Optimal 3D-UAV Trajectory and Resource Allocation of DL UAV-GE Links with Directional Antennas

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Abstract— **Unmanned Aerial Vehicle (UAV) is a promising technology to solve many new challenging problems. It provides high maneuverability and control, low manufacturing cost with respect to other flying technologies and many other features. In particular, there is an increasing interest in UAVs in the field of wireless communications, due to their capacity to carry on transceivers and establish communication among UAVs, or between UAVs and ground base stations/users. In this work, we investigate a UAV deployment in which each flying vehicle serves a set of users, carrying directional antennas. To do so, we maximize the minimum downlink rate among the users that a UAV serves. Due to the non-convexity of the problem, we will divide it into four sub-problems. Afterwards, an iterative algorithm is proposed to optimize the four sub-problems by using the block coordinate descent method, successive convex approximation and sequential quadratic programming. Simulation results show that the addition of directional antennas results in a better performance in terms of throughput compared with omni-directional benchmarks.**

Index Terms— **UAV, trajectory optimization, directional antennas, resource allocation, successive convex approximation, sequential quadratic programming, block coordinate descent**

I. INTRODUCTION

Due to the low-cost of producing Unmanned Aerial Vehicles (UAVs), many fields are taking benefit from their features. Their easy deployment, control and maneuverability, make UAV technology a promising tool. In particular, there are many advantages in using UAVs in a communication system. UAVs can be equipped with wireless transceivers allowing them to establish communication with Ground Equipments (GEs), static Base Stations (BSs) or between them. For example, UAVs can act as relay networks for ground users [1] or may be used as mobile base stations to multicast information [2]. Other promising approaches use UAVs to collect data from ground sensors [3] or to offload ground base stations [4]. The environment and the application determine the static or dynamic nature of the UAV deployment. In this work, we allow UAV movements, i.e., dynamic UAV deployment, and consider optimizing such movements/trajectories. Another advantage of UAV technology is the dominance of the line-of-sight (LoS) channel component [5], making the analysis simpler.

Besides, the UAV deployment problem has significantly attracted the scientific community due to the appearance of 5G and Internet of Things (IoT) scenarios. One of the premises in 5G is the use of higher frequency bands and the availability of more bandwidth. Consequently, the number of users in each cell can be increased at a price of increasing interference. There are many different approaches to mitigate interference in multi-user systems; look at [6] [7] and the references therein. One appealing alternative is the use of directional antennas to avoid the interference to begin with. While there has been some UAV work that utilize directional antennas in the system model, to the best of our knowledge, with the exception of our group's recent work [8], a constant-gain directional antenna channel model has been adopted. However, more realistic directional antenna channel models suggest that the antenna gain within the dominant direction is not constant [9], which may result in complex non-convex optimization problems. To overcome such complexity, we use the successive convex approximation algorithm [10].

In this paper, we study a new downlink (DL) UAV-to-GE system where UAVs are equipped with directional antennas as shown in Fig. 1. The main motivation behind these arrays is that they provide a higher gain and do not create as much interference as omni-directional antennas. In addition, we allow UAVs to share the same frequency band in a multi-user set-up, despite causing interference. We also assume that UAVs serve as mobile base stations and connect to the backbone of a network. We consider the design of the downlink where UAVs transmit data to different users. To maintain fairness in the network, we maximize the minimum user rate as the main objective of our work.

The rest of the paper is organized as follows: We introduce a realistic and mathematically tractable model for our problem in terms of channel and directional antennas in Section II. We formulate a constraint optimization problem that takes into account physical features like power and UAV movement characteristics in Section III. In Section IV, we propose a solution to the resulting non-convex optimization problem. Furthermore, we provide numerical results in Section V.

II. SYSTEM MODEL

Fig. 1 depicts a system with M UAVs carrying directional antennas. Let us assume that the UAVs' ground projections are given by $\boldsymbol{Q}(t) = \{\boldsymbol{q}_m(t) \in \mathbb{R}^2\ ,\ m = 1, 2, ...M\ ,\ t \in [0, T]\}$ and fly at altitudes $\boldsymbol{H}(t) = \{h_m(t) \in \mathbb{R}\ ,\ m = 1, 2, ...M\ ,\ t \in [0, T]\}$, where T represents the flying time of the UAVs. While UAVs with directional antennas have been studied in the literature to improve the performance, in most cases, the antenna gain is assumed to be a constant within

*This work was supported in part by the NSF Award CCF-1815339.

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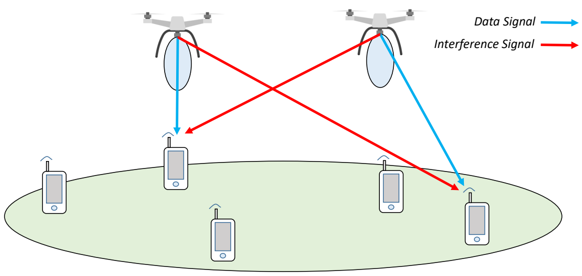

Fig. 1. Multi-UAV scenario with directional antennas.

the 3dB beamwidth and 0 otherwise. Such a model ignores the strong dependency between the gain and the LoS angle [8], which is taken into account in this work.

