A 1D Gaussian Function for Efficient Generation of Plane Waves in 1D, 2D, and 3D FDTD

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***Abstract*—A 1D Gaussian expression is derived and used as the 1D E/H incident field in the TF/SF formulation to efficiently generate plane waves in 1D, 2D, and 3D FDTD simulations. The analytic expression is simple, and it eliminates the need for computational resources to store and compute the E/H-field incident arrays and their associated absorbing boundaries. FDTD simulation results at the magic time-step in 1D, 2D, and 3D FDTD show good correlation between plane waves generated by the 1D analytic Gaussian function vs. those generated by 1D FDTD incident arrays.**

I. Introduction

The Finite-Difference Time-Domain (FDTD) [1] is a well-known method for solving Maxwell's electromagnetic equations in 1D, 2D, and 3D space. For certain electromagnetic structures, it may be required to perform scattering analysis [2] due to an incident plane wave which must be generated in FDTD. Previous works have presented methods for generating plane waves; for example, by running a parallel 1D FDTD simulation of the incident electric field (E-field) and magnetic field (H-field) [3].

In this work, we derive a simple analytic expression for generating a Gaussian plane wave in 1D, and use it as the E/H incident fields in the total-field/scattered-field (TF/SF) [3], [4] formulation, to generate plane waves in 1D, 2D, and 3D FDTD.

The proposed analytic expression is simple and quite easy to implement in the traditional Yee-based FDTD algorithm, while eliminating the need for computational resources to store and compute the E/H-field incident arrays and their associated perfectly matched layer (PML) [5] absorbing arrays. Simulations show that the plane waves generated by the 1D analytic Gaussian expression perform just as well as those generated by 1D FDTD incident arrays, in 1D, 2D, and 3D FDTD.

In section II, the analytic expression of the 1D Gaussian plane wave is derived. In section III, results of simulations in 1D, 2D, and 3D FDTD are presented, comparing plane waves generated by the analytic Gaussian vs. 1D FDTD incident arrays. The paper is concluded with remarks are in section IV.

II. Formulation

We begin by defining the 1-dimensional (1D) Gaussian input function $g[t]$ in time-domain (TD), where t is time, t_s is the pulse spread, and t_p is the time at which the pulse reaches its peak amplitude a.

$$g[t] = a e^{-\frac{(t-t_p)^2}{2{t_s}^2}} \tag{1}$$

Taking the Fourier transform of $g[t]$ leads to the frequency-domain (FD) response $G[\omega]$ of the system [6], where ω is the angular frequency, and $\jmath = \sqrt{-1}$, below.

$$G[\omega] = \frac{a t_s}{\sqrt{2\pi}} e^{-\frac{t_s^2 w^2}{2} + \jmath t_p w} \tag{2}$$

The expression of a time-harmonic plane wave [2] travelling in the $+z$ direction in space at a single harmonic frequency ω_0, is given below.

$$p[t,z] = e^{\jmath t \omega_0 - \jmath \beta z} \tag{3}$$

Noting that in free-space the phase constant $\beta = \omega_0 / c_0$, the Fourier transform of (3) may be obtained, below.

$$P[\omega, z] = e^{-\frac{\jmath \omega_0 z}{c_0}} \delta[\omega + \omega_0], \tag{4}$$

where c_0 is the phase velocity in free-space, and $\delta[]$ is the Dirac Delta impulse function. Note that the phase velocity may be modified in terms of material properties, as appropriate.

The FD response to a Gaussian pulse may be obtained, below.

$$\begin{aligned} PG[\omega, z] &= P[\omega, z] \times G[\omega] \\ &= \frac{a t_s}{\sqrt{2\pi}} \delta[\omega + \omega_0] e^{-\jmath \frac{\omega_0 z}{c_0} + \jmath t_p \omega - \frac{1}{2} t_s^2 \omega^2} \end{aligned} \tag{5}$$

Applying the inverse Fourier transform to (5) yields the complex response of the Gaussian input at a single harmonic ω_0, below.

$$pg_{\omega_0}[t,z] = \frac{a t_s}{\sqrt{2\pi}} e^{\frac{\jmath \omega_0 \left(-2z + c_0\left(2t - 2t_p + \jmath t_s^2 \omega_0\right)\right)}{2c_0}} \tag{6}$$

Finally, integrating (6) over all frequencies ω_0, gives the TD system response to the Gaussian input excitation (1), as shown below. Equation (7) is the 1D analytic Gaussian expression which may be used as the E/H incident source in the TF/SF formulation of 1D, 2D, and 3D FDTD simulations.

$$pg[t,z] = \int_{-\infty}^{+\infty} pg_{\omega_0}[t,z]\, d\omega_0 = a e^{-\frac{(c0(t_p - t) + z)^2}{2 c_0^2 t_s^2}} \tag{7}$$

III. Results and Discussion

The results of applying the 1D analytic Gaussian function (7) to the TF/SF regions of 1D, 2D, and 3D FDTD simulations, are shown in Figures 1, 2, 3. In each figure, the E-field is plotted at an arbitrary point in time, as it propagates through space. In each plot, we compare the recorded E-field generated by the 1D analytic Gaussian vs. that generated by the 1D FDTD incident array. The time-step used in each FDTD simulation is set to the magic time-step in 1D ($\Delta t = \Delta x/c_0$), in 2D ($\Delta t = \Delta x/\sqrt{2}c_0$), and in 3D ($\Delta t = \Delta x/\sqrt{3}c_0$); where Δx is the spatial discretization and Δt is the temporal discretization in FDTD.

