Single-Particle Enhancement of the Quantum Multiple Access Channel

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A multiple access channel describes a situation in which multiple senders are trying to forward messages to a single receiver using some physical medium. In this paper we consider scenarios in which this medium consists of just a single classical or quantum particle. To make the comparison between quantum and classical channels precise, we introduce an operational framework in which all possible encoding strategies consume no more than a single particle. When used for the purpose of communication, this setup embodies a multiple-access channel (MAC) built with a single particle.

The task of multi-party communication consists of N spatially-separated senders $(\mathsf{A}_1, \mathsf{A}_2, \cdots \mathsf{A}_N)$ and one receiver (B) (see Fig. 1(a)), in which sender A_i sits along path i and wishes to send a classical message a_i drawn from set $\mathcal{A}_i$, The receiver B obtains some output data b belonging to set $\mathcal{B}$ that depends on the collection of messages $(a_1, a_2, \cdots, a_N)$ chosen by the senders. Ideally, b would be a perfect copy of all the N messages, $b = (a_1, a_2, \cdots, a_N)$. However in practice there are some physical limitations that prevent perfect communication. In such scenarios, the communication is described by the transition probabilities $p(b|a_1, \cdots a_N)$. The distributions $p(b|a_1, \cdots a_N)$ collectively represent a MAC [1], or an uplink channel, as its known in wireless communication [2]. Ultimately, the probabilities $p(b|a_1, \cdots a_N)$ are determined by the particular physical system used to transmit the information. The question we raise here is what MACs can be generated under the restriction that the communication channel be implemented using only a single particle, with none of its internal degrees of freedom being accessible. More precisely, information is only allowed to be encoded in external relational degrees of freedom, such as what particular points in space-time the particle occupies. We are interested in comparing the MACs that can be realized when a quantum versus classical particle is used to transmit information in this way.

Before making a comparison between the classical and quantum MACs, we defined and compared different classical MACs based on different levels of shared randomnesses the system possesses. These classical MACs are denoted as $\mathcal{C}_N$, $\mathcal{C}'_N$ and $\text{conv}[\mathcal{C}_N]$ representing the cases with no shared randomness, partial shared randomness and full shared randomness respectively (depicted in Fig 1). We show that these MACs are identical in the communication scenario with binary input and output, that is when $|\mathcal{A}_i| = |\mathcal{B}| = 2$, while they are completely different for more general cases. To facilitate our discussion, we also introduce a superset of all these MACs, which we call separable MACs, $\mathcal{C}_N^{(\text{sep})}$, and it is formed by MACs having probability decomposition $p(b|a_1, \cdots, a_N) = \sum_{i=1}^N p_i g_i(b|a_i)$. We analyze the structure of these MACs and show that they are identical to the more restricted families in the binary case.

Our main results involve providing a full characterization of the N-party classical MACs that can be built from a single classical particle and restricted local number-preserving (NP) operation. Simply put, the NP operations have a dilation in which the overall particle number is persevered. The main finding is that these MACs are characterized entirely in terms of vanishing second-order interference terms. More precisely, the particular linear combination

$$I_2 = p(b|a_i, a_j) + p(b|a'_i, a'_j) - p(b|a'_i, a_j) - p(b|a_i, a'_j) \tag{1}$$

must equal zero for any output b and arbitrary inputs a_i, a'_i, a_j, a'_j chosen by parties A_i and A_j. This quantity is also well-known in the study of double-slit experiments. In contrast, every quantum state with a non-zero

