A model-free method for learning flexibility capacity of loads providing grid support

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***Abstract*—Flexible loads are a resource for the Balancing Authority (BA) of the future to aid in the balance of power supply and demand. In order to be used as a resource, the BA must know the capacity of the flexible loads to vary their power demand over a baseline without violating consumers' quality of service (QoS). Existing work on capacity characterization is model-based: They need models relating power consumption to variables that dictate QoS, such as temperature in the case of an air conditioning system. However, in many cases the model parameters are not known or are difficult to obtain. In this work, we pose a data driven capacity characterization method that does not require model information, it only needs access to a simulator. The capacity is characterized as the set of feasible spectral densities (SDs) of the demand deviation. The proposed method is an extension of our recent work on SD-based capacity characterization that was limited to the case where the loads dynamic model is completely known. Numerical evaluation of the method is provided, which compares our approach to the model-based solution of our past work.**

I. Introduction

The future of the power grid is green: an increased reliance on renewable generation. This poses a challenge to Balancing Authorities (BAs) to balance demand and supply. Most loads have some flexibility in power demand: they can deviate their power demand from a baseline value without violating their quality of service (QoS). The baseline consumption is then power consumption in absence of any requests from the BA. Examples of flexible loads include pumps and water heaters [1], residential air conditioners [2], and commercial HVAC systems [3].

Many loads are needed to provide a meaningful service. A BA would request a desired demand deviation from a collection of loads, which we call the *reference signal*, to help balance demand and supply, and the load ensemble is expected to track this reference signal accurately. Tracking the reference must not cause individual loads to violate their QoS. From the viewpoint of the grid operator, poor reference tracking makes the loads an unreliable resource. From the viewpoint of a load, a reference signal that continually requires it to violate its QoS provide incentive for it to stop providing grid support. In either case, avoidance of the above scenarios is paramount to the long term success of grid support from flexible loads. In other words, reference signals must be designed to respect the *capacity* of flexible loads, which informally represents limitations in the ability to track a demand deviation reference signal by a collection due to QoS requirements at the individual.

Prior work on capacity estimation of a collection of flexible loads can be characterized into time domain and frequency domain approaches. Within time domain approaches, which include [4]–[12], a popular approach is to develop ensemble level necessary conditions, e.g., [4], [6]. If a reference signal does not satisfy these conditions, then at least some loads will not be able to satisfy their individual QoS in order for the ensemble to track that reference.

The frequency domain approaches seek to obtain constraints on the statistics of the reference signal, especially *spectral density* (SD) of the reference signal, than the reference signal itself [13]–[15]. These methods aim to precisely quantify the regions shown in Figure 1 based on the QoS of the loads considered. One particular advantage of frequency domain characterization is that it is better suited to answering questions such as "how many flexible loads will a BA require for the next year?", which are useful for long term resource planning. To answer such questions using time domain approaches is challenging; the BA would have to predict demand-supply imbalance many months in advance to see if flexible loads can track the corresponding reference signal. At this time scale, predicting statistics of demand-supply imbalance is far more tractable.

Irrespective of statistical or time domain characterizations, many of the listed works have one thing in common: they are model-based. Meaning, they need a simple model that relates demand deviation of the collection of flexible load(s) to individuals' QoS. The computed capacity of the load(s) depend on the model/model parameters. These parameters of the flexible load model are typically unknown or require estimation from experimental data or high-fidelity simulations. In the spirit of model free control, one might wonder then, is it possible to directly estimate a characterization of flexible load capacity from data?

In this work, we develop a data-driven (model free) frequency domain framework for capacity characterization of a load collection. The proposed method does not need a simple model of the collection; rather it needs data from a simulator of individual loads or from large scale experiments. This framework builds off of our past work [15], where we characterize the capacity of flexible load(s) as constraints on the SD of their power deviation. To obtain a SD we set up an optimization problem: the BA projects its needs (roughly, the SD of net demand, e.g., shown in Figure 1) onto the constraint set of feasible of SDs' of the load collection.

