

# Towards Energy-Aware Feedback Planning for Long-Range Autonomous Underwater Vehicles

Tauhidul Alam<sup>1,\*</sup>, Abdullah Al Redwan Newaz<sup>2</sup>, Leonardo Bobadilla<sup>3</sup>,  
Wesam H. Alsabban<sup>4</sup>, and Ryan N. Smith<sup>5</sup>, and Ali Karimodini<sup>2</sup>

<sup>1</sup>*T. Alam is with the Department of Computer Science, Louisiana State University Shreveport, Shreveport, LA 71115, USA*

<sup>2</sup>*A. A. R. Newaz and A. Karimodini are with the Department of Electrical and Computer Engineering, North Carolina A&T State University, Greensboro, NC 27411, USA*

<sup>3</sup>*L. Bobadilla is with the School of Computing and Information Sciences, Florida International University, Miami, FL 33199, USA*

<sup>4</sup>*W. H. Alsabban is with the College of Computer and Information Systems, Umm Al-Qura University, Makkah, Saudi Arabia*

<sup>5</sup>*R. N. Smith is with the Institute of Environment, Florida International University, Miami, FL 33199, USA*

Correspondence\*:  
Tauhidul Alam  
talam@lsus.edu

## 2 ABSTRACT

3 Ocean ecosystems have spatiotemporal variability and dynamic complexity that require a long-  
4 term deployment of an autonomous underwater vehicle for data collection. A new generation of  
5 long-range autonomous underwater vehicles (LRAUVs), such as the Slocum glider and Tethys-  
6 class AUV, has emerged with high endurance, long-range, and energy-aware capabilities. These  
7 new vehicles provide an effective solution to study different oceanic phenomena across multiple  
8 spatial and temporal scales. For these vehicles, the ocean environment has forces and moments  
9 from changing water currents which are generally on the order of velocity of the operational  
10 vehicle velocity. In this scenario, it is not practical to generate a simple trajectory from an initial  
11 location to a goal location in an uncertain ocean, as the vehicle can deviate significantly from the  
12 prescribed trajectory.

13 Since state estimation remains challenging in underwater conditions, feedback planning must  
14 incorporate state uncertainty that can be framed into a stochastic energy-aware path planning  
15 problem. This article presents an energy-aware feedback planning method for an LRAUV utilizing  
16 its kinematic model in an underwater environment under motion and sensor uncertainties. Our  
17 approach uses ocean dynamics from a predictive ocean model to understand the water flow  
18 pattern and introduces a goal-constrained belief space to make the feedback plan synthesis  
19 computationally tractable. Energy-aware feedback plans for different water current layers are  
20 synthesized through sampling and ocean dynamics. The synthesized feedback plans provide  
21 strategies for the vehicle that drives it from an environment's initial location toward the goal



**Figure 1.** Two images of a Tethys-class vehicle deployed in the ocean (MBARI, 2009).

22 location. We validate our method through extensive simulations involving the Tethys vehicle's  
 23 kinematic model and incorporating actual ocean model prediction data.

24 **Keywords:** feedback planning, energy-aware, long-range autonomous underwater vehicles, predictive ocean model, kinematic model,  
 25 observation model

## 1 INTRODUCTION

26 Ocean ecosystems are complex and have high variability in both time and space. Consequently, ocean  
 27 scientists must collect data over long periods to obtain a synoptic view of ocean ecosystems and understand  
 28 their spatiotemporal variability. To support data collection, autonomous underwater vehicles (AUVs) are  
 29 increasingly being used for studying different oceanic phenomena such as oil spill mapping (Kinsey et al.,  
 30 2011), harmful algal blooms (Das et al., 2010), phytoplankton and zooplankton communities (Kalmbach  
 31 et al., 2017), and coral bleaching (Manderson et al., 2017). These AUVs can be classified into two categories:  
 32 (i) propeller-driven vehicles, such as the Dorado class, which can move fast and gather numerous sensor  
 33 observations but are limited in deployment time to multiple hours; and (ii) minimally-actuated vehicles  
 34 such as drifters, profiling floats, and gliders that move slower, but can remain on deployment for tens of  
 35 days to multiple weeks.

36 A new breed of the long-range autonomous underwater vehicle (LRAUV), *i.e.*, Tethys, combines the  
 37 advantages of both minimally-actuated and propeller-driven AUVs (Hobson et al., 2012). The LRAUV  
 38 can move quickly for hundreds of kilometers, float with water currents, and carry a broad range of data  
 39 collection sensors. It can also control its buoyancy for changing depths in the water and the angle at which  
 40 it moves through the water. By mixing modalities, this vehicle can be deployed in the water for weeks at a  
 41 time and navigate challenging ocean current conditions for short time periods. Two images of deployed  
 42 Tethys AUVs are shown in Figure 1. A planning and control technique for this vehicle is critical to increase  
 43 its autonomy and generate mission trajectories during long-range operations. The execution of a planned  
 44 trajectory for this vehicle is also challenging due to ocean currents' variability and uncertainty. Thus, it  
 45 is not practical to generate a simple navigation trajectory from an initial location to a goal location in a  
 46 dynamic ocean environment because the vehicle can deviate from its trajectory due to motion noise and  
 47 cannot estimate its state accurately in underwater environments due to sensor noise.

48 As such, we consider the use of feedback motion planning for an LRAUV by combining its kinematic  
 49 modeling and an ocean dynamic model while also incorporating motion and sensor uncertainties. A  
 50 feedback plan is calculated over each ocean current layer in an underwater environment for a vehicle  
 51 inspired by our previous work (Alam et al., 2020) so that the vehicle can adapt its trajectory from  
 52 any deviated state in the presence of any noise or modeling errors. Furthermore, this feedback plan

53 is crucial when the vehicle state is not fully observable from sensor readings. For such vehicles with  
54 partially observable states, a Partially Observable Markov Decision Process (POMDP) provides a standard  
55 mathematical model for vehicle motion planning under uncertainties. Two major factors make solving our  
56 problem particularly difficult: (a) for the POMDP formulation, finding the optimal solution is formally hard  
57 (NP-hard or PSPACE-hard), and (b) our objective is to compute stochastic energy-aware feedback plans  
58 using ocean dynamics in contrast to other prior POMDP feedback planning methods that calculate the  
59 stochastic shortest path. A large body of existing research focuses on the stochastic shortest path problem  
60 without considering energy constraints. However, it may be unrealistic to assume that the vehicle has  
61 unlimited resources in many applications. A more realistic model would consider that an autonomous  
62 vehicle has limited stored energy, which continually depletes as it operates. Here, we address this constraint  
63 and propose an extension to the POMDP framework that includes energy awareness. Although energy  
64 awareness should take into account an initial energy condition, the efficiency of actuation, and the drag  
65 effect, our method mostly utilizes ocean currents in our calculations.

66 **Contributions:** In this article, we present a method to synthesize feedback plans for an LRAUV in an  
67 underwater environment under motion and sensor uncertainties. First, we develop an ocean dynamic model  
68 from ocean current prediction data. Second, a goal-constrained belief space is introduced to make the  
69 feedback plan synthesis computationally tractable. Finally, energy-aware feedback plans for several water  
70 current layers are synthesized by utilizing sampling and the ocean dynamic model.

