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Muon $g-2$ discrepancy within D-brane string compactifications

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Abstract

Very recently, the Muon $g-2$ experiment at Fermilab has confirmed the E821 Brookhaven result, which hinted at a deviation of the muon anomalous magnetic moment from the Standard Model (SM) expectation. The combined results from Brookhaven and Fermilab show a difference with the SM prediction $\delta a_\mu = (251 \pm 59) \times 10^{-11}$ at a significance of 4.2σ, strongly indicating the presence of new physics. Motivated by this new result we reexamine the contributions to δa_μ from both: *(i)* the ubiquitous $U(1)$ gauge bosons of D-brane string theory constructions and *(ii)* the Regge excitations of the string. We show that, for a string scale $\mathcal{O}(\mathrm{PeV})$, the contribution from anomalous $U(1)$ gauge bosons which couple to hadrons could help to reduce (though not fully eliminate) the discrepancy reported by the Muon $g-2$ Collaboration. Consistency with null results from LHC searches of new heavy vector bosons imparts the dominant constraint. We demonstrate that the contribution from Regge excitations is strongly suppressed as it was previously conjectured. We also comment on contributions from Kaluza-Klein (KK) modes, which could help resolve the δa_μ discrepancy. In particular, we argue that for 4-stack intersecting D-brane models, the KK excitations of the $U(1)$ boson living on the lepton brane would not couple to hadrons and therefore can evade the LHC bounds while fully bridging the δa_μ gap observed at Brookhaven and Fermilab.

I. INTRODUCTION

The gyromagnetic factor g is defined by the relation between the particle's spin $\vec{s}$ and its magnetic moment $\vec{\mu} = g\, e\, \vec{s}/(2m)$, where e and m are the charge and mass of the particle. In Dirac's theory of charged point-like spin-1/2 particles, $g = 2$. Quantum electrodynamics (QED) predicts deviations from Dirac's value, as the charged particle can emit and reabsorb virtual photons. These QED effects slightly increase the value of g. It is conventional to express the difference of g from 2 in terms of the value of the so-called anomalous magnetic moment, a dimensionless quantity defined as $a_l = (g-2)/2$, with $l = e, \mu$. Over the last decade, the muon magnetic dipole moment has maintained a long-standing discrepancy of about 3.7σ between the Standard Model (SM) prediction and the Brookhaven E821 experimental measurement [1–3]. Very recently, the Muon $g-2$ Experiment at Fermilab released its first results, which in combination with the previous E821 measurement lead to a new experimental average of the muon anomalous magnetic dipole moment of $a_\mu^{\rm exp} = 116592061(41) \times 10^{-11}$ [4]. The difference $\delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = (251 \pm 59) \times 10^{-11}$ has a significance of 4.2σ. In this paper we investigate whether this discrepancy can be explained in the context of low-mass-scale strings [5]. Before proceeding, it is important to stress that the absence of anomalous magnetic moment in a supersymmetric abelian gauge theory [6] does not connect to the models discuss herein since the brane configuration is not supersymmetric.

Our calculations are framed in the context of intersecting D-brane models; namely, we consider extensions of the SM based on open strings ending on D-branes, with gauge bosons due to strings attached to stacks of D-branes and chiral matter due to strings stretching between intersecting D-branes [7–15]. Intersecting D-brane models encase a set of building block ground rules, which can be used to assemble UV completions of the SM with the effective low energy theory inherited from properties of the overarching string theory. For these models, the elemental unit of gauge invariance is a $U(1)$ field, and therefore a stack of N identical D-branes sequentially gives rise to a $U(N)$ theory with the associated $U(N)$ gauge group. If many types of D-brane are present in the model, the gauge group grows into product form $\prod U(N_i)$, where N_i specifies the number of D-branes in each stack. For $N = 2$, the gauge group can be $Sp(1) \cong SU(2)$ rather than $U(2)$.[1] For further details, see e.g. [16, 17].

The minimal embedding of the SM particle spectrum requires at least three brane stacks [18] leading to three distinct models of the type $U(3) \times U(2) \times U(1)$ that were classified in [18, 19]. Only one of them (model C of [19]) has baryon number as a gauge symmetry that guarantees proton stability (in perturbation theory), and can be used in the framework of low mass scale string compactifications. Besides, since the charge associated to the $U(1)$ of $U(2)$ does not participate in the hypercharge combination, $U(2)$ can be replaced by the symplectic $Sp(1)$ representation of Weinberg-Salam $SU(2)_L$, leading to a model with one extra $U(1)$ added to the hypercharge [20]. The SM embedding in four D-brane stacks leads to many more models that have been classified in [21, 22]. Whether low-mass-scale strings are realized in nature is yet to be answered, and the search for new physics signals of intersecting D-brane models is still one of the goals of current day research [23–44].

In D-brane models there are three types of contribution to the anomalous magnetic moment of the muon: the one from anomalous massive $U(1)$ gauge bosons, the one from excited

[1] In the presence of orientifolds, one also obtains orthogonal and symplectic gauge groups.

string states (excitations of l^* are expected to appear in the s-channel, the poles occur at $\sqrt{s} = \sqrt{n}\, M_s$, with $n = 1, 2, \ldots$ and M_s the string scale [45]), and that of Kaluza-Klein (KK) modes. While the anomalous $U(1)$ Lagrangian permits a direct one loop calculation of the anomalous magnetic moment [46], a direct calculation of the contribution from Regge recurrences is not possible due to the nonrenormalizability of the theory. However, it was conjectured in [46] that the contributions from string oscillators and KK states must be largely suppressed. In this paper we check thoroughly the various contributions to the anomalous magnetic moment of the muon in three and four D-brane stacks realizations of the SM. The layout of the paper is as follows. In Sec. II we reconsider the contribution from anomalous $U(1)$ gauge bosons and derive new constraints on the parameter space imposed by recent LHC data. In Sec. III we use sum rules methods to calculate the contribution from Regge excitations, and we verify the strong suppression of this contribution conjectured in [46]. The paper wraps up with some conclusions presented in Sec. IV. Before proceeding we note that the latest Fermilab data already lead to several new physics interpretations with connections to other fundamental problems in particle physics, astrophysics, and cosmology [47–55].

II. CONTRIBUTIONS FROM ANOMALOUS MASSIVE $U(1)$ GAUGE BOSONS

To develop our program in the simplest way, we work within the construct of minimal models with 3 and 4 stacks of D-branes.

A. 3 stack models

For 3 stack models, the canonical gauge group is $U(3) \times U(2) \times U(1)$, with stacks labeled a, b, and c, respectively [18]. In the bosonic sector, the open strings terminating on the QCD stack a contain the standard $SU(3)_C$ octet of gluons g^a_μ and an additional $U(1)_a$ gauge boson C_μ, most simply the manifestation of a gauged baryon number symmetry: $U(3) \sim SU(3)_C \times U(1)_a$. On the $U(2)$ stack the open strings correspond to the electroweak gauge bosons A^a_μ, and again an additional $U(1)_b$ gauge field X_μ. So the associated gauge groups for these stacks are $SU(3)_C \times U(1)_a$, $SU(2)_L \times U(1)_b$, and $U(1)_c$, respectively. The quantum numbers of quarks and leptons in each family are given by

$$\begin{array}{ll} Q & (\mathbf{3},\mathbf{2};1,1+2z,0)_{1/6} \\ u^c & (\bar{\mathbf{3}},\mathbf{1};-1,0,0)_{-2/3} \\ d^c & (\bar{\mathbf{3}},\mathbf{1};-1,0,1)_{1/3} \\ L & (\mathbf{1},\mathbf{2};0,1,z)_{-1/2} \\ l^c & (\mathbf{1},\mathbf{1};0,0,1)_{1} \end{array} \tag{1}$$

where $z = 0, -1$. The charge assignments for the two Higgs doublets read

$$H \quad (\mathbf{1},\mathbf{2};0,1+2z,1)_{1/2} \qquad H' \quad (\mathbf{1},\mathbf{2};0,-(1+2z),0)_{1/2} \tag{2}$$

The relations for $U(N)$ unification, $g'_a = g_a/\sqrt{6}$ and $g'_b = g_b/2$, hold only at M_s because the $U(1)$ couplings (g'_a, g'_b, g'_c) run differently from the non-Abelian $SU(3)$ (g_a) and $SU(2)$ (g_b) [33].