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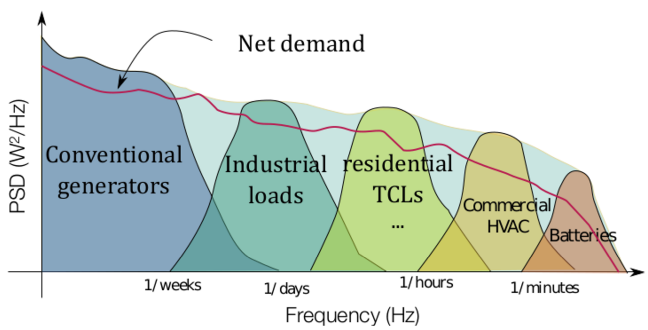

Fig. 1: An example spectral allocation of resources to meet the grids needs.

In [15], the models relating demand deviation to QoS variables were assumed LTI with known parameter values. In this work we solve the projection problem without having access to model information, but only from data from a simulator. The key insight that allows for this model free construction is the choice of approximation architecture when discretizing the infinite dimensional optimization problem. The proposed data driven framework is validated by comparing its capacity estimate to the model based framework of our previous work [15].

The paper proceeds as follows. In Section II we introduce the method from our prior work. In Section III we introduce our data-driven method. Numerical experiments are conducted in Section IV and we conclude in Section V

II. Spectral Characterization of QoS Constraints

The symbol t is used to denote the continuous time while k is used to denote a discrete time index. The sampling interval is Δt. Additionally we define $\lfloor m \rfloor \triangleq \{1,\dots,m\}$ for $m \in \mathbb{N}$.

A. Deterministic QoS constraints

Denote by $P[k]$ the power consumption of a flexible load at time index k, and let $P^b[k]$ its baseline demand. The demand deviation is $\tilde{P}[k] := P[k] - P^b[k]$. The load provides grid support service by controlling the deviation $\tilde{P}[k]$ to track a desired deviation signal, called a reference, while maintaining its own QoS. The first QoS constraint is simply an actuator constraint:

$$\text{QoS-1:} \quad \left|\tilde{P}[k]\right| \le c_1, \quad \forall\ k, \tag{1}$$

where the constant c_1, the maximum possible deviation of power consumption, depends on the rated power and the baseline demand. Second, define the demand increment $\tilde{P}_\delta[k] := \tilde{P}[k] - \tilde{P}[k-\delta]$, where $\delta > 0$ is a predetermined (small) integer time interval. The second constraint is a ramping rate constraint:

$$\text{QoS-2:} \quad \left|\tilde{P}_\delta[k]\right| \le c_2, \quad \forall\ k. \tag{2}$$

Third, define the additional energy use during any integer time interval of length T:

$$\tilde{E}[k] = \sum_{\sigma=k-T+1}^{k} \tilde{P}[\sigma]. \tag{3}$$

The third QoS constraint is that

$$\text{QoS-3:} \quad \left|\tilde{E}[k]\right| \le c_3, \quad \forall\ k. \tag{4}$$

The parameter T in (3) can represent the length of a billing period. Ensuring (4) ensures that the energy consumed during a billing period is close to the nominal energy consumed.

To define the fourth and last QoS constraint, we associate with the VES system a *storage variable* $\tilde{x}[k]$ that is related to the demand deviation, and impose the constraint

$$\text{QoS-4:} \quad |\tilde{x}[k]| \le c_4, \quad \forall\ k. \tag{5}$$

1) Understanding QoS-4: To understand the storage variable, imagine a flexible HVAC system providing VES. We first present a model of the HVAC systems internal temperature T_z in continuous time, as it is more naturally presented in this setting. This model is:

$$C\frac{dT_z(t)}{dt} = \frac{1}{R}\left(T_a(t) - T_z(t)\right) + \dot{q}_{\text{int}}(t) + Q(t), \tag{6}$$

where C and R are thermal capacitance and resistance, $T_a(t)$ is the ambient temperature, and $\dot{q}_{\text{int}}(t)$ is an exogenous disturbance. The quantity $Q(t)$ is the rate of heat delivered to the building by the HVAC system (negative if cooling).

