Wideband RFI Cancellation Using True Time Delays and a Hadamard Projection Operator

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Abstract—**Radio frequency interference (RFI) in a devastating problem for high-sensitivity phased arrays. This paper explores a method of mitigating RFI in a receiving array using a combination of true-time delay with a truncated Hadamard projection that can place a wide-band spatial null over the RFI. The operations involved can be performed with analog circuity before sampling for the digital signal processing engine in order to enhance dynamic range. The modified beamformer solution is briefly derived and performance is compared to the existing maximum SIR beamformer using analytical phasor domain models. The results show successful null placement at the expense of control of the main lobe shape and side lobe levels.**

I. MOTIVATION

Radio frequency interference (RFI) is a serious problem for passive spectrum users in remote sensing and radio astronomy. Time and frequency blanking, beam pattern nulling and spatial filtering, adaptive filters, and many other methods have been used for RFI mitigation. Many of these methods are inherently narrowband or lead to data loss, so new methods are needed for dealing with wideband RFI. An architecture was proposed in [1] the reported a high degree of RFI mitigation using a novel architecture comprised of a true time delay (TTD) stage [2] followed by a truncated Hadamard matrix projection operator. The combination of both techniques resembles wide-band implementation of the subspace rotation and null-projection techniques [3] familiar in the array signal processing literature. The key benefit of this architecture for high sensitivity applications is that moving traditionally digital domain operations to the analog circuitry buys higher dynamic range for the analog to digital converter.

The work in [1] originally reported results in the context of MIMO operation, but opened up questions if the TTD and truncated Hadamard projection (here on referred to as "the cancellation technique" for convenience) could be viable for high sensitivity phased array technology. This paper investigates the cancellation technique by comparing its performance to the well known maximum signal-to-interference ratio (SIR) digital beamformer [3]. The SIR beamformer is a special case of the general class of maximum signal-to-noise ratio (SNR) beamformer that incorporates statistics of the interferer.

In the cancellation technique, the TTD is designed to create coherence in signals arriving in the direction of RFI. After time aligning the RFI, the RFI appears like a common mode bias to each port. The truncated Hadamard projection matrix then removes the common bias among all ports to cancel the RFI [4] [5]. The cancellation technique may be intuited from the perspective of the subspace projection operator [3] familiar in array signal processing. The true time delay is a wide-band rotation operation of the spatial/time basis vectors of the incoming signal, and the Hadamard projection operator is changing to a $N-1$ rank basis with null space equal to the common mode of the N ports.

Removing the common mode imposes a significant transformation that must be corrected with a secondary digital beamformer. The secondary beamformer is tuned to produce coherence for the signal of interest (SOI) after the cancellation transformation. The purpose of this study is to evaluate the quality of the reconstruction in the secondary beamformer after applying the RFI cancellation.

II. SYSTEM MODELING

The truncated Hadamard matrix $\mathbf{H}_p$ is the 2^N-1 by 2^N traditional Hadamard matrix sans the top row of ones. For example, the matrix H_3 derived from a length 4 Hadamard matrix is the following.

$$\begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \tag{1}$$

In this study, a 8 by 8 square array is projected from rank 64 to to rank 63 space by the matrix $\mathbf{H}_{63}$. The loss in rank represents a projection onto a null space of the full rank system.

The null space of $\mathbf{H}_p$ is the common mode signals shared by all the ports. The balanced number of ±1 in each row mean any shared signal common to each port is removed on average by the balanced additions and subtraction along each row of the projection matrix.

A TTD circuit is one which implements the transformation $s(t) \rightarrow s(t+\Delta)$ for some small time delay Δ on some time domain waveform $s(t)$. The time delays for port n are depended on array geometry, and are designed to align signals from a given direction of arrival (DoA). When implemented

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with an analog circuit at baseband, the local oscillator phase of the mixer must also be compensated in the down-conversion.

The TTD and $\mathbf{H}_p$ can be modeled as linear transformations on the phasor voltages in an array response column vector $\mathbf{v}$ [3] at any frequency of interest. Although the TTD technique is inherently a wide-band operation, a collection of narrow band analysis vectors taken at the center and extremes of the operating band provides insight to the performance of the operator and leads to analytic solutions. These linear operators allow easy modification to the maximum SIR beamformer [3] to incorporate the transformations.

$$\mathbf{w}_{\text{max SIR}} = \arg\max_{\mathbf{w}} \frac{\mathbf{w}^{\mathrm{H}} \mathbf{R}_{\text{signal}} \mathbf{w}}{\mathbf{w}^{\mathrm{H}} \left(\mathbf{R}_{\text{noise}} + \mathbf{R}_{\text{interferer}} \right) \mathbf{w}} \tag{2}$$

$$\mathbf{w}_{\text{cancellation}} = \arg\max_{\mathbf{w}} \frac{\mathbf{w}^{\mathrm{H}} \mathbf{H}_{\mathrm{p}} \mathbf{T} \mathbf{R}_{\text{signal}} \mathbf{T}^{\mathrm{H}} \mathbf{H}_{\mathrm{p}}^{\mathrm{H}} \mathbf{w}}{\mathbf{w}^{\mathrm{H}} \mathbf{H}_{\mathrm{p}} \mathbf{T} \left(\mathbf{R}_{\text{noise}} + \mathbf{R}_{\text{interferer}} \right) \mathbf{T}^{\mathrm{H}} \mathbf{H}_{\mathrm{p}}^{\mathrm{H}} \mathbf{w}} \tag{3}$$

where $\mathbf{T} = \operatorname{diag}\left(\exp(j\omega\Delta_n)\right)$ is diagonal matrix of true time delay phase shifts for phasor frequency ω and delay of the n^{th} antenna Δ_n, $\mathbf{H}_p$ is the truncated Hadamard projection, $\mathbf{R}$ is the port correlation matrix for the signal of interest, noise, and interferer respectively.

