeV for various scenarios [65] consistent with the LEP data. Scenario 1 has only polarized *s* quark nonvanishing, scenario 2 has *u* and *d* quark equal, but with opposite sign from *s* quark, scenario 3 has *u*, *d*, and *s* quark equal to each other. In all scenarios, gluon vanishes at the input scale and is generated entirely from QCD evolution.

$$F_{UU,U} \sim f_{a/A} \mathcal{D}_1^{h/c}, \ F_{UU,T}^{\sin(\phi_h - \phi_{S_h})} \sim f_{a/A} \mathcal{D}_{1T}^{\perp h/c},$$
(22a)

$$F_{LU,L} \sim g_{a/A} \mathcal{G}_{1}^{h/c}, \ F_{LU,T}^{\cos(\hat{\phi}_{h} - \hat{\phi}_{S_{h}})} \sim g_{a/A} \mathcal{G}_{1T}^{h/c},$$
(22b)

$$F_{TU,U}^{\sin(\phi_{S_{A}}-\hat{\phi}_{h})} \sim h_{a/A} \mathcal{H}^{\perp h/c}, \ F_{TU,L}^{\cos(\phi_{S_{A}}-\hat{\phi}_{h})} \sim h_{a/A} \mathcal{H}_{1L}^{h/c}, \ F_{TU,T}^{\cos(\phi_{S_{A}}-\hat{\phi}_{S_{h}})} \sim h_{a/A} \mathcal{H}_{1}^{h/c}, \ F_{TU,T}^{\cos(2\hat{\phi}_{h}-\hat{\phi}_{S_{h}}-\phi_{S_{A}})} \sim h_{a/A} \mathcal{H}_{1T}^{\perp h/c},$$
(22c)

where we have suppressed the collinear unpolarized PDFs $f_{b/B}$ in the unpolarized incoming proton. Detailed expressions of different structure functions will be given in the forthcoming long paper [61].

It is instructive to point out that the azimuthal angle ϕ_{S_A} of the transverse spin S_T of the incoming proton is measured in the centerof-mass (CM) frame. As shown in Fig. 2, in the CM frame, we align the jet momentum in the *x*-*z* plane. At the same time, we choose another reference frame in which the jet momentum aligns in $+z_J$ direction, with the x_J axis in the *x*-*z* plane. We then measure the azimuthal angle of the relative momentum \mathbf{j}_{\perp} and the transverse spin vector $\mathbf{S}_{h\perp}$ of the hadron in the jet in this $x_J - y_J - z_J$ reference frame, and they are denoted as $\hat{\phi}_h$ and $\hat{\phi}_{S_h}$, respectively.

If one integrates over \mathbf{j}_{\perp} and thus only measures the z_h -distribution of hadrons inside the jet, one would have

$$\frac{d\sigma^{p(S_A)+p/e \to (jeth(S_h))X}}{dp_{JT}d\eta_J dz_h} = F_{UU,U} + \Lambda_h \lambda F_{LU,L} + |\mathbf{S}_{h\perp}| |\mathbf{S}_T| \cos(\phi_{S_A} - \hat{\phi}_{S_h}) F_{TU,T}^{\cos(\phi_{S_A} - \hat{\phi}_{S_h})},$$
(23)

where $F_{AB,C}$ are the collinear version of the structure functions given in eq. (20). Of course if we choose to polarize both incoming particles, there would be more terms in both eqs. (20) and (23). In the remaining section, as key examples of application, we study longitudinal spin transfer as encoded in the \mathbf{j}_{\perp} -integrated version of $F_{LU,L}$, as well as the transverse polarization of Λ production as contained in the structure function $F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{s_h})}$. The unpolarized hadron distribution in jets as encoded in $F_{UU,U}$ [7,10] and the hadron Collins asymmetry as encoded in $F_{TU,U}^{\sin(\hat{\phi}_h - \hat{\phi}_{s_h})}$ [48] have been explored previously.

3.2. Example 1: longitudinally polarized Λ

As their polarizations can be determined via dominant weak decay channel $\Lambda \rightarrow p \pi (\bar{\Lambda} \rightarrow \bar{p} \pi)$, $\Lambda(\bar{\Lambda})$ -hyperons are particularly suited for studying the spin-dependent fragmentation. There have been many different measurements of polarized $\Lambda(\bar{\Lambda})$ -hyperons, and we give predictions for longitudinally polarized Λ inside jets in this section.