We use a linear model for relating the electrical power deviation to the indoor temperature. The temperature deviation will play the role of the storage variable $\tilde{x}[k]$. Suppose $Q(t) = -\eta_0 P(t)$ where η_0 is the coefficient of performance (COP) under design conditions. In general, the baseline power consumption for a HVAC system is the value $P^b(t)$ that keeps the internal temperature of the load at a fixed value $\bar{T}$, which for (6) is

$$P^b(t) = -\frac{\left(T_a(t) - \bar{T}\right)}{\eta_0 R} - \frac{\dot{q}_{\text{int}}(t)}{\eta_0}. \tag{7}$$

Since we are concerned with the flexibility in the load, we linearize (6) about the thermal setpoint $\bar{\theta}$ and the baseline power $P^b(t)$ yielding,

$$\dot{\tilde{T}}_z(t) = -\gamma \tilde{T}_z(t) + \beta \tilde{P}(t), \quad \gamma = \frac{1}{RC}, \quad \beta = \frac{\eta_0}{C}, \tag{8}$$

where $\tilde{T}_z \triangleq T_z(t) - \bar{T}$ is the internal temperature deviation and $\tilde{P}$ is as defined at the beginning of this Section. The corresponding discrete-time dynamic model relating $\tilde{P}[k]$ to $\tilde{T}_z[k]$ is

$$\tilde{T}_z[k+1] = a\tilde{T}_z[k] + b\tilde{P}[k], \tag{9}$$

(where $a = e^{-\gamma\Delta t}$ and $b = \beta\int_0^{\Delta t} e^{-\gamma\tau} d\tau$), which is also a first order linear time invariant (LTI) model.

B. Mathematical Preliminaries

In our prior work [15], we had developed a methodology that characterizes the capacity of a flexible load in the frequency domain. We briefly discuss this prior work here. Denote the power consumption of a flexible load as $\tilde{P}[k]$, where we model $\tilde{P}$ as a stochastic process. The mean and autocorrelation function for $\tilde{P}$ are,

$$\mu_{\tilde{P}}[k] \triangleq \mathsf{E}[\tilde{P}[k]], \qquad \forall\ k, \tag{10}$$

$$R_{\tilde{P}}[s,k] \triangleq \mathsf{E}[\tilde{P}[s]\tilde{P}[k]], \quad \forall\ s,\ k, \tag{11}$$

where $\mathsf{E}[\cdot]$ denotes mathematical expectation. In the past work, we made the following assumption about the stochastic process $\tilde{P}$.

Assumption 1. *The stochastic process $\tilde{P}$ is wide sense stationary (WSS) with mean function $\mu_{\tilde{P}}[k] = 0$ for all k.*

Under this assumption we have that the autocorrelation function will solely be a function of $\tau = s - k$. In this case, the autocorrelation function is an asymmetric Fourier transform pair with the Spectral Density:

$$R_{\tilde{P}}(\tau) = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{\tilde{P}}(\Omega)e^{j\Omega\tau}d\Omega, \text{ and} \tag{12}$$

$$S_{\tilde{P}}(\Omega) = \sum_{\tau=-\infty}^{\infty} R_{\tilde{P}}[\tau]e^{-j\Omega\tau}, \tag{13}$$

where $S_{\tilde{P}}(\Omega)$ is the (power) Spectral Density (SD) of $\tilde{P}$, $\omega \in [-\pi, \pi]$ is the frequency variable, and j is the imaginary unit. The above is based on the general definition of the SD of the signal $\tilde{P}$,

$$S_{\tilde{P}}(\Omega) \triangleq \lim_{N\to\infty} \frac{1}{N}\mathsf{E}\left[\left|\sum_{k=1}^{N}\tilde{P}[k]e^{-j\Omega k}\right|^2\right] \tag{14}$$

the equivalence of definitions (14) and (13) for a WSS process is the Wiener-Khinchin theorem. Since the mean function of $\tilde{P}$ is zero for all time we have

$$\sigma_{\tilde{P}}^2 = R_{\tilde{P}}(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{\tilde{P}}(\Omega)d\Omega, \tag{15}$$

that is, the variance $\sigma_{\tilde{P}}^2$ of $\tilde{P}$ is the integral of the (power) SD. To illustrate our method from prior work we will also make use of the Chebyshev inequality for a r.v. X:

$$\mathsf{P}\big(|X - \mu_X| \geq C\big) \leq \frac{\sigma_X^2}{C^2}, \quad \forall\ C > 0, \tag{16}$$

where $\mathsf{P}(\cdot)$ denotes probability. Another useful relation is the following:

Proposition 1. *If the input $x[k]$ to a linear time invariant system with frequency response $H(e^{j\Omega})$ is WSS and has SD Φ_x, then the output $y[k]$ is also WSS and its SD Φ_y is given by $\Phi_y(\Omega) = \Phi_x(\Omega)|H(e^{j\Omega})|^2$.*

C. Probabilistic QoS Constraints and SD-based Capacity Characterization

Each QoS constraint potentially involves a distinct signal. In QoS-1, the signal is the power deviation $\tilde{P}[k]$ itself. In QoS-4, it is the storage variable $\tilde{x}[k]$. *We denote by $Z_\ell[k]$ the signal relevant for the ℓ-th QoS constraint.* In each QoS, the relevant signal is related to the power deviation $\tilde{P}[k]$, and we denote by $\mathcal{G}_\ell$ the (potentially dynamic) linear system that relates the input $\tilde{P}$ to the output Z_ℓ.

Next we illustrate how to pose the QoS constraints as constraints on SDs. We start by considering the ℓ-th QoS, which we re-formulate as

$$\mathsf{P}\left(|Z_\ell[k]| \geq c_\ell\right) \leq \varepsilon_\ell, \quad \forall k \tag{17}$$

where $\varepsilon_\ell \ll 1$ is the tolerance. From the Chebyshev inequality (16) and the equation (15) we have:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} S_{Z_\ell}(\Omega)d\Omega \leq c_\ell^2\varepsilon_\ell \implies \mathsf{P}\left(|Z_\ell[k]| \geq c_\ell\right) \leq \varepsilon_\ell. \tag{18}$$

Thus, the probabilistic constraint (17) can be assured by asking for the following constraint involving SD of Z_ℓ to be satisfied:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} S_{Z_\ell}(\Omega)d\Omega \leq b_\ell =: c_\ell^2\varepsilon_\ell. \tag{19}$$

Since the dynamic system $\mathcal{G}_\ell$ relating the input $\tilde{P}$ and output Z_ℓ is a linear time invariant system, then (19) can be translated to a constraint on the power deviation SD:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} S_{\tilde{P}}(\Omega)|G_\ell(e^{j\Omega})|^2 d\Omega \leq b_\ell \tag{20}$$

where $G_\ell(e^{j\Omega})$ is the frequency response of the dynamic system $\mathcal{G}_\ell$. In the general case with m constraints, the constraint set for the SD $S_{\tilde{P}}$ is

$$\mathcal{S} \triangleq \left\{ S_{\tilde{P}} \;\middle|\; \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{\tilde{P}}(\Omega)|G_\ell(e^{j\Omega})|^2 d\Omega \leq b_\ell,\ \ell \in \lfloor m \rfloor \right\}. \tag{21}$$

As long as the SD of the demand deviation $\tilde{P}[k]$ belong to the set $\mathcal{S}$, each of the ℓ probabilistic QoS constraints - such as QoS 1-4 described in Section II-A - holds. Thus, the set $\mathcal{S}$ also represents the demand deviation capacity of the flexible load.