71 A preliminary version of this article appeared in (Orioke et al., 2019). This article is fundamentally  
72 different in that it extends (Orioke et al., 2019) by incorporating motion uncertainty and sensor uncertainty  
73 coupled with energy awareness from water flow of an underwater environment within a modified POMDP  
74 framework.

## 2 RELATED WORK

75 The feedback mission control of autonomous underwater vehicles in dynamic and spatiotemporal aquatic  
76 environments has attracted a great deal of interest. A feedback trajectory tracking scheme was developed for  
77 an AUV in a dynamic oceanic environment with modeled and unmodeled uncertainties (Sanyal and Chyba,  
78 2009). An informative feedback plan was generated for AUVs to visit essential locations by estimating  
79 Kriging Errors from spatiotemporal fields (Reis et al., 2018). An obstacle avoidance method (Kawano,  
80 2006) is presented, where an MDP-based re-planner considers only the geometrical properties of obstacles  
81 and the dynamics and kinematics of an AUV to find and track its target path. An adaptive mission plan  
82 for an AUV according to its available resources, such as battery and memory usage, is proposed to add or  
83 remove locations for data collection tasks in underwater environments (Harris and Dearden, 2012).

84 A finite-state automata-based supervisory feedback control (Xu and Feng, 2009) is presented for obstacle  
85 avoidance by an AUV. A temporal plan is calculated in (Cashmore et al., 2014) for AUV mission  
86 control that optimizes the time taken to complete a single inspection tour. A feedback and replanning  
87 framework (Cashmore et al., 2014) is integrated along with the temporal plan in the Robot Operating  
88 System (ROS). Sampling Based Model Predictive Control (SBMPC) (Caldwell et al., 2010) is utilized to  
89 simultaneously generate control inputs and feasible trajectories for an AUV in the presence of nonlinear  
90 constraints.

91 Open-loop trajectory design methods (Smith et al., 2010; Chyba et al., 2009) drive an AUV from a given  
92 initial location to the desired goal location, minimizing a cost in terms of energy and time taken by the  
93 vehicle. The implementation of open-loop trajectories for AUVs works well in environments without any  
94 model uncertainties. In our previous work (Alam et al., 2018a, 2020), we have proposed an open-loop

95 approach for solving the problem of deploying a set of minimally-actuated drifters for persistent monitoring  
 96 of an aquatic environment. In our another work (Alam et al., 2018b), we predicted the localized trajectory  
 97 of a drifter for a sequence of compass observations during its deployment in a marine environment. We  
 98 presented a closed-loop approach (Alam et al., 2018b) when an AUV has a considerable unpredictability of  
 99 executing its action in a dynamic marine environment. Moreover, the previous studies (Hobson et al., 2012;  
 100 Bellingham et al., 2010) on the Tethys AUV described the mission and other capabilities of the vehicle.  
 101 However, there is no work on the development of a planning algorithm for controlling the vehicle.

102 Various types of rewards modification in POMDPs have been investigated in previous research  
 103 efforts (Kim et al., 2019; Lee et al., 2018). Typically, the reward function in POMDPs is designed  
 104 to solve the stochastic shortest path problem, where the goal is to compute a feedback plan that reaches a  
 105 target state from a known initial state by maximizing the expected total reward. From a motion planning  
 106 point of view, the reward can be replaced by a cost, where the goal is to minimize the expected total cost.  
 107 In both cases, the sequence of rewards or costs, however, can be aggregated by considering the discounted  
 108 reward (cost) or the average reward (cost).

109 A point-based algorithm to calculate approximate POMDP solutions is presented combining the full  
 110 and partial observable components of an AUV's state to reduce the dimension of its belief space (Ong  
 111 et al., 2009). An efficient point-based POMDP algorithm for AUV navigation (Kurniawati et al., 2008)  
 112 exploiting the optimally reachable states is developed to improve computational efficiency. A point-based  
 113 POMDP approach (Kurniawati and Patrikalakis, 2013) is presented, where the original solution is updated  
 114 by modifying a set of sample beliefs. The planning for hydrothermal vent mapping problems using  
 115 information from plume detections is modeled as a POMDP utilizing the reachable states as the current  
 116 state of an AUV (Saigol et al., 2009). In this work, an information likelihood algorithm is proposed turning  
 117 the POMDP into an information state MDP. An online POMDP solver (Kurniawati and Yadav, 2016)  
 118 based on an adaptive belief tree is proposed to improve the existing solution and update the solution when  
 119 replanning is needed in dynamic environments.

120 To the best of our knowledge, this is the first work for synthesizing energy-aware feedback plans from a  
 121 POMDP solution for an underwater vehicle using water flow under motion and sensor uncertainties. In  
 122 our work, we utilize an LRAUV's sensor readings to control its mission operation, taking into account its  
 123 several drifting and actuation capabilities.

### 3 PRELIMINARIES

124 In this section, we describe a representation of an underwater environment and motion and observation  
 125 (sensor) models for our vehicle with relevant definitions. Then, we formulate our problem of interest.

126 First, we consider a 3-D environment where a workspace is an ocean environment denoted as  $\mathcal{W} \subset \mathbb{R}^3$ .  
 127 The workspace is divided into a set of 2-D water current layers at different depths of the environment  
 128 which are represented by the third dimension. Let  $L$  be the total number of water current layers in the  
 129 environment.

130 **DEFINITION 3.1 (Workspace).** *The workspace is defined as  $\mathcal{W} = W_1 \cup W_2 \cup \dots \cup W_L$ . At each current*  
 131 *layer, we model the workspace  $W_l \subset \mathbb{R}^2$ , where  $l \in \{1, \dots, L\}$ , as a polygonal environment. Let  $O_l \subset \mathbb{R}^2$*   
 132 *be the land and littoral region of the environment at each layer which is considered an inaccessible region*  
 133 *for the vehicle. The free water space at each current layer is composed of all navigable locations for the*  
 134 *vehicle, and it is defined as  $E_l = W_l \setminus O_l$ . The free water space in the whole workspace is denoted by*  
 135  *$\mathcal{E} = E_1 \cup E_2 \cup \dots \cup E_L$ . We discretize each workspace layer  $W_l$  as a 2-D grid. Each grid point or location,*

136 denoted as  $q$ , has a geographic coordinate in the form of longitude, latitude, and depth (water current  
137 layer)  $q = (x, y, l)$ , where  $x, y \in \mathbb{R}$  and  $l \in \{1, \dots, L\}$ .

138 Second, in our vehicle motion model, we incorporate noise and uncertainty in the vehicle's movement to  
139 account for the modeling error and unmodeled dynamics.