To overcome the difficulty of an optimization problem whose variables are continuous in time, we discretize the time index. To simplify the problem, the time horizon T is equally divided into N time slots, such that $T = \delta N$. We also introduce the discrete-time index $n = \frac{t}{\delta}$. To ensure the accuracy of the time quantization, the following inequality must be met: $\max\{V_{m,q}\delta\ ,\ V_{m,h}\delta\} << H_{min}$ where $V_{m,q}$ and $V_{m,h}$ represent the maximum velocity in the horizontal and vertical axis, respectively, and H_{min} is the minimum UAV altitude. As such, the continuous-time representation of the horizontal and vertical UAV trajectory, $\boldsymbol{q}_m(t)$ and $h_m(t)$, can be approximately denoted by $\boldsymbol{q}_m[n]$ and $h_m[n]$. Finally, we can re-write the discrete set of 2-D UAV locations as $\boldsymbol{Q} = \{\boldsymbol{q}_m[n] \in \mathbb{R}^2\ ,\ m = 1, 2, ..., M\ ,\ n = 1, 2, ..., N\}$. Proceeding similarly with the altitude of each UAV, we define the set: $\boldsymbol{H} = \{h_m[n] \in \mathbb{R}\ ,\ m = 1, 2, ..., M\ ,\ n = 1, 2, ..., N\}$. As previously mentioned, in this work we assume all UAVs share the same frequency band, and therefore GEs could suffer from interference as depicted in Fig. 1. Moreover, as we focus on the DL, UAVs serve GEs using a time-division-multiple-access (TDMA) scheme, so that there is a need to introduce a set of variables that determine the UAV-GE association at each time instant. Let us define the following binary variables $\boldsymbol{A} = \{a_{k,m}[n]\ \forall\, k = 1, 2, ..., K\ ,\ m = 1, 2, ..., M\ ,\ n = 1, 2, ..., N\}$ as:

$$a_{k,m}[n] = \begin{cases} 1 & \text{if, at } n\text{, UAV } m \text{ is associated to GE } k \\ 0 & \text{otherwise} \end{cases}$$

To physically ensure that each UAV is only associated to one GE and vice-versa, variables in $\boldsymbol{A}$ must satisfy $\sum_k a_{k,m}[n] \leq 1\, \forall m, n$ and $\sum_m a_{k,m}[n] \leq 1\, \forall k, n$. Without loss of generality, we consider the static position of the GEs as $\boldsymbol{w}_k \in \mathbb{R}^2$. A common channel model approach for these scenarios, is using a free space path-loss model. The reason why many authors use such model is because as shown in [11], air-to-ground channels are mainly dominated by the line-of-sight (LoS) component. Also, both 3GPP and ITU [12] recommend using such air-to-ground models for UAV base stations. Finally, the channel gain between UAV m and GE k at time n, is given by $P_{L,dB} = 10 \log_{10} A - 10\beta \log_{10}\left(\frac{d_{k,m}[n]}{d_0}\right)$ where A is a unit-less constant depending on the antenna characteristics, d_0 is a reference distance, $d_{k,m}[n] = \sqrt{||\boldsymbol{q}_m[n] - \boldsymbol{w}_k||^2 + h_m^2[n]}$ is the Euclidean distance between the m-th UAV and the k-th GE, while $\beta \geq 1$ refers to the terrestrial path-loss exponent. With this, we can express the received power at GE k from UAV m, transmiting at power $p_m[n]$, as: $p_{k,m}[n] = p_m[n] G_m(\boldsymbol{w}_k) G_k\, A\, d_0^{\beta} d_{k,m}^{-\beta}[n]$, where $G_m(\boldsymbol{w}_k)$ and G_k are the antenna gains of the UAV and GE, respectively. Here, we assume perfect omni-directional patterns at the receiver side, $G_k = 1$. On the other hand, we introduce the notion of angle-dependent antenna gains for UAV trajectory optimization problems in the following manner. We model the UAV directional antenna gains as:

$$G_m(\boldsymbol{w}_k) = D_o(r_m) cos^{r_m}(\theta_m(\boldsymbol{w}_k)) = D_0(r_m) \frac{h_m^{r_m}[n]}{d_{k,m}^{r_m}[n]} \quad (1)$$

The model depends on $r_m \geq 1$, which defines the maximal directivity of the antenna at $\theta = 0$ as: $D_0(r_m) = 2(r_m + 1)$ [9]. Note that $r_m = 0$ is the same as having an isotropic antenna. For simplicity, we ignore side-lobes, which are represented by $cos(l\theta)$ patterns. The larger the parameter r_m, the narrower the beam and therefore the directivity of the antenna would increase. That is why if we are interested in covering a precise area, it is better to use narrow beams. To avoid overloading the notation, we define $\rho_0(r_m) = D_0(r_m)\, A\, d_0^{\beta}$ and $\gamma_m = \frac{r_m + \beta}{2}$. Finally, the signal to interference and noise ratio (SINR) of the k-th user, receiving information from the m-th UAV can be calculated as:

$$\Gamma_{k,m}[n] = \frac{\frac{p_m[n]\rho_0(r_m) h_m^{r_m}[n]}{(||\boldsymbol{q}_m[n] - \boldsymbol{w}_k||^2 + h_m^2[n])^{\gamma_m}}}{\sum\limits_{i \neq m} \frac{p_i[n]\rho_0(r_i) h_i^{r_i}[n]}{(||\boldsymbol{q}_i[n] - \boldsymbol{w}_k||^2 + h_i^2[n])^{\gamma_i}} + \sigma^2}\ \forall k, m, n \quad (2)$$

where σ^2 is the noise power at the receiver side, following a circularly symmetric complex Gaussian distribution $\mathcal{CN}(0, \sigma^2)$. Thus, the instantaneous achievable rate of User k is:

$$R_k[n] = \sum_{m=1}^{M} a_{k,m}[n] \log_2(1 + \Gamma_{k,m}[n]) \quad (3)$$

and therefore the average achievable rate over the N time slots is: $\bar{R}_k = \frac{1}{N} \sum\limits_{n=1}^{N} R_k[n]$

