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Highlights:

- Importance of inspections in binary systems based on the Value of Information;
- Two metrics for component-level and for system-level maintenance actions;
- Inspection priorities for series and parallel systems;
- A heuristic proposed for reducing the computation complexity in general systems;

Optimal Inspection of Binary Systems via Value of Information Analysis

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Abstract

We develop computable metrics to assign priorities for information collection on binary systems composed of binary components. Components are worth inspecting because their condition states are uncertain, and system functioning depends on them. The Value of Information (VoI) enables assessment of the impact of information in decision making under uncertainty, including the component's reliability and role in the system, the precision of the observation, the available maintenance actions and the expected economic loss. We introduce the VoI-based metrics for system-level ("global") and component-level ("local") maintenance actions, analyze the properties of these metrics, and apply them to series and parallel systems. We discuss their computational complexity in applications to general network systems and, to tame the complexity for the local metric assessment, we present a heuristic and assess its performance on some case studies.

Keywords:

Binary Networks, Importance Measure, Inspections, Value of Information

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1. Introduction

Many civil infrastructures (e.g. transportation and gas pipeline networks) consist of multiple binary components, arranged in a system to fulfill various functions [21] [23] [28]. The binary states of the components, either intact or damaged, determine the system condition. The belief of the agent controlling the maintenance process can be described by a probabilistic distribution on the possible states of the components. Maintenance actions are selected to trade the risk of system malfunctioning for the cost of maintenance (including repair and retrofitting actions). Observations of the components' states can improve decision making and reduce the uncertainty and the maintenance cost. However, because of budget constraints, it is often impossible to inspect all components in a system. Therefore it is important to assign inspection priorities for the components. Intuitively, many factors can affect the inspection preferences, such as the probabilities of failure events, the maintenance costs and the role of each component in the system. These factors can be integrated in an Importance Measure (IM) for inspections, i.e. a value assigned to each component to summarize the benefit of observing the state of a component.

To introduce the problem, consider a binary system composed of N components: $\{c_1, c_2, \cdots, c_N\}$. Let $s = [s_1, s_2, \cdots, s_N] \in S$ denote the states of the components, with $s_j = 1$ indicating that component c_j is working, and $s_i = 0$ that it fails, where $S = \mathbb{B}^N$ and $\mathbb{B} = \{0, 1\}$. The system state $u = \phi(s)$ is also a binary variable, where $\phi : S \rightarrow \mathbb{B}$ is the component-to-system function. State s is unknown to the agent who manages the system. Instead, the agent optimizes the measurement and maintenance plans on the basis of her belief of s. The prior probability distribution of s is denoted as $p_s : S \to [0, 1]$, and $p_i = \mathbb{P}[s_i = 0]$ indicates the prior marginal failure probability of c_i. The failure probability of the system is $p_u = \mathbb{P}[u = 0]$, and we use p_π and $p_{\omega|E}$ for the prior value of p_u and its posterior value given event E, respectively.

In this paper, we develop metrics to assess the importance of inspecting any component. We assume that the outcome of the inspection is also binary. If component c_i is inspected, $y_i = 0$ indicates an "alarm", i.e. a symptom that c_i is not working, whereas $y_i = 1$ indicates that c_i seems to work, and we define this outcome as a "silence". If the inspection is perfect, then $y_i = s_i$.

On the basis of the measurement outcome, we can update the prior distribution of random variables s to posterior distribution $p_{s|y_i}$ and, the system level failure probability to $p_{\omega|y_i}$. When the components are interdependent, the measurement of one component may also affect the failure probability of other components.

Birnbaum was first to introduce Importance Measures (IMs) [4] to evaluate the contribution of each component to a system's performance, such as the system connectivity. Birnbaum's Measure (BM) [4] evaluates the importance of a component by the difference in the posterior system failure probability when it is damaged or intact (i.e., in our framework, when the inspection outcome is alarm or silence):

$$\mathrm{BM}(i) = p_{\omega|y_i=0} - p_{\omega|y_i=1} \tag{1}$$

Other IMs are discussed in Appendix B. Most of them focus on the marginal or conditional probability of the failure events, and they do not explicitly include any evaluation of the maintenance cost and risk. In maintenance problems, a component need high attention because of its topological function in the system and because of its high probabilities of failure. To assess this need of attention, Wu and Coolen [29] extended the BM to a cost-based IM. Zio and Podofillini [30] presented an approach for optimizing multiple objectives (such as system risk and maintenance costs), and they developed generic algorithms to reduce the computation time. Der Kiureghian et al. [6] modeled the component failures as independent Poisson events and developed IMs for long-term maintenance of series, parallel and general systems based on the system unavailability, mean rate of failure and mean duration of downtime.

To compare and rank the impact of inspections, one can assess their Value of Information (VoI). VoI assessment is based on Bayesian pre-posterior analysis, as introduced by [10], who integrated the probabilistic knowledge about the system with the economic factors related to the available actions. In the maintenance process of infrastructure systems, the economic costs are related to the system malfunctioning, the execution of inspections, and repair or replacement actions.

VoI has been studied intensively in the area of Structural Health Monitoring (SHM). Straub and Faber [26] integrated VoI for risk-based inspection scheduling and maintenance planning of structural systems. Pozzi and Der Kiureghian [18] provided a framework for assessing VoI for the long-term SHM, and proposed a Monte Carlo approach to reduce the computation complexity. They also investigated how the imperfect measurements affected the posterior decisions. Straub et al. [25] illustrated how to model the stochastic dependencies of component deterioration, the failure consequences and the inspection cost. The VoI has also been applied to long-term decision making problems. Miller [17] extended VoI analysis to optimize not only static one-shot inspection, but also to optimize sequentially dependent observations. Srinivasan and Parlikad [24], Memarzadeh and Pozzi [16] and Andriotis et al. [1] applied the component-wise VoI metric to sequential decision making in the management of infrastructure systems, modeled by Partially Observable Markov Decision Process (POMDP). Thöns [27] used decision trees to assess long-term VoI. Bensi et al. [3] developed Bayesian Networks and Influence Diagrams to evaluate post-event inspections, and they proposed VoI-based heuristic for optimal inspection sequences. Sensitivity analysis of the process parameters with respect to the optimal maintenance actions was presented by [31] and [5]. The complexity of computing VoI can grow exponentially with the number of components in a system [14]. Even worse, the VoI generally lacks the property of submodularity [15], so that the application of greedy approaches does not provide certain guarantees of near-optimal solutions [20]. Effective strategies have been proposed for efficient VoI computation in some special cases [12].

In this paper, we investigate VoI-based metrics related to system-level ("global") and component-level ("local") decision making after component inspections, for systems with various topologies, and compare these results with traditional IMs. A recent paper [8] also focuses on inspections for networked systems, developing an approach to identify the components most in need of inspection, similar to what we define as the local metric. We also derive simple optimal rules for series and parallel systems. For general systems, we discuss the computational complexity of the problem and provide a heuristic approach. In Section 2, we introduce the global and local metrics for evaluating the components' VoI. Section 3 describes rules for optimizing these metrics to typical systems such as series and parallel systems. In Section 4, we propose approximated approaches to simplify the optimization complexity, and in Section 5 we examine different applications of global, local and heuristic approaches to some system examples.

2. Global and local VoI metrics

2.1. Principles of VoI

Fig 1a illustrates the decision graph for the process of inspecting and maintaining the system. Continuous arrows from one set of nodes to one node indicate that the probability of latter variable is defined conditional to the former ones. Double arrows indicate deterministic relations. Dashed arrows from random variables to decision variables indicate that the former ones are observed before the latter is selected. Let $\mathcal{A}$ denote the set of all possible maintenance plans, that we simply call "actions". Action $A \in \mathcal{A}$ transforms current components' state $s \in S$ into state $s' \in S$, via transition distribution $p_{s'|s,A} : S \times \mathcal{A} \times S \to [0,1]$. Loss function $\mathcal{L}(s', A) = \mathcal{L}_{\mathrm{I}}(\phi(s')) + \mathcal{L}_{\mathrm{II}}(A) : S \times \mathcal{A} \to \mathbb{R}$ summarizes the overall cost: $\mathcal{L}_{\mathrm{I}}(\phi(s')) = C_F(1 - u')$ adds failure costs C_F if the system is not functioning, depending on system state u' after taking action A, which is associated with implementing cost $\mathcal{L}_{\mathrm{II}}(A)$.

The prior loss L_π is the minimum expected cost among all possible actions, before any inspection:

$$L_\pi = \min_A \mathbb{E}_s \mathbb{E}_{s'|s,A} \mathcal{L}(s', A) = \min_A \mathbb{E}_{s'|A} \mathcal{L}(s', A) \tag{2}$$

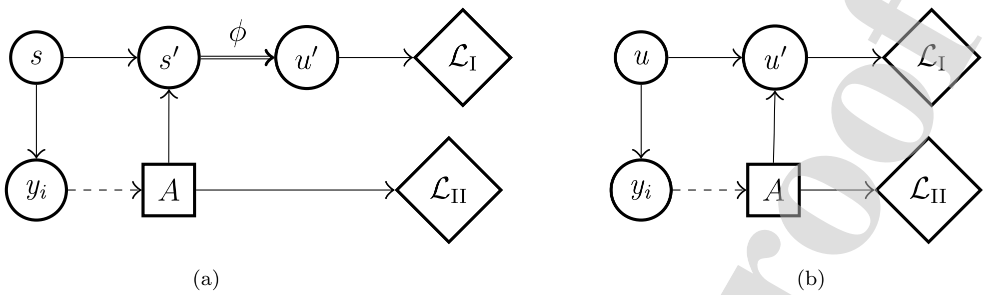


Figure 1: Decision graph for the general problem (a), and for the global metric (b).

where $\mathbb{E}_{s'|A}[\cdot] = \mathbb{E}_s\mathbb{E}_{s'|s,A}[\cdot]$ denotes the statistical expectation depending on distributions $p_{s'|s,A}$ and p_s.

Inspecting component c_i, the agent collects observation y_i distributed according to function $p_{y_i} : \mathbb{B} \to [0, 1]$, and the belief of the components' state s is updated to posterior distribution $p_{s|y_i} : S \times \mathbb{B} \to [0, 1]$. These functions are obtained by Bayes' rule:

$$p_{y_i} = \sum_s p_{y_i|s}p_s \quad p_{s|y_i} = \frac{p_{y_i|s}p_s}{p_{y_i}} \tag{3}$$

where $p_{y_i|s} : \mathbb{B} \times S \to [0, 1]$ is the likelihood function related to observation y_i.

The corresponding expected posterior loss is:

$$L_\omega(i) = \mathbb{E}_{y_i} \min_A \mathbb{E}_{s'|y_i,A}\mathcal{L}(s', A) \tag{4}$$

where $\mathbb{E}_{s'|y_i,A}[\cdot] = \mathbb{E}_{s|y_i}\mathbb{E}_{s'|s,A}[\cdot]$ is the posterior expectation, related to distribution $p_{s|y_i}$, and $\mathbb{E}_{y_i}[\cdot]$ is related to distribution p_{y_i}.