140 **DEFINITION 3.2 (Motion Model).** *The state space for the vehicle is defined as  $X = \mathcal{E} \times \Theta$  in which  $\Theta$   
141 is the set of angles such that  $\theta \in \Theta$ , and  $\theta$  represents the vehicle's orientation. At time  $t$ , the vehicle state  
142 in the state space is represented by  $\mathbf{x}_t = (x_t, y_t, l_t, \theta_t)$  in which  $(x_t, y_t, l_t)$  denotes the vehicle's position in  
143 the free water space, and  $\theta_t$  provides the vehicle's orientation.*

144 The motion model  $\mathbf{f}$  of the vehicle can be written as

$$\mathbf{x}_{t+1} = \mathbf{f}(\mathbf{x}_t, u_t, d_t), \quad (1)$$

145 where  $x_t$  is the vehicle state,  $d_t$  is motion noise, and  $u_t$  is the action belonging to a set of admissible actions  
146  $U$  such that  $u_t \in U$ .

147 Third, it is assumed that our vehicle can observe its positions and the goal location with uncertainties due  
148 to imperfect sensor readings and the dynamic nature of an underwater environment.

149 **DEFINITION 3.3 (Observation Model).** *Let  $Y$  be the observation space, which is the set of all possible  
150 sensor observations  $\mathbf{y} \in Y$ , the vehicle receives. The observation model  $\mathbf{h}$  of the vehicle can be represented  
151 as below.*

$$\mathbf{y}_t = \mathbf{h}(\mathbf{x}_t, w_t), \quad (2)$$

152 where  $w_t$  denotes sensor noise.

153 It is challenging to plan in an uncertain, stochastic environment when there are motion and observation  
154 uncertainties in a vehicle model. To formulate this planning problem, it is necessary to connect hidden  
155 states and observations of our vehicle. A generic model in this context is Partially Observable Markov  
156 Decision Processes (POMDPs).

157 **DEFINITION 3.4 (POMDP).** *A POMDP is defined by a tuple  $\mathcal{P} = (X, U, \mathbf{f}, R, Y, \mathbf{h}, \gamma)$ , where*

- 158 •  $X$  is a finite set of states.
- 159 •  $U$  is a finite set of actions, available to the vehicle.
- 160 •  $\mathbf{f}(\mathbf{x}, u, d, \mathbf{x}') = p(\mathbf{x}' | \mathbf{x}, u, d)$  is a probabilistic transition function, which defines the probability of  
161 moving to a state  $\mathbf{x}' \in X$  after taking an action  $u \in U$  and sustaining a noise  $d$  in a state  $\mathbf{x} \in X$ .
- 162 •  $R(\mathbf{x}, u)$  is a reward function, which defines a real-valued reward after taking an action  $u \in U$  in a  
163 state  $\mathbf{x} \in X$ .
- 164 •  $Y$  is a finite set of observations for the vehicle.
- 165 •  $\mathbf{h}(\mathbf{x}', u, \mathbf{y}) = p(\mathbf{y} | \mathbf{x}', u)$  is a probabilistic observation function, which defines the probability of  
166 observing  $\mathbf{y} \in Y$  after taking an action  $u \in U$  and reaching a state  $\mathbf{x}' \in X$ .
- 167 •  $\gamma \in [0, 1)$  is a discount factor.

168 Due to sensor noise, observations of our vehicle provide only partial information over the states. Planning  
 169 with partial information can be framed as a search problem in a belief space. Let  $B$  be the belief space.

170 DEFINITION 3.5 (Belief). A belief state  $b_t \in B$  of the vehicle is defined as a posterior distribution over  
 171 all possible states given the past actions and sensor observations  $b_t = (\mathbf{x}_t | u_0, \dots, u_{t-1}, \mathbf{y}_0, \dots, \mathbf{y}_t)$ . The  
 172 belief state  $b_t$  can be recursively updated with the following transition function  $\tau$  (Kim et al., 2019)

$$b_t = \tau(b_{t-1}, u_{t-1}, \mathbf{y}_t), \quad (3)$$

173 in which the next belief state depends only on the current belief state, action, and observation.

174 Typically, the POMDP solution can be found by solving the equivalent belief MDP where every belief is  
 175 a state.

176 DEFINITION 3.6 (Belief MDP). An equivalent belief MDP is defined by a tuple  $\mathcal{P} = (B, U, \tau, R, \gamma)$ ,  
 177 where

- 178 •  $B$  is the set of belief states over the POMDP states.
- 179 •  $U$  is a finite set of actions, available to the vehicle as for the original POMDP.
- 180 •  $\tau$  is the belief state transition function.
- 181 •  $R(b, u)$  is the reward function on belief states.
- 182 •  $\gamma \in [0, 1]$  is a discount factor equivalent to the  $\gamma$  in the original POMDP.

183 A feedback plan is called a *solution* to a belief MDP problem if it causes the goal state to be reached  
 184 from every belief state in  $B$ . Let  $b_g \in B$  be a goal belief state of the vehicle at any water current layer of  
 185 the environment. Our objective of the article is to compute a feedback plan for our vehicle.

186 DEFINITION 3.7 (Feedback Plan). A feedback plan  $\pi$  is defined as a function over the belief space  
 187  $\pi : B \rightarrow U$  to produce an action  $\pi(b) = u \in U$ , for a belief state  $b \in B$ , to reach the goal belief state  $b_g$ .

188 The value function of a feedback plan  $\pi$  is computed from the expected discounted reward at the current  
 189 belief state  $b$  as follows:

$$V_\pi(b) = E \left( \sum_{t=0}^{\infty} \gamma^t R(b_t, \pi(b_t) | b_0) \right), \quad (4)$$

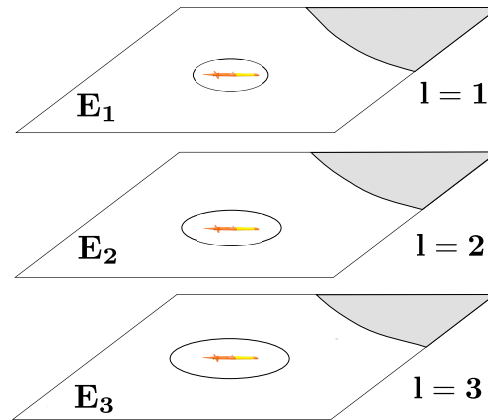
190 where  $\gamma$  is the discount factor, and  $b_0$  is the initial belief state. This value function is maximized for the  
 191 optimal feedback plan  $\pi^*$  as follows:

$$\pi^*(b) = \arg \max_{\pi} V_\pi(b), \quad \forall b \in B. \quad (5)$$

### 192 3.1 Problem Formulation

193 In our 3-D workspace  $\mathcal{W}$ , we account for different localization uncertainties due to sensor noise for its  
 194 divided 2-D water current layer at different depths. Specifically, we consider an almost reliable localization  
 195 on the water surface layer (first water current layer) since the GPS information is accessible to the vehicle  
 196 on the water surface. As the vehicle goes deeper in the water column, its localization uncertainty is assumed

197 to increase due to the implied time increase between potential GPS fixes, as illustrated in Figure 2. In  
 198 that circumstance, the vehicle's state is estimated using dead-reckoning only, and the vehicle is required  
 199 to navigate to the water surface periodically to keep the localization uncertainty tractable. Thus, the  
 200 localization uncertainty for the vehicle decreases with its upward motion in the water column; it could  
 conceivably *quickly* surface for a GPS fix with minimal time and/or energy consumption.