III. Problem Formulation

Our aim is to find a 3D-trajectory such that we maximize the minimum average rate to the GEs using directional antennas at the transmitter side. The problem may be subject to different constraints, as the ones related to the velocity of the UAVs or their initial and final positions. In addition, not only we optimize with respect to the set $\boldsymbol{Q}$ and $\boldsymbol{H}$, but also we take into account the user scheduling, i.e., $\boldsymbol{A}$ and the power set, defined by $\boldsymbol{P} = \{p_m[n]\ \forall\, m, n\}$. Therefore, the overall optimization problem (P1) can be formulated as follows:

$$\underset{\eta, \boldsymbol{A}, \boldsymbol{Q}, \boldsymbol{H}, \boldsymbol{P}}{\text{maximize}} \quad \eta \tag{4a}$$

$$\text{subject to} \quad \bar{R}_k \geq \eta \quad \forall k \tag{4b}$$

$$\sum_k a_{k,m}[n] \leq 1 \; \forall m, n \tag{4c}$$

$$\sum_m a_{k,m}[n] \leq 1 \; \forall k, n \tag{4d}$$

$$0 \leq a_{k,m}[n] \leq 1 \; \forall m, n, k \tag{4e}$$

$$p_m[n] \leq P_{max} \; \forall m, n \tag{4f}$$

$$||\boldsymbol{q}_m[n+1] - \boldsymbol{q}_m[n]||^2 \leq (V_{m,q}\delta)^2 \;\; \forall m, n \tag{4g}$$

$$||h_m[n+1] - h_m[n]||^2 \leq (V_{m,h}\delta)^2 \; \forall m, n \tag{4h}$$

$$\boldsymbol{q}_m[1] = \boldsymbol{q}_m[N] \qquad \forall m \tag{4i}$$

$$h_m[1] = h_m[N] \qquad \forall m \tag{4j}$$

$$||\boldsymbol{q}_m[n] - \boldsymbol{q}_j[n]||^2 \geq d_{min}^2 \; \forall \, n, m, j \neq m \tag{4k}$$

$$H_{min} \leq h_m[n] \leq H_{max} \quad \forall \, n, m \tag{4l}$$

As previously mentioned, we aim to maximize the minimum average rate among all users. The combination of (4a) and (4b) achieves that goal. (4c) and (4d) take into account the one-to-one mapping between a UAV and a GE. Constraint (4e) is a relaxed version of the initial binary assumption made for $\boldsymbol{A}$. Such a relaxation simplifies the problem formulation significantly. Once the relaxed problem is solved, we can recover the binary values as thoroughly discussed in the literature [13]. In addition, (4f) guarantees that UAVs do not transmit beyond their maximum power. We have also added realistic constraints related to the trajectory. Constraints (4g) and (4h) represent the maximum velocity of the UAVs along the horizontal and vertical planes, respectively. Constraints (4i) and (4j) enforce the initial and final positions of the UAVs to be the same, while (4k) prevents from the collision between UAVs, as guarantees a minimum safety distance. Finally, (4l) determines the range for the UAV altitudes.

The addition of directional antennas makes the optimization problem even more challenging. (P1) is highly non-convex because of constraints (4b) (both with respect to $\boldsymbol{q}_m[n]$, $h_m[n]$ and $p_m[n]$), (4g) and (4k) (with respect to $\boldsymbol{q}_m[n]$). To solve it, we decompose the problem into four sub-problems: (i) scheduling optimization with fixed UAV trajectory, altitude and transmit power (ii) UAV trajectory optimization with fixed scheduling, altitude and transmit power (iii) UAV altitude optimization with fixed scheduling, trajectory and transmit power (iv) power optimization with fixed scheduling, UAV trajectory and altitude. Once the solution of each problem is obtained separately, we apply a block coordinate descent method to iteratively maximize the minimum user rate until convergence [14].

IV. Proposed Algorithm

A. Scheduling Optimization

First, we solve the UAV-GE association problem, given by the set of variables $\boldsymbol{A}$. For fixed $\boldsymbol{Q}$, $\boldsymbol{H}$ and $\boldsymbol{P}$, the UAV-GE association problem (P2) can be formulated as:

$$\underset{\eta, \boldsymbol{A}}{\text{maximize}} \quad \eta \tag{5a}$$

subject to (4b), (4c), (4d) and (4e). Since the objective function and the constraints are linear with respect to the optimization variables, we can efficiently solve it using standard linear programming (LP) techniques, such as the interior point method [15].