Now, denote by $\mathcal{H}$ the set of SDs defined over $[-\pi, \pi)$, and define the function $\bar{B}_\ell : \mathcal{H} \to \mathbb{R}^+$ as

$$\bar{B}_\ell(S_{\tilde{P}}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{\tilde{P}}(\Omega)|G_\ell(e^{j\Omega})|^2 d\Omega \tag{22}$$

and the array $\bar{B}(S_{\tilde{P}}) = [\bar{B}_1(S_{\tilde{P}}), \ldots, \bar{B}_m(S_{\tilde{P}})]^T \in \mathbb{R}^m$. By denoting $\mathsf{b} = [\varepsilon_1 c_1^2, \ldots, \varepsilon_m c_m^2]^T \in \mathbb{R}^m$ the constraint $S_{\tilde{P}} \in \mathcal{S}$ can be represented as

$$\bar{B}(S_{\tilde{P}}) \leq \mathsf{b}. \tag{23}$$

Now consider the following optimization problem:

$$\begin{aligned} \min_{S} \quad & \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(S(\Omega)-S^{\text{BA}}(\Omega)\right)^2 d\Omega \\ \text{s.t.} \quad & \bar{B}(S_{\tilde{P}}) \le \mathsf{b} \ \text{ and } S(\Omega)\ge 0 \quad \forall\, \Omega\in[-\pi,\pi), \end{aligned} \tag{24}$$

where $S^{\text{BA}}(\Omega)$ is the spectral density of the stochastic process that generates the reference signals from the BA. How to determine $S^{\text{BA}}(\Omega)$ is discussed in the next section. *We define the capacity of the flexible load as the solution $S^*_{\tilde{P}}$ of the optimization problem* (24).

1) BA's spectral needs: The *total* needs of the BA is encapsulated by the SD of the net demand signal, an example of which is shown in Figure 1. With historical data, a BA can estimate the SD of the net demand signal, which we denote as $S^{\text{ND}}(\Omega)$. Any well posed estimation technique can be applied. All controllable resources, including generators, flywheels, batteries, and flexible loads, together have to supply $S^{\text{ND}}(\Omega)$. To determine solely the portion of $S^{\text{ND}}(\Omega)$ that flexible loads should contribute to we "filter" $S^{\text{ND}}(\Omega)$. That is, with $F(e^{j\Omega})$ an appropriate filter we have

$$S^{\text{BA}}(\Omega) = \left|F(e^{j\Omega})\right|^2 S^{\text{ND}}(\Omega). \tag{25}$$

The quantity S^{BA} is the frequency domain analog of the reference signal $r[k]$ that will be asked from the loads, and will be referred to as the *reference SD* in the sequel.

D. Model based approach: prior work

The constraints in Problem (24) are all linear in the decision variable $S_{\tilde{P}}(\Omega)$ with a quadratic objective function. Hence the problem is a quadratic problem, although infinite dimensional.

Remark 1. *The problem* (24) *can be reduced to a tractable finite dimensional optimization problem by discretizing the continuous frequency Ω into N points on the unit circle. The decision vector of the optimization problem becomes N. The resulting problem is a finite dimensional quadratic program (QP) that can be efficiently solved using readily available solvers.* In all such problems in the rest of the paper that involve functions of continuous frequency Ω over $[-\pi,\pi]$, we assume that such a discretization is done to convert the problem to a finite dimensional problem. □

The finite dimensional QP alluded to in Remark 1 is the problem posed and solved in our prior work [15]. Thus, the optimization problem needed to characterize capacity is fairly straightforward to solve, *as long as the models $\mathcal{G}_\ell$'s are LTI and the model parameters are known*. There is, at least, one weakness: obtaining the model parameters is not an easy task. Take the LTI model (9) of temperature deviation in a building. This equation alone is actually quite merited for this particular application, and there is a plethora of work spanning back to the 1980's [16] on using ODEs of this form to model the dynamics in certain flexible loads. These works focus on *estimating* the parameters of the model such as (9). These parameters are challenging to estimate. Still, many current capacity characterizations explicitly depend on the parameters such as R and C.

III. Proposed Data Driven Method

The goal of this section will be to develop an algorithm that can solve the problem (24) using data that can come from experiments or simulations, but without requiring any knowledge of the models $\mathcal{G}_\ell$'s. Only a simulator that can simulate $\mathcal{G}_\ell$'s for various inputs is needed.

To facilitate our algorithm, we first elect a function approximation architecture for the decision variable S in the optimization problem (24). With our form of function approximation, we show how to obtain an estimate of all of the ingredients needed to solve (24) with solely data.