**Figure 2.** Localization uncertainty of a vehicle increases as it goes down along different water current layers.

201

202 When the vehicle is uncertain about its state due to sensor noise and has also motion uncertainty, it is  
 203 crucial to compute a feedback plan that maps every belief state to an action. In computing a feedback  
 204 plan, we take the environmental water flow into account as an ocean dynamic model. We assume that  
 205 this ocean dynamic model and the reward function are known *a-priori*. Our reward function is strictly  
 206 positive, monotonically increasing toward the goal belief state, and additive. Unlike many prior POMDP  
 207 feedback planning algorithms that compute the stochastic shortest path, our goal is to compute the stochastic  
 208 energy-aware path using the ocean dynamic model. Due to the curse of dimensionality of the belief space,  
 209 it is computationally intractable to synthesize feedback plans for multiple water current layers concurrently.  
 210 Therefore, we assume that a high-level planner provides an intermediate goal at each water current layer.  
 211 This motivates us to formulate the following problem to synthesize water current layer-wise feedback plans  
 212 for our vehicle.

213 **Problem Statement:** *Given an ocean environment  $\mathcal{E}$  and its dynamic model, the action set of our vehicle*  
 214  *$U$ , the vehicle motion model, and a goal belief state  $b_g$ , compute a feedback plan  $\pi$  for each water current*  
 215 *layer that drives the vehicle from a belief state  $b$  of the environment to reach the goal belief state  $b_g$  of the*  
 216 *same water current layer.*

## 4 METHODOLOGIES

217 In this section, we detail an energy-aware feedback planning method that utilizes sampling and the ocean  
 218 dynamic model for solving the problem formulated in Section 3.

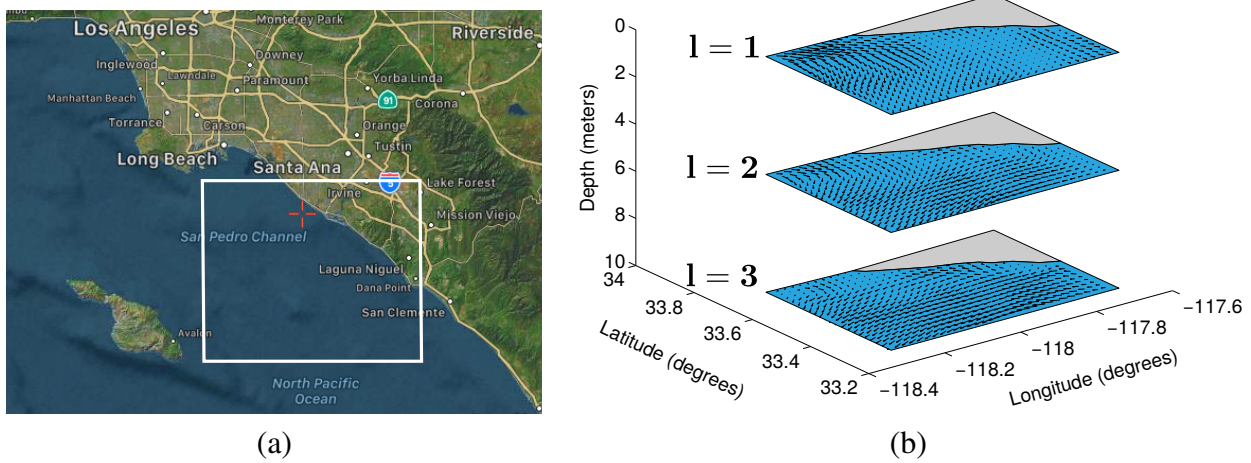
### 4.1 Ocean Dynamic Model

#### 4.1.1 Data Acquisition

221 We utilize the Regional Ocean Modeling System (ROMS) (Shchepetkin and McWilliams, 2005) predicted  
 222 oceanic current data in the Southern California Bight (SCB) region, CA, USA, as illustrated in Figure 3(a),

223 which is contained within  $33^{\circ}17'60''$  N to  $33^{\circ}42'$  N and  $-117^{\circ}42'$  E to  $-118^{\circ}15'36''$  E. ROMS is a free-  
 224 surface, split-explicit, terrain-following, nested-grid mode, and an extensively used ocean model. ROMS  
 225 is also an open-source ocean model that is widely accepted and supported throughout the oceanographic  
 226 and modeling communities. ROMS primarily assimilates surface velocities from HF radar data, and it is  
 227 assumed that the forecasting for near-surface velocities is accurate in direction and magnitude.

228 The four dimensions of 4-D ROMS current prediction data are longitude, latitude, depth, and time. The  
 229 ROMS current prediction data are given at depths from 0 m to 125 m and with 24 hours forecast for each  
 230 day. Each ROMS current velocity prediction is given at depths from 0 m to 4000 m, with a 12-h hindcast, a  
 231 12-h nowcast, and a 48-h forecast each day. The first 24-h comprising hindcasts and nowcasts of each day  
 232 are the most accurate ocean current prediction in the ROMS model. In our work, we utilize a concatenation  
 233 of the earliest 24-h of each prediction for each day for 30 days of predictions. The three components of  
 234 oceanic currents are northing current ( $\alpha$ ), easting current ( $\beta$ ), and vertical current ( $\lambda$ ). These components  
 235 are given based on the four dimensions (time, depth, longitude, and latitude).



**Figure 3.** (a) The area of interest in the SCB region, California. (b) Flow fields generated from ROMS oceanic current prediction data.

#### 236 4.1.2 Water Flow Characterization

237 We create flow fields at several water current layers from the ROMS ocean current prediction data. Ocean  
 238 current prediction data for a specific time and at a particular water current layer can be represented as a  
 239 flow field. Let the flow field on a location  $q$  at a particular water current layer of the environment  $E_l$  be  
 240  $F(q)$ . For a location  $q$  at a particular water current layer, the easting component along the latitude axis is  
 241 denoted by  $\alpha(q)$ , the northing component along the longitude axis is denoted by  $\beta(q)$ , and the vertical  
 242 component at that water current layer is denoted by  $\lambda(q)$ . The flow field based on two components for a  
 243 location  $q$  at that water current layer is specified as:

$$F(q) = \alpha(q)i + \beta(q)j, \tag{6}$$

244 where  $i$  and  $j$  are unit vectors along the latitude and longitude axes, respectively.

245 The vertical component of the ocean current  $\lambda(q)$  at several water current layers is considered zero. Thus,  
 246 we create flow fields for three water current layers as illustrated in Figure 3(b). Then, we find flow lines of



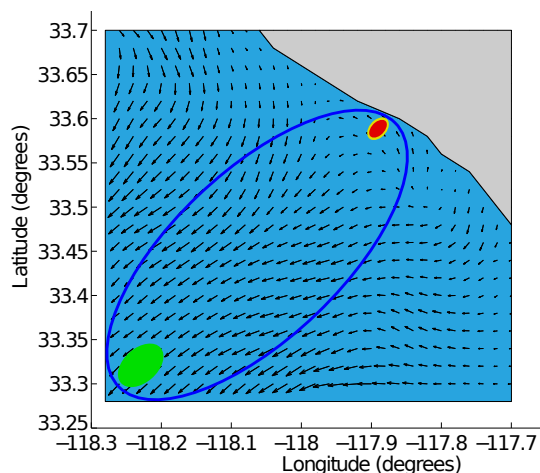
247 the water flow from these flow fields. Flow lines of the water flow over the flow field  $F$  are the trajectories  
 248 or paths traveled by an omnidirectional vehicle at the given water current layer whose vector field is the  
 249 flow field.