The VoI for inspecting c_i is the expected loss reduction due to the inspection, i.e. the difference between the prior and posterior loss functions [10]:

$$\mathrm{VoI}(i) = L_\pi - L_\omega(i) \tag{5}$$

Loss function $\mathcal{L}$ does not include the cost of monitoring, and the VoI is always not negative. However, if such cost is uniform among components, the VoI is a rational IM that assesses the relevance of inspections. The optimal component to inspect, $c_{i^\star}$, is the argument that maximizes Eq.(5):

$$i^\star = \arg\max_i \mathrm{VoI}(i) \tag{6}$$

The VoI depends on the specific number N of components, the action domain $\mathcal{A}$, the loss function $\mathcal{L}$ (in turn defined by the component-to-system function ϕ, the failure cost C_F, and the implementing cost $\mathcal{L}_{\mathrm{II}}$), the prior probability p_s, the transition probability $p_{s'|s,A}$ and the likelihood function $p_{y_i|s}$ adopted, as apparent in Fig 1a. In the following Sections, we describe a form of the likelihood function for binary components, and then we focus on two classes of losses and transitions, related to global and local decision making.

2.2. Modeling imperfect inspections

The VoI analysis also depends on the specific assumed likelihood function. If the binary outcome y_i, of inspecting component c_i, depends only on the state s_i of that component, likelihood function $p_{y_i|s}$ in Eq.(3) is reduced to a 4-entry emission table $p_{y_i|s_i} : \mathbb{B} \times \mathbb{B} \to [0,1]$, shown in Table 1.

Observations of components' states are prone to error, and the inaccuracy can be formulated by two parameters $\epsilon_{\mathrm{FA}} = \mathbb{P}[y_j = 0|s_j = 1]$ and $\epsilon_{\mathrm{FS}} = \mathbb{P}[y_i = 1|s_i = 0]$, which are the probability of type I error: having an "alarm" when the component is undamaged, and of type II error, a silence when the component is damaged. Although these probabilities can depend on

Actual state \ Observation	Silence $y_i = 1$	Alarm $y_i = 0$
Undamaged $s_i = 1$	$1 - \epsilon_{\text{FA}}$	ϵ_{FA}
Damaged $s_i = 0$	ϵ_{FS}	$1 - \epsilon_{\text{FS}}$

Table 1: Emission probability table for observation y_i given state s_i.

the specific component, in the following discussion, we assume that all the components have identical ϵ_{FS} and ϵ_{FA}.

Inspection outcomes probability function $p_{y_i} : \mathbb{B} \to [0, 1]$, is related to a single value: the probability $h_i = \mathbb{P}[y_i = 0]$ of receiving an alarm on c_i, which is:

$$h_i = (1 - \epsilon_{\text{FS}})p_i + \epsilon_{\text{FA}}(1 - p_i) = \epsilon_{\text{FA}} + Kp_i \tag{7}$$

where constant $K = 1 - \epsilon_{\text{FA}} - \epsilon_{\text{FS}}$ is strictly positive, because we assume that both ϵ_{FA} and ϵ_{FS} are less than $1/2$.

2.3. Global metric

We define the global metric assuming that action A affects the system state u. In this setting, for any of the two values of the binary variable u, an expected loss value can be assigned to any action A, regardless of the details of components' conditions (e.g., the damage location) described by variable s. Fig 1b shows the corresponding decision graph, in which the loss is a function of system state u' after the taken action: $l(u', A) = \mathcal{L}(s', A)$, with $u' = \phi(s')$. Transition function $p_{s'|s,A}$ is now converted into function $p_{u'|u,A}$: $\mathbb{B} \times \mathcal{A} \times \mathbb{B} \to [0, 1]$, in turn defined by a pair of values: $p'_{\omega|A,u=0}$ and $p'_{\omega|A,u=1}$,

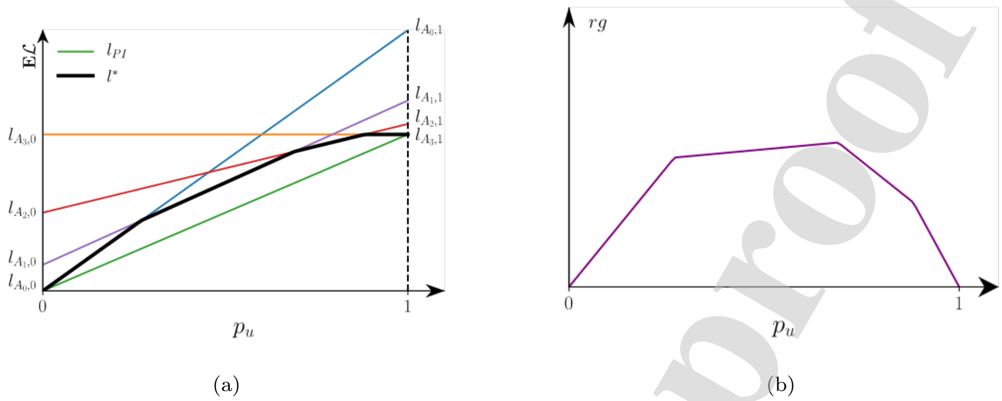


Figure 2: Expected loss function (a) and corresponding regret (b) for a global problem with 4 possible actions.

which are the probabilities that $u' = 0$ given action A and given $u = 0$ or $u = 1$, respectively. Then, $l_{A,0} = p'_{\omega|A,u=0}C_F + C_A$ and $l_{A,1} = p'_{\omega|A,u=1}C_F + C_A$, with $C_A = \mathcal{L}_{\text{II}}(A)$, represent the expected losses when u is zero and one (i.e. when the system is not working and is working), respectively, for action A. For each pair of losses $l_{A,0}$ and $l_{A,1}$, with $0 \leq l_{A,0} - l_{A,1} \leq C_F$, one can find a pair of values $C_A = l_{A,1}$ and $p'_{\omega|A,u=0} = (l_{A,0} - l_{A,1})/C_F$, to represent the target losses, assuming that no maintenance action makes the system degrade, so $p'_{\omega|A,u=1} = 0$. In this interpretation C_A is the cost for repairing, and $p'_{\omega|A,u=0}$ is the probability that the repair is ineffective. The agent has to find an optimal trade-off between implementing more expensive actions related to a low risk, and less expensive actions related to a higher risk.

The corresponding expected loss under action A is a linear function of the system failure probability p_u:

$$l_A(p_u) = \mathbb{E}_u\mathbb{E}_{u'|u,A}l(u', A) = p_u l_{0,A} + (1 - p_u)l_{1,A} \quad (8)$$

By taking the minimum among available actions in domain $\mathcal{A}$, we define the optimal loss by concave function $l^*(p_u) = \min_A l_A(p_u)$. Thus, the prior expected loss of Eq.(2) for the global metric is $L^{\mathrm{G}}_{\pi} = l^*(p_\pi)$ and, following Eq.(2), the posterior loss inspecting c_i is:

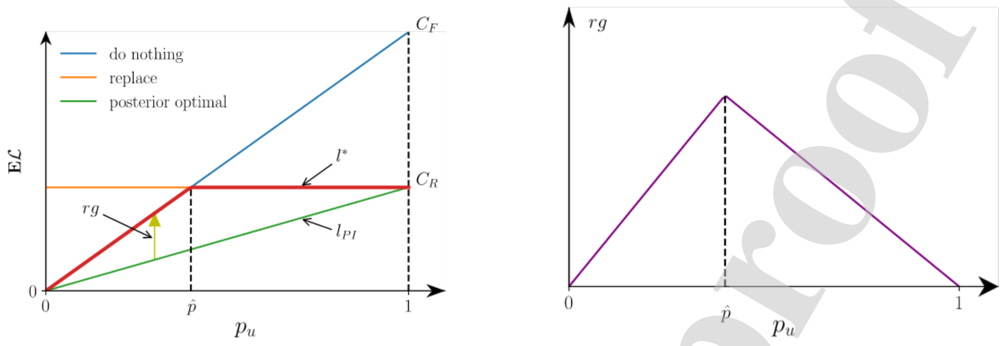
$$L^{\mathrm{G}}_{\omega}(i) = h_i l^*(p_{\omega|y_i=0}) + (1 - h_i) l^*(p_{\omega|y_i=1}) \tag{9}$$

and the VoI, following Eq.(5), is $\mathrm{VoI}_{\mathrm{G}}(i) = L^{\mathrm{G}}_{\pi} - L^{\mathrm{G}}_{\omega}(i)$.

As a function of p_u, the expected loss with perfect information of u is the linear function $l_{\mathrm{PI}}(p_u) = p_u l^*_0 + (1 - p_u) l^*_1$, with $l^*_0 = \min_A l_{0,A} = l^*(1)$ and $l^*_1 = \min_A l_{1,A} = l^*(0)$, and the "regret" is the concave function $rg(p_u) = l^*(p_u) - l_{\mathrm{PI}}(p_u)$, with $rg(0) = rg(1) = 0$. The corresponding prior regret is $\mathrm{RG}_\pi = rg(p_\pi)$. Because function l_{PI} is linear, the expected posterior loss with perfect information is $L_{\mathrm{PI}} = l_{\mathrm{PI}}(p_\pi)$, and expected posterior regret inspecting c_i is $\mathrm{RG}_\omega(i) = L^{\mathrm{G}}_{\omega}(i) - L_{\mathrm{PI}} = -\mathrm{VoI}_{\mathrm{G}}(i) + L^{\mathrm{G}}_{\pi} - L_{\mathrm{PI}}$. Hence, component c_{i^*}, that maximizes the VoI identified in Eq.(6), also minimizes the expected posterior regret:

$$i^* = \arg\min_i \mathrm{RG}_\omega(i) \tag{10}$$

The global metric depends on the set of pairs of expected losses for all actions $\{l_{0,A_0}, l_{0,A_0}, l_{0,A_1}, l_{1,A_1}, \cdots l_{A_{0,|\mathcal{A}|}}, l_{1,A_{|\mathcal{A}|}}\}$, where $|\mathcal{A}|$ is the cardinality of set $\mathcal{A}$, or, equivalently, on the concave function l^*. Fig 2 shows an example with $|\mathcal{A}| = 4$ actions available. The binary case is when only $|\mathcal{A}| = 2$ actions are available: doing-nothing, accepting the risk of paying cost C_F if the system is not working, with $A = 0$, or repairing the system at cost C_R, with $A = 1$. As shown in Fig 3a, this setting is defined by $l_{1,0} = 0, l_{0,1} = C_F, l_{0,1} = l_{1,1} = C_R$, and the corresponding normalized regret function rg/C_R is bi-linear, with peak $(1 - \tilde{p})$ at $p_u = \tilde{p} = C_R/C_F$.



(a) (b)

Figure 3: Expected loss (a) and corresponding regret (b) for the binary actions case.

2.4. Local metric

The local metric refers to actions at component level, whose effects depend on components' state s. For this approach, we define each action A as a vector $\{a_1, a_2, \cdots, a_N\}$ of N binary entries, where $a_i = 1$ if the agent repairs c_i, and $a_i = 0$ otherwise. Hence the cardinality of the action set is $|\mathcal{A}| = 2^N$. We assume that the components' repairs are perfect so that transition function $p_{s'|s,A}$ is defined as follows: in the vector $s' = [s'_1, s'_2, \cdots, s'_N]$ of states after maintenance, $s'_i = 1$ if $a_i = 1$, and $s'_i = s_i$ if $a_i = 0$. Function $\mathcal{L}_{\mathrm{I}}(\phi(s'))$ is defined as in Section 2.1, while $\mathcal{L}_{\mathrm{II}}(A) = C_R^\top \cdot A$, where $C_R = [C_{R,1}, C_{R,2}, \cdots, C_{R,N}]^\top$ is the repair cost vector and $C_{R,i}$ is the cost of repairing c_i. This model assumes that the accumulated cost is the sum of repair costs for the individual components. Other cost models, assuming a more complex cost interaction among component' costs, can also be implemented.