A. Function approximation

We consider linear function approximations, that is, we approximate the decision variable S in (24) through

$$S^\theta(\Omega) = \sum_{i=1}^{d}\psi_i(\Omega)\theta_i = \Psi^T(\Omega)\theta, \tag{26}$$

where each *basis* $\psi_i(\Omega)$ is a SD, and $\theta \ge 0$. The number of basis functions, d, is a design choice. *We use $\Psi^T\theta$ to denote the entire trajectory $\{\Psi^T(\Omega)\theta)\}_{\Omega=-\pi}^{\Omega=\pi}$.*

We then transform the optimization problem (24) over S to one over the finite dimensional vector $\theta \in R^d$. The problem (24) is transformed to a finite dimensional convex program:

$$\theta^* = \arg\min_{\theta\ge 0}\ f(\theta), \quad \text{s.t.} \quad B\theta \le \mathsf{b}, \tag{27}$$

where $f(\theta)$ is the transformation of the objective in (24), and it is expressed as the quadratic form

$$f(\theta) = \theta^T A\theta + C\theta + d, \tag{28}$$

where

$$A = \frac{1}{2\pi}\int_{-\pi}^{\pi} A(\Omega)d\Omega, \ \text{with } A(\Omega) = \Psi(\Omega)\Psi^T(\Omega), \tag{29}$$

$$C = \frac{1}{2\pi}\int_{-\pi}^{\pi} C(\Omega)d\Omega, \ \text{with } C(\Omega) = \Psi(\Omega)S^{\text{BA}}(\Omega), \tag{30}$$

$$d = \frac{1}{2\pi}\int_{-\pi}^{\pi} d(\Omega)d\Omega, \ \text{with } d(\Omega) = \left(S^{\text{BA}}(\Omega)\right)^2. \tag{31}$$

The matrix B is constructed as follows, since $S_{\tilde{P}} = \Psi^T\theta$, it follows from (22) that

$$\bar{B}_\ell(\theta) = \bar{B}_\ell(\Psi^T\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi}|G_\ell(e^{j\Omega})|^2\Psi^T(\Omega)\theta d\Omega. \tag{32}$$

Since θ does not depend on Ω we have from (32) that

$$\bar{B}_\ell(\theta) = \left[\frac{1}{2\pi}\int_{-\pi}^{\pi}|G_\ell(e^{j\Omega})|^2\Psi^T(\Omega)d\Omega\right]\theta. \tag{33}$$

Stacking these $\bar{B}_\ell(\theta)$'s, we obtain

$$\bar{B}(\theta) = B\theta \tag{34}$$

where $B \in \mathbb{R}^{m\times d}$ is

$$B = \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathsf{G}(\Omega)\Psi^T(\Omega)d\Omega, \quad \text{with} \tag{35}$$

$$\mathsf{G}(\Omega) = [|G_1(e^{j\Omega})|^2,\ldots,|G_m(e^{j\Omega})|^2]^T.$$

Since $\psi_i(\Omega) \geq 0$ for each i, requiring $\theta \geq 0$ in (27) ensures that $\Psi^T(\Omega)\theta$ satisfies the properties of SDs (non-negativity and even) and so the search is limited to SDs, and the solution obtained by solving the problem (27), $\Psi^T(\Omega)\theta^*$, is guaranteed to be a SD.

B. Estimating B from data

The (ℓ, i^{th}) element of the matrix B is of the form $B_{\ell,i} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |G_\ell(e^{j\Omega})|^2\psi_i(\Omega)d\Omega$. Hence, for each (ℓ, i) we estimate the quantity $B_{\ell,i}$ independently. This is done in two steps.