Longitudinal spin dependent $\Lambda/\bar{\Lambda}$ fragmentation functions were first determined (see also [66] for earlier study) by analyzing the LEP data [67] to NLO [65], which was not able to constrain the valence fragmentation functions for all flavors; vastly different helicity FFs motivated by different scenarios in Fig. 3 were able to describe the LEP data equally well. As demonstrated by [68], however, study of the longitudinal spin transfer D_{LL} in longitudinally polarized proton-proton collisions, $\vec{p}p \rightarrow \vec{\Lambda}X$, as a function of rapidity can discriminate the different scenarios of valence spin dependent fragmentation functions. The positive rapidity region (corresponding to the forward region of the polarized proton) corresponds to the valence region of the polarized proton, and thus spin transfers are dominated by u and d quarks inside the polarized proton. Since these polarized u and d quarks then fragment, D_{LL} plotted as a function of rapidity can distinguish different scenarios of helicity FFs of u and d quark. For the past decade, there have been many measurements and analyses done for such longitudinal spin transfer to Λ and $\bar{\Lambda}$ hyperons by the STAR collaboration at RHIC [69–71]. However, the measurements are only binned in two rapidity bins, the negative ($-1.2 < \eta_J < 0$) and positive ($0 < \eta_J < 1.2$) bins (at the most recent measurement [71]), and are not yet able to discriminate the different scenarios proposed in [65].

As the measurement can be binned more continuously in the transverse momentum $p_{\Lambda T}$ of the Λ particle, we propose using the relevant collinear helicity JFF to scan through the helicity FFs more directly by studying $z_{\Lambda} = p_{\Lambda T}/p_{JT}$ distribution of the longitudinally polarized Λ particles inside a jet in longitudinally polarized proton-proton collisions, $\vec{p} + p \rightarrow (\text{jet}\vec{\Lambda}) + X$. Analogous longitudinal spin transfer $D_{LL}^{\text{jet}\Lambda}$ for Λ polarization in the jet can be defined as follows

$$D_{LL}^{\text{jet}\Lambda} = \frac{d\Delta\sigma^{pp \to (\text{jet}\Lambda)X}}{dp_{JT}d\eta_{J}dz_{\Lambda}} \left/ \frac{d\sigma^{pp \to (\text{jet}\Lambda)X}}{dp_{JT}d\eta_{J}dz_{\Lambda}} = \frac{F_{LU,L}}{F_{UU,U}},$$
(24)



Fig. 4. Predictions for the asymmetry D_{LL}^{jetA} at the RHIC (left) and EIC (right) for various scenarios, which have been proposed in [65] and shown in Fig. 3. Our predictions show that D_{LL}^{jetA} is capable of discriminating different scenarios.

where the last equation uses the structure functions in eq. (23). For the unpolarized cross section in the denominator, factorization is given by $[7]^1$

$$\frac{d\sigma^{pp\to(jet\Lambda)X}}{dp_{JT}d\eta_{J}dz_{\Lambda}} = \sum_{a,b,c} \int_{x_{a}^{\min}}^{1} \frac{dx_{a}}{x_{a}} f_{a}(x_{a},\mu) \int_{x_{b}^{\min}}^{1} \frac{dx_{b}}{x_{b}} f_{b}(x_{b},\mu) \int_{z_{c}^{\min}}^{1} \frac{dz_{c}}{z_{c}^{2}} H_{ab}^{c} \mathcal{D}_{1}^{h/c}(z_{c},z_{\Lambda},p_{JT}R,\mu).$$

$$(25)$$

On the other hand, the numerator of eq. (24) is defined as $d\Delta\sigma = [d\sigma(+, +) - d\sigma(+, -)]/2$ with the first and second index representing the helicities λ and Λ_{Λ} , respectively, and it can be written in the following factorized form

$$\frac{d\Delta\sigma^{\vec{p}p\to(jet\vec{h})X}}{dp_{JT}d\eta_{J}dz_{\Lambda}} = \sum_{a,b,c_{x_{a}^{\min}}} \int_{x_{a}}^{1} \frac{dx_{a}}{x_{a}} g_{a}(x_{a},\mu) \int_{x_{b}^{\min}}^{1} \frac{dx_{b}}{x_{b}} f_{b}(x_{b},\mu) \int_{z_{c}^{\min}}^{1} \frac{dz_{c}}{z_{c}^{2}} \Delta_{LL} H_{ab}^{c} \mathcal{G}_{1}^{h/c}(z_{c},z_{\Lambda},p_{JT}R,\mu).$$

$$(26)$$

Here $f_a(x_a, \mu)$ and $g_a(x_a, \mu)$ are the unpolarized PDFs and helicity parton distribution functions, and H_{ab}^c (or $\Delta_{LL}H_{ab}^c$) are the corresponding hard functions, respectively. Finally, $\mathcal{D}_1^{h/c}$ and $\mathcal{G}_1^{h/c}$ are the relevant unpolarized collinear JFFs and helicity JFFs defined in the previous section. Once again, $\mathcal{G}_1^{h/c}$ can be matched onto the standard helicity FFs as in eq. (13).