After the inspection, the agent selects the optimal subset of components to repair. When the inspection outcome is $y_i = c$, the corresponding posterior expected loss is:

$$L^{\mathrm{L}}_{\omega|y_i=c} = \mathbb{E}_{s|y_i=c} \min_{A} \mathbb{E}_{s'|s,A} \mathcal{L}(s', A) \tag{11}$$

Following Eq.(4), the corresponding expected posterior loss is:

$$L^{\mathrm{L}}_{\omega}(i) = (1 - h_i) L^{\mathrm{L}}_{\omega|y_i=1} + h_i L^{\mathrm{L}}_{\omega|y_i=0} \tag{12}$$

and the VoI according to the local metric is $\mathrm{VoI}_{\mathrm{L}}(i) = L^{\mathrm{L}}_{\pi} - L^{\mathrm{L}}_{\omega}(i)$, where prior loss L^{L}_{π} is computed as in Eq.(2).

3. Metric properties and inspection priorities on typical systems

3.1. Nested posterior intervals for global metric

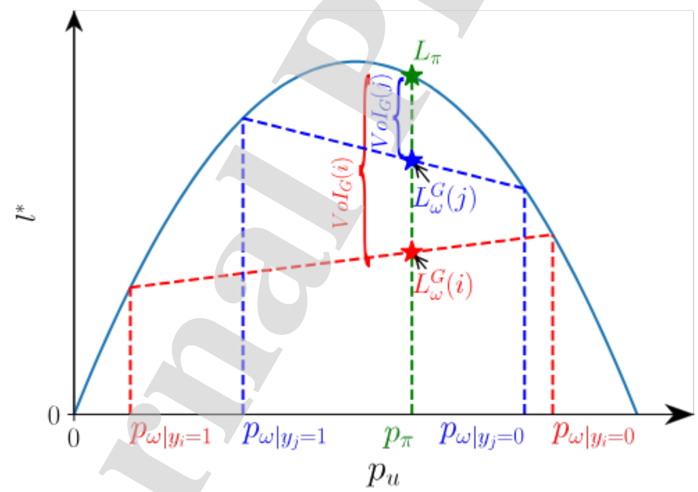


Figure 4: Example of expected loss for the global metric, with nested posterior intervals.

As we discussed in Section 2.3, the global metric adopts a univariate concave function l^*, or rg, of p_u. An example of such a function is shown in Fig 4, which can also be interpreted as regret, because it is zero at the limits of the probability domain. Inspecting every component c_i, the posterior system failure probability after an alarm is higher than the priori, which is in turn higher than the posterior system failure probability after a silence: $p_{\omega|y_i=1} \leq p_\pi \leq p_{\omega|y_i=0}$.

Now consider two components c_i and c_j. Suppose that a silence on c_i is more reassuring than a silence on c_j and an alarm from c_i is more worrying than an alarm from c_j, i.e. $p_{\omega|y_i=1} \leq p_{\omega|y_j=1}$ and $p_{\omega|y_i=0} \geq p_{\omega|y_j=0}$. Then, for any concave function l^* (or rg), the posterior loss of inspecting c_i is lower than the loss of inspecting c_j and the VoI of inspecting c_i is higher than that of inspecting c_j i.e. $L_\omega^{\mathrm{G}}(i) \leq L_\omega^{\mathrm{G}}(j)$ and $\mathrm{VoI}_{\mathrm{G}}(i) \geq \mathrm{VoI}_{\mathrm{G}}(j)$. The proof of this implication is intuitive by examining Fig 4, and it is given formally in Appendix C.

We can also reformulate the implication in terms of "posterior intervals". Let us define the posterior interval for c_i as $I_i = [p_{\omega|y_j=1}, p_{\omega|y_j=0}]$. If that posterior interval contains the corresponding interval for c_j, i.e. if $I_i \supseteq I_j$, then $\mathrm{VoI}_{\mathrm{G}}(i) \geq \mathrm{VoI}_{\mathrm{G}}(j)$. Hence, the importance ranking is invariant with respect to the choice of l^*, and all possible global metrics prioritize the component with larger interval to inspect, consistently with BM defined in Eq.(1), i.e. $I_i \supseteq I_j \Rightarrow \mathrm{BM}(i) \geq \mathrm{BM}(j)$. However, the reverse implication is not guaranteed, and Birnbaum's measure is not necessarily consistent with the global metric.

Moreover, if the posterior intervals are not nested, one can always find a pair of loss functions $\{l_\alpha^*, l_\beta^*\}$, so that c_i has a higher VoI than c_j under l_α^*, but a lower VoI under l_β^*. For proof, refer to the bi-linear loss function plotted in Fig 3. If probability $\tilde{p}$ is not in posterior interval I_i (i.e., I_i is on one side of $\tilde{p}$), then the corresponding VoI, $\text{VoI}_{\text{G}}(i)$, is zero, because the loss function is linear in that range. If intervals I_i and I_j are not nested, we can find two disjoint intervals: interval $I_{i\setminus j}$ belongs to I_i but not to I_j, interval $I_{j\setminus i}$ belongs to I_j but not to I_i. If $\tilde{p}$ is in $I_{i\setminus j}$, then $\text{VoI}_{\text{G}}(i) \geq \text{VoI}_{\text{G}}(j) = 0$, while if $\tilde{p}$ is in $I_{j\setminus i}$, then $\text{VoI}_{\text{G}}(j) \geq \text{VoI}_{\text{G}}(i) = 0$. This argument shows that, for not nested posterior intervals, the priority order depends on the adopted loss function.

3.2. *Global metric for parallel systems*

A parallel system will function if at least one of its components is intact. For such systems, the global metric will always give the highest priority to the most reliable component (i.e., to the one with the lowest marginal failure probability), independent of the specific loss function l^* adopted, when the inspection quality is the same for all components. The proof is simple for the special case of perfect sensors, i.e. when ϵ_{FA} and ϵ_{FS} are zero. In that case, if a silence is detected for any component, then the posterior system failure probability is zero. Because the failure of the system implies the failure of all components, after an alarm on component c_i, p_u becomes $p_{\omega|s_j=0} = p_\pi/p_i$. Hence, if $p_i \leq p_j$, then $I_i \supseteq I_j$ and, according to the rule illustrated in Section 3.1, we conclude that $\text{VoI}_{\text{G}}(i) \geq \text{VoI}_{\text{G}}(j)$.

When sensors are imperfect, the proof is still based on Bayes' formula (i.e., on the ratio between joint and marginal probabilities). After a silence on c_i, p_u becomes:

$$p_{\omega|y_i=1} = \frac{p_\pi \epsilon_{\text{FS}}}{1-h_i} = \frac{p_\pi \epsilon_{\text{FS}}}{1-\epsilon_{\text{FA}}-Kp_i} \tag{13}$$

where the second identity follows from Eq.(7), and we note again that K is strictly positive. The corresponding probability after an alarm is:

$$p_{\omega|y_i=0} = \frac{p_\pi(1-\epsilon_{\text{FS}})}{h_i} = \frac{p_\pi(1-\epsilon_{\text{FS}})}{\epsilon_{\text{FA}} + Kp_i} \quad (14)$$

The denominator of Eq.(13) decreases monotonically with p_i, and the denominator of Eq.(14) increases monotonically with p_i. Hence, as in the case of perfect sensors, if $p_i \leq p_j$, then $I_i \supseteq I_j$ and, $\text{VoI}_{\text{G}}(i) \geq \text{VoI}_{\text{G}}(j)$.

In summary, the ranking of importance measures follows the opposite of the marginal failure probability of the components (i.e., the ranking follows component reliability). Hence, in a parallel system, the component, c_{i^*}, with highest VoI is the most reliable component. This result holds for any interdependence between components' states, that is for any distribution p_s, when the inspection quality, defined by parameters ϵ_{FA} and ϵ_{FS}, is the same for all components.

3.3. Global metric for series systems

A series system works only if all components function properly. In that case, the global metric always prioritizes the most vulnerable component, i.e. the component with the highest prior failure probability, regardless of the adopted function l^* or the interdependence among components. The proof is similar to that related to parallel systems. Let us start with the case of perfect sensors. The posterior system failure probability will become 1 after an alarm on any component, and will become $p_{\omega|s_{j=1}} = 1-(1-p_\pi)/(1-p_i)$ after a silence on component c_i, which monotonically increases with marginal component failure probability p_i. Hence the most vulnerable component should be inspected.

For imperfect sensors, after a silence on c_i, p_u is (again using Eq.(7)):

$$p_{\omega|y_i=1} = 1 - \frac{(1-p_\pi)(1-\epsilon_{\text{FA}})}{1-h_i} = 1 - \frac{(1-p_\pi)(1-\epsilon_{\text{FA}})}{1-\epsilon_{\text{FA}} - Kp_i} \quad (15)$$

After an alarm, that probability is:

$$p_{\omega|y_i=0} = 1 - \frac{(1-p_\pi)\epsilon_{\text{FA}}}{h_i} = 1 - \frac{(1-p_\pi)\epsilon_{\text{FA}}}{\epsilon_{\text{FA}} + Kp_i} \quad (16)$$

The denominator of the fraction in Eq.(15) monotonically decreases with p_i, and the denominator of Eq.(16) monotonically increases with p_i. Hence, if $p_i \geq p_j$, then $I_i \supseteq I_j$ and $\text{VoI}_{\text{G}}(i) \geq \text{VoI}_{\text{G}}(j)$, as in the case of perfect sensors. So, in a series system, regardless of the interdependence between components, the inspection ranking follows the marginal component failure probability, and c_{i^*} is the most vulnerable component.

In other words, the most vulnerable component, c_{i^*}, is the one to inspect because detecting a silence on that component (i.e. $y_{i^*} = 1$) induces the highest reduction of p_u, and an alarm (i.e. $y_{i^*} = 0$) induces the highest increment in that probability. Although the former property is almost trivial, the latter may be less intuitive. After all, c_{i^*} was (relatively) likely to be damaged; thus, why does an alarm on that component produce the more "surprising" result on the system reliability (compared with alarms on less vulnerable components)? For imperfect inspections, two factors affect the posterior probability. On one hand, after detecting an alarm on c_{i^*}, the system can still count on the other components, which are more reliable than c_{i^*} (instead, after an alarm on a safer component, the system can only count on more vulnerable components). Hence, this factor suggests that an alarm of c_{i^*} is less worrying that an alarm on others. Conversely, following Bayes' rule, an alarm on c_{i^*} produces a relatively high posterior failure probability

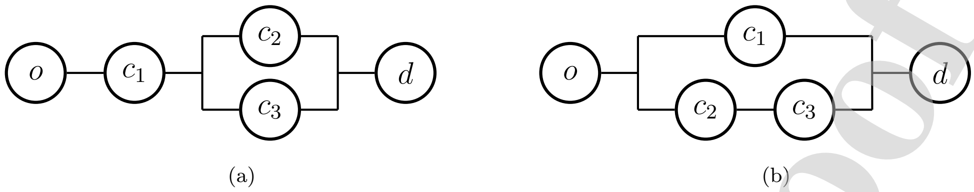


Figure 5: Block diagrams for series-parallel (a) and parallel-series (b) 3-component system.