1) Generate samples of the ℓ-th QoS signal, $Z_\ell[k]$, when power deviation $\tilde{P}[k]$ has SD $\psi_i(\Omega)$. This is done in two steps:
   a) Input generation: For each i ($i = 1, \ldots, d$) generate a colored noise sequence $\varphi_i[k]$ with SD $\Psi_i(\Omega)$. This can be done in many ways. One possibility is to perform a spectral factorization of Ψ_i to obtain a filter $H(e^{j\Omega})$ so that $|H(e^{j\Omega})|^2 = \Psi_i(\Omega)$. Passing a zero mean unit variance white noise through will generate a WSS process with SD $\Psi_i(\Omega)$ due to Prop. 1.
   b) Output generation: Use a simulator of the system $\mathcal{G}_\ell$ to generate $Z_{\ell,i}[k]$, the output of the system $\mathcal{G}_\ell$ by using the input $\varphi_i[k]$, for each i and ℓ.
2) Estimate the value $B_{\ell,i}$ from the samples $Z_{\ell,i}[k]$ by utilizing the Wiener-Khinchin theorem. Namely,

$$\overline{|G_\ell(e^{j\Omega})|^2\psi_i(\Omega)} = \hat{\mathsf{E}}\Big[\frac{1}{N}\Big|\sum_{k=1}^{N} Z_{\ell,i}[k]e^{-j\Omega k}\Big|^2\Big], \quad (36)$$

$$\hat{B}_{\ell,i} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \overline{|G_\ell(e^{j\Omega})|^2\psi_i(\Omega)}d\Omega \quad (37)$$

where $\hat{\mathsf{E}}[\cdot]$ is a shorthand for an estimate of the expectation $\mathsf{E}[\cdot]$. In particular, the estimate is obtained by performing multiple simulations, computing the quantity inside the square braces in the right hand side of (36) in each simulatin, and averaging over those simulations.

To completely specify the problem (27), the quantity $S^{\text{BA}}(\Omega)$ is required. As mentioned in section II-C1, it can be estimated using net load data.

Remark 2. *Key in our ability to remove dependence on the model knowledge is the requirement that each basis function Ψ_i is in fact a SD. Without this form of function approximation it may be difficult to develop a truly model free form of the problem (24). This model free dependence rids us of the limitations of the past work discussed in Section II-D.*

IV. Numerical Experiments

A numerical example of using the proposed data driven method to determine the capacity is illustrated in this section. The flexible loads considered are a collection of commercial building HVAC systems. We consider a homogeneous collection in this preliminary work. Each HVAC system in the collection has QoS 1-4 listed in Section II-A. The parameters are displayed in Table I, and are chosen so that the HVAC systems are representative of those in large commercial buildings (hence the large superscripts in Table I).

TABLE I: Simulation parameters

Par.	Unit	Value	Par.	Unit	Value
R	°C/kW	8	C	kWh/°C	22
T	hours	5	$\{\epsilon_i\}_{i=1}^4$	N/A	0.05
c_1	kW	40	c_2	kW	8
c_3	°C	1	c_4	kWh	8
η_0	N/A	3.5	T_a	°C	30
δ	sec.	20	$\dot{q}_{\text{int}}$	kW	0

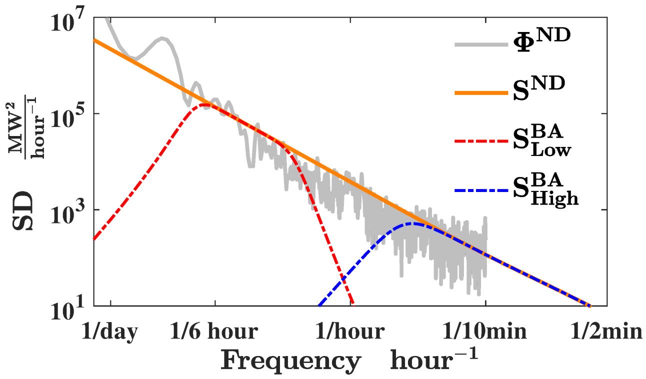

Fig. 2: Empirical net demand SD, modeled SD for BPA's net demand, and the two reference SD's for the high and low frequency passband.

We validate the proposed method by comparing it to the model based approach from our prior work [15]. All relevant simulation parameters, if not specified otherwise, can be found in Table I. Note that the method does *not* have access to the R, C, and η_0 parameter values.