B. UAV Trajectory Optimization

For any fixed $\boldsymbol{A}$, $\boldsymbol{H}$ and $\boldsymbol{P}$, the UAV trajectories $\boldsymbol{q}_m[n]$ can be optimized solving the following problem:

$$\underset{\eta, \boldsymbol{Q}}{\text{maximize}} \quad \eta \tag{6a}$$

subject to (4b), (4g), (4i) and (4k). Since constraints (4b) and (4k) are non-convex, there is no efficient way to optimally solve it. That is why we resort to Successive Convex Approximation (SCA) techniques [10]. SCA methods alternate between two steps: (i) approximate the original non-convex functions by its first-order Taylor expansion and (ii) find the optimal solution of the approximated convex problem.

At this point, we decompose the instantaneous rate, $R_{k,m}[n]$, as the difference between the following two terms:

$$R_{k,m}[n] = \hat{R}_{k,m}[n] - \tilde{R}_{k,m}[n] \tag{7}$$

where:

$$\hat{R}_{k,m}[n] = \log_2\left(\sum_{i=1}^{M} \frac{p_i[n]\rho_0(r_i)h_i^{r_i}[n]}{(||\boldsymbol{q}_i[n] - \boldsymbol{w}_k||^2 + h_i^2[n])^{\gamma_i}} + \sigma^2\right) \tag{8}$$

and

$$\tilde{R}_{k,m}[n] = \log_2\left(\sum_{i\neq m}^{M} \frac{p_i[n]\rho_0(r_i)h_i^{r_i}[n]}{(||\boldsymbol{q}_i[n] - \boldsymbol{w}_k||^2 + h_i^2[n])^{\gamma_i}} + \sigma^2\right) \tag{9}$$

Although the term $\hat{R}_{k,m}[n]$ is non-convex with respect to $\boldsymbol{q}_m[n]$, it is convex with respect to $||\boldsymbol{q}_m[n] - \boldsymbol{w}_k||^2$. Therefore, we can apply the fact that any convex function is lower-bounded by its first-order Taylor expansion at any point of its domain [10]. By doing so, we obtain a lower bound for $\hat{R}_{k,m}[n]$ around $||\boldsymbol{q}_i^p[n] - \boldsymbol{w}_k||^2$ as follows:

$$\hat{R}_{k,m}[n] \geq \sum_{i=1}^{M} -A_{k,i}^p[n]\Big(||\boldsymbol{q}_i[n] - \boldsymbol{w}_k||^2 - ||\boldsymbol{q}_i^p[n] - \boldsymbol{w}_k||^2\Big) + B_{k,m}^p[n] = \hat{R}_{k,m}^{lb}[n] \tag{10}$$

where coefficients $A_{k,i}^p[n]$ and $B_{k,m}^p[n]$ are:

$$A_{k,i}^p[n] = \frac{\frac{(\,p_i[n]\rho_0(r_i)h_i^{r_i}[n]\,)\gamma_i}{(||\boldsymbol{q}_i^p[n] - \boldsymbol{w}_k||^2 + h_i^2[n])^{\gamma_i+1}}}{\left(\sum\limits_{j=1}^{M} \frac{p_j[n]\rho_0(r_j)h_j^{r_j}[n]}{(||\boldsymbol{q}_j^p[n] - \boldsymbol{w}_k||^2 + h_j^2[n])^{\gamma_j}} + \sigma^2\right)} \log_2(e) \tag{11}$$

and

$$B^p_{k,m}[n] = \log_2\left(\sum_{i=1}^{M} \frac{p_i[n]\rho_0(r_i)h_i^{r_i}[n]}{(||\boldsymbol{q}_i^p[n]-\boldsymbol{w}_k||^2 \,+\, h_i^2[n])^{\gamma_i}} \,+\, \sigma^2\right) \tag{12}$$

Note that $\hat{R}^{lb}_{k,m}[n]$ is a concave function of UAV positions. For $\tilde{R}_{k,m}[n]$, by introducing the following slack variables $\boldsymbol{S} = \{S_{k,i}[n] = (||\boldsymbol{q}_i[n]-\boldsymbol{w}_k||^2 + h_i^2[n])^{\gamma_i}\ \forall n,\ i \neq m\}$, we obtain a more tractable constraint by making $\tilde{R}_{k,m}[n]$ convex with respect to $S_{k,i}[n]$.

$$\tilde{R}_{k,m}[n] = \log_2\Big(\sum_{i\neq m}^{M} \frac{p_i[n]\rho_0(r_i)h_i^{r_i}[n]}{S_{k,i}[n]} \,+\, \sigma^2\Big)\ \ \forall k \tag{13}$$

As a result, a new non-convex constraint appears: $S_{k,i}[n] \leq (||\boldsymbol{q}_i[n]-\boldsymbol{w}_k||^2 \,+\, h_i^2[n])^{\gamma_i}\ \forall\, n,\ i \neq m$. It can be verified, without loss of optimality, that all constraints in the previous inequality can be met with equality. Applying SCA to the new non-convex constraint and to (4k), the final approximated problem (P3) can be re-written as:

$$\begin{aligned}
\underset{\eta, \boldsymbol{Q}, \boldsymbol{S}}{\text{maximize}}\quad & \eta \\
\text{subject to}\quad & \frac{1}{N}\sum_{\forall m,n} a_{k,m}[n]\Bigg(\hat{R}^{lb}_{k,m}[n] - \\
& \log_2\Big(\sum_{i\neq m}^{M}\frac{p_i[n]\rho_0(r_i)}{S_{k,i}[n]} \,+\, \sigma^2\Big)\Bigg) \geq \eta\ \ \forall k \\
& S_{k,i}[n] \leq (||\boldsymbol{q}_i^p[n]-\boldsymbol{w}_k||^2 \,+\, h_i^2[n])^{\gamma_i} + \\
& \quad 2\gamma_i(||\boldsymbol{q}_i^p[n]-\boldsymbol{w}_k||^2 \,+\, h_i^2[n])^{\gamma_i-1} \\
& \quad (\boldsymbol{q}_i^p[n]-\boldsymbol{w}_k)^T(\boldsymbol{q}_i[n]-\boldsymbol{q}_i^p[n])\ \ \forall i \neq m \\
& d_{min}^2 \leq 2(\boldsymbol{q}_m^p[n]-\boldsymbol{q}_j^p[n])^T(\boldsymbol{q}_m[n]-\boldsymbol{q}_j[n]) - \\
& \quad ||\boldsymbol{q}_m^p[n] \,-\, \boldsymbol{q}_j^p[n]||^2 \quad \forall\, n, m, j \neq m \\
& ||\boldsymbol{q}_m[n+1] \,-\, \boldsymbol{q}_m[n]||^2 \leq (V_{m,q}\delta)^2\ \ \forall m, n \\
& \boldsymbol{q}_m[1] \,=\, \boldsymbol{q}_m[N] \quad \forall m
\end{aligned} \tag{14}$$

It can be verified that both the objective function and the constraints are convex; thus, (P3) can be solved by standard optimization solvers. Since we obtain a lower bound on the original problem, the solution to (P3) is also a solution to the original non-convex problem.

C. Altitude Optimization

Next, we consider the height optimization. For any fixed $\boldsymbol{A}$, $\boldsymbol{Q}$ and $\boldsymbol{P}$, the UAV altitudes $h_m[n]$ can be optimized solving the following problem:

$$\underset{\eta, \boldsymbol{H}}{\text{maximize}}\quad \eta \tag{15a}$$

subject to (4h), (4j) and (4l). We propose to approach this problem by means of the so-called Sequential Quadratic Programming (SQP). The objective function and the constraints are three times continuously differentiable in their domain, and therefore, we can compute the Gradient and Hessian with respect to the altitudes. The SQP method is an iterative way to solve non-convex problems in which the solution for the next iteration is obtained by means of solving a quadratic problem. Apart from being easy to solve, SQP can reflect the non-linearities of the original problem. An important issue in SPQ methods is the choice of the appropriate quadratic functions. To take into account non-linearities, SQP methods use a quadratic model of the Lagrangian function as the objective. First, we define $\boldsymbol{\theta} = [\eta\ h_1[1]\ h_1[2], \ldots, h_1[N]\ h_2[1], \ldots, h_2[N], \ldots, h_M[N]]^T$. Then, the Lagrangian can be written as:

$$\begin{aligned}
\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\lambda}, \boldsymbol{\nu}) \,=\, & \eta + \sum_{k=1}^{K}\lambda_k\Bigg(\frac{1}{N}\sum_{n=1}^{N}\sum_{m=1}^{M} a_{k,m}[n]\log_2\Bigg(1 + \\
& + \frac{\frac{p_m[n]\rho_0(r_m)h_m^{r_m}[n]}{(||\boldsymbol{q}_m[n]-\boldsymbol{w}_k||^2 + h_m^2[n])^{\gamma_m}}}{\sum\limits_{j\neq m}\frac{p_j[n]\rho_0(r_j)h_j^{r_j}[n]}{(||\boldsymbol{q}_j[n]-\boldsymbol{w}_k||^2 + h_j^2[n])^{\gamma_j}} \,+\, \sigma^2}\Bigg) - \eta\Bigg) + \\
& \sum_{m=1}^{M}\sum_{n=1}^{N}\lambda_{(m-1)N+n+K}\Big((V_{m,h}\delta)^2 - ||h_m[n+1] \,-\, h_m[n]||^2\Big) \\
& + \sum_{m=1}^{M}\sum_{n=1}^{N}\lambda_{(m-1)N+n+K+MN}\big(H_{max} - h_m[n]\big) \\
& + \sum_{m=1}^{M}\sum_{n=1}^{N}\lambda_{(m-1)N+n+K+2MN}\big(h_m[n] - H_{min}\big) \\
& + \sum_{p=1}^{M}\nu_p\big(h_p[N] - h_p[1]\big)
\end{aligned} \tag{16}$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$ refer to the inequality and equality Lagrangian multipliers. Note that the original problem, (15a), and maximizing $\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)$ with respect to $\boldsymbol{\theta}$ subject to (4h) and (4j) are equivalent problems, where the symbol * denotes optimal. Since we do not know the value of the multipliers, estimating $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$ can be included in the iterative process. Finally, the second order Taylor expansion of the Lagrangian around $\boldsymbol{\theta}^p$ is given by:

$$\begin{aligned}
\mathcal{L}^Q(\boldsymbol{\theta}, \boldsymbol{\lambda}, \boldsymbol{\nu}) \approx\, & \mathcal{L}(\boldsymbol{\theta}^p, \boldsymbol{\lambda}^p, \boldsymbol{\nu}^p) \,+\, \nabla\mathcal{L}_{\boldsymbol{\theta}}(\boldsymbol{\theta}^p, \boldsymbol{\lambda}^p, \boldsymbol{\nu}^p)(\boldsymbol{\theta}-\boldsymbol{\theta}^p) \\
& + \frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\theta}^p)^T\boldsymbol{H}_{\mathcal{L}_{\boldsymbol{\theta}}}(\boldsymbol{\theta}^p, \boldsymbol{\lambda}^p, \boldsymbol{\nu}^p)(\boldsymbol{\theta}-\boldsymbol{\theta}^p)
\end{aligned}$$

where $\boldsymbol{H}_{\mathcal{L}_{\boldsymbol{\theta}}}$ refers to the Hessian of the Lagrangian with respect to the parameter vector $\boldsymbol{\theta}$. To avoid overloading with formulas, we call $g_i(\boldsymbol{\theta})\ i \,=\, 1, 2, \ldots, K+3MN$ and $h_j(\boldsymbol{\theta})\ j \,=\, 1, 2, \ldots, M$ the inequality and equality constrained functions associated to their respective multiplier. Then, our new QP problem, called (P4), is:

$$\begin{aligned}
\underset{\boldsymbol{\theta}}{\text{maximize}}\quad & \mathcal{L}^Q(\boldsymbol{\theta}, \boldsymbol{\lambda}, \boldsymbol{\nu}) \\
\text{subject to}\quad & \nabla g_i(\boldsymbol{\theta}^p)(\boldsymbol{\theta}-\boldsymbol{\theta}^p) + g_i(\boldsymbol{\theta}^p) \geq 0\ \ , 1 \leq i \leq K+3MN \\
& \nabla h_j(\boldsymbol{\theta}^p)(\boldsymbol{\theta}-\boldsymbol{\theta}^p) + h_j(\boldsymbol{\theta}^p) \,=\, 0\,, 1 \leq j \leq M
\end{aligned} \tag{17}$$

For (17) to be efficiently solved by standard optimization solvers, we actually need a PSD Hessian. In this work, we use the BFGS method to create a positive definite quasi-Newton matrix approximation of the Hessian [16]. Authors in [17] provide more details on the BFGS method and a convergence analysis of the SQP methods.

D. Power Optimization

Last but not least, there is a need to optimize with respect to the transmit power of the UAVs. For any fixed $\boldsymbol{A}$, $\boldsymbol{Q}$ and $\boldsymbol{H}$, the power optimization problem can be written as:

$$\underset{\eta, \boldsymbol{Q}}{\text{maximize}} \quad \eta \tag{18a}$$

subject to (4b) and (4f). Again, to tackle the non-convexity of the instantaneous rate with respect to the power, we resort to SCA. Proceeding in a similar manner as for the trajectory optimization problem, we obtain the following approximated convex problem (P5):

$$\begin{aligned} &\underset{\eta, \boldsymbol{P}}{\text{maximize}} \quad \eta \\ &\text{subject to} \quad \frac{1}{N}\sum_{n,m} a_{k,m}[n]\Big(\hat{R}_{k,m}[n] - \tilde{R}^{ub}_{k,m}[n]\Big) \geq \eta \; \forall k \\ &\qquad\qquad\quad p_m[n] \leq P_{max} \; \forall m, n \end{aligned} \tag{19}$$

where we have approximated $\tilde{R}_{k,m}[n]$ by its first order upper bound due to its concavity with respect to $p_m[n]$:

$$\tilde{R}_{k,m}[n] \leq \sum_{i \neq m}^{M} E^p_{k,i}[n]\Big(p_i[n] - p^p_i[n]\Big) + F^p_{k,m}[n] = \tilde{R}^{ub}_{k,m}[n] \tag{20}$$

where:

$$E^p_{k,i}[n] = \frac{\frac{\rho_0(r_i)h_i^{r_i}[n]}{(||\boldsymbol{q}_i[n]-\boldsymbol{w}_k||^2 + h_i^2[n])^{\gamma_i}}}{\Big(\sum\limits_{l \neq m}^{M} \frac{p^p_l[n]\rho_0(r_l)h_l^{r_l}[n]}{(||\boldsymbol{q}_l[n]-\boldsymbol{w}_k||^2 + h_l^2[n])^{\gamma_l}} + \sigma^2\Big)} \log_2(e) \tag{21}$$

and

$$F^p_{k,m}[n] = \log_2\left(\sum_{i \neq m}^{M} \frac{p^p_i[n]\rho_0(r_i)h_i^{r_i}[n]}{(||\boldsymbol{q}_i[n] - \boldsymbol{w}_k||^2 + h_i^2[n])^{\gamma_i}} + \sigma^2\right) \tag{22}$$

Once the solutions to the four sub-problems are obtained, we alternatively optimize them until some convergence criterion is met, given the convergence analysis provided in [13] and [17].