Let us now present the numerical results for longitudinally polarized Λ production inside jets at the RHIC and EIC kinematics. We use the polarized and unpolarized NNPDF sets [72,73] for collinear helicity parton distributions and unpolarized PDFs, respectively. For both unpolarized and longitudinally polarized fragmentation functions for Λ , we use the leading-order set from [65], whose three different scenarios are plotted in Fig. 3.

Following the recent STAR measurement at RHIC [71], we first study Λ polarization inside jets that are reclustered through the anti- k_T algorithm with a jet radius R = 0.4 in proton-proton collisions at $\sqrt{s} = 200$ GeV. We plot our predictions for $D_{LL}^{\text{jet}\Lambda}$ in Fig. 4a as a function of z_{Λ} , while integrating over the jet transverse momentum and rapidity: $10 < p_{JT} < 15$ GeV and $|\eta_J| < 1.2$. We find that the size of the asymmetry is on the order of a few percents, and polarized FFs in different scenarios lead to quite different behavior in the spin asymmetry. Such behavior can be understood from the fact that contributions from u and d quarks dominate in this kinematic region, and thus $D_{LL}^{\text{jet}\Lambda}$ follows the signs of the corresponding helicity FFs $G_1^{\Lambda/u,d}$. In other words, as z_{Λ} directly probes the polarized FFs in Fig. 3, no other differential information is required to discriminate the three scenarios. This new observable is thus capable of discriminating different scenarios without binning in η_I , as in the case of single inclusive Λ production [68].

Next, we study longitudinally polarized Λ for the EIC at a CM energy of $\sqrt{s} = 89$ GeV. We measure R = 0.4 anti- k_T jets with transverse momentum and rapidity in the range 10 $< p_{JT} < 15$ GeV and $|\eta_J| < 2$, respectively. We notice that with quark contributions enhanced by the LO $eq \rightarrow eq$ process, $D_{LL}^{\text{jet}\Lambda}$ becomes order of tens of percents at the EIC as shown in Fig. 4b and will be able to clearly discriminate different scenarios of helicity FFs.

3.3. Example 2: transversely polarized Λ from unpolarized scatterings

As a phenomenological application of polarized TMDJFFs, we consider production of a transversely polarized Λ inside a jet from an unpolarized fragmenting parton. The corresponding standard TMDFFs are the TMD PFFs $D_{1T}^{\perp h/c}$, as mentioned in section 2.2. The Belle

¹ We will write factorization expressions for *pp*, but similar expressions for *ep* follow simply.

Collaboration has recently measured transversely polarized Λ in the back-to-back production of Λ and a light hadron in e^+e^- collisions, $e^+e^- \rightarrow \Lambda^{\uparrow} + h + X$ [74], from which the TMD PFFs have been extracted in [75,76].

We study the transversely polarized Λ production inside a jet in unpolarized proton-proton and lepton-proton collisions, $p + e/p \rightarrow \infty$ $(\text{jet } \Lambda^{\uparrow}) + X$. This has been suggested as a promising measurement at the LHC [77,78]. We measure the longitudinal momentum fraction z_{Λ} of the jet carried by the Λ particle, transverse momentum j_{\perp} with respect to the jet direction, and the transverse spin $S_{h\perp}$ of the Λ particle. As emphasized already in section 3.1, the transverse momentum j_{\perp} and the transverse spin $S_{h\perp}$ of the Λ particle correlate with each other, generating a $\sin(\hat{\phi}_h - \hat{\phi}_{S_h})$ azimuthal dependence as in

$$\frac{d\sigma}{dp_{JT}d\eta_J dz_\Lambda d^2 \boldsymbol{j}_\perp} = F_{UU,U} + |\boldsymbol{S}_{h\perp}| \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} + \cdots, \qquad (27)$$

where " \cdots " represent other terms in eq. (20). We define the so-called Λ transverse polarization observable inside the jet as follows

$$P_{\Lambda} = \frac{F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})}}{F_{UU,U}}.$$
(28)