A. BA's spectral needs

The net demand data is collected from BPA (a BA in the pacific northwest United States). The empirical SD of the net demand is determined using the method described in Section II-C1. We then fit an ARMA(2,1) model to the empirically estimated SD. Since the estimate Φ^{ND} will cap out at the Nyquist frequency 1/10min, we extrapolate the net demand SD to the higher frequencies. The empirical SD (denoted Φ^{ND}) and the extrapolated net demand SD (denoted S^{ND}) are shown in Figure 2.

We then choose two passbands to filter S^{ND}: (i) a low passband [1/6,1/2] (1/hour) and (ii) a high passband [1/30,1] (1/min). The results of "filtering" (see eq. (25)) S^{ND} are also shown in Figure 2. The low passband SD is termed $S^{\text{BA}}_{\text{Low}}$ and roughly corresponds to the region for TCLs in Figure 1. The high passband SD is termed $S^{\text{BA}}_{\text{High}}$ and roughly corresponds to the region for HVAC systems in Figure 1.

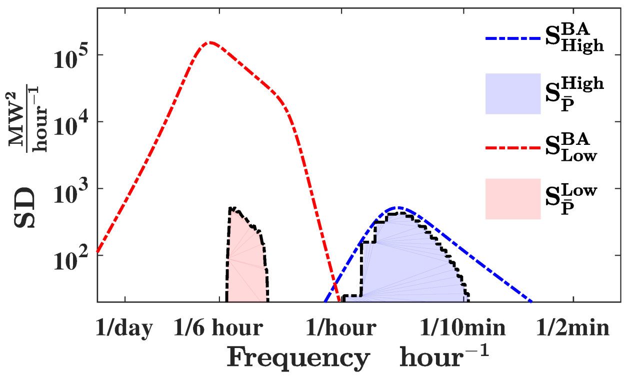

Fig. 3: The two reference SDs and the corresponding capacity SDs (boundary of the shaded regions) obtained from the proposed method for a homogeneous collection of $n = 2000$ loads. Black dashed lines represent model based solution.

B. Method Evaluation

In this section we compare the data-driven method with the quadratic program (27) to full model knowledge, both to obtain θ^*. In both cases we use CVX [17] to solve the QP. We use the following basis SD's

$$\psi_i(\Omega) = \begin{cases} 1, & \text{if } \Omega \in [\hat{\Omega}_{i-1}, \hat{\Omega}_i). \\ 0, & \text{otherwise.} \end{cases} \tag{38}$$

for $1 \leq i \leq d$. The set of points $\{\hat{\Omega}_i\}_{i=1}^d$ is a subset of the linearly spaced discrete frequency points on the unit circle. We consider $n = 2000$ large commercial buildings as one large flexible load. The idea is to illustrate how much of the grids needs can be met by the collection. To do this, the two reference SDs obtained from the previous section are projected onto the *same* ensemble constraint set.

The results of this are shown in Figure 3, where the black dashed lines represent the model based solution. The two SD's are nearly identical. This provides confidence in the method as it is able to reproduce results for the case of LTI loads with known paramaters.

V. Conclusion

We presented a data driven method to estimate the capacity of flexible load(s) as the optimal spectral density of demand deviation. Optimal here refers to being close to what the power grid needs while ensuring feasibility, that loads do not have to violate their local quality of service constraints in tracking such a reference. The method builds on our prior work [15] which was model-based. The method proposed here is does not need model knowledge. It only needs access to a simulator (or experimental measurements of relevant data). The core of the algorithm is a function approximation architecture with basis functions that are chosen to be spectral densities. In simulations, our proposed data-driven method is validated against the scenario in which load models are linear and models are known; the results are positive.

In future work we will leverage the proposed data driven framework to: (i) estimate the capacity of heterogeneous ensembles of flexible loads (ii) estimate the capacity under time varying weather conditions/disturbances, and (iii) estimate the capacity of loads with non-linear dynamics. Verifying that the resulting capacity characterization, the optimal SD, provided by the method will require large scale Monte Carlo simulations to check potential QoS violations.

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