These quadratic form optimizations are typically solved with largest λ eigen solution to the generalized eigenvalue problem $\mathbf{wA} = \lambda\mathbf{wB}$ where $\mathbf{A} = \mathbf{H}_{\mathrm{p}}\mathbf{T}\mathbf{R}_{\text{signal}}\mathbf{T}^{\mathrm{H}}\mathbf{H}_{\mathrm{p}}^{\mathrm{H}}\mathbf{w}$ and $\mathbf{B} = \mathbf{H}_{\mathrm{p}}\mathbf{T}\mathbf{R}_{\text{noise}}\mathbf{T}^{\mathrm{H}}\mathbf{H}_{\mathrm{p}}^{\mathrm{H}}$.

III. SIMULATION RESULTS

With the analytic models in Equations 2 and 3, a map of the receiver array gain can be formed for a given field-of-view. This involves scanning the array response vectors for each pixel in the field of view, forming rank 1 correlation matrices $\mathbf{R} = \mathbf{v}\mathbf{v}^{\mathrm{H}}$ (remembering the transformations in the cancellation model) and evaluating the power under the beamformer weights and normalizing for gain.

Two gain slices are shown in Figure 1 for an 8 x 8 element array of isotropic elements spaced half-wavelength for a center frequency of 1.55 GHz. The noise field is assumed uniform. Mutual coupling is not modeled. The RFI is located a two locations along the 45 deg ϕ plane. The top plot shows the RFI at 10 degrees θ, and the bottom plot at 25 degrees ϕ. The SOI is at boresight.

The results show that the secondary beamformer defined in Equation 3 is effective at placing a null over the RFI, but struggles to suppress the side lobes. Reducing the rank may be sacrificing degrees of freedom for shaping the beam response leading to divergence from the maximum SIR beamformer.

IV. CONCLUSION

The true-time-delay and truncated Hadamard projection operator provide an opportunity to increase dynamic range before analog-to-digital converter in high sensitivity phased arrays. The secondary beamformer can reconstruct the transformed beam with a null over the RFI at cost of less control in the pattern side lobes and degradation to the main beam. Without adjustment, this study suggest the beamformer of Equation 3 is unacceptable for high-sensitivity radio astronomy applications. An open question is if another criteria or joint optimization scheme of the TTD weights alongside the secondary beamformer weights could balance control of the main beam and side-lobe levels and make wide-band analog RFI cancellation with true time delay and truncated Hadamard projection practical.

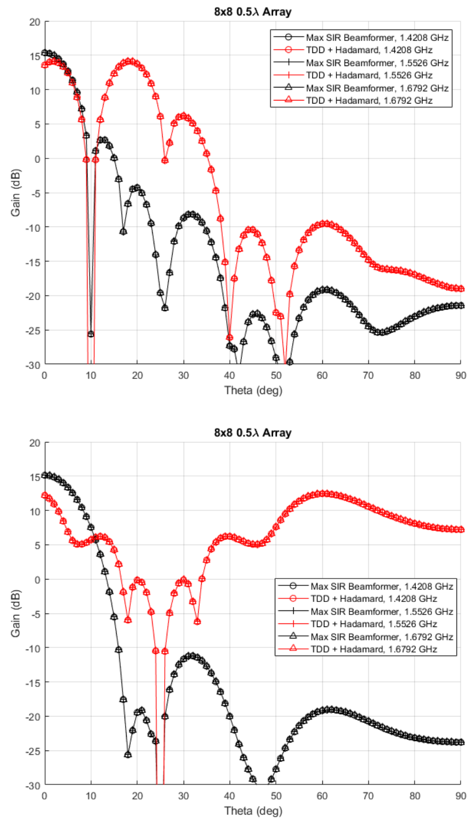

Fig. 1. Comparing the gain pattern of two null placing beamformer algorithms for RFI located at 10 degrees (top) 25 degrees (bottom) theta along the 45 degree phi plane. The secondary beamformer after the cancellation method struggles to constrain low side-lobes and emphasis the main lobe like the maximum SIR beamformer.

REFERENCES

[1] E. Ghaderi, A. S. Ramani, A. A. Rahimi, D. Heo, S. Shekhar, and S. Gupta, "Four-element wide modulated bandwidth mimo receiver with¿ 35-db interference cancellation," *IEEE Transactions on Microwave Theory and Techniques*, vol. 68, no. 9, pp. 3930–3941, 2020.

[2] ——, "An integrated discrete-time delay-compensating technique for large-array beamformers," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 66, no. 9, pp. 3296–3306, 2019.

[3] K. F. Warnick, R. Maaskant, M. V. Ivashina, D. B. Davidson, and B. D. Jeffs, *Phased arrays for radio astronomy, remote sensing, and satellite communications*. Cambridge University Press, 2018.

[4] R. R. Yarlagadda and J. E. Hershey, "The hadamard transform and error-correction coding," in *Hadamard Matrix Analysis and Synthesis*. Springer, 1997, pp. 39–50.

[5] G. Zhu and K. Huang, "Analog spatial cancellation for tackling the near-far problem in wirelessly powered communications," *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 12, pp. 3566–3576, 2016.