We can perform a unitary transformation on the gauge fields $A_i = U_{ij}\widetilde{A}_j$ (with $A_Y = \widetilde{A}_1,\ A_\alpha = \widetilde{A}_2,\ A_\beta = \widetilde{A}_3$) to diagonalize their mass matrix. The $U(1)$ brane can be independent of the other branes and has in general a different gauge coupling g_c. In the model of [18], however, the $U(1)$ brane was located on top of either the color or the weak D-branes, and consequently g_c being equal to either g_a or g_b. Here we relax the additional constraint imposed in [18] and instead set $g_a = yg_c$ at the string scale M_s. As a result there are two free parameters y, θ in U_{ij}. A_α, A_β are the anomalous $U(1)$ gauge fields, whose charges depend on z. For $z = 0$, the anomaly free hypercharge is given by

$$Q_Y = Q_c - \frac{Q_b}{2} + \frac{2Q_a}{3}\,, \tag{3}$$

yielding

$$\begin{aligned} Q_\alpha = -Q_c\frac{1}{\sqrt{2}y}\left(16+9x^2\right)\sin\theta + \frac{\sqrt{2}}{\sqrt{3}x}Q_b\left(2\cos\theta\sqrt{16+9x^2+12y^2} - 3\sqrt{3}x\,y\sin\theta\right) \\ +\frac{1}{\sqrt{6}}Q_a\left(3x\cos\theta\sqrt{16+9x^2+12y^2} + 8\sqrt{3}y\sin\theta\right)\,, \end{aligned} \tag{4}$$

and

$$\begin{aligned} Q_\beta = \frac{1}{\sqrt{2}y}Q_c\left(16+9x^2\right)\cos\theta + \frac{\sqrt{2}}{\sqrt{3}x}Q_b\left(2\sin\theta\sqrt{16+9x^2+12y^2} + 3\sqrt{3}x\,y\cos\theta\right) \\ +\frac{1}{\sqrt{6}}Q_a\left(3x\sin\theta\sqrt{16+9x^2+12y^2} - 8\sqrt{3}y\cos\theta\right)\,, \end{aligned} \tag{5}$$

while for $z = -1$, the hypercharge is found to be

$$Q_Y = Q_c + \frac{Q_b}{2} + \frac{2Q_a}{3}\,, \tag{6}$$

yielding

$$\begin{aligned} Q_\alpha = -\frac{1}{\sqrt{2}y}Q_c\left(16+9x^2\right)\sin\theta + \frac{\sqrt{2}}{\sqrt{3}x}Q_b\left(2\cos\theta\sqrt{16+9x^2+12y^2} + 3\sqrt{3}x\,y\sin\theta\right) \\ +\frac{1}{\sqrt{6}}Q_a\left(-3x\cos\theta\sqrt{16+9x^2+12y^2} + 8\sqrt{3}y\sin\theta\right) \end{aligned} \tag{7}$$

and

$$\begin{aligned} Q_\beta = \frac{1}{\sqrt{2}y}Q_c\left(16+9x^2\right)\cos\theta + \frac{\sqrt{2}}{\sqrt{3}x}Q_b\left(2\sin\theta\sqrt{16+9x^2+12y^2} - 3\sqrt{3}x\,y\cos\theta\right) \\ -\frac{1}{\sqrt{6}}Q_a\left(3x\sin\theta\sqrt{16+9x^2+12y^2} + 8\sqrt{3}y\cos\theta\right)\,, \end{aligned} \tag{8}$$

with

$$x = \frac{g_a/\sqrt{3}}{g_b/\sqrt{2}}\,. \tag{9}$$

Throughout we adopt the convention for which the $U(1)$ coupling is $g'_c = g_c/\sqrt{2}$. The three coupling constants are related to g_Y at the string scale M_s by [33]

$$\frac{2}{g_c^2} + \frac{4\left(\frac{1}{2}\right)^2}{g_b^2} + \frac{6\left(\frac{2}{3}\right)^2}{g_a^2} = \frac{1}{g_Y^2}\,. \tag{10}$$

The anomalous magnetic moment of the muon in the D-brane realization of the SM, $a_\mu^{U(3)\times U(2)\times U(1)}$, can be computed from one-loop correction to the muon vertex. Generalizing the procedure described in [46] for $y \neq 1$, we calculate the contribution to $a_\mu^{U(3)\times U(2)\times U(1)}$ from the anomalous $U(1)$ exchanged diagrams as well as the axion diagrams. When these contributions are added to the SM prediction we obtain

$$a_\mu^{U(3)\times U(2)\times U(1)} = a_\mu^{\rm SM} + g^2 \sum_{i=\alpha,\beta} \frac{Q_{iL}^2 - 3Q_{iL}Q_{iR} + Q_{iR}^2}{12\pi^2}\left(\frac{m_l}{\mu_i}\right)^2 + \frac{h^2}{16\pi^2}\,, \tag{11}$$

where Q_{iL} and Q_{iR} are the $U(1)$ charges for the left and right-handed muons respectively, g is the effective coupling for the anomalous $U(1)$ gauge fields and its value is

$$g = \frac{g_c}{\sqrt{16+9x^2}\sqrt{16+9x^2+12y^2}}\,, \tag{12}$$

h is the Planck constant, and μ_α, μ_β are the masses of the anomalous $U(1)$ fields. Substituting in Q_α and Q_β, we get the following equation relating μ_α and μ_β to the difference δa_μ of anomalous magnetic moment of the muon for $z=0$,

$$\begin{aligned} g_3^2 m^2 \Big\{ & t^2 \left[16\mu_\alpha^2 y^2\left(9x^2+12y^2+16\right) + 3\mu_\beta^2 x^2\left(-18\left(9x^2+16\right)y^2 + \left(9x^2+16\right)^2 + 36y^4\right)\right] \\ & +3\mu_\alpha^2 x^2\left(-18\left(9x^2+16\right)y^2 + \left(9x^2+16\right)^2 + 36y^4\right) + 16\mu_\beta^2 y^2\left(9x^2+12y^2+16\right) \\ & -12txy(\mu_\alpha^2-\mu_\beta^2)\left(9x^2-4y^2+16\right)\sqrt{27x^2+36y^2+48}\Big\} \\ & -72\pi^2\mu_\alpha^2\mu_\beta^2(\delta a_\mu - a_{\mu\phi'})\left(t^2+1\right)x^2\left(9x^2+16\right)y^2\left(9x^2+12y^2+16\right) = 0, \end{aligned} \tag{13}$$