## 250 4.2 Goal-Constrained Belief Space

251 It is computationally expensive to compute a feedback plan for a given goal belief state  $b_g$  of a water  
 252 current layer under a finite horizon because of the high dimensional belief space  $B$  (Papadimitriou and  
 253 Tsitsiklis, 1987). Therefore, we utilize a reachable belief space  $\mathcal{R}(b_0)$  containing belief states from an  
 254 initial belief state  $b_0$  to compute the plan for the water current layer  $W_l$ . The reachable belief space  $\mathcal{R}(b_0)$   
 255 is much smaller than  $B$  in terms of the number of belief states. Then, we construct a goal-constrained  
 256 belief space  $\mathcal{R}^*(b_0, b_g)$  containing belief states from an initial belief state  $b_0$  that drive the AUV to the goal  
 257 belief state  $b_g$  of the same water current layer  $W_l$ . The goal-constrained belief space  $\mathcal{R}^*(b_0, b_g)$  is much  
 258 smaller than the reachable belief space  $\mathcal{R}(b_0)$  since  $\mathcal{R}^*(b_0, b_g)$  is pruned from  $B$ . This goal-constrained  
 259 belief space  $\mathcal{R}^*(b_0, b_g)$  leads to a computationally efficient synthesis of the optimal feedback plan  $\pi^*$   
 260 for the water current layer  $W_l$  because any vehicle state sample  $x$  in  $\pi^*$  is taken within  $\mathcal{R}^*(b_0, b_g)$ . The  
 261 representation of  $\mathcal{R}^*(b_0, b_g)$  is represented as an ellipse with  $x_0 \sim b_0$  and  $x_g \sim b_g$  as focal points. This  
 262  $\mathcal{R}^*(b_0, b_g)$  can be expressed as

$$\mathcal{R}^*(b_0, b_g) = \{b \in B \mid \|x_0 - x\|_2 + \|x_g - x\|_2 < \delta\}, \quad (7)$$

263 where  $x_0 \sim b_0$ ,  $x_g \sim b_g$ ,  $x \sim b$ , and  $\delta$  is a threshold value which can be tuned to obtain a desired  $\mathcal{R}^*(b_0, b_g)$ .  
 264 An example  $\mathcal{R}^*(b_0, b_g)$  is illustrated in Figure 4.



**Figure 4.** The blue elliptical goal-constrained belief space  $\mathcal{R}^*(b_0, b_g)$  is given as prior knowledge for the green goal belief state  $b_g$  from the red initial belief state  $b_0$  of the vehicle.

## 265 4.3 Energy-Aware Feedback Plan Synthesis

266 We develop our energy-aware feedback planning algorithm based on the Partially Observable Monte  
 267 Carlo Planning (POMCP) algorithm (Silver and Veness, 2010). The POMCP algorithm assumes that the  
 268 optimal plan can be synthesized by aggregating rewards of the available actions from each state using the  
 269 Monte-Carlo Tree Search (MCTS) algorithm. It is an approximate method that does not consider energy  
 270 awareness, but it is known to extract near-optimal policies in finding the stochastic shortest path where  
 271 optimal rewards depend on the distance from the goal state. Furthermore, the POMCP algorithm allows us

**Algorithm 1:** PREFERRED\_ACTION( $h, x, U, F, \mathcal{R}^*$ )

---

**Input:**  $h, x, U, F, \mathcal{R}^*$  – History of belief states, State, Actions, Flow field, Goal-constrained belief space  
**Output:**  $\mathcal{A}$  – A set of preferred actions

```

1  $\mathcal{A} \leftarrow \emptyset$ 
2 for each  $u \in U$  do
3    $(x', y, r) \sim \mathcal{G}(x, a, F)$  // enforcing the goal-constrained belief space
4   if  $x' \in \mathcal{R}^*$  then
5      $\mathcal{A} \leftarrow \mathcal{A} \cup \{u\}$ 
6 return  $\mathcal{A}$ 

```

---

**Algorithm 2:** SEARCH( $h, F, \mathcal{R}^*$ )

---

**Input:**  $h, F, \mathcal{R}^*$  – History of belief states, Flow field, Goal-constrained belief space  
**Output:**  $\pi^*$  – An optimal feedback plan

```

1 for  $t \leftarrow 1$  to  $\mathcal{T}$  do
2   if  $h == \emptyset$  then
3      $x \sim b_0$ 
4   else
5      $x \sim B(h)$ 
6 Simulate ( $x, h, 0, F, \mathcal{R}^*$ )
7  $\pi^* = \arg \max_b V(b)$ 
8 return  $\pi^*$ 

```

---

272 to utilize the domain knowledge. In our work, we use the domain knowledge of the reachable belief space  
273  $\mathcal{R}^*$  to reduce the search space for choosing actions. Instead of searching actions over all possible events  
274 that could happen with low probabilities, the reachable belief space constraints the action search space for  
275 the most likely events.

276 To overcome the challenges associated with solving belief space planning, we first define a set of discrete  
277 actions and a set of discrete outcomes. For an LRAUV planning to reach a goal location, we consider nine  
278 actions that include actions toward eight compass directions, *i.e.*, N, NE, E, SE, S, SW, W, NW along  
279 with drift (idle). The outcomes of actions could be three observations, *i.e.*, goal, intermediate, and outside.  
280 In other words, the goal observation refers to the vehicle reaches to the goal location, the intermediate  
281 observation refers to it moves toward the goal location, and the outside observation refers to it goes beyond  
282 the goal-constrained belief space. Since the outcome of any action is not deterministic, the LRAUV must  
283 consider all three observations when simulating an action. For a given state  $x$ , Algorithm 1 provides a set  
284 of preferred actions  $\mathcal{A}$  based on the goal-constrained belief state. Algorithm 2 returns the optimal feedback  
285 plan  $\pi^*$  for a water current layer from a history of belief states.