(at component and at system level), because of the high prior probability that c_{i^*} is damaged. For safer components, the impact of the alarm is diluted by the more optimistic prior information, and the posterior failure probability after an alarm is lower at component level (obviously) and at system level, as formally proved by Eq.(16). Hence, this latter factor dominates the former factor, and c_{i^*} has the highest VoI. This result depends on the assumption that the sensor accuracy is uniform among components. If the accuracy was higher for a specific component, that component could have the highest VoI, even if it was not the most vulnerable component.

3.4. Global metric for general systems

If the posterior probability interval related to one component nests all the others, then the rule of Section 3.1 identifies the optimal component to inspect. For general systems, the global metric does not always select the most vulnerable or the most reliable component, because the posterior intervals may not be nested, and the rule does not apply.

We illustrate this by discussing two simple examples of 3-component systems, with perfect sensors as shown in Fig 5. The system functions if there is an intact path from origin node o to destination node d. Fig 5a shows a system in which component c_1 is in series with a parallel subsystem composed of components c_2 and c_3. Intuitively, component c_1 should be inspected, because it is a "bottleneck" of the system; thus, it seems topologically more important. Detecting that c_1 is not working takes p_u from one, i.e., to a higher value related to an alarm on c_2 or on c_3. A silence detected on c_1 takes p_u to the joint failure probability, $p_{\omega|s_1=1}$, which is determined by components c_2 and c_3, and is $(p_2 p_3)$ if they are independent. Instead, a silence detected on c_2 (or on c_3) takes p_u to $p_{\omega|s_2=1} = p_1$. Hence, posterior interval I_1 contains the other two if $p_{\omega|s_1=1}$ is less than p_1. Conversely, if p_1 is less than $p_{\omega|s_1=1}$, the posterior intervals are not nested, and the priority depends on the selected loss function l^*. This result confirms the intuition that if c_1 is much safer than the other components, it may not be the most important component to inspect (trivially, if p_1 is zero while p_2 and p_3 are positive, then c_1 has the lowest priority).

In the example of Fig 5b, component c_1 is parallel with a series subsystem composed of components c_2 and c_3. Again, c_1 seems topologically more important. After a silence on c_1, p_u is zero, a value lower than the value related to silence on c_2 or on c_3. An alarm on c_1 takes p_u to $1 - r_{2,3}$, where $r_{2,3}$ is the joint survival probability of the other two components, that is $(1 - p_2)(1 - p_3)$ for independent components, whereas an alarm on c_2 (or on c_3) takes p_u to p_1. Hence, posterior interval I_1 contains the others if p_1 is less than $1 - r_{2,3}$ i.e., for independent components, if p_1 is less than $p_2 + p_3 - p_2 p_3$. Approximating this latter value with $p_2 + p_3$, we conclude that the global metric gives higher priority to c_1 when p_1 is less than $(p_2 + p_3)$. If p_1 is higher than $(p_2 + p_3)$, priority depends on the selected loss function l^*. This conclusion confirms the intuition that, if c_1 is much more vulnerable than the other components, it is better to inspect others (in the limit case where p_1 is one, $\text{VoI}_\text{G}(1)$ is zero). These two examples illustrate how the topological role of a component matters, but also its failure probability: in some schemes a high failure probability guarantees a high priority.

We discuss now a more general example, focusing on two components, c_1 and c_2. The components' roles are described completely by the system failure probability for each of the 2^2 joint conditions of the pair of components, that we assume as $p_{\omega|s_1=1,s_2=1} = 0.5\%$, $p_{\omega|s_1=1,s_2=0} = p_{\omega|s_1=0,s_2=1} = 2.5\%$, $p_{\omega|s_1=0,s_2=0} = 90\%$ (so the roles played by the two components are the same). We also assume that $p_1 = 1\%$ and $p_2 = 20\%$ (so that the c_1 is significantly more reliable than c_2), the states of all components are independent, and inspections are perfect (i.e., $y_i = s_i$). Fig 6 shows the diagram of a system consistent with these values. The interval of posterior probabilities I_i is $[0.90\%, 20.0\%]$ for $i = 1$ and $[0.52\%, 3.38\%]$ for $i = 2$, whereas p_π is 1.09% (these results are directly related to the assumed values, e.g. $p_{\omega|y_2=1} = p_{\omega|s_1=0,s_2=1}p_1 + p_{\omega|s_1=1,s_2=1}(1-p_1)$). The intervals are not nested; hence, the rule in Section 3.1 does not apply, and the VoI depends on the specific function l^*. Fig 7 refers to the bi-linear regret function for binary actions plotted in Fig 3, and mentioned in Section 2.2, with peak at $\tilde{p}$. The figure shows how the VoI related to each component, normalized by prior regret RG_π, varies as a function of $\tilde{p}$. When $\tilde{p}$ is below $p_{\omega|y_2=1} = 0.52\%$ (i.e., when C_R is below 0.52% of C_F), the VoI of each component is nil, because the posterior decision is always to repair. Then, $\text{VoI}_G(2)$ increases up to about 42% of RG_π when $\tilde{p} = p_\pi$ (i.e., for that condition observing y_2 is worth 42% of the value of observing u), then it decreases to zero at $\tilde{p} = p_{\omega|y_2=0} = 3.38\%$.

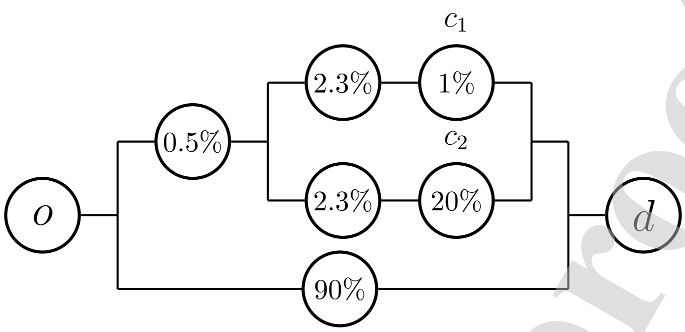


Figure 6: Block diagram for a system where posterior probability intervals for components c_1 and c_2 are not nested

For $\tilde{p}$ higher than $p_{\omega|y_2=0}$, the posterior decision is always to accept the risk. The behavior of $\text{VoI}_\text{G}(1)$ is similar; it is zero outside I_1, and it peaks at p_π, where it is about 17% of RG_π. Clearly, the optimal inspection decision depends on $\tilde{p}$, i.e. on the decision-making problem shaping function l^*. This conclusion is apparent in Fig 7, if the repair cost is cheaper than 2.5% of the cost of failure, it is more convenient to inspect the more reliable c_2, whereas it is better to inspect the less reliable c_1 for a higher repair cost.

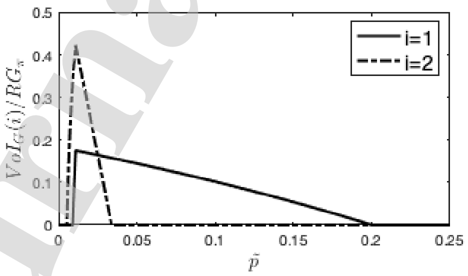


Figure 7: Normalized VoI depending on peak probability $\tilde{p}$.

3.5. *Local metric on parallel systems*

The local metric, as defined in Section 2.4, will select the most reliable component in a parallel system, consistently with the global metric. This selection behavior is because, in a parallel system, repairing one component guarantees the functioning of the system. Hence, the agent faces a binary decision: do nothing or repair the less expensive component, at cost $\min_i C_{R,i}$. This problem setting is equivalent to that of the global metric, with the bi-linear function l^* of Fig 3a. Therefore, the local and global metrics have identical conclusions about the optimal inspection.

3.6. *Local metric on series systems*

With the local metric, the optimal component to inspect in a series system is not always the most vulnerable component, i.e. the component identified by the global metric. We start discussing the case of a system with two components, c_1 and c_2, with identical repair costs, $C_{R_1} = C_{R_2} = C_R$, and equipped with perfect sensors. Let us also assume that $C_R \leq C_F/2$, so that the cost for repairing both components is less than the failure cost. Hence, if any component c_i is detected as damaged, it is necessary to repair it ($A_i = 1$), to avoid paying the failure cost. After the repair, the system failure probability is the posterior failure probability of the uninspected component. That uninspected component should be also repaired if the corresponding risk is above the repair cost, so that the posterior expected maintenance cost for that component is $R(i, x) = \min\{C_R, p_{\omega|s_i=x,A_i=1-x}C_F\}$, with $x = 0$. Instead, if the inspected component works, it has not to be repaired, and the state of the uninspected component is decided by comparing repair cost and system failure risk, so that the expected posterior cost is $R(i, 1)$. Hence, the expected posterior loss is $L^{\mathrm{L}}_{\omega}(i) = p_i C_R + p_i R(i,0) + (1-p_i)R(i,1)$. In the special case of independent components, for any outcome x, probability $p_{\omega|s_i=x}$ is identical to the prior failure probability p_j of the uninspected component c_j, so that $R(i,0) = R(i,1)$, and $L^{\mathrm{L}}_{\omega}(i) = p_i C_R + \min\{C_R, p_j C_F\}$. If we refer to bi-linear regret function rg of Fig 5b, we conclude that, for each component c_i, $\mathrm{VoI}_{\mathrm{L}}(i) = \mathrm{RG}_{\omega}(i)$; thus, we should inspect the component with the higher value of $\mathrm{RG}_{\omega}(i)$. If both prior failure probabilities are below $\tilde{p} = C_R/C_F$, the local metric will prioritize the more vulnerable component. However, if the failure probability of a component is above $\tilde{p}$, then the higher that probability, the lower the corresponding VoI. Fig 8 shows the optimal inspection policies for $\tilde{p} = 0.2$, $p_1 \geq p_2$, and different correlation coefficient ρ between variables s_1 and s_2. The joint probability can be defined given the correlation coefficient ρ and the marginal probability p_1 and p_2. For example, the joint probability of both components work is:

$$\mathbb{P}[s_1 = 1, s_2 = 1] = \rho\sqrt{p_1(1-p_1)p_2(1-p_2)} + (1-p_1)(1-p_2) \tag{17}$$

. We have discussed the case when ρ is zero. When it is positive, the domain of feasible pairs (p_1,p_2) shrinks but, inside the feasible domain, the region expands where it is more convenient to inspect the more vulnerable component. When the correlation is negative, for any feasible pair $\{p_1, p_2\}$, the VoI is the same for both components. We can provide a simple approximation for series system if their states are independent and the failure probabilities are relatively low. In that case, the risk $\mathbb{E}[\mathcal{L}_I]$ can be approximated as that of a "cumulative system"[13]. In a cumulative system, individual costs are associated with the failure of each component, and the costs are accumulated to obtain the system-level cost (hence, no component-to-system function ϕ