where $t = \tan\theta$ (θ is the angle that appears in U_{ij}) and $a_{\mu\phi'}$ is the contribution from the axion (which is proportional to h [46]). Likewise, for the $z=-1$ model we obtain

$$\begin{aligned} g_3^2 m^2 \Big\{ & t^2 \left(16\mu_\alpha^2 y^2\left(9x^2+12y^2+16\right) + 3\mu_\beta^2 x^2\left(30\left(9x^2+16\right)y^2 + 5\left(9x^2+16\right)^2 + 36y^4\right)\right) \\ & +3\mu_\alpha^2 x^2\left(30\left(9x^2+16\right)y^2 + 5\left(9x^2+16\right)^2 + 36y^4\right) + 16\mu_\beta^2 y^2\left(9x^2+12y^2+16\right) \\ & -4txy(\mu_\alpha^2-\mu_\beta^2)\left(45x^2+12y^2+80\right)\sqrt{27x^2+36y^2+48}\Big\} \\ & -72\pi^2\mu_\alpha^2\mu_\beta^2(\delta a_\mu - a_{\mu\phi'})\left(t^2+1\right)x^2\left(9x^2+16\right)y^2\left(9x^2+12y^2+16\right) = 0. \end{aligned} \tag{14}$$

We note that $\tan\theta$ has to be real and therefore the discriminant of (13) (or (14)) has to be positive definite. The values of x, y are determined by g_b, g_c, g_Y at the string scale M_s and we have three free parameters μ_α, μ_β, M_s. For simplicity, to analyze the (μ_β, M_s) parameter space we set $\mu_\alpha = 1$ TeV. The corresponding allowed regions are given in Fig. 1.

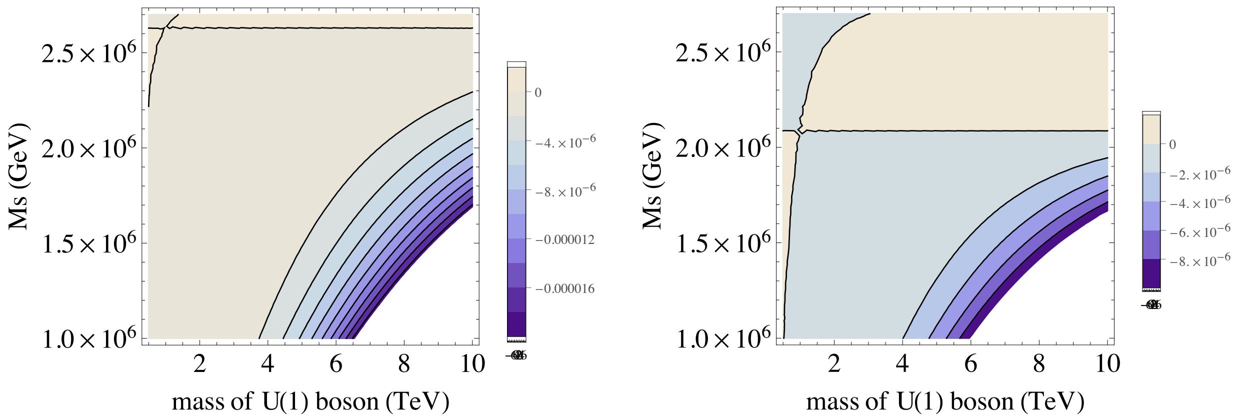

FIG. 1: Allowed regions for 3-stack models. The left panel is for $z = 0$ while the right panel is for $z = -1$. By setting $\mu_\alpha = 1$ TeV, we are left with two parameters M_s and μ_β (x-axis). The contours are for the values of the discriminant of equations (13) and (14) (more precisely we multiply the discriminant by μ_β^4). The allowed regions are where the discriminant is positive and we can have some combination of μ_α, μ_β, M_s so that the deviation of anomalous magnetic moment is from the contribution of the anomalous $U(1)$ bosons.

TABLE I: Chiral spectrum.

Fields	Sector	Representation	Q_B	Q_L	Q_{I_R}	Q_Y
U_R	$3 \leftrightharpoons 1^*$	$(3,1)$	-1	0	-1	$\frac{2}{3}$
D_R	$3 \leftrightharpoons 1$	$(3,1)$	-1	0	1	$-\frac{1}{3}$
L_L	$4 \leftrightharpoons 2$	$(1,2)$	0	-1	0	$-\frac{1}{2}$
E_R	$4 \leftrightharpoons 1$	$(1,1)$	0	-1	1	-1
Q_L	$3 \leftrightharpoons 2$	$(3,2)$	-1	0	0	$\frac{1}{6}$
N_R	$4 \leftrightharpoons 1^*$	$(1,1)$	0	-1	-1	0
H	$2 \leftrightharpoons 1$	$(1,2)$	0	0	-1	$\frac{1}{2}$
H''	$4 \leftrightharpoons 1$	$(1,1)$	0	1	1	0

B. 4 stack models

Now, let us consider models with 4 stacks of D-branes. If we consider next-to-minimal constructs where in the b stack we choose projections leading to the symplectic $Sp(1)$ representation of Weinberg-Salam, the gauge extended sector, $U(3)_B \times SU(2)_L \times U(1)_L \times U(1)_{I_R}$, has two additional $U(1)$ symmetries. A schematic representation of the D-brane construct is shown in Fig. 2 and the quantum numbers of the chiral spectrum are summarized in Table I.

The resulting $U(1)$ content gauges the baryon number B [with $U(1)_B \subset U(3)_B$], the lepton number L, and a third additional abelian charge I_R which acts as the third isospin component of an $SU(2)_R$. Contact with gauge structures at TeV energies is achieved by a

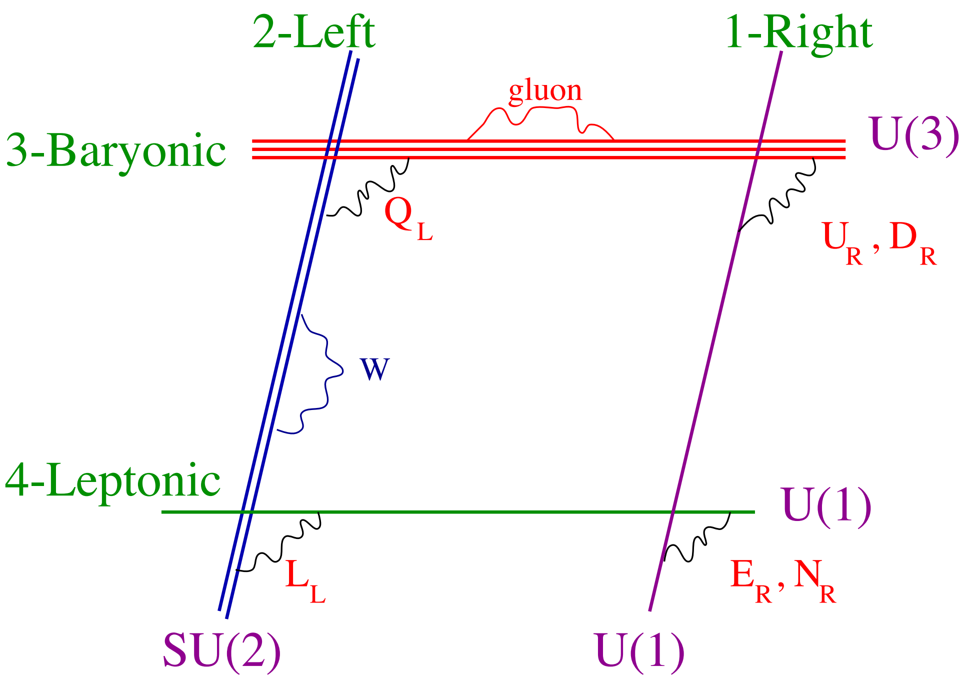

FIG. 2: Pictorial representation of the $U(3)_B \times SU(2)_L \times U(1)_L \times U(1)_{I_R}$ D-brane model, for stacks a, b, c, d, respectively.