286 Algorithm 3 simulates an action and keeps track of its outcome. We refer to a complete simulated trial as  
287 a rollout where we keep track of actions and their outcomes as history  $h$ . To plan with energy-awareness,  
288 we incorporate the ocean dynamic model  $F$  in Algorithm 4 as a prior to the simulator  $\mathcal{G}$ . Therefore, during  
289 a rollout, the set of available preferred actions and their outcomes take advantage of the prior knowledge.  
290 In Algorithm 4, we compute the reward values of actions by considering the flow field. The reward value is  
291 calculated high when a simulated action takes advantage of the flow field. Otherwise, the reward value  
292 is calculated low. For instance, if the vehicle simulates a particular action in a rollout, using transition  
293 probabilities and the ocean dynamic model, we first generate a simulated trajectory and then evaluate the

**Algorithm 3:** SIMULATE ( $x, h, \beta, F, \mathcal{R}^*$ )

---

**Input:**  $x, h, \beta, \mathcal{R}^*$  – State, History of belief states, Depth, Flow field, Goal-constrained belief space  
**Output:**  $R$  – Reward

```

1 if  $\gamma^\beta < \epsilon$  then
2   return 0
3 if  $h \notin T$  then
4    $\mathcal{A} \leftarrow \text{PREFERRED\_ACTION}(h, x, U, F, \mathcal{R}^*)$ 
5   for each  $a \in \mathcal{A}$  do
6      $T(ha) \leftarrow (N_i(ha), V_i(ha), \emptyset)$ 
7   return ROLLOUT( $x, h, \beta, F$ )
8  $a \leftarrow \arg \max_b V(hb) + c \sqrt{\frac{\log N(h)}{N(hb)}}$ 
9  $(x', y, r) \leftarrow \mathcal{G}(x, a, F)$  // considering energy awareness from the flow field
10  $R \leftarrow r + \gamma \cdot \text{SIMULATE}(x', hay, \beta + 1, F, \mathcal{R}^*)$ 
11  $B(h) \leftarrow B(h) \cup \{x\}$ 
12  $N(h) \leftarrow N(h) + 1$ 
13  $N(ha) \leftarrow N(ha) + 1$ 
14  $V(ha) \leftarrow V(ha) + \frac{R - V(ha)}{N(ha)}$ 
15 return  $R$ 

```

---

**Algorithm 4:** ROLLOUT ( $x, h, \beta, F$ )

---

**Input:**  $x, h, \beta, F$  – State, History of belief states, Depth, Flow field  
**Output:**  $r$  – Step reward

```

1 if  $\gamma^\beta < \epsilon$  then
2   return 0
3  $a \sim \pi_{\text{rollout}}(h, \cdot)$ 
4  $(x', y, r) \leftarrow \mathcal{G}(x, a, F)$  // evaluating an action with energy awareness from the flow field
5 return  $r + \gamma \cdot \text{ROLLOUT}(x', hay, \beta + 1, F)$ 

```

---

294 trajectory with respect to the goal location. To evaluate a simulated trajectory, we employ the particle  
 295 filter, where each state on the trajectory is considered as a particle and the goal location can be thought  
 296 of as a landmark (see this work (Kim et al., 2019) for a detailed explanation of particle filter in the robot  
 297 localization). When considering the next step of this rollout, the LRAUV knows which action from the set  
 298 of available actions is more likely to drive it to the goal location by computing the reward associated with  
 299 each action. The changes from the standard POMCP are highlighted in blue in our algorithms.

## 5 EXPERIMENTAL RESULTS

300 In this section, we examine a Tethys-like LRAUV's kinematic model and evaluate its navigation solution in  
 301 an underwater environment under motion and sensing uncertainties. The experiments are conducted on a  
 302 Unix/Linux computer with Intel Core i7 4.5GHz processor and 32GB memory.

### 303 5.1 LRAUV Kinematic Model

304 The vehicle motion is noisy due to the inherent dynamic nature of water flow of the underwater  
 305 environment. The vehicle observation model suffers uncertainty in measuring distances and locations in

306 sensor-denied, such as GPS, underwater environments. We modeled our vehicle motion and observation  
307 models under Gaussian noise. This setup also makes our Tethys navigation problem a POMDP problem.

308 Let  $E_l \in \mathbb{R}^n$  be the state space of a water current layer  $W_l$  and  $U \in \mathbb{R}^m$  be the action space of the vehicle,  
309 where  $m \leq n$ . Let  $Y \in \mathbb{R}^p$  be the observation space of the vehicle sensors. The state transition model of  
310 our vehicle similar to a unicycle-model can be written as

$$x_{t+1} = x_t + u_t \cos(\theta_t) \quad (8)$$

$$y_{t+1} = y_t + u_t \sin(\theta_t) \quad (9)$$

$$\dot{\theta}_t = \omega_t. \quad (10)$$

311 We incorporate water flow fields as prior knowledge in our motion model for the vehicle. In other words,  
312 the next transition state of the vehicle is influenced by the water flow field of a current layer as well as its  
313 actions. The unicycle motion and observation models for the vehicle can be expressed as

$$\dot{x} = \mathbf{f}(x_t, u_t, d_t) = A_t x_t + B_t u_t + d_t \quad d_t \sim \mathcal{N}(0, D_t) \quad (11)$$

$$\dot{y} = \mathbf{h}(x_t, w_t) = C_t y_t + w_t \quad w_t \sim \mathcal{N}(0, W_t), \quad (12)$$

314 in which  $A$  is the state transition matrix of dimension  $n \times n$ ,  $B$  is the action transition matrix of dimension  
315  $n \times m$ ,  $C$  is the sensor observation matrix of dimension  $p \times n$ , and  $d_t$  and  $w_t$  represent the motion and  
316 sensor noise from a zero-mean Gaussian with variance  $D_t$  and  $W_t$  respectively.

317 The importance of incorporating water flow fields as the ocean dynamics in our motion model is that  
318 a Tethys-like vehicle is deployed to navigate through the water flow. However, the vehicle can leverage  
319 pressure, velocity, and acceleration of flow fields at times to perform a drifting action and save energy  
320 in its long-term mission. It is also important to note that motion and sensor noises provide motion and  
321 observation uncertainties but flow fields can be utilized for performing a passive action (drift) with no  
322 actuation and thus saving energy.

323 The updated observation model with energy awareness from the ocean dynamics can be expressed as

$$\dot{y} = \mathbf{h}(x_t, w_t) = C_t y_t + w_t + D\tilde{u}, \quad (13)$$

324 in which the energy awareness  $\tilde{u} = [\phi, \psi]$  and its weight  $D = \text{diag}(k_u, k_w)$ , where  $k_u, k_w > 0$ . The energy  
325 awareness for a specific location  $q$  on the water current is expressed as

$$\psi = \arctan(\beta(q), \alpha(q)) \quad (14)$$

$$\phi = \tanh(x^2 + y^2), \quad (15)$$

326 where  $\phi$  is the angular velocity and  $\psi$  is the linear velocity of the flow field.

## 327 5.2 Simulation Results

328 A simulated Tethys-like LRAUV with the above kinematics model can take nine actions that include  
329 actions toward eight compass directions, *i.e.*, N, NE, E, SE, S, SW, W, NW along with drift (idle). The  
330 task for the vehicle is to reach a designated goal state with an energy-aware trajectory by utilizing water

331 currents as much as possible. In our simulation, when LRAUV takes an action, the outcome of that action  
 332 could be any of three observations, *i.e.*, goal, intermediate, and outside.

333 To incorporate the water flow pattern in our simulation, we used the ROMS (Shchepetkin and McWilliams,  
 334 2005) predicted ocean current data observed in the SCB region. The 3-D ocean environment was taken  
 335 into account as a simulated environment for the Tethys movements having six 2-D ocean surfaces at six  
 336 different water current layers or depths (e.g., 0 m, 5 m, 10 m, 15 m, 20 m, and 25 m). Each 2-D ocean  
 337 current layer is tessellated into a grid map. Each tessellated water current layer is a  $21 \times 29$  grid map with  
 338 a spatial resolution of  $1 \text{ km} \times 1 \text{ km}$ .