V. Numerical Results

In this section, we provide numerical results evaluating the performance of our proposed algorithms. To justify the use of directional antennas instead of isotropic patterns, we first consider a Single-UAV scenario. Our aim is to show that before extending the problem to a Multi-UAV scenario, we are capable of outperforming the rates that isotropic patterns provide to the GEs. To do so, we first consider a scenario with $K = 6$ GEs randomly generated in a $1000 \times 1000\, m^2$ area. The path loss exponent is set to $\beta = 2$, and therefore $\gamma_1 = \frac{r_1+2}{2}$. We fix $d_0 = -30$ dB and $\sigma^2 = -110$ dBm with a $P_{max} = 0.1$W and $A = 1$. To maintain fairness in the comparison, we fix the altitude to the same value $h_1[n] = 100$m $\forall n$. The maximum UAV velocity is set to $V_{m,q} = 50$ m/s and we sample every $\delta = 0.2$s. To initialize the algorithm and define $\boldsymbol{q}^p_1[n]$, we first calculate the mass center of the users $\boldsymbol{c}_K = \frac{1}{K}\sum \boldsymbol{w}_k \; \forall n$. Afterwards, we distribute each point in $\boldsymbol{q}^p_1[n]$ around a circle of radius $\sqrt{V_{m,q}\delta}$ and center $\boldsymbol{c}_K$. Fig. 2 provides the rates for different UAV antenna setups and flying time (T) as a function of the number of iterations, k. A larger T results in a higher rate. The main message of the Single-UAV scenario is that after convergence, the use of directional patterns, e.g. $r \geq 1$, can greatly increase the rates compared with omni-directional antenna UAV systems ($r = 0$).

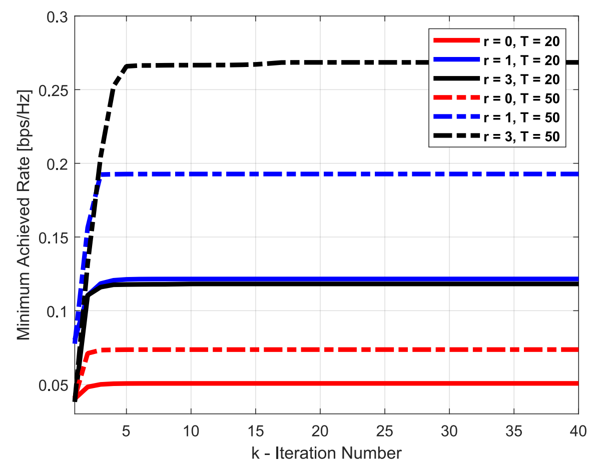

Fig. 2. Minimum achieved rates as a function of the Iteration number for different values of T and r, in a Single-UAV scenario with 6 GEs.

We also provide simulation results for $M = 2$ UAVs, where the altitude optimization problem is taken into account. We select $H_{min} = 100$m, $H_{max} = 300$m and $K = 6$ GEs in the same squared area as the previous case. We set the minimum safety distance between UAVs to $d_{min} = 20$m. For both UAVs, we set $r_i = 2$ and the flying time to $T = 20$s. Again, $P_{max} = 0.1$W, $A = 1$, $d_0 = -30$ dB, $V_{m,q} = V_{m,h} = 50$ m/s and $\sigma^2 = -110$ dBm. To initialize the altitudes, we set them to $h_m[n] = \frac{H_{min}+H_{max}}{2} \; \forall n, m$. For the initial trajectories, we first run K-Means to obtain M centroids. After computing the covariance matrix of each cluster, we create the ellipsoids associated to a Mahalanobis radius of $s = -2\log(1-p)$, where we select $p = 0.3$, to construct the initial ellipsoidal trajectories. Fig. 3 shows the initial trajectories for both UAVs (dashed-red and dashed-blue) and the ones obtained after iteratively solving the four sub-problems (solid-red and solid-blue). We also include the union of the -3 dB coverage areas at each time, taking into account the optimal altitude and power obtained at that time. As a result of trajectory optimization, UAVs choose paths and heights to avoid interfering with each other. We present the time index of some trajectory points as a reference to indicate how the two UAVs coordinately move to increase their distance and reduce the interference. The solution to the optimal altitudes provides more insight on how UAVs should move. Fig. 4 demonstrates the tendency to fly at H_{min} when being near a GE. As a consequence, the rate is increased as the antenna is pointing directly to the user under the UAV. However, during transitions between GEs, both UAVs tend to increase their altitudes and therefore increase the beamwidth/coverage areas to provide service to more than one GE. In fact, the minimum rate we obtain when including the altitude optimization problem is: $\eta = 0.204$ bps/Hz whereas if UAVs stay at an altitude of 200m, $\eta = 0.105$ bps/Hz, meaning that the use of directional antennas with 3D-trajectory optimization reduces interference without compromising the rate. We also include Fig. 5, showing the 2D-velocity variation of each UAV, respectively. Fig. 5 shows that UAVs tend to decrease their speed at some intervals. Those intervals correspond to the times where the UAV is near a GE and therefore it spends more time flying around to provide more service. After that, UAVs increase their velocity, up to $V_{max,q}$, to reach the next GE as fast as possible.

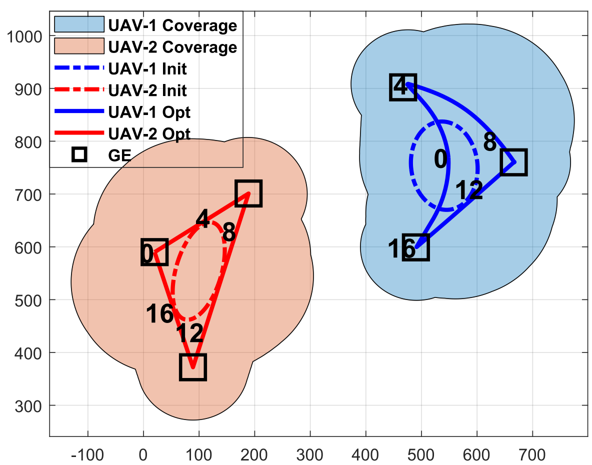

Fig. 3. 2-UAV 6-GE scenario with the covered areas by each UAV.