field rotation to couple diagonally to hypercharge Y_μ. Two of the Euler angles are determined by this rotation, and the hypercharge is given by

$$Q_Y = \frac{Q_c}{2} - \frac{Q_a}{6} - \frac{Q_d}{2}\,, \tag{15}$$

The charges Q_α and Q_β of the two anomalous $U(1)$ are given by

$$\begin{aligned} Q_\alpha \;&=\; \frac{1}{\sqrt{2}} g_c Q_c \cos\theta \sin\phi + \frac{1}{\sqrt{2}} g_d Q_d (\sin\theta \sin\psi \sin\phi + \cos\psi \cos\phi) \\ &\quad + \frac{1}{\sqrt{6}} g_a Q_a (\sin\theta \cos\psi \sin\phi - \sin\psi \cos\phi), \end{aligned} \tag{16}$$

$$\begin{aligned} Q_\beta \;&=\; \frac{1}{\sqrt{2}} g_c Q_c \cos\theta \cos\phi + \frac{1}{\sqrt{2}} g_d Q_d (\sin\theta \sin\psi \cos\phi - \cos\psi \sin\phi) \\ &\quad + \frac{1}{\sqrt{6}} g_a Q_a (\sin\theta \cos\psi \cos\phi + \sin\psi \sin\phi), \end{aligned} \tag{17}$$

where g_c, g_d are the coupling constants of the two $U(1)$ branes and we again include the $1/\sqrt{2}$ in the coupling. The gauge couplings are related to g_Y by [33]

$$\frac{2\left(\frac{1}{2}\right)^2}{g_c^2} + \frac{2\left(\frac{1}{2}\right)^2}{g_d^2} + \frac{6\left(\frac{1}{6}\right)^2}{g_a^2} = \frac{1}{g_Y^2}\,. \tag{18}$$

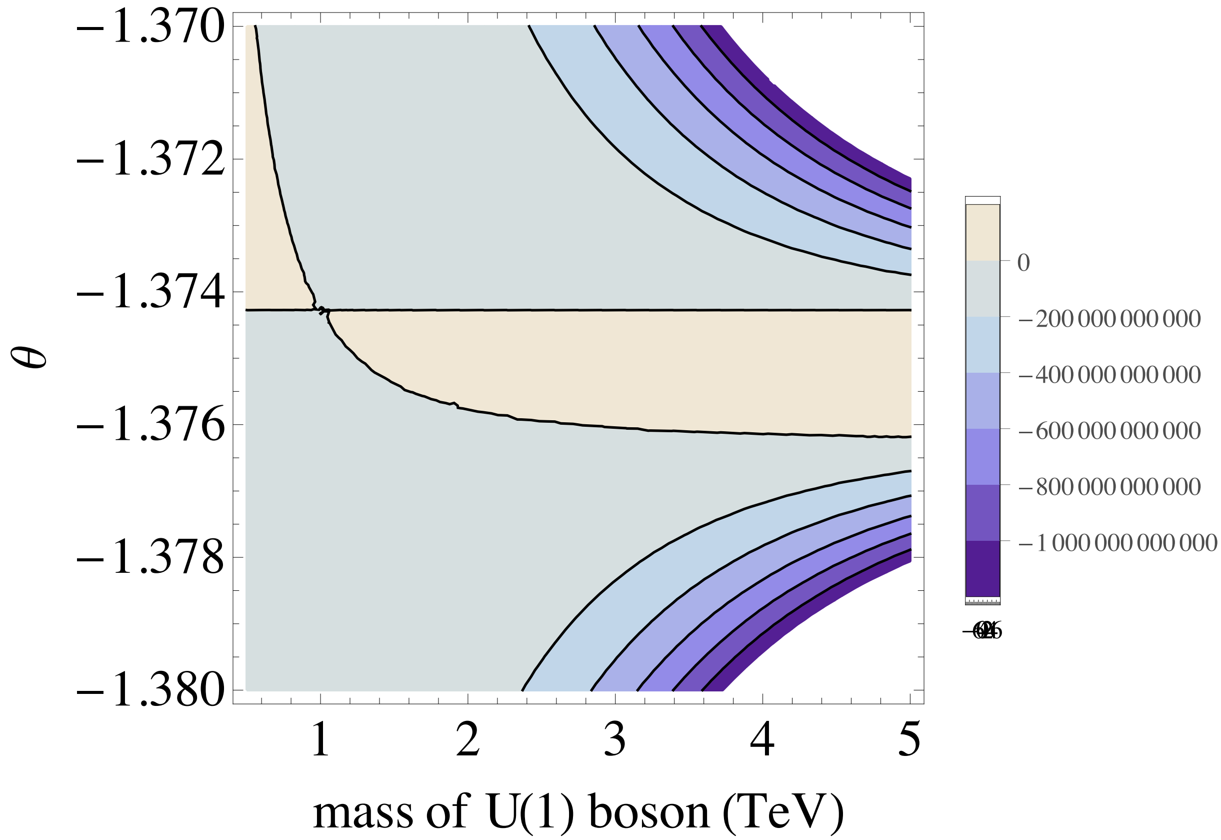

FIG. 3: Allowed regions for 4-stack model. By setting $\mu_\alpha = 1$TeV and $M_s = 2\times10^3$, we are left with two parameters μ_β (x-axis) and θ (y-axis). The contours are again for the values of the discriminant of a quadratic equation set up by requiring measured anomalous magnetic moment given by (11). As we can see there are regions with positive discriminant where we can find appropriate ϕ so that the $U(1)$ anomalous bosons can account for the deviation of anomalous magnetic moment.

As previously noted, the Euler angles θ, ψ, ϕ parameterize the unitary transformation $A_i = U_{ij}\widetilde{A}_j$. We can parametrize ψ in terms of the g_d coupling constant and θ,

$$\psi \to -\arcsin\left(-\frac{\sqrt{2}g_Y}{2g_d\cos\theta}\right). \tag{19}$$

For later convenience, we can also parameterize the coupling g_c using the angle θ

$$g_c \to -\frac{\sqrt{2}g_Y}{2\sin\theta}. \tag{20}$$

The anomalous magnetic moment is again given by (11), which leads to a quadratic equation of $t = \tan\phi$. To analyze the (μ_β, M_s) parameter space we set $\mu_\alpha = 1$ TeV. Now we have one more parameter (θ) than before and following the same philosophy the string scale M_s is picked to be 2×10^3 TeV. The allowed regions of the parameter space, parameterized by θ, μ_β, are encapsulated in Fig. 3.