339 The feedback plan synthesis using the MCTS algorithm depends not only on the distance between initial  
 340 and goal locations but also on the ocean dynamics. In our experiments during the rollout step of the MCTS  
 341 algorithm, we use 50 trials for each action over an approximated belief state. We then employ the particle  
 342 filter to evaluate the rollout outcomes with respect to the goal location. When selecting the next best action  
 343 using Algorithm 3, we utilize a simple PID controller to follow the high-level action.

344 We implement our energy-aware feedback planning algorithm for many water current layers from our  
 345 ROMS ocean current prediction data. We obtain a set of feedback plans as an output from our layer-  
 346 wise feedback plan synthesis. Figure ?? illustrates the executed trajectories of the vehicle applying the  
 347 synthesized feedback plans for the same pair of given initial and goal locations. For these experiments,  
 348 we use longitude and latitude coordinates to represent the vehicle locations. We first set the vehicle's  
 349 initial location at  $(-117.84, 33.54)$  and the vehicle needs to reach within 1 km radius of the goal location  
 350  $(-118.22, 33.54)$ . We then show the results for the different water current layers subject to time-varying  
 351 ocean currents taking 3 hours of water currents into account. A couple of videos related to these experiments  
 352 can be found at <https://youtu.be/FEk6QghDwgI> and at <https://youtu.be/9dnCam8JFTg>. Table 1 shows the  
 353 execution statistics of our synthesized feedback plans in terms of trajectory lengths and plan synthesis  
 354 times. We assume that our vehicle operates at a constant velocity of 4.5 km/h.

355 We also execute trajectories applying the synthesized feedback plan for the same water current layer  
 356 for the varying pairs of initial and goal locations that are illustrated in Figure ?. We observe that the  
 357 trajectories of our feedback plans are not straight lines. This is because our energy-aware feedback plan  
 358 chooses an action using the ocean dynamics in Algorithm 4. Therefore, the actions are selected to facilitate  
 359 drifting through water currents, as mentioned in Section 4.3.

Water current layer	Hour	Initial location (longitude, latitude)	Goal location (longitude, latitude)	Trajectory length (km)	Plan synthesis time (s)
2	1	(-117.84, 33.54)	(-118.22, 33.54)	4.33	0.61
	2	(-117.84, 33.54)	(-118.22, 33.54)	4.05	0.49
4	1	(-117.84, 33.54)	(-118.22, 33.54)	4.06	0.36
	3	(-117.84, 33.54)	(-118.22, 33.54)	3.44	0.42
6	1	(-117.84, 33.54)	(-118.22, 33.54)	4.38	0.41
	2	(-117.84, 33.54)	(-118.22, 33.54)	4.38	0.45

**Table 1.** Comparison of executed trajectory lengths using synthesized feedback plans for several water current layers along with plan synthesis times for a number of hours.

## 6 CONCLUSION AND DISCUSSION

360 This article presents an energy-aware feedback planning method for an LRAUV utilizing its kinematic  
361 model in an underwater environment under motion and sensor uncertainties. First, we generated flow fields  
362 for several water current layers from a concatenated ROMS ocean current prediction data to introduce  
363 the ocean dynamic model. Our method then synthesizes energy and computationally efficient feedback  
364 plans on goal-constrained belief spaces for many water current layers using the ocean dynamic model and  
365 sampling. Our simulation results of the execution of synthesized feedback plans demonstrated our method's  
366 practical and potential application. There are several exciting directions to follow up on this research.

367 Our POMDP solution approach uses nine actions (eight neighboring cells and drift) for planning, which  
368 fits the scales of the ROMS resolutions (kilometers) and allows us to treat the LRAUV as a unicycle vehicle.  
369 We believe that our approach can be easily generalized to incorporate modeling AUV dynamics in shorter  
370 spatial scales. We are currently using our planner, but a realistic AUV simulator (Manhães et al., 2016),  
371 could be used as a black box to generate the next states. Paring our planner with a physically realistic  
372 simulation will help us avoid complicated system identification issues and extend our methodology's  
373 range of applications. Additionally, we would like to incorporate an initial amount of available energy, the  
374 actuator efficiency, and the drag effect in our energy model.

375 One desirable feature of AUV deployments in many scenarios is avoiding constant resurfacing due to  
376 energy, *stealth*, and collision safety constraints. The vehicle can collide with ships and jeopardize its  
377 mission. We are currently extending our framework to incorporate dynamic obstacles on the surface,  
378 representing, for example, boats and other vessels. We are interested in the short term to generalize this idea  
379 to other external motion fields that can be used by autonomous vehicles to use their resources efficiently.  
380 Aerial platforms such as blimps and balloons (Wolf et al., 2010; Das et al., 2003) can provide another  
381 exciting study case for our ideas.

## CONFLICT OF INTEREST STATEMENT

382 The authors declare no conflict of interest.

## AUTHOR CONTRIBUTIONS

383 The first two authors have an equal contribution to this manuscript. Other authors have contributed to  
384 develop ideas and write the paper.

## FUNDING

385 This work is supported in part by the Louisiana Board of Regents Contract Number LEQSF(2020-21)-RD-  
386 A-14 and by the U.S. Department of Homeland Security under Grant Award Number 2017-ST-062000002.  
387 This work is also supported in part by the Office of Naval Research Award Number N000141612634,  
388 and by the National Science Foundation awards IIS-2034123, IIS-2024733, and the MRI Award Number  
389 1531322.

## REFERENCES

- 390 Alam, T., Reis, G. M., Bobadilla, L., and Smith, R. N. (2018a). A data-driven deployment approach for  
391 persistent monitoring in aquatic environments. In *Proceedings of the IEEE International Conference on*  
392 *Robotic Computing (IRC)*. 147–154
- 393 Alam, T., Reis, G. M., Bobadilla, L., and Smith, R. N. (2018b). An underactuated vehicle localization  
394 method in marine environments. In *Proceedings of the MTS/IEEE OCEANS Charleston*. 1–8