As can be seen, in each case the plane waves generated by the analytic Gaussian correlate those generated by the 1D FDTD incident array. In the case of the analytic Gaussian, as the pulse approaches the far side of the TF/SF boundary in 2D and 3D FDTD, we observed a small amount of the incident wave being transmitted into the SF region; however, this wave is completely absorbed by the PML region.

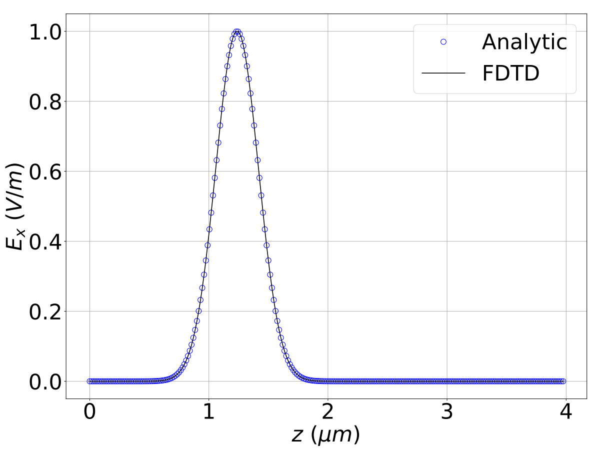

Fig. 1. Simulation results in 1D FDTD. E_x is plotted vs. distance, as the plane wave propagates along the z-axis.

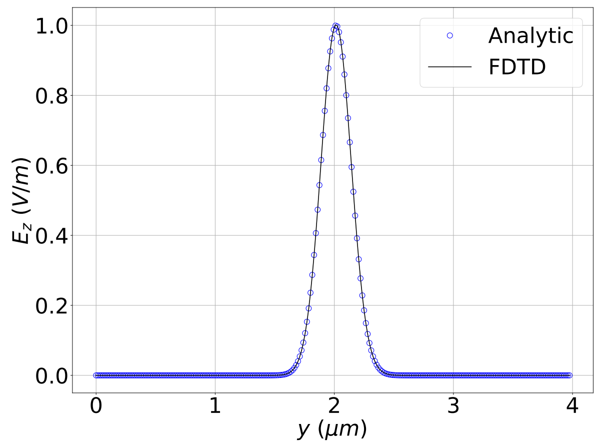

Fig. 2. Simulation results in 2D FDTD. E_z is plotted vs. distance, as the plane wave propagates along the y-axis.

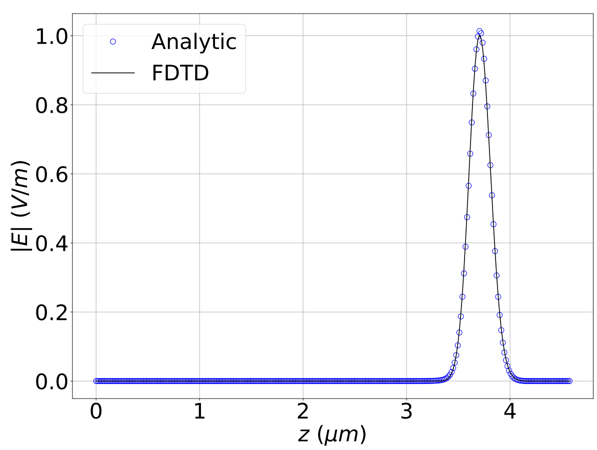

Fig. 3. Simulation results in 3D FDTD. Magnitude of $|E| = \sqrt{E_x^2 + E_y^2 + E_z^2}$ is plotted vs. distance, as the plane wave propagates along the z-axis.

IV. Conclusion

In this work we derived a simple 1D analytic Gaussian function for generating plane waves. The function was used as the incident source in the TF/SF formulation to generate plane wave in 1D, 2D, and 3D FDTD simulations. Results show that plane waves generated by analytic Gaussian expression are virtually indistinguishable from those generated by the 1D FDTD incident array at the magic time-step for 1D, 2D, and 3D FDTD. In addition to its simplicity, the analytic function eliminates the need for computational resources to store and compute 1D E-field and H-field incident arrays and their associated PML arrays.

A topic of future research may be to investigate the extension of the above 1D Gaussian function to 2D and 3D Gaussian functions with arbitrary angles of incidence in 2D and 3D FDTD.

References

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