C. LHC constraints

We now turn to confront our findings with null results from LHC searches of new heavy vector bosons [66, 67]. To this end we duplicate the procedure outlined above but scanning on μ_α. The possible highest mass scale for μ_α is in general around 1 TeV. For 3 stack models, it is 0.6 TeV for $z = 0$, and 1.4 TeV for $z = -1$. The 4-stack model does not help much as the upper bound of the mass scale is also around 1.5 TeV. The dominant constraint comes from the requirement that neither $U(1)$ coupling is above 2π: g_c becomes $\mathcal{O}(1)$ at the string scale; its running is between the anomalous $U(1)$ mass and the string scale. The 4-stack model has some extra freedom in that the string scale can vary (in the 3-stack case the string scale is fixed by the coupling). Since the couplings of the anomalous $U(1)$ bosons are in general much larger than the SM coupling (especially when their masses are large), we can compare the constraints on μ_α with LHC limits on the generalized sequential model [68], containing the sequential SM boson gauge boson that has SM-like couplings to SM fermions [69]. We can see that the discrepancy δa_μ reported by Fermilab cannot be accommodated within the 1σ contours without being in tension with LHC data [66, 67], unless the massive $U(1)$ gauge boson Z' is leptophilic [55].

We consider the resonant production cross section of $\sigma(pp \to Z' \to \ell\ell)$. Under the narrow width approximation, the cross section can be written in the form of $c_u w_u + c_d w_d$, where w_u, w_d are given by model-independent parton distribution functions [68]. The coupling of Z' with up and down quarks (assuming same coupling to three families) are encoded in c_u, c_d. More precisely, for a generic coupling between Z' and fermion f

$$Z'_\mu \gamma^\mu (\bar{f}_L \epsilon_L^f f_L + \bar{f}_R \epsilon_R^f f_R)\,, \tag{21}$$

the coefficients c_u and c_d take the following form

$$c_f = ({\epsilon_L^f}^2 + {\epsilon_R^f}^2)\mathrm{Br}(\ell^+\ell^-)\,. \tag{22}$$

We compute the branching faction $\mathrm{Br}(\ell^+\ell^-)$ by including only the decay channels to leptons and quarks. The total decay rate is given by

$$\Gamma_{Z'} = \frac{1}{24\pi} M_{Z'} \left[9 \sum_{q=u,d} ({\epsilon_L^q}^2 + {\epsilon_R^q}^2) + 3 \sum_{\ell=e,\nu} ({\epsilon_L^\ell}^2 + {\epsilon_R^\ell}^2) \right]. \tag{23}$$

Because of the constraint (10), there are two free parameters (for a given string scale M_s): ϕ and $g_d(M_s)$. Setting the mass of Z' to 2 TeV, we then search over the parameter space to get the smallest possible values of c_u, c_d. For simplicity, the combination of $\sqrt{c_u^2 + c_d^2}$ is considered. We find that the optimal value of ϕ generally suppresses the couplings to left-handed quarks and the remaining couplings to the right-handed quarks are controlled by g_d. In the best case scenario, $g_c(M_s)$ is set to 2π at $M_s = 10$ TeV, the corresponding cross section (or rather $\sqrt{c_u^2 + c_d^2} \sim 8.4 \times 10^{-5}$) is roughly 2 percent of that given by the sequential standard model boson, saturating the LHC limit [67]. We note that the branching fraction to leptons is close to 1 due to the small coupling to quarks. The signal can be further reduced by including other decay channels. Moreover, the largest possible $g_c(M_s)$ also gives the most contribution to a_μ.

From (11) we see that each anomalous $U(1)$ in the 4-stack model provides the following contribution to a_μ

$$a_\mu^{(i)} = -\frac{Q_{iL}^2 - 3Q_{iL}Q_{iR} + Q_{iR}^2}{12\pi^2}\left(\frac{m_l}{\mu_i}\right)^2 , \quad i = \alpha, \beta\,. \tag{24}$$

We can choose A_α to be the leptophilic Z' and plug into (16) the charges from Table I. The L and R charges then read

$$Q_{\alpha L} = -\frac{g_c \cos\theta \sin\phi}{\sqrt{2}} \tag{25}$$

and

$$Q_{\alpha R} = \frac{g_d(\sin\theta\sin\phi\sin\psi + \cos\phi\cos\psi) - g_c\cos\theta\sin\phi}{\sqrt{2}}\,. \tag{26}$$

Such a Z' boson gives $a_\mu^{(\alpha)} = 9.9\times 10^{-11}$, which is still not enough to explain the observed discrepancy. A_β shall be much heavier to avoid the LHC bound and its contribution is negligible. If we require $\mu_\alpha \gtrsim 5$ TeV and $\mu_\beta \gtrsim 5$ TeV, then $\delta a_\mu \lesssim 3\times 10^{-11}$, with $g_c \sim 1$.

III. CONTRIBUTIONS FROM EXCITED STRING STATES

The low energy limit of the spin-flip forward Compton amplitude in the laboratory frame of the target lepton l is related to the square of its anomalous magnetic moment. The assumption of analyticity and sufficient convergence permits an unsubtracted dispersion relation for this amplitude and, together with the optical theorem, a sum rule for a_l^2 [56, 57], where

$$(a_l^{\rm QED} + a_l^{\rm non-QED})^2 = \frac{m_l^2}{2\pi^2\alpha}\int_{s_{\rm th}}^\infty \frac{ds}{s}\,\Delta\sigma\,, \tag{27}$$

α is the QED fine-structure constant, s is the square ceneter-of-mass energy, and where in obvious notation [58]

$$\Delta\sigma = \frac{1}{2}\left[\left(\sigma_{1,1/2} - \sigma_{-1,1/2}\right) + \left(\sigma_{1,-1/2} - \sigma_{-1,-1/2}\right)\right]\,. \tag{28}$$

The first of the 3 terms on the LHS of the equation is canceled on the RHS by the integral containing only $\Delta\sigma_{\rm QED}$, so that the truncated sum rule is [59–61]

$$2\,(a_{\rm QED})\,(a_{\rm non-QED}) \;+\; (a_{\rm non-QED})^{\,2} = \frac{m_l^2}{2\pi^2\alpha}\int_{s_{\rm th}}^\infty \frac{ds}{s}\,\Delta\sigma_{\rm non-QED}\,. \tag{29}$$

With the use of collinear string amplitudes [25], a straightforward calculation shows that the contribution of a single spin 1/2 or 3/2 resonance of mass M_s to the right-hand side of (29) is given by

$$\Delta\sigma_{\rm non-QED} = 2\pi e^2\,\delta(s - M_s^2)\,, \tag{30}$$

which gives a contribution $4(m_l/M_s)^2$. However, in the string spectrum, there is the possibility of cancelation between the different spin contributions to the RHS of (27).