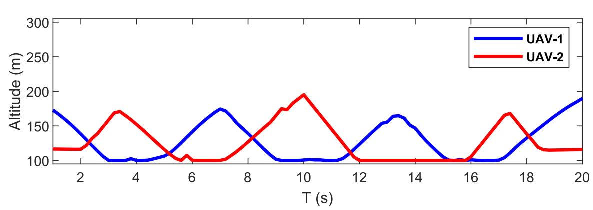

Fig. 4. UAV altitudes corresponding to the optimal solution in Fig. 3.

VI. Conclusion

This paper has introduced the notion of UAV trajectory optimization with directional antennas. By exploiting the UAV mobility, a more general formulation has been proposed to maximize the minimum rate GEs can achieve. We have included the solution with respect to the scheduling, 3D-trajectory and power, which are iteratively optimized by means of block coordinate methods, SCA and SQP. Simulation results show that the trajectory with directional patterns outperforms that of using isotropic antennas in terms of supported rate.

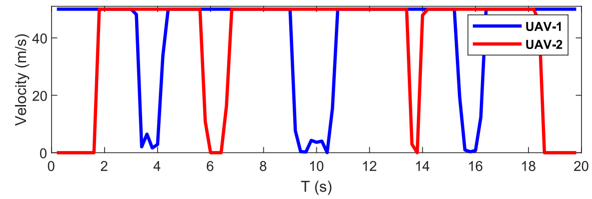

Fig. 5. UAV velocities corresponding to the optimal solution in Fig. 3.

References

[1] P. Zhan, K. Yu, and A. L. Swindlehurst, "Wireless Relay Communications with Unmanned Aerial Vehicles: Performance and Optimization," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 47, pp. 2068–2085, Jul. 2011.

[2] Y. Zeng, X. Xu, and R. Zhang, "Trajectory Design for Completion Time Minimization in UAV-Enabled Multicasting," *IEEE Transactions on Wireless Communications*, vol. 17, pp. 2233–2246, Apr. 2018.

[3] E. Koyuncu, M. Shabanighazikelayeh, and H. Seferoglu, "Deployment and Trajectory Optimization of UAVs: A Quantization Theory Approach," *IEEE Wireless Communications and Networking Conference (WCNC)*, Jun. 2018.

[4] F. Cheng, S. Zhang, Z. Li, Y. Chen, N. Zhao, F. R. Yu, and V. C. Leung, "UAV Trajectory Optimization for Data Offloading at the Edge of Multiple Cells," *IEEE Transactions on Vehicular Technology*, vol. 67, pp. 6732–6736, Jul. 2018.

[5] A. A. Khuwaja, Y. Chen, N. Zhao, M. S. Alouini, and P. Dobbins, "A Survey of Channel Modeling for UAV Communications," *IEEE Communications Surveys and Tutorials*, vol. 20, pp. 2804–2821, Oct. 2018.

[6] J. Kazemitabar and H. Jafarkhani, "Performance Analysis of Multiple Antenna Multi-user Detection," in *2009 Information Theory and Applications Workshop*, pp. 150–159, Jan. 2009.

[7] E. Koyuncu and H. Jafarkhani, "Distributed Beamforming in Wireless Multiuser Relay-Interference Networks With Quantized Feedback," *IEEE Transactions on Information Theory*, vol. 58, pp. 4538–4576, Jul. 2012.

[8] J. Guo, P. Walk, and H. Jafarkhani, "Optimal Deployments of UAVs With Directional Antennas for a Power-Efficient Coverage," *IEEE Transactions on Communications*, vol. 68, pp. 5159–5174, Aug. 2020.

[9] C. A. Balanis, *Antenna Theory : Analysis and Design*. Wiley, 4th ed., 2016.

[10] M. Diehl, F. Glineur, E. Jarlebring, and W. Michiels, *Recent Advances in Optimization and its Applications in Engineering*. Springer Berlin Heidelberg, 2010.

[11] "LTE Unmanned Aircraft Systems Trial Report," tech. rep., Qualcomm, San Diego, CA, USA, 2017.

[12] "Guidelines for evaluation of radio interface technologies for IMT-2020 M Series Mobile, radiodetermination, amateur and related satellite services," tech. rep., ITU-R, Oct. 2017.

[13] Q. Wu, Y. Zeng, and R. Zhang, "Joint Trajectory and Communication Design for Multi-UAV Enabled Wireless Networks," *IEEE Transactions on Wireless Communications*, vol. 17, pp. 2109–2121, Mar. 2018.

[14] M. Hong, M. Razaviyayn, Z.-Q. Luo, and J.-S. Pang, "A Unified Algorithmic Framework for Block-Structured Optimization Involving Big Data," *arXiv:1511.02746v1*, Nov. 2015.

[15] S. P. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge University Press, 2004.

[16] R. Fletcher, *Practical Methods of Optimization*. Wiley, 2nd ed., 2013.

[17] P. T. Boggs and J. W. Tolle, "Sequential Quadratic Programming," *Acta numerica*, vol. 4, pp. 1–51, Jan. 1995.