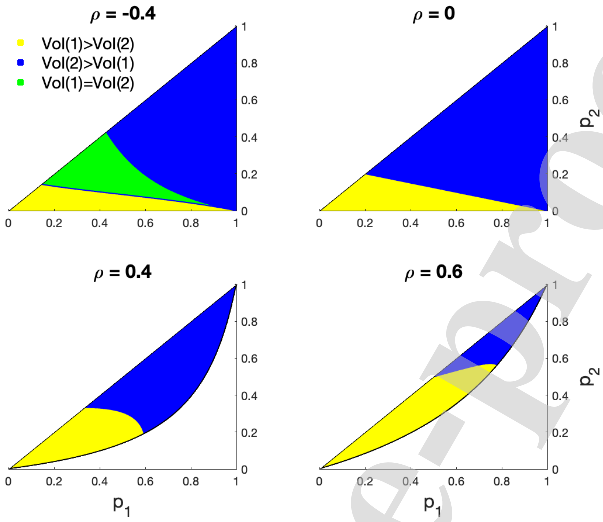


Figure 8: Optimal inspection i^* when two components are dependent, depending on correlation ρ.

is defined for these systems). To illustrate this approach, we recall that, for a series system with independent components, the risk is:

$$\mathbb{E}_{\text{ser.}}[\mathcal{L}_I] = C_F[1 - \prod_i (1-p_i)^{1-A_i}] \tag{18}$$

For a cumulative system with component failure cost C_F, it is:

$$\mathbb{E}_{\text{cum.}}[\mathcal{L}_I] = C_F \sum_i (1-A_i)p_i \tag{19}$$

By linearizing the former expression (neglecting higher order terms), the two risks become identical. For a cumulative system, it is straightforward to evaluate the benefit of inspecting component c_i which is related to the selection of action A_i, and when sensors are perfect, it is $\text{VoI}_\text{L}(i) = \text{RG}_\omega(i)$ (for imperfect sensors, one has to subtract the posterior regret). These results are also consistent with the case in which $N = 2$.

3.7. Connections and differences between local and global metrics

Whereas the local metric follows the traditional Bayesian pre-posterior analysis for a selected class of actions and losses, the global metric takes inspiration from the BM, and it focuses on the impact of information on the system failure probability, neglecting the impact on damage localization. This latter approach allows for a simpler optimization and VoI analysis, and the intuition supporting it is that a component should receive a high priority if its inspection outcome is highly informative on the system state.

The two metrics refer to different problem classes, which are not nested one into another (given the restrictive rules we impose to the local metric). On the one hand, information in the local metric keeps referring to local quantities, i.e. the damage condition of individual components, and therefore is key to supporting local repairing, i.e. the repairing of those components. In the global metric approach, local information is neglected, and the optimal posterior action is only a function of the posterior system failure probability. This indicates that the local metric cannot be generally reduced to an equivalent global one. On the other hand, the global metric cannot be generally reduced to the local one either. This is because, for example, in the local metric we assume that each action identifies whether to repair or not for each component; hence, for a system with N components, there are 2^N available actions in local metric, whereas the global metric can define an arbitrary number of actions (each associated with different repair cost C_A and posterior probability $p'_{\omega|A,u=0}$). Also, the assumptions of perfect repairs and of additive repair costs impose constraints to the available actions in the local metric, while the global metric is not subject to these limitations.

We note that for both metrics, following Section 2.1, one can define a concave function l^* on the belief p_s of the joint condition state s of all components, i.e. on a N dimensional domain (with the linear constraint that $\sum_j p_s(j) = 1$). However, this function can be transformed into a univariate function of the system failure probability p_u, as illustrated in Section 2.3, only in the the global metric.

These major difference between the local and global metrics can be highlighted on a paradigmatic case. Consider a two-component series sub-system in a larger system, where one of the two components is working and the other is not, and the working component is one (or the other), with probability 1/2. Hence, the states of the two components are perfectly negatively correlated. The perfect inspection of one component enables the agent to identify the malfunctioning component (it is the inspected one, if the outcome is "alarm", or the other, if the outcome is "silence"). From the local metric perspective, this information is relevant because it enables repairing the malfunctioning component with perfect information. However, from the perspective of the global metric, the VoI of inspecting any of the two components is nil, because the posterior system failure probabilities are identical to the prior one. From this latter perspective information has value only if the inspection outcome has an impact on the system failure probability: a component is important if silence is good news and alarm bad news for system reliability. The example shows how information can have value for supporting local maintenance, regardless of its impact on system reliability.

4. Computational complexity and Heuristic

4.1. Complexity of VoI computation

The computational complexity of solving Eq.(6) varies with different metrics, but is generally intimidating for large systems. The core step of the computational process is solving the reliability problem, identifying the system failure probability p_u, depending on actions and observations. This analysis is nested into the optimization of the maintenance actions.

The general network reliability problem is NP-hard [2] [19], but numerous approximations and bounds have been proposed. To compute the risk $\mathbb{E}[\mathcal{L}_{\mathrm{I}}]$, one has to assess the system connectivity for each of the 2^N system states. A matrix-based method was proposed to compute system reliability based on a components' condition matrix with each row representing one state, and a binary condition vector with each entry representing whether the system is functioning at that specific components' state [21]. The general computation complexity of the method is $\mathcal{O}(N \times 2^N)$. When the joint distribution of the components' states is known, with the components' condition matrix and the binary system condition vector previously computed, the computational complexity of system reliability is linear with respect to the system states. An approximate estimation can also be achieved based on Monte Carlo simulations [11].

For the global metric, $\mathbb{E}[\mathcal{L}_{\mathrm{II}}]$ can be determined in $\mathcal{O}(1)$ time once we compute the posterior system failure probabilities. Therefore, to select the component with highest VoI among N components will cost $\mathcal{O}(N \times 2^N)$.

For the local metric, there is an additional computation step before assessing the VoI. $\mathbb{E}[\mathcal{L}]$ is optimized among 2^N combinations of maintenance actions. Suppose that, on the basis of different inspection outcomes, the agent can select an arbitrary subset of the components to repair; then, the computation complexity is generally $\mathcal{O}(N \times 2^N \times 2^N) = \mathcal{O}(N \times 2^{2N})$.

4.2. Approximation for local metric

In this section, we propose a simple heuristic approach for approximating the local metric, to reduce the computational complexity related to the optimization of maintenance actions depending on the inspection outcome.

Let us define $A_\pi = \{a_{\pi,1}, a_{\pi,2}, \cdots, a_{\pi,N}\}$ as the prior maintenance plan, $A_\omega = \{a_{\omega,1}, a_{\omega,2}, \cdots, a_{\omega,N}\}$ as the posterior one, and L_π is the prior optimal loss related to A_π, as defined in Section 2.1. We assume that A_π and L_π have been identified. Consider inspecting component c_i. The proposed heuristic assumes that the agent confirms all actions for uninspected components (i.e., $\forall j \neq i, a_{\omega,j} = a_{\pi,j}$). Only the posterior action on the inspected component, $a_{\omega,i}$, depends on the inspection's outcome, y_i. If the prior action for c_i is to do-nothing (i.e., if $a_{\pi,i} = 0$) and the inspection's outcome is silence (i.e., if $y_i = 1$), or if the prior action is to repair (i.e., if $a_{\pi,i} = 1$) and the inspection produces an alarm (i.e., if $y_i = 0$), then the agent will confirm the prior action also for the inspected component (i.e., if $y_i \neq a_{\pi,i}$, then $A_\omega = A_\pi$). Instead, if an alarm is detected on a previously unrepaired component, or if a silence is detected on a previously repaired component (i.e., if $y_i = a_{\pi,i}$), then the agent considers the two alternatives: repair or not repair c_i. One of the two alternatives is, again, to completely confirm the prior plan (i.e. $A_\omega = A_\pi$); thus, the prior loss L_π associated with this option is already known. The agent computes the expected cost of the alternative plan (in which only action $a_{\omega,i}$ is reversed), and executes the best option, i.e. the option related to the minimum expected cost. The computational saving is related to the avoidance of searching for optimal posterior action in the full set $\mathcal{A}$.

One argument supporting the choice of this heuristic is the consistency with the optimal behavior in some special cases,for example, when the high penalty of a system collapse forces the agent to be conservative. To model this scenario, suppose that (i) the prior decision is to do-nothing (i.e., $\forall i, a_{\pi,i} = 0$), that (ii) a detected silence cannot increase the system failure probability (i.e. $\forall i, p_{\omega|y_i=1} \leq p_\pi$), that (iii) the do-nothing option is still optimal when the probability of failure decreases and that (iv) a component sending an alarm must be repaired, because its posterior failure probability is too high to be tolerated. Condition (iii) is not obviously satisfied even if the first two conditions are satisfied, because the prior decision might also be doing nothing for another reason, i.e., the agent is pessimistic about the components' conditions. For such a pessimistic agent, it is not worth repairing any set of components; repairing few components may be ineffective, and repairing many components may be too expensive. However, detecting a functioning component may improve the expectation of the system and persuade the pessimistic agent to invest in repairing other components. Condition (iii) forbids the occurrence of this process, by assuming the agent's optimism about the system condition. To prove that the heuristic is optimal under conditions (i-iv), we must show that the optimal response to an alarm on component c_i cannot be to repair any other component. Because of (iv), c_i must be repaired. Now suppose that component c_j is also to be repaired.

This condition implies the following inequality:

$$C_{R,i} + C_{R,j} + C_F p_{\omega|y_i=0,a_i=1,a_j=1} \leq C_{R,i} + C_F p_{\omega|y_i=0,a_i=1} \tag{20}$$

If, as assumed before, repairs are perfect and components' states are independent, then $p_{\omega|y_i=0,a_i=1} = p_{\omega|s_i=1} = p_{\omega|y_i=1}$, and $p_{\omega|y_i=0,a_i=1,a_j=1} = p_{\omega|s_i=1,s_j=1} = p_{\omega|y_i=1,a_j=1}$, so the inequality Eq.(20) can be re-written, subtracting $C_{R,i}$ from both terms as:

$$C_{R,j} + C_F p_{\omega|y_i=1,a_j=1} \leq C_F p_{\omega|y_i=1} \tag{21}$$

This equation indicates that repairing c_j should be the optimal response to a silence on c_i, but this response violates conditions (i-iii), which show that only c_i should be repaired after receiving an alarm on it. Of course, if conditions (i-iv) are not satisfied, there is no guarantee that the heuristic is truly optimal.

The VoI defined by the heuristic is certainly non-negative, as the prior maintenance plan can be confirmed, if the collected observations do not suggest any improvement. Moreover, given that the heuristic limits the domain of the posterior actions, the corresponding VoI cannot be higher than the VoI assessed by the local metric.

5. Examples of System Analysis

We analyze three examples of systems. The first example is the 6-component system in Fig 9, in which the failure probability of each component is listed inside the corresponding node. We start by considering perfect inspections and independent components. The corresponding values of the BMs are shown in Fig 10a, and c_2 has the highest importance in BM.