Further insights into the problem are as follows: the tree level contribution to $\Delta\sigma_{\rm non-QED}$ from right handed muons is proportional to

$$\sigma_{\gamma_L\mu_R} - \sigma_{\gamma_R\mu_R} \equiv \sum_n \left(\sigma^n_{\gamma_L\mu_R} - \sigma^n_{\gamma_R\mu_R}\right), \tag{31}$$

where $\sigma^n_{\gamma_L\mu_R}$ is the total cross section for outgoing μ^*_R Regge excited states at level n. Note that the muon has its momentum along $+z$ direction. For $n=1$, the right hand side can be simplified to

$$\sigma^1_{B_L\mu_R} - \sigma^1_{B_R\mu_R} \propto \left(\left|F^{1,J=3/2}_{B_L\mu_R}\right|^2 - \left|F^{1,J=1/2}_{B_R\mu_R}\right|^2\right), \tag{32}$$

where $F^{n,J}_{B_{L,R}\mu_{L,R}}$ are the collinear amplitudes of the μ^*_R Regge excitation of spin J (to simplify notation, gauge group indices have been omitted) and level n. If $\Delta\sigma = 0$, then the relation

$$F^{1,J=1/2}_{B_R\mu_R} = F^{1,J=3/2}_{B_L\mu_R} \tag{33}$$

should hold. Using [25]

$$F^{\alpha J=1/2}_{+\frac{1}{2}+1\alpha_3 a_4} = F^{\alpha J=1/2}_{-\frac{1}{2}-1\alpha_3 a_4} = F^{\alpha J=3/2}_{+\frac{1}{2}-1\alpha_3 a_4} = F^{\alpha J=3/2}_{-\frac{1}{2}+1\alpha_3 a_4} = \sqrt{2}\, g\, M\, T^{a_4}_{\alpha_3\,\alpha} \tag{34}$$

it is straightforward to see that (33) is satisfied. The amplitude for $\mu_L\gamma \to \mu^*_L$ is proportional to a linear combination of $F^{n,J}_{X_L\mu_L}$ and $F^{n,J}_{A^3_L\mu_L}$. More explicitly, we have

$$\sigma^1_{\gamma_L\mu_L} \propto \left|\eta\; C_W\; F^{n,J=1/2}_{X_L\mu_L} + S_W\; F^{n,J=1/2}_{A^3_L\mu_L}\right|^2, \tag{35}$$

where $C_W \equiv \cos\theta_W$, $S_W = \sin\theta_W$, and θ_W is the Weinberg angle. The cross section $\sigma^1_{\gamma_R\mu_L}$ can be expressed in a similar form (with $J=1/2$ replaced by $J=3/2$). Then $\sigma^1_{\gamma_R\mu_L} - \sigma^1_{\gamma_L\mu_L} = 0$ follows from Eq. (34).

For $n>1$, it is actually more convenient to show the cancelations in terms of the helicity amplitudes, i.e., $A_{-1,-1/2} = A_{+1,-1/2}$. These amplitudes are given by [25],

$$A_{+1,-1/2} \sim \mathcal{M}_{\mu_L(1)X_R(2)\to\mu_L(3)X_R(4)} = g_b'^2 \frac{\langle 14\rangle^2}{\langle 12\rangle\langle 23\rangle}\left(\frac{s}{t}V_s + \frac{u}{t}V_u\right) \tag{36}$$

and

$$A_{-1,-1/2} \sim \mathcal{M}_{\mu_L(1)X_L(2)\to\mu_L(3)X_L(4)} = g_b'^2 \frac{\langle 12\rangle^2}{\langle 14\rangle\langle 34\rangle}\left(\frac{s}{t}V_s + \frac{u}{t}V_u\right). \tag{37}$$

The amplitude $\mathcal{M}_{\mu_L A^3\to\mu_L A^3}$ can be obtained by replacing $g_b'^2$ by g_b^2. Note that we restore the form factors V_u, V_s, which gives all the pole terms. They are defined as $V_t = V(s,t,u)$, $V_u = V(t,u,s)$, $V_s = V(u,s,t)$, with

$$V(s,t,u) = \frac{\Gamma(1-s/M_s^2)\;\Gamma(1-u/M_s^2)}{\Gamma(1+t/M_s^2)}. \tag{38}$$

For forward scattering, we have the photon momenta $k_4 = -k_2$ and it is not difficult to see that the tree level contribution to $\Delta\sigma_{\rm non-QED}$ gets canceled, i.e., $A_{-1,-1/2} = A_{+1,-1/2}$.

The cancelation may be violated at order α by differing mass shifts for different spins. Support for this is manifest by the different widths (imaginary parts of the bubble) for $J = 1/2$, $J = 3/2$ [38], which will imply that the mass shifts will differ. In passing we note that resonance production accompanied by single photon emission from the electron line, $\gamma l \to \gamma l^*$, does not contribute to $\Delta\sigma_{\rm non-QED}$ [62].

We now turn to the explicit mass shift calculation. The Breit-Wigner form for unstable particles $\sigma^{J=1/2,3/2}$ is given by

$$\sigma(E) = \frac{2J+1}{(2S_1+1)(2S_2+1)} \frac{4\pi}{k^2} \left[\frac{\Gamma^2/4}{(E-E_0)^2+\Gamma^2/4}\right] B_{\rm in} B_{\rm out}, \tag{39}$$

where $B_{\rm in} = \Gamma_{\rm in}/\Gamma$ and $B_{\rm out} = \Gamma_{\rm in}/\Gamma$. The out states need to be integrated and therefore $B_{\rm out} = 1$. Since $\Gamma_{\rm in}^{J=1/2} = 2\Gamma_{\rm in}^{J=3/2}$ we have $(2J+1)\Gamma_{\rm in}^{J=3/2} = (2J+1)\Gamma_{\rm in}^{J=1/2}$. We denote the energy of the photon in the rest frame of the electron by ν, and it is related to s by $(\nu + m_l)^2 - \nu^2 = s \equiv E^2$. The integration over $d\nu$ then gives

$$\begin{aligned} \frac{m_l^2}{2\pi^2\alpha}\int \frac{d\nu}{\nu}\sigma(E) &= \frac{m_l^2}{2\pi^2\alpha}\frac{16\pi(2J+1)\Gamma_{in}}{3}\int \frac{2EdE}{E^2-m_l^2}\frac{1}{\Gamma E^2}\left[\frac{\Gamma^2/4}{(E-M_s)^2+\Gamma^2/4}\right] \\ &= \frac{64 m_l^2(2J+1)\Gamma_{in}}{3\alpha\Gamma^3}\int \frac{dx}{\pi\,(x^2-x_1^2)\,x}\left[\frac{1}{(x-x_0)^2+1}\right], \end{aligned} \tag{40}$$

where $x = 2E/\Gamma$. To avoid the infrared divergence at $x = x_1$ we set a cutoff and integrate over the energy range $(M_s - 2\Gamma, M_s + 2\Gamma)$, that is $(x_0 - 4, x_0 + 4)$. The integral in the second line of (40) is found to be

$$\int_{-4}^{+4} \frac{dx}{\pi\,(x^2-x_1^2)\,x}\left[\frac{1}{(x-x_0)^2+1}\right] = \frac{2\tan^{-1}(4)}{\pi}\epsilon^3 - \frac{12\,(\tan^{-1}(4)-4)}{\pi}\epsilon^5 + \mathcal{O}(\epsilon^7)\,, \tag{41}$$

where $\epsilon = 1/x_0 = \Gamma/(2M_s)$. The difference between cross sections into $J = 1/2$ and $J = 3/2$ reads,

$$\frac{m_l^2}{2\pi^2\alpha}\int\frac{d\nu}{\nu}\Delta\sigma \sim \frac{2m_l^2\times 2\Gamma_{\rm in}^{J=1/2}}{3\alpha M_s^3}\frac{12\,(4-\tan^{-1}4)}{\pi}\left(\frac{\Delta\Gamma^2}{M_s^2}\right). \tag{42}$$

For 3-stack models, the $J = 1/2$ decay width of the left-handed leptons is

$$\Gamma_L^{J=1/2} = \frac{1}{8\times 1}\frac{{g'_c}^2}{4\pi}M_s + \frac{2^2-1}{8\times 2}\frac{{g_b}^2}{4\pi}M_s + \frac{1}{8\times 2}\frac{{g'_b}^2}{4\pi}M_s\,, \tag{43}$$

while for right-handed leptons we have

$$\Gamma_R^{J=1/2} = \frac{2^2}{8\times 1}\frac{{g'_c}^2}{4\pi}M_s\,. \tag{44}$$

For 4-stack models, we obtain

$$\Gamma_L^{J=1/2} = \frac{1}{8\times 1}\frac{{g'_c}^2}{4\pi}M_s + \frac{2^2-1}{8\times 2}\frac{{g_b}^2}{4\pi}M_s\,, \tag{45}$$

and

$$\Gamma_R^{J=1/2} = \frac{1}{8\times 1}\frac{{g'_d}^2}{4\pi}M_s + \frac{1}{8\times 1}\frac{{g'_c}^2}{4\pi}M_s\,. \tag{46}$$

Before proceeding we note again that $\Gamma_{L,R}^{J=3/2} = \frac{1}{2}\Gamma_{L,R}^{J=1/2}$.