Here the structure functions in the numerator and denominator can be written as the following factorized form²

$$F_{UU,U} = \sum_{a,b,c} \int_{x_{a}^{\min}}^{1} \frac{dx_{a}}{x_{a}} f_{a}(x_{a},\mu) \int_{x_{b}^{\min}}^{1} \frac{dx_{b}}{x_{b}} f_{b}(x_{b},\mu) \int_{z_{c}^{\min}}^{1} \frac{dz_{c}}{z_{c}^{2}} H_{ab}^{c} \mathcal{D}_{1}^{\Lambda/c}(z_{c},z_{\Lambda},\boldsymbol{j}_{\perp},p_{JT}R,\mu),$$
(29)

$$F_{UU,T}^{\sin(\hat{\phi}_{h}-\hat{\phi}_{S_{h}})} = \sum_{a,b,c} \int_{x_{a}^{\min}}^{1} \frac{dx_{a}}{x_{a}} f_{a}(x_{a},\mu) \int_{x_{b}^{\min}}^{1} \frac{dx_{b}}{x_{b}} f_{b}(x_{b},\mu) \int_{z_{c}^{\min}}^{1} \frac{dz_{c}}{z_{c}^{2}} H_{ab}^{c} \mathcal{D}_{1T}^{\perp\Lambda/c}(z_{c},z_{\Lambda},\boldsymbol{j}_{\perp},p_{JT}R,\mu).$$
(30)

Note that the expression for the unpolarized structure $F_{UU,U}$ has been given previously in [10], following which one can derive the formalism for the spin-dependent structure function $F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})}$. In other words, they are sensitive to the unpolarized TMDJFFs $\mathcal{D}_1^{\Lambda/c}$ and the spin-dependent TMDJFFs $\mathcal{D}_{1T}^{\perp\Lambda/c}$, respectively. Their RG evolution equations and connections to the standard TMDFFs are given in section 2.2. In particular, $\mathcal{D}_{1T}^{\perp\Lambda/c}$ is related to the TMD PFFs $D_{1T}^{\perp\Lambda/c}(z_h, j_{\perp}; \mu_J)$ in eq. (17). In order to provide some estimate for Λ transverse polarization inside the jet, we implement the following model for the TMD PFFs.

Using the so-called b_* prescription [79], we combine TMD evolution with the recent gaussian fit of the Belle data to parametrize $D_{1T}^{\perp\Lambda/c}$ at the jet scale $\mu_I \sim p_{IT} R$ as

$$D_{T}^{\perp\Lambda/c}(z_{\Lambda}, \boldsymbol{j}_{\perp}; \mu_{J}) = \frac{1}{z_{\Lambda}^{2}} \left(\frac{1}{2z_{\Lambda}}\right) \int_{0}^{\infty} \frac{b^{2} db}{2\pi} J_{1}\left(\frac{j_{\perp}b}{z_{\Lambda}}\right) F_{c}(z_{\Lambda}, \mu_{b_{*}}) e^{-S_{\text{pert}}^{i}(b_{*}, \mu_{J}) - S_{\text{NP}}^{i}(b, \mu_{J})},$$
(31)

where S_{pert}^{i} are the usual perturbative Sudakov factors [80], and $F_{c}(z_{\Lambda}, \mu_{b_{*}})$ is fitted from the recent Belle data and has the following functional form

$$F_c(z_\Lambda, \mu_b^*) \equiv \mathcal{N}_c(z_\Lambda) D_1^{\Lambda/c}(z_\Lambda, \mu_b^*), \qquad (32)$$

where $D_1^{\Lambda/c}(z_\Lambda, \mu_b^*)$ are the unpolarized collinear $c \to \Lambda$ fragmentation functions, and the parametrization of $\mathcal{N}_c(z_\Lambda)$ for different quark flavors can be found in [75]. Note that at the moment, there is no existing extraction for the gluon TMD PFF $D_{1T}^{\perp \Lambda/g}$ and we thus do not include it in the numerical study below. However, we do include the unpolarized gluon TMDFF $D_1^{\Lambda/g}$ in the calculation of $F_{UU,U}$. The scale $\mu_{b_*} = 2e^{-\gamma_E}/b_*$ with $b_* = b/\sqrt{1 + b^2/b_{max}^2}$. We then choose to parametrize the non-perturbative Sudakov factor for quark TMD PFFs as [10,81,82]

$$S_{\rm NP}^{q}(b,\mu_{J}) = \frac{g_2}{2} \ln\left(\frac{b}{b_*}\right) \ln\left(\frac{p_{JT}R}{Q_0}\right) + \frac{\langle M_D^2 \rangle}{4z_{\Lambda}^2} b^2, \qquad (33)$$