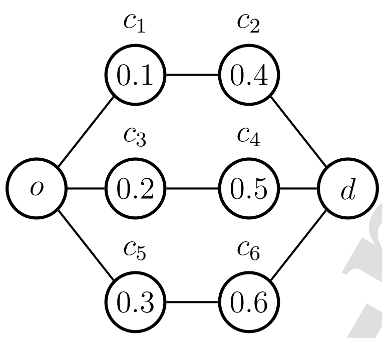


Figure 9: Block diagram for the counter-intuitive example 1

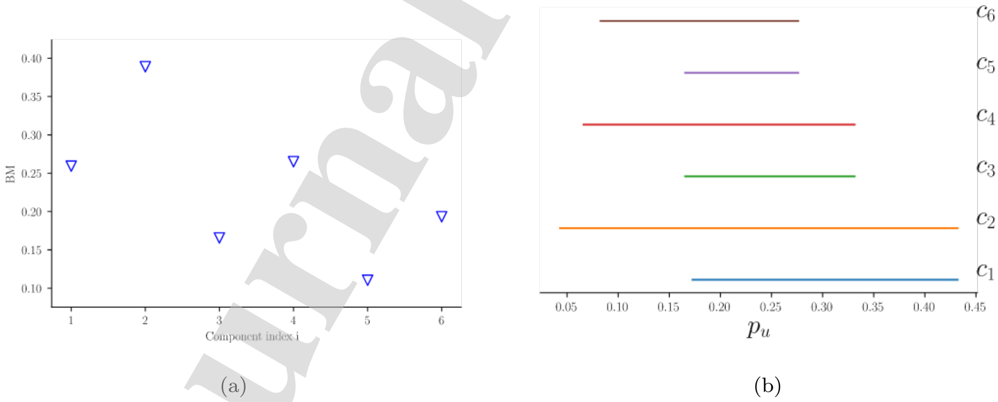


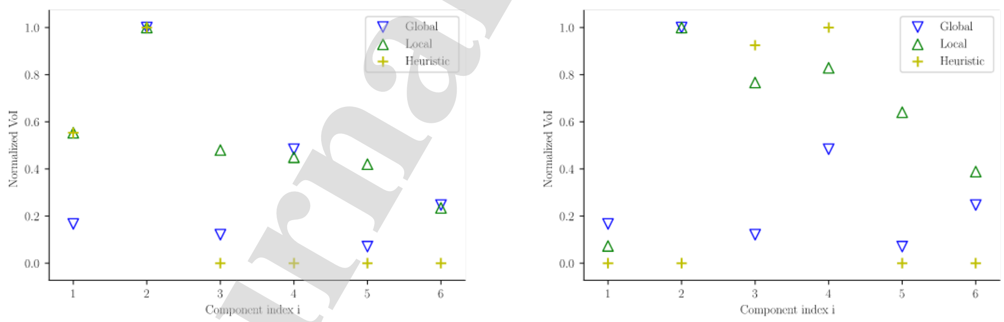
Figure 10: BM for the system in Fig 9 (a), and corresponding posterior intervals (b).

Fig 10b shows the posterior probabilities intervals I_i for all components. All intervals are nested in I_2. Thus, component c_2 has the highest VoI, according to the global metric, regardless of the loss function we adopt (and so it has also the highest BM, as noted above). We divide the VoI of each component by the maximum VoI of all the components under the same metric obtain a normalized VoI. The normalized VoI under the global metric, for loss function $l^*(p_u) = p_u(1 - p_u)$, is shown in Fig 11. For the local metric, we assume that $C_F/C_{R,i} = 10$, for every component c_i, i.e. the cost of system failure is ten times the cost of repairing one component. The optimal prior maintenance action is to repair component c_2. As shown in Fig 11a, the local metric and the heuristic both identify c_2 as the component with the highest VoI.

However, if the maintenance cost for c_2 increases to $C_F/C_{R,2} = 5$ while the cost for the others remains the same, the optimal prior action becomes repairing c_4. Table 2 reports the optimal posterior actions depending on the inspection outcome, for this new assumption on the costs. As shown in Fig 11b, the local metric still gives the highest inspection priority to c_2 (as the global metric does), but the heuristic selects c_4 instead. The difference between the local metric and heuristic approach is because the posterior optimal action may not include repairing c_4 (e.g. after a silence on c_2), or it may include repairing uninspected components (e.g. after an alarm on c_1, c_3 is to be repaired). Though the VoI calculated from the heuristic approach is no higher than that from the true optimal solution, the heuristic approach overestimates the priority of inspecting c_4 over c_2, which is inconsistent with the local metric.

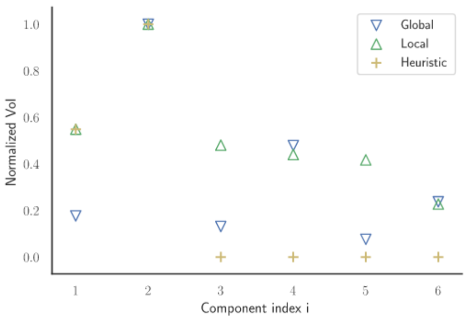
Inspected component \ Inspection outcome	Silence ($y_i = 1$)	Alarm ($y_i = 0$)
c_1	$\{c_4\}$	$\{c_3, c_4\}$
c_2	$\emptyset$	$\{c_3, c_4\}$
c_3	$\{c_4\}$	$\{c_3, c_4\}$
c_4	$\emptyset$	$\{c_4\}$
c_5	$\{c_6\}$	$\{c_4\}$
c_6	$\{\emptyset\}$	$\{c_6\}$

Table 2: Posterior subset of components to be repaired for the system in Fig 9.

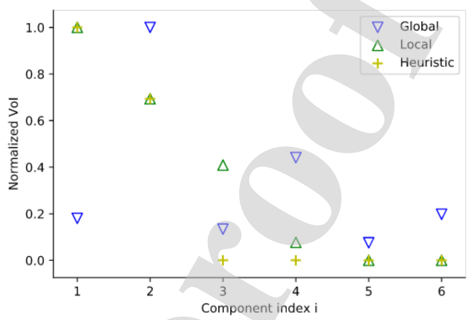


(a) (b)

Figure 11: Normalized VoI for the system in Fig 9, with $C_F/C_{R_i} = 10$ (a), and with $C_F/C_{R_2} = 5, C_F/C_{R_i} = 10, \forall i \neq 2$ (b).



(a)



(b)

Figure 12: Normalized VoI for the system in Fig 9, with $\epsilon_{FA} = \epsilon_{FS} = 0.01$ (a), and $\epsilon_{FA} = 0.01, \epsilon_{FS} = 0.40$ (b).

Error rates in imperfect inspections also affect the optimal decision. We now assume, again, that $C_F/C_{R,i} = 10$ for every component c_i, but inspections are imperfect; when $\epsilon_{FA} = \epsilon_{FS} = 0.01$, the corresponding VoI, shown in Fig 12a, is similar to the perfect inspection case shown in Fig 11a, and c_2 has the highest VoI. But when the type II error rate ϵ_{FS} is increased to 0.40, the VoI changes to Fig 12b, and component c_1 gains the highest priority according to the local metric and heuristic approach.

The second example is a 16-component system represented in Fig 13. The components have different topological importance: components c_1, c_4 and c_8, and the ones symmetric to them, can be considered as “bottlenecks”, with respect to other components.

We assume that the marginal probability of failure is $p_i = 0.01$ for every component c_i. For the global metric, we use $l^*(p_u) = p_u(1 - p_u)$ as a loss function, and the corresponding VoI is shown in Fig 14a. For the local metric, we assume the cost ratio between system failure penalty and component

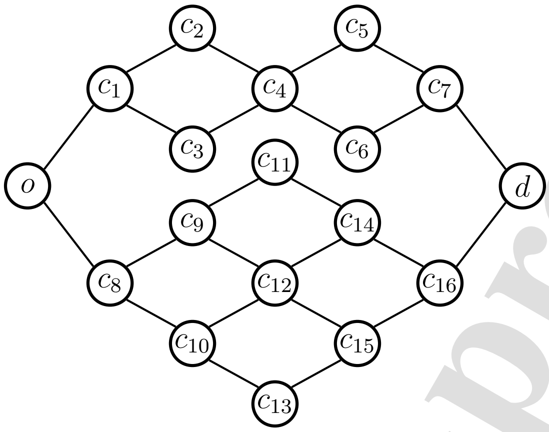
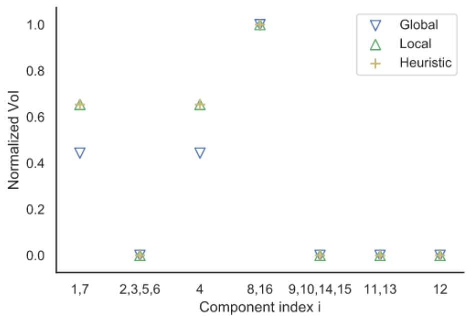


Figure 13: Block diagram of 16 component system

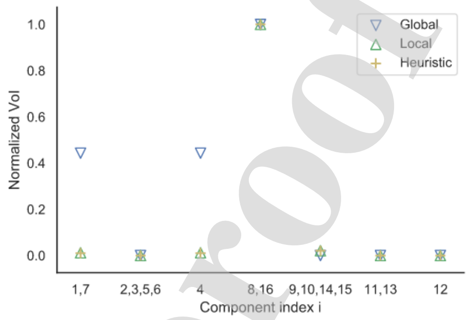
maintenance is $C_F/C_{R,i} = 10^3$ for every component c_i, so that the resulting optimal prior maintenance action is to repair no component. Under the local metric, c_8 and c_{16} have the highest VoI, followed by c_1, c_4 and c_7. The heuristic approach gives the same result as the local metric and the global metric.

However, if $C_F/C_{R,i}$ increases to 10^4 for every component c_i, the new optimal prior maintenance action becomes repair the symmetric bottlenecks c_8 and c_{16}. The VoI for the global and local metrics and the heuristic with this new assumption on costs is illustrated is Fig 14b. The local VoI of inspecting c_2, c_9 and the components symmetric to them is now nil, because the cost for system failure is so (relatively) high, that the agent will not alter the prior action even if a silence is received on these components.

Depending on the setting, the bottleneck components may not always have the highest VoI. If $p_{11} = 0.5, p_{12} = 0.4, p_{13} = 0.3$, $p_i = 0.01, i \neq 11, 12, 13$ and $C_F/C_{R,i} = 1000$ for every component c_i, the VoI is that shown in Fig 15.



(a)



(b)

Figure 14: Normalized VoI for the system in Fig 13 with different maintenance cost, with and $C_F/C_{R,i} = 10^3$ (a) and $C_F/C_{R,i} = 10^4$ (b)

Now the optimal prior action is to repair c_{12}. The global metric prioritizes c_1, c_4 and c_7 for inspection, but the local metric prioritizes c_{13}, even though it is not the most vulnerable component (which is c_{11}). After c_{13}, the components with high VoI under local metric will be c_{12} and c_{11}. Instead, the heuristic approach assigns the highest VoI to c_{12}. This assignment occurs because, when the inspection of c_{11} or c_{13} receives silence, the optimal action is to do nothing, but the heuristic approach forces the agent to at least execute the prior plan.