- 395 Alam, T., Reis, G. M., Bobadilla, L., and Smith, R. N. (2020). A data-driven deployment and planning  
396 approach for underactuated vehicles in marine environments. *IEEE Journal of Oceanic Engineering*
- 397 Bellingham, J. G., Zhang, Y., Kerwin, J. E., Erikson, J., Hobson, B., Kieft, B., et al. (2010). Efficient  
398 propulsion for the Tethys long-range autonomous underwater vehicle. In *Proceedings of the IEEE/OES*  
399 *Autonomous Underwater Vehicles (AUV)*. 1–7
- 400 Caldwell, C. V., Dunlap, D. D., and Collins, E. G. (2010). Motion planning for an autonomous underwater  
401 vehicle via sampling based model predictive control. In *Proceedings of the MTS/IEEE OCEANS Seattle*.  
402 1–6
- 403 Cashmore, M., Fox, M., Larkworthy, T., Long, D., and Magazzeni, D. (2014). AUV mission control via  
404 temporal planning. In *Proceedings of the IEEE International Conference on Robotics and Automation*  
405 *(ICRA)*. 6535–6541
- 406 Chyba, M., Haberkorn, T., Singh, S., Smith, R., and Choi, S. (2009). Increasing underwater vehicle  
407 autonomy by reducing energy consumption. *Ocean Engineering* 36, 62–73
- 408 Das, J., Rajany, K., Frolovy, S., Pyy, F., Ryany, J., Caronz, D. A., et al. (2010). Towards marine bloom  
409 trajectory prediction for AUV mission planning. In *Proceedings of the IEEE International Conference*  
410 *on Robotics and Automation (ICRA)*. 4784–4790
- 411 Das, T., Mukherjee, R., and Cameron, J. (2003). Optimal trajectory planning for hot-air balloons in linear  
412 wind fields. *Journal of Guidance, Control, and Dynamics* 26, 416–424
- 413 Harris, C. and Dearden, R. (2012). Contingency planning for long-duration AUV missions. In *Proceedings*  
414 *of the IEEE/OES Autonomous Underwater Vehicles (AUV)*. 1–6
- 415 Hobson, B. W., Bellingham, J. G., Kieft, B., McEwen, R., Godin, M., and Zhang, Y. (2012). Tethys-class  
416 long range AUVs-extending the endurance of propeller-driven cruising AUVs from days to weeks. In  
417 *Proceedings of the IEEE/OES Autonomous Underwater Vehicles Symposium (AUV)*. 1–8
- 418 Kalmbach, A., Girdhar, Y., Sosik, H. M., and Dudek, G. (2017). Phytoplankton hotspot prediction with  
419 an unsupervised spatial community model. In *Proceedings of the IEEE International Conference on*  
420 *Robotics and Automation (ICRA)*. 4906–4913
- 421 Kawano, H. (2006). Real-time obstacle avoidance for underactuated autonomous underwater vehicles  
422 in unknown vortex sea flow by the mdp approach. In *Proceedings of the IEEE/RSJ International*  
423 *Conference on Intelligent Robots and Systems (IROS)*. 3024–3031
- 424 Kim, S.-K., Thakker, R., and Agha-Mohammadi, A.-A. (2019). Bi-directional value learning for risk-aware  
425 planning under uncertainty. *IEEE Robotics and Automation Letters* 4, 2493–2500
- 426 Kinsey, J. C., Yoerger, D. R., Jakuba, M. V., Camilli, R., Fisher, C. R., and German, C. R. (2011). Assessing  
427 the deepwater horizon oil spill with the sentry autonomous underwater vehicle. In *Proceedings of the*  
428 *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. 261–267
- 429 Kurniawati, H., Hsu, D., and Lee, W. S. (2008). Sarsop: Efficient point-based POMDP planning by  
430 approximating optimally reachable belief spaces. In *Proceedings of the Robotics: Science and Systems*  
431 *(RSS)*
- 432 Kurniawati, H. and Patrikalakis, N. M. (2013). Point-based policy transformation: adapting policy to  
433 changing POMDP models. In *Proceedings of Algorithmic Foundations of Robotics (WAFR) X*. 493–509
- 434 Kurniawati, H. and Yadav, V. (2016). An online POMDP solver for uncertainty planning in dynamic  
435 environment. In *Proceedings of the International Symposium on Robotics Research (ISRR)*. 611–629
- 436 Lee, J., Kim, G.-H., Poupart, P., and Kim, K.-E. (2018). Monte-Carlo tree search for constrained POMDPs.  
437 In *Proceedings of the Advances in Neural Information Processing Systems (NIPS)*. 7923–7932
- 438 Manderson, T., Li, J., Dudek, N., Meger, D., and Dudek, G. (2017). Robotic coral reef health assessment  
439 using automated image analysis. *Journal of Field Robotics* 34, 170–187

- 440 Manhães, M. M. M., Scherer, S. A., Voss, M., Douat, L. R., and Rauschenbach, T. (2016). UUV simulator:  
441 A gazebo-based package for underwater intervention and multi-robot simulation. In *Proceedings of the*  
442 *MTS/IEEE OCEANS-Monterey*. 1–8
- 443 [Dataset] MBARI (2009). Autonomous underwater vehicles. Available at [https://www.mbari.org/at-](https://www.mbari.org/at-sea/vehicles/autonomous-underwater-vehicles/)  
444 [sea/vehicles/autonomous-underwater-vehicles/](https://www.mbari.org/at-sea/vehicles/autonomous-underwater-vehicles/)
- 445 Ong, S. C., Png, S. W., Hsu, D., and Lee, W. S. (2009). POMDPs for robotic tasks with mixed observability.  
446 In *Proceedings of the Robotics: Science and Systems (RSS)*. vol. 5, 4
- 447 Orioke, O. S., Alam, T., Quinn, J., Kaur, R., Alsabban, W. H., Bobadilla, L., et al. (2019). Feedback  
448 motion planning for long-range autonomous underwater vehicles. In *Proceedings of the MTS/IEEE*  
449 *OCEANS-Marseille*. 1–6
- 450 Papadimitriou, C. H. and Tsitsiklis, J. N. (1987). The complexity of Markov decision processes.  
451 *Mathematics of Operations Research* 12, 441–450
- 452 Reis, G. M., Alam, T., Bobadilla, L., and Smith, R. N. (2018). Feedback-based informative AUV planning  
453 from Kriging errors. In *Proceedings of the IEEE/OES Autonomous Underwater Vehicles (AUV)*
- 454 Saigol, Z. A., Dearden, R., Wyatt, J. L., and Murton, B. J. (2009). Information-lookahead planning for  
455 AUV mapping. In *Proceedings of International Joint Conference on Artificial Intelligence (IJCAI)*.  
456 1831–1836
- 457 Sanyal, A. K. and Chyba, M. (2009). Robust feedback tracking of autonomous underwater vehicles with  
458 disturbance rejection. In *Proceedings of the American Control Conference (ACC)*. 3585–3590
- 459 Shchepetkin, A. F. and McWilliams, J. C. (2005). The Regional Oceanic Modeling System (ROMS):  
460 a split-explicit, free-surface, topography-following-coordinate oceanic model. *Ocean Modelling* 9,  
461 347–404
- 462 Silver, D. and Veness, J. (2010). Monte-Carlo planning in large POMDPs. In *Proceedings of the Advances*  
463 *in Neural Information Processing Systems (NIPS)*. 2164–2172
- 464 Smith, R. N., Chao, Y., Li, P. P., Caron, D. A., Jones, B. H., and Sukhatme, G. S. (2010). Planning  
465 and implementing trajectories for autonomous underwater vehicles to track evolving ocean processes  
466 based on predictions from a regional ocean model. *The International Journal of Robotics Research* 29,  
467 1475–1497
- 468 Wolf, M. T., Blackmore, L., Kuwata, Y., Fathpour, N., Elfes, A., and Newman, C. (2010). Probabilistic  
469 motion planning of balloons in strong, uncertain wind fields. In *Proceedings of the IEEE International*  
470 *Conference on Robotics and Automation (ICRA)*. 1123–1129
- 471 Xu, H. and Feng, X. (2009). An AUV fuzzy obstacle avoidance method under event feedback supervision.  
472 In *Proceedings of the MTS/IEEE OCEANS*. 1–6