The sum rule applies for either the left or right handed leptons. The correction to a_l^2 (from mass shift) are proportional to the difference in the square of widths of left- and right-handed excited leptons respectively. This implies that the magnetic moment is not proportional to the spin.

It is convenient to parametrize the RHS of (42) as follows

$$\frac{m_l^2}{M_s^2}\frac{4\Gamma_{in}(J=\frac{1}{2})}{3\alpha M_s}\frac{12\,(4-\tan^{-1}4)}{\pi}\left(\frac{\Delta\Gamma^2}{M_s^2}\right) = 4\left(\frac{\alpha}{\pi}\right)\left(\frac{m_l^2}{M_s^2}\right)\kappa_{L,R}\,. \tag{47}$$

Using the expressions for the total decay widths given above and $\Gamma_{\rm in}^{J=1/2} = \alpha M_s/8$ [25] we evaluate (42) to obtain $\kappa_{L,R}$. For 3-stack models, $\kappa_L = 0.0064$ and $\kappa_R = 0.00022$. For 4-stack models, the ratio g_c'/g_d' is driven by g_Y [38]. By requiring both g_c' and g_d' to be less than 1, we constrain the couplings to be within $(0.183, 1)$. Note that $\kappa_{L,R}$ and the constraint are invariant under a swap of g_c' and g_d'. In this coupling range, the interval for kappa becomes $0.0063 < \kappa_L < 0.037$ while $0.00023 < \kappa_R < 0.015$.

Now we would like to estimate the contribution from higher resonance states, $n > 1$. For simplicity, we consider the case in which the photon and the muon have parallel spins. The amplitude of two gauge bosons and two fermions is given by

$$\mathcal{M}(g_1^-, f_2^-, g_3^+, \bar{f}_4^+) \;=\; 2\,g^2\,\delta_{\beta_2}^{\beta_4}\,\frac{\langle 12\rangle^2}{\langle 23\rangle\langle 34\rangle}\,\Big[(T^{a_1}T^{a_3})_{\alpha_4}^{\alpha_2}\,\frac{s}{t}\,V_s + (T^{a_3}T^{a_1})_{\alpha_4}^{\alpha_2}\,\frac{u}{t}\,V_u\Big]\,, \tag{48}$$

where g is the $U(N)$ coupling constant, $\langle ij\rangle$ are the standard spinor products written in the notation of [63–65], and the $U(N)$ generators are normalized according to $\mathrm{Tr}\,(T^aT^b) = \delta^{ab}/2$ [26]. The function V_u has poles at $s = nM_s^2$,

$$V_u(n) = V(s,u,t) \approx \frac{1}{s-nM_s^2} \times \frac{M_s^{2-2n}}{(n-1)!}\prod_{J=0}^{n-1}(t+M_s^2 J)\,, \tag{49}$$

and near the poles the amplitude takes the following form

$$\mathcal{M}(g_1^-, f_2^-, g_3^+, \bar{f}_4^+) \to 2\,g^2\,(T^{a_3}T^{a_1})_{\alpha_4}^{\alpha_2}\,\delta_{\beta_2}^{\beta_4}\,\frac{M_s^2}{s-nM_s^2}\sum_{j=0}^{n-1} c_j^{(n)} d^{j+\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2}}(\theta)\,. \tag{50}$$

The partial width of the nth resonance with angular momentum J decaying into $g+f$ with parallel spins reads

$$\Gamma(R_{n,J} \to g^\pm f^\pm) = g^2\delta\,\frac{c_j^{(n)} M_s^2}{16(2J+1)\pi\sqrt{n}M_s}\,. \tag{51}$$

To compute the coefficients $c_k^{(n)}$, we first write the resonance amplitude as

$$\begin{aligned}\mathcal{M}(g_1^-, f_2^-, g_3^+, \bar{f}_4^+) \;&\to\; 2\,g^2\,(T^{a_3}T^{a_1})_{\alpha_4}^{\alpha_2}\,\delta_{\beta_2}^{\beta_4}\,\sqrt{-\frac{s}{u}}\;\frac{u}{t}\;\hat{V}_u \\ &\equiv\; \frac{M_s^2}{s-nM_s^2}\sqrt{-\frac{u}{s}}f(-x,n)\,, \end{aligned}\tag{52}$$

where

$$f(x,n) = \left[\frac{n}{(n-1)!}\right] \underbrace{[\frac{nx}{2} - (\frac{n}{2}-1)][\frac{nx}{2} - (\frac{n}{2}-2)]\cdots[\frac{nx}{2} + (\frac{n}{2}-2)][\frac{nx}{2} + (\frac{n}{2}-1)]}_{n-1 \text{ factors}}$$
$$= \left[\frac{n}{(n-1)!}\right] \left(\frac{nx}{2} - \frac{n}{2} + 1\right)_{(n-1)}. \tag{53}$$

The Pochhammer Symbol is defined as follows,

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)...(x+n-1) = \sum_{k=0}^{n} (-1)^{n-k} s(n,k) x^k, \tag{54}$$

where $s(n,k)$ is the Stirling number of the first kind. For odd n, we obtain[2]

$$f(x,n) = \left[\frac{n}{(n-1)!}\right] \left(\frac{nx}{2} - \frac{n}{2} + 1\right)_{(n-1)}$$
$$= \frac{n}{(n-1)!} (a_{n-1}^{(n)} x^{n-1} + a_{n-3}^{(n)} x^{n-3} + \cdots + a_2^{(n)} x^2 + a_0^{(n)}), \tag{55}$$

with

$$a_k = \sum_{i=0}^{n-1-k} \frac{(-1)^{n-1-k}}{2^{k+i}} s(n-1, k+i) \begin{pmatrix} k+i \\ i \end{pmatrix} (n)^k (n-2)^i. \tag{56}$$