The third example is taken from [21] and it represents a two-line electrical substation with 12 components with 6 different functions as illustrated in Fig 16: DS - Disconnect Switch, CB - Circuit Breaker, PT - Power Transformer, DB - Drawout Breaker, TB - Tie Breaker, FB - Feeder Breaker. We assume that the marginal failure probability of the components with function DS, CB or DB is 9.53×10^{-3}, and that of components with function FB, PT and TB is 2.32×10^{-3}. For every component c_i, costs are defined by ratio

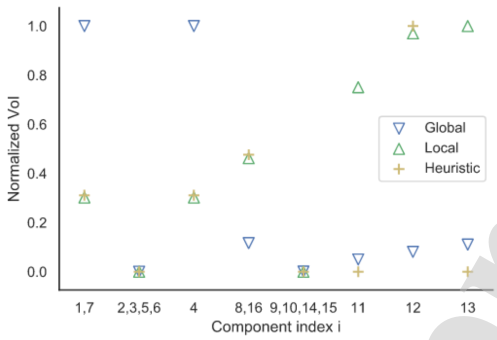


Figure 15: Normalized VoI for the system in Fig 13 with $p_i = 0.01, i \neq 11, 12, 13$, $p_{11} = 0.5, p_{12} = 0.4, p_{13} = 0.3$ and $C_F/C_{R,i} = 10^3$

$C_F/C_{R_i} = 1000$. In this example, we investigate how the correlation between a component's state affects the VoI. If all the components are statistically dependent, complexity of computing the system failure probability may become intractable. Conditional independence between component events given the outcomes of a few random variables representing the source of common effects was assumed in [22], and a matrix-based method based on this assumption was developed to compute the system reliability. Following that work, we assume interdependence among the components' states, but only for components with the same function. For group k of components with the same function, let x_k denote a binary variable which indicates the occurrence of an external event relevant for the group if $x_k = 1$, and it is $x_k = 0$ otherwise. The Bernoulli probability of such variable is defined by probability $\alpha_k = \mathbb{P}[x_k = 0]$. For any component c_i within that group, if $x_k = 1$, then the component is surely functioning, i.e. $\mathbb{P}[s_i = 1|x_k = 1] = 1$; while if $x_k = 0$ component c_i fails with conditional probability $\beta_k = \mathbb{P}[s_i = 0|x_k = 0]$. Let ρ_k be the correlation coefficient between the states of any pair of components

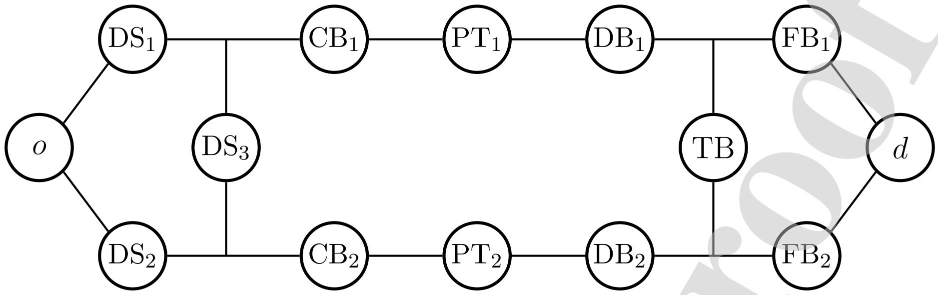


Figure 16: Block diagram of a two-transmission-line substation system

within group k, and $\overline{p}_k$ the marginal failure probability for any component in the group. The corresponding factors are:

$$\begin{cases} \beta_k = \rho_k(1-\overline{p}_k) + \overline{p}_k \\ \alpha_k = \overline{p}_k/\beta_k \end{cases} \tag{22}$$

The example of Fig 16 is defined by 6 pairs of features $\{\overline{p}_1, \rho_1, \ldots \overline{p}_6, \rho_6\}$ corresponding to coefficients $\{\alpha_1, \beta_1, \ldots, \alpha_6, \beta_6\}$.

When all the components are independent, the prior action is to do nothing, and the optimal posterior action is to repair the inspected component after an alarm, except for DS$_3$ and TB. Thus, the local metric and heuristic give identical results. Although CB and DB have relatively higher failure probability compared with other components, the cost reduction by repairing the damaged components CB or DB is significantly higher than repairing others. This is why CB and DB components have the highest VoI according to the local metric and the heuristic, as shown in Fig 17a. For the global metric, with loss function $l^*(p_u) = p_u(1-p_u)$, the posterior system failure probability given an alarm from components CB or DB is the highest, and the probability given a silence from those components is the lowest, i.e. posterior intervals $I_{\mathrm{CB}} = I_{\mathrm{DB}}$ contain the corresponding intervals of all the others; thus, those components have the highest VoI, according to the global metric.

When the correlation among states in DS components grows, while other groups remain independent (and the marginal probability remains the same), the VoI favors the group of correlated components. The prior action becomes repairing DS_1 or DS_2 when the correlation coefficient ρ is above 0.4. The optimal action is shown in Table 3. Components DS_1 or DS_2 should be kept functioning, depending on which link set the inspected component is in. One exception is DS_3, which has different VoI for the local metric and the heuristic. As shown in Fig 17b, when the correlation coefficient ρ for the states of components DS increases, inspecting one of them reveals additional information about the other two, making the VoI of inspecting DS components higher than the VoI of other components with different functions. When ρ is close to one, DS_1 and DS_2 act like one bottleneck component, which dominates the VoI as shown in Fig 17c.

6. Discussion and Conclusions

We have derived metrics based on the VoI to assign priorities among component inspections in networked systems. The VoI analysis can be applied to any setting, but its computational complexity depends on the complexity of the ingredients that define the problem. We have restricted the attention to binary components, binary inspection outcomes in a binary system. In this setting, we have introduced two metrics, local and global, that assume different sets of available actions and different loss functions. The problems modeled by the global metric do not form a strict subclass of that mod-

Insp. outcome / Component	Silence ($y_i = 1$)	Alarm ($y_i = 0$)
DS_1	$\emptyset$	DS_1
DS_2	$\emptyset$	DS_2
DS_3	$\emptyset$	DS_3
CB_1	DS_1	DS_1, CB_1
CB_2	DS_2	DS_2, CB_2
PT_1	DS_1	DS_1, PT_1
PT_2	DS_2	DS_2, PT_2
DB_1	DS_1	DS_1, DB_1
DB_2	DS_2	DS_2, DB_2
TB	DS_1	DS_1
FB_1	DS_1	DS_1, FB_1
FB_2	DS_2	DS_2, FB_2

Table 3: Optimal posterior action for the system in Fig 16 when $\rho = 0.4$

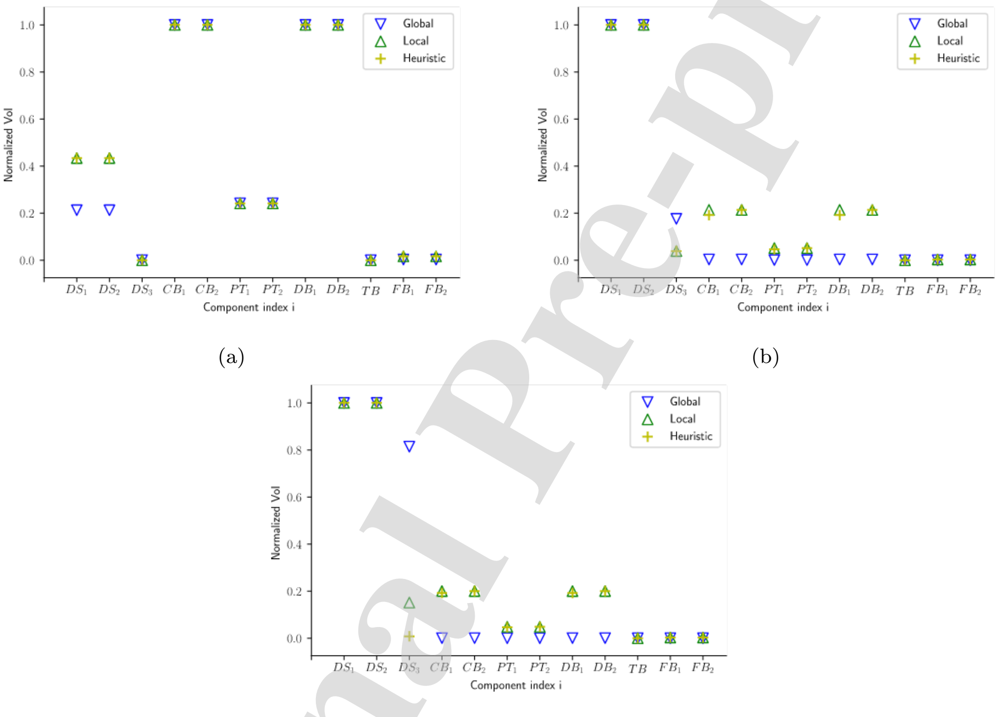


Figure 17: Normalized VoI for the system in Fig 16, with correlation among the DS component of $\rho = 0$ (a), $\rho = 0.4$ (b), $\rho = 0.9$ (c)

eled by the local metric. We have proven general rules for identifying what components have higher importance for those metrics in series and parallel systems. The evaluation of the global metric is generally less complex than evaluation of the local metric, because of the underlying optimization of the maintenance actions of different scales. The selection of the appropriate metric should be based on the actual set of actions available. However, when only limited computational resources are available, a simpler metric such as the global metric or the heuristic approach may be appropriate.

We have proposed a heuristic approach to approximate the local metric, by simplifying the corresponding optimization of maintenance actions. We have illustrated the heuristic's performance in some examples, but there is no guarantee that the heuristic captures the exact local metric. The VoI assessed by the heuristic is surely non-negative, and no higher than the VoI of the original local metric; however, the ranking can be arbitrarily different.

The distinction between local and global metrics can be extended to the case of multiple values (more than binary) for the state of the components and of the system, and for inspection outcomes. However, some concepts are defined only for the binary case, e.g. the posterior intervals in the global metric are defined only for binary inspection outcomes in a binary system (if the system state dimension is higher than the component state dimension, then the posterior interval can be generalized into the concept of "posterior polyhedron").

We have limited the analysis to the "static" optimization of the inspection of one component. Several more complex problems can be built on this optimization. One complex problem is the off-line or on-line optimization of multiple inspections for a system with static condition states [13], that can be based on a greedy sequential approximation. The same global and local approaches can be adopted in the greedy approach. Among these two options, the on-line setting is generally simpler; the off-line option is generally more expensive because M binary inspections produce 2^M joint outcomes that must be analyzed in an exhaustive pre-posterior analysis. Another extension is related to temporal problems, in which the components' condition degrades in time, and they can be periodically and sequentially inspected and repaired [15, 16]. Given the complex interplay between present and future decisions and costs, we cannot predict the effectiveness of the metrics proposed when applied to those dynamic settings.