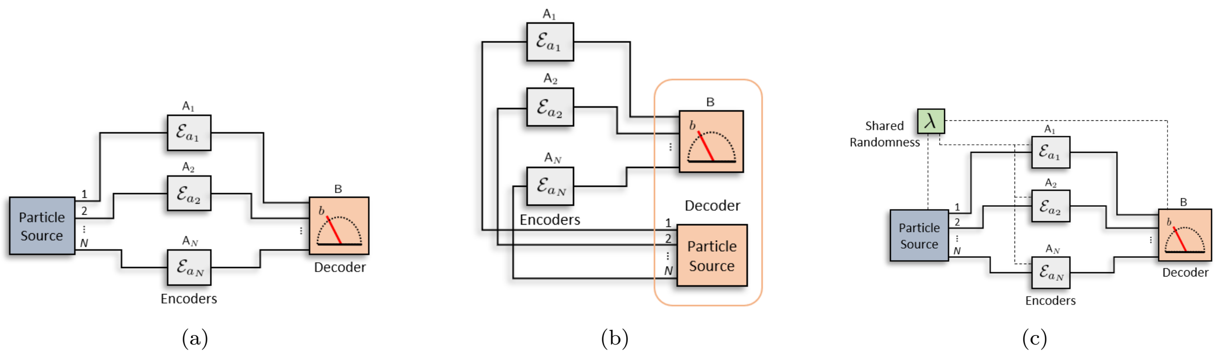

Figure 1: Physical implementations of MAC using different level of share randomness: (a) no share randomness; (b)share randomness between the source and decoder; (b)share randomness among the source, encoders and decoder

off-diagonal term in the path basis can generate $I_2 \neq 0$. Therefore, a clear distinction between quantum particle and classical particle in the multiple access communication scenario is presented.

To further distinguish the capabilities of a single classical and quantum particle, we relax the locality constraint and allow for joint encodings by subsets of $1 < K \leq N$ parties. This generates a richer family of classical MACs whose polytope dimension we compute which depends on the cardinality of the input set $\mathcal{A}_i$ and $\mathcal{B}$ as $(|\mathcal{B}|-1)\sum_{k=0}^{K}(|\mathcal{A}|-1)^k\binom{N}{k}$ (Assuming $|\mathcal{A}_i| = |\mathcal{A}|$ for simplicity). In low dimensions, the resulting classical polytope is fully analyzed, while for arbitrary N and K, we show that the classical polytope exhibits a tight facet inequality, which we call the generalized fingerprinting inequality [3]:

$$-p(b|\overbrace{0,\cdots,0}^{N}) + \sum_{i=1}^{K+1} p(b|\overbrace{0,\cdots,1_i,\cdots,0}^{K+1},\overbrace{0,\cdots,0}^{N-K-1}) \leq K. \tag{2}$$

We verify that a quantum particle equally distributed among N separated parties can violate this inequality without joint operation even when $K = N-1$, the violation of given as $\frac{2}{N}$ if $N = 2, 3$ and $\frac{1}{N(N-2)(N-3)}$ if $N > 3$. Moreover, we draw a connections between the single-particle framework and multi-level coherence theory. We proof that every pure state with K-level coherence can violate one of the above inequality, thus, its K-level coherence can be detected in a semi-device independent manner, with the only assumption being conservation of particle number.

Except for the above generalized fingerprinting inequality, a family of high-order interference equalities is also showed to be valid constraints for the classical MACs with at most K parties performing joint encoding:

$$I_K = \sum_{a_1,\cdots,a_K\in\{0,1\}} \prod_{i=1}^{K}(-1)^{a_i} p(0|a_1,\cdots a_K), \qquad K = 2,\cdots,N-1. \tag{3}$$

Notice that $K = 2$ corresponds to the second-order interference given in Eq. (3). The quantity I_K arises naturally in K-slit interference experiments, and while quantum mechanics can generate $I_2 \neq 0$, it is known that $I_K = 0$ for $K > 2$ in all standard single-particle quantum mechanical setups, an intriguing fact that has motivated multiple studies into the nature of higher-order interference. However, it is showed in our work when K parties are allowed to jointly encode on a quantum system of just one particle using operations constrained to particle-number conservation, I_{2K} can be nonzero, but $I_{K'}$ vanishes for all $K' > 2K$.

References

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