with $Q_0^2 = 2.4 \text{ GeV}^2$, $b_{\text{max}} = 1.5 \text{ GeV}^{-1}$, $g_2 = 0.84$, and $\langle M_D^2 \rangle = 0.118 \text{ GeV}^2$ [75]. We now present our predictions for Λ polarization P_{Λ} at the LHC, RHIC and the future EIC. We use anti- k_T jets with R = 0.5 for the LHC kinematics and R = 0.4 for the RHIC and EIC kinematics. We again use LO NNPDF [73] as the PDF sets and AKK08 [83] parametrization of the Λ fragmentations to be consistent with the extraction of TMD PFFs we are using [75]. For the LHC, we implement the kinematics used on the recent LHCb measurements for the distribution of charged hadrons in Z-tagged jets [35] in proton-proton collisions at the CM energy $\sqrt{s} = 8$ TeV in the forward rapidity regions $2.5 < \eta_J < 4$. Fig. 5a shows z_{Λ} distribution of P_{Λ} with j_{\perp} integrated from $0 < j_{\perp} < 1$ GeV for three different ranges of jet transverse momenta: $20 < p_{JT} < 30$ GeV, $30 < p_{JT} < 50$ GeV and $50 < p_{JT} < 100$ GeV. The asymmetry P_{Λ} starts out positive and becomes negative around $z_{\Lambda} \sim 0.3$. This behavior can be explained by the fact that PFF for the *u* quark is positive from the extraction [75] and dominates in the small z_{Λ} region. When $z_{\Lambda} \gtrsim 0.3$, negative PFF of the *d* quark

² Once again, expressions for *ep* follow simply.



Fig. 5. Predictions for the asymmetry P_{Λ} for the LHC (left), RHIC (middle) and EIC (right) kinematics.

begins dominating. As the quark jet fraction increases with p_{JT} , we observe that asymmetry P_{Λ} is enhanced as jet transverse momenta are increased. At the RHIC kinematics, we choose the transverse momentum of the jets to be $10 < p_{JT} < 15$ GeV and study two different ranges of rapidity, $0 < \eta_J < 1$ and $1 < \eta_J < 2.2$, where the latter may be available once a forward detector upgrade is made available at sPHENIX [84]. In Fig. 5b, we present our results differential in z_{Λ} again with j_{\perp} integrated over $0 < j_{\perp} < 1$ GeV. Just as P_{Λ} at the LHC, u quark PFF dominates when $z_{\Lambda} \leq 0.3$, but is taken over by negative d quark PFF at the larger values of z_{Λ} . We also note that valence contributions are enhanced when we look at more forward rapidity region of $1 < \eta_J < 2.2$. Since valence quarks, especially uand d, have the largest PFFs, size of P_{Λ} is enhanced for the more forward rapidity region. Nevertheless, the Λ polarization P_{Λ} in protonproton collisions at both LHC and RHIC is about 2% level in magnitude, which is similar in size to the recent ATLAS measurement for the transverse polarization of single inclusive Λ production in $pp \rightarrow \Lambda^{\uparrow} + X$ [85].

Finally, we present our prediction for P_{Λ} as a function of z_{Λ} for the future EIC at a CM energy of $\sqrt{s} = 89$ GeV in Fig. 5c. We choose the transverse momentum and rapidity of the jets to be within the intervals $10 < p_{JT} < 15$ GeV and $|\eta_J| < 2$, respectively, and we again integrate j_{\perp} over $0 < j_{\perp} < 1$ GeV. While the result remains qualitatively similar to the ones for the LHC and RHIC kinematics, u quark PFF dominates over a larger region of z_{Λ} and P_{Λ} remains positive until $z_{\Lambda} \lesssim 0.4$. Since quark PFFs dominate in the lepton-proton collisions, the polarization is larger in size with $P_{\Lambda} \sim 10\%$ at $z_{\Lambda} \sim 0.5$. Thus this would be a feasible jet substructure observable at the future EIC [86].

4. Conclusion

In this paper, we provide the general theoretical framework to study the distribution of hadrons inside the jet by taking full advantage of the polarization effects. We define the polarized jet fragmentation functions, in which both the parton that initiates the jet and the hadron inside the jet can have the general polarization. For single inclusive jet production in both proton-proton and lepton-proton collisions, we write down the general structure functions for hadron distributions inside the jet with different characteristic modulation in the azimuthal angles. We then use the developed frameworks to study both longitudinally polarized collinear Λ production and transversely polarized transverse momentum dependent Λ production. By carrying out phenomenological studies in the context of both *pp* collider like LHC and RHIC and *ep* collider like the EIC, we demonstrate that sizeable asymmetry can be measured at different collider facilities. Our results especially show the importance of studying these observables at the future Electron-Ion Collider designed to shed light on hadron structures. We expect our work will open new and exciting opportunities in the direction of studying spin-dependent hadron structures using hadrons inside jets.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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