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Appendix A. Notation table

Notation	Meaning
N	Number of components
$c_i, i = 1, \cdots, N$	Component index
$s_i \in \mathbb{B}, i = 1, \cdots, N$	Binary component state

$s = [s_1, s_2, \cdots, s_N]$	System state vector
$u \in \mathbb{B} = \phi(s)$	Binary system state
p_s	Probability distribution of s
$p_i, i = 1, \cdots, N$	Marginal failure probability of component c_i
p_u	Failure probability of the system
$y_i \in \mathbb{B}, i = 1, \cdots, N$	Inspection outcome on component c_i
p_{y_i}	Prior probability of receiving y_i
$p_{s\|y_i}$	Posterior probability distribution of s given y_i
$p_{\omega\|y_i}$	Posterior system failure probability given y_i
A	Maintenance action that changes the state s
$p_{s'\|s,A}$	Posterior probability distribution of s' given s and A
$\mathcal{L}(s', A)$	Expected loss given action A and posterior state s'
$\mathcal{L}_{\mathrm{I}}(s')$	Expected system failure penalty
$\mathcal{L}_{\mathrm{II}}(A)$	Expected action cost
L_π	Minimum expected cost before inspection
$L_\omega(i)$	Minimum expected cost after inspecting c_i
$\mathrm{VoI}(i)$	Value of Information for inspecting c_i
$\epsilon_{FS}, \epsilon_{FA}$	False silence and alarm inspection error rate
h_i	Probability of receiving alarm on c_i
$l_{A,u}$	Cost given prior state u and action A
$q_{u,A}$	System failure probability given prior state u and action A
$p'_{\omega\|A,u}$	Posterior probability of system failure given prior state u and action A
C_A	Action cost
C_F	System failure cost

$l_A(p_u)$	Expected loss when taking action A given prior belief p_u
$L^G_\omega(i)$	VoI of inspecting c_i under global metric
$L^L_\omega(i)$	VoI of inspecting c_i under local metric
$l_{PI}(p_u)$	Value of Perfect Information (VoPI) given prior belief p_u
$rg(p_u)$	Difference between the VoI and VoPI

Table A.4: Major notations

Appendix B. Importance Measures

Similar to the Birnbaum's measure, the Criticality IM [7], evaluates the importance of c_i with the approximated conditional component failure probability given that the system has failed:

$$\mathrm{CRT}(i) = (p_{\omega|y_i=0} - p_{\omega|y_i=1})\frac{p_i}{p_\pi} \propto \mathrm{BM}(i) \cdot p_i \tag{B.1}$$

Some IMs emphasize on the topology structure of the system. Based on the cut sets, [9] evaluates the importance of c_i by the number of cut sets it belongs to and the accumulated appearance probability of such cut sets.

To use IMs as utility-based applications, the risk achievement worth (RAW) and the risk reduction worth (RRW) are developed. RAW evaluates the component with the contributions of maintaining a certain level of reliability of the component to the system reliability, i.e. for component c_j, its importance can be measured as:

$$\mathrm{RAW}(i) = \frac{1 - p_{\omega|y_i=1}}{p_\pi} \tag{B.2}$$

So, between two components c_i and c_j, $\mathrm{RAW}(i) \geq \mathrm{RAW}(j) \Leftrightarrow p_{\omega|y_i=1} \leq p_{\omega|y_j=1}$. RRW evaluates a component by the decrease of system failure risk given that the component is intact:

$$\mathrm{RRW}(j) = \frac{p_\pi}{1 - p_{\omega|y_i=0}} \tag{B.3}$$

So $\mathrm{RRW}(i) \geq \mathrm{RRW}(j) \Leftrightarrow p_{\omega|y_i=0} \geq p_{\omega|y_j=0}$.

Appendix C. Nested posterior intervals in the global metric

To prove the lemma in Section 3.1, we now write $p_{\omega|y_a=b}$ as $x_{a,b}$ for simplicity. We assume that $I_i \supseteq I_j$, we have that $0 \leq x_{i,1} \leq x_{j,1} \leq x_{j,0} \leq x_{i,0} \leq 1$. Because of the law of expectation, we have:

$$p_\pi = p_1 x_{i,1} + (1 - p_1) x_{i,0} = p_2 x_{j,1} + (1 - p_2) x_{j,0} \tag{C.1}$$

We prove that:

$$L^{\mathrm{G}}_\omega(1) = p_1 l(x_{i,1}) + (1 - p_1) l(x_{i,0}) \leq p_2 l(x_{j,1}) + (1 - p_2) l(x_{j,0}) = L^{\mathrm{G}}_\omega(2) \tag{C.2}$$

Because $x_{j,1} = \frac{x_{i,0}-x_{j,1}}{x_{i,0}-x_{i,1}} x_{i,1} + \frac{x_{j,1}-x_{i,1}}{x_{i,0}-x_{i,1}} x_{i,0}$ and $x_{j,0} = \frac{x_{i,0}-x_{j,0}}{x_{i,0}-x_{i,1}} x_{i,1} + \frac{x_{j,0}-x_{j,1}}{x_{i,0}-x_{i,1}} x_{i,0}$, and l is a concave function, we have:

$$\begin{aligned} p_2 l(x_{j,1}) + (1 - p_2) l(x_{j,0}) \geq & p_2 [\frac{x_{i,0} - x_{j,1}}{x_{i,0} - x_{i,1}} l(x_{i,1}) + \frac{x_{j,1} - x_{i,1}}{x_{i,0} - x_{i,1}} l(x_{i,0})] \\ & + (1 - p_2) [\frac{x_{i,0} - x_{j,0}}{x_{i,0} - x_{i,1}} l(x_{i,1}) + \frac{x_{j,0} - x_{j,1}}{x_{i,0} - x_{i,1}} l(x_{i,0})] \\ = & p_1 l(x_{i,1}) + (1 - p_1) l(x_{i,0}) \end{aligned} \tag{C.3}$$

References

[1] Andriotis, C., Papakonstantinou, K., Chatzi, E., 2019. Value of structural health monitoring quantification in partially observable stochastic environments. arXiv preprint arXiv:1912.12534 .

[2] Ball, M.O., 1986. Computational complexity of network reliability analysis: An overview. IEEE Transactions on Reliability 35, 230–239.

[3] Bensi, M., Kiureghian, A.D., Straub, D., 2015. Framework for post-earthquake risk assessment and decision making for infrastructure systems. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering 1, 04014003.

[4] Birnbaum, Z.W., 1968. On the importance of different components in a multicomponent system. Technical Report. Washington Univ Seattle Lab of Statistical Research.

[5] Daneshkhah, A., Stocks, N.G., Jeffrey, P., 2017. Probabilistic sensitivity analysis of optimised preventive maintenance strategies for deteriorating infrastructure assets. Reliability Engineering & System Safety 163, 33–45.

[6] Der Kiureghian, A., Ditlevsen, O.D., Song, J., 2007. Availability, reliability and downtime of systems with repairable components. Reliability Engineering and System Safety 92, 231–242. doi:10.1016/j.ress.2005.12.003.

[7] Espiritu, J.F., Coit, D.W., Prakash, U., 2007. Component criticality importance measures for the power industry. Electric Power Systems Research 77, 407–420.

[8] Fauriat, W., Zio, E., 2018. An importance measure to assess the value of a component inspection policy, in: 2018 3rd International Conference on System Reliability and Safety (ICSRS), IEEE. pp. 368–375.

[9] Fussell, J., 1975. How to hand-calculate system reliability and safety characteristics. IEEE Transactions on Reliability 24, 169–174.

[10] Howard, R.A., 1966. Information value theory. IEEE Transactions on systems science and cybernetics 2, 22–26.

[11] Kroese, D.P., Taimre, T., Botev, Z.I., 2013. Handbook of monte carlo methods. volume 706. John Wiley & Sons.

[12] Malings, C., Pozzi, M., 2016a. Conditional entropy and value of information metrics for optimal sensing in infrastructure systems. Structural Safety 60, 77 – 90. doi:https://doi.org/10.1016/j.strusafe.2015.10.003.

[13] Malings, C., Pozzi, M., 2016b. Value of information for spatially distributed systems: Application to sensor placement. Reliability Engineering & System Safety 154, 219–233. doi:10.1016/j.ress.2016.05.010.

[14] Malings, C., Pozzi, M., 2018. Value-of-information in spatio-temporal systems: Sensor placement and scheduling. Reliability Engineering & System Safety 172, 45–57.

[15] Malings, C., Pozzi, M., 2019. Submodularity issues in value-of-information-based sensor placement. Reliability Engineering and System Safety 183, 93–103. doi:10.1016/j.ress.2018.11.010.

[16] Memarzadeh, M., Pozzi, M., 2016. Value of information in sequential decision making: Component inspection, permanent monitoring and system-level scheduling. Reliability Engineering & System Safety 154, 137–151.

[17] Miller, A.C., 1975. The value of sequential information. Management Science 22, 1 – 11.

[18] Pozzi, M., Der Kiureghian, A., 2011. Assessing the value of information for long-term structural health monitoring. Health Monitoring of Structural and Biological Systems 2011 7984, 79842W. doi:10.1117/12.881918.

[19] Rosenthal, A., 1977. Computing the reliability of complex networks. SIAM Journal on Applied Mathematics 32, 384–393.

[20] Shamaiah, M., Banerjee, S., Vikalo, H., 2010. Greedy sensor selection: Leveraging submodularity, in: 49th IEEE conference on decision and control (CDC), IEEE. pp. 2572–2577.

[21] Song, J., Der Kiureghian, A., 2003. Bounds on System Reliability by Linear Programming. Journal of Engineering Mechanics 129, 627–636. doi:10.1061/(asce)0733-9399(2003)129:6(627).

[22] Song, J., Kang, W.H., 2009. System reliability and sensitivity under statistical dependence by matrix-based system reliability method. Structural Safety 31, 148–156. doi:10.1016/j.strusafe.2008.06.012.

[23] Song, J., Ok, S.Y., 2010. Multi-scale system reliability analysis of lifeline networks under earthquake hazards. Earthquake engineering & structural dynamics 39, 259–279.

[24] Srinivasan, R., Parlikad, A.K., 2013. Value of condition monitoring in infrastructure maintenance. Computers & Industrial Engineering 66, 233–241.

[25] Straub, D., Chatzi, E., Bismut, E., Courage, W., Döhler, M., Faber, M.H., Köhler, J., Lombaert, G., Omenzetter, P., Pozzi, M., et al., 2017. Value of information: A roadmap to quantifying the benefit of structural health monitoring, in: ICOSSAR-12th International Conference on Structural Safety & Reliability.

[26] Straub, D., Faber, M.H., 2005. Risk based inspection planning for structural systems. Structural Safety 27, 335 – 355. doi:https://doi.org/10.1016/j.strusafe.2005.04.001.

[27] Thöns, S., 2018. On the value of monitoring information for the structural integrity and risk management. Computer-Aided Civil and Infrastructure Engineering 33, 79–94.

[28] Tien, I., Der Kiureghian, A., 2013. Compression algorithm for bayesian network modeling of binary systems.

[29] Wu, S., Coolen, F.P., 2013. A cost-based importance measure for system components: An extension of the Birnbaum importance. European Journal of Operational Research 225, 189–195. doi:10.1016/j.ejor.2012.09.034.

[30] Zio, E., Podofillini, L., 2007. Importance measures and genetic algorithms for designing a risk-informed optimally balanced system. Reliability Engineering & System Safety 92, 1435–1447.

[31] Zitrou, A., Bedford, T., Daneshkhah, A., 2013. Robustness of maintenance decisions: Uncertainty modelling and value of information. Reliability Engineering & System Safety 120, 60–71.

Authorship Conformation Form

For the manuscript “Optimal inspection of network systems via Value of Information Analysis”, by Chaochao Lin, Junho Song and Matteo Pozzi, submitted to RESS, the authors agree that:

- All authors have participated in (a) conception and design, or analysis and interpretation of the data; (b) drafting the article or revising it critically for important intellectual content; and (c) approval of the final version.

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Conflict of Interest

For the manuscript “Optimal inspection of network systems via Value of Information Analysis”, by Chaochao Lin, Junho Song and Matteo Pozzi, submitted to RESS, the authors agree that:

- The authors have no affiliation with any organization with a direct or indirect financial interest in the subject matter discussed in the manuscript.

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