It is convenient to rewrite $\sqrt{-\frac{u}{s}} f(-x,n)$ as

$$\sqrt{-\frac{u}{s}} f(-x,n)$$
$$= \frac{n}{(n-1)!} \left(\frac{x+1}{2}\right)^{\frac{1}{2}} (a_{n-1}^{(n)} (-x)^{n-1} + a_{n-3}^{(n)} (-x)^{n-3} + \cdots + a_2^{(n)} (-x)^2 + a_0^{(n)})$$
$$= \left(\frac{x+1}{2}\right)^{-\frac{1}{2}} \left(c_n^{(n)} P_n^{(0,-1)} + c_{n-1}^{(n)} P_{n-1}^{(0,-1)} + \cdots + c_1^{(n)} P_1^{(0,-1)}\right). \tag{57}$$

where in the last line we have used $d^{j+\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2}}(\theta) = \left(\cos\frac{\theta}{2}\right)^{-1} P_j^{(0,-1)}(\cos\theta)$, with $P_j^{(\alpha,\beta)}(x)$ the Jacobi Polynomials. Note that $(x+1)x^m/2$ can be expanded in terms of the Jacobi Polynomials,

$$\left(\frac{x+1}{2}\right) x^m = \sum_{i=m+1,m,\cdots} {d_i}^m P_i^{(0,-1)}(x), \tag{58}$$

where

$${d_i}^m = 2i \int_{-1}^{+1} x^m P_i^{(0,-1)}(x) dx$$
$$= i \left(\frac{2^{\frac{i-m}{2}} \left((-1)^i + (-1)^m\right) m!}{\left(\frac{m-i}{2}\right)! \, (m+i+1)!!} + \frac{2^{\frac{i-m-1}{2}} \left((-1)^i - (-1)^m\right) m!}{\left(\frac{m-i+1}{2}\right)! \, (m+i)!!} \right). \tag{59}$$

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[2] The computation for even n is similar.

We can then obtain $c_j^{(n)}$ as

$$c_j^{(n)} = \frac{n}{(n-1)!} \sum_{k=0}^{\frac{n-j}{2}} a_{j-1+2k} d_j^{j-1+2k} \,. \tag{60}$$

Using (53), (54), and (57) the sum of $c_j^{(n)}$ can be obtained

$$\sum_j c_j^{(n)} = \frac{f(1,n)}{P_j^{(0,-1)}(1)} = n, \tag{61}$$

where we also used the fact that $P_j^{(\alpha,\beta)}(1) = C_n^{n+\alpha}$. As a result, the partial width (51) satisfies the following relation

$$\sum_J \frac{(2J+1)\Gamma_{\rm in}}{\sqrt{n} M_s} = g^2 \delta \, \frac{1}{16\pi} \,. \tag{62}$$

We note that this n-independent quantity appears in the Breit-Wigner form(39). In the narrow width approximation, it is the factor multiplying the delta function $\delta(s - nM_s^2)$. In this case, we can see that the contribution to a_l^2 from level n is proportional to n^{-1}. In other words, particles of masses M together contribute $\sim (m_l/M)^2$ to a_l^2. Assuming that the ratio of the total decay width and mass

$$\frac{\Gamma_{\rm tot}}{M} = \frac{g^2}{4\pi} \frac{\mathcal{C}}{4} \,, \tag{63}$$

remains a constant, it is straightforward to see that the same behavior applies for a Breit-Wigner form.

TABLE II: Lower limits on M_s/TeV.

chirality	3-stack	4-stack
L	0.66	0.66 to 1.6
R	0.12	0.13 to 1.0

With $a_l^{\rm QED} \simeq \alpha/(2\pi)$, we find on solving Eq. (29)

$$a_l^{\rm non-QED} \simeq 4 \left(\frac{m_l}{M}\right)^2 \; \kappa \, b(n_0) \,, \tag{64}$$

where $b(n_0)$ denotes the total contribution from levels $n < n_0$. As we discussed elsewhere [38], for $n_0 \gtrsim 40$, the total decay width grows bigger than the spacing of mass levels and the resonance picture breaks down. Specializing now to the muon, we see that $a_\mu = 1.16 \times 10^{-3}$ and $a_\mu^{\rm non-QED} < 3 \times 10^{-9}$ [4] translates to a lower limit on the string scale, that is:

$$M_s > 4 \text{ TeV} \sqrt{\kappa \; b(n_0)} \,. \tag{65}$$

The bounds for 3- and 4-stack models summarized in Table II show that the contributions from string excitations to the anomalous magnetic moment of the muon are largely suppressed, as conjectured in [46].

IV. CONCLUSIONS

Very recently, the Fermilab Muon $g-2$ experiment released its first measurement of the anomalous magnetic moment, which is in full agreement with the previous measurement at Brookhaven and pushes the world average deviation from the SM expectation δa_μ to a significance of 4.2σ [4]. Galvanized by this brand new result we have reexamined the contributions to δa_μ from anomalous $U(1)$ gauge bosons and excitations of the string. We have shown that while the contribution from Regge recurrences is strongly suppressed, the contribution from the heavy vector bosons can help ameliorate (though not fully eliminate) the δa_μ discrepancy.

In closing, we note that KK winding modes could provide a non-negligible contribution to δa_μ. It was conjectured in [46] that the KK contribution would be also suppressed. However, we argue herein that this speculation is model dependent, because it is based on the statement that the compactification scale is of order the string scale. An order of magnitude estimate can be obtained by using the truncated sum rule: setting $a_{\rm QED} = \alpha/(2\pi)$ in (29) we obtain

$$\left(a_{\rm non-QED}\right) + \frac{\pi}{\alpha} \left(a_{\rm non-QED}\right)^2 = \frac{m_l^2}{2\pi\alpha^2} \int_{s_{\rm th}}^\infty \frac{ds}{s} \, \Delta\sigma_{\rm non-QED} \,. \tag{66}$$

If $a_{\rm non-QED} \ll \alpha/\pi$, then the left-hand-side of (66) is dominated by its first term,

$$\left(a_{\rm non-QED}\right) \sim \frac{m_l^2}{2\pi\alpha^2} \int_{s_{\rm th}}^\infty \frac{ds}{s} \, \Delta\sigma_{\rm non-QED} \,. \tag{67}$$

Assuming that both the left- and right-hand-side of (67) can be written as power laws in α (or loop number), on the left-hand-side we expect $a^{\rm KK}_{\rm non-QED}$ to be $\mathcal{O}(\alpha)$, as seen from triangle diagrams containing heavy stuff. Then, on the right-hand-side $\Delta\sigma^{\rm KK}_{\rm non-QED}$ (for excitation M_j) needs to be of $\mathcal{O}(\alpha^3/M_j^2)$, which yields

$$a^{\rm KK}_{\rm non-QED} \sim \sum_j \left(\frac{m_l}{M_j}\right)^2 \alpha \,. \tag{68}$$

As we have shown in Fig. 1, if $\mu_\alpha \sim 1$ TeV, for particular choice of gauge couplings, we can bridge the δa_μ gap reported by the Muon $g-2$ Collaboration. However, LHC experiments have set a 95% CL bound $\mu_\alpha < 5$ TeV [66, 67]. We can infer from (68) that if the $U(1)$ retains the same gauge couplings but $\mu_a \sim 5$ TeV, then the contribution of each KK excitation will be suppressed by about two orders of magnitude. Summation over all the KK modes may allow recovering the suppression factor [70]. Note that the KK of other gauge bosons with different couplings (e.g. hypercharge) would also contribute. In addition, it is important to stress that *for the 4 stack model exhibited in Fig. 2, the KK excitations of the $U(1)$ which lives in the lepton brane do not have tree level couplings to hadrons and therefore their production at the LHC would be suppressed, but these excitations could still yield the dominant contribution to δa_μ.* The future muon smasher will provide the final verdict on such leptophilic KK excitations [71]. A direct string calculation of the KK contribution is under way and will be presented elsewhere.

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