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Conceptualizing Stable States in Origami-Based Devices Using an Energy Visualization Approach

In many origami-based applications, a device needs to be maintained in one or more fold states. The origami stability integration method (OSIM) presented in this paper provides an approach for graphically combining various techniques to achieve stability. Existing stability techniques are also categorized into four groups based on whether they are intrinsic or extrinsic to the origami pattern and whether they exhibit gradual or non-gradual energy storage behaviors. These categorizations can help designers select appropriate techniques for their application. The paper also contains design considerations and resources for achieving stability. Finally, two case studies are presented which use the OSIM and the technique categorization to conceptualize stability in origami-based devices.
[DOI: 10.1115/1.4046437]

Keywords: origami-based design, compliant mechanisms, conceptual design, design visualization

1 Introduction

Many advantages of origami capitalize on its shape-changing ability—how origami moves as it folds. Many origami-based devices can be manufactured starting in a planar state then folding into a desired shape [1–4]. Origami can provide complex motions with simple actuation and low degrees-of-freedom (DOF) [5–7]. This ability to quickly transform from a compact shape to a deployed shape has inspired the design of many mechanisms such as spherical overconstrained mechanisms [8], conceal-and-reveal systems [9], medical stents [10], origami shelters [11], and deployable solar arrays [12].

However, the ability of origami-based devices to fold can also make them mechanically unstable and lead to undesired motion or behaviors. The balance between retaining foldability and providing adequate stability is a fundamental challenge for origami-based design. While many techniques exist for creating stability in origami-based devices, major difficulties for designers lie in (1) determining which techniques to use and (2) how they will interact with the chosen origami pattern, the loading conditions, and other techniques.

This paper addresses these difficulties by presenting a method for classifying origami stability and by introducing the *origami stability integration method* (OSIM), a design tool for visualizing and predicting how stability can be achieved in origami devices with the combination of techniques and loading conditions. The paper is outlined as follows: review of background material (Sec. 2), description of the OSIM (Sec. 3), discussion on stability technique categorization (Sec. 4), description of select stability techniques (Sec. 5), and presentation of two case studies (Sec. 6).

2 Background

This work builds on the concepts of viewing origami as a linkage and visualizing energy storage behavior graphically. It also references a variety of existing origami stability techniques.

Contributed by the Mechanisms and Robotics Committee of ASME for publication in the Journal of Mechanical Design. Manuscript received July 29, 2019; final manuscript received January 14, 2020; published online February 24, 2020. Assoc. Editor: Joo H. Kim.

2.1 Origami as a Kinematic Linkage. The motion of origami can be modeled as a kinematic linkage [8,13,14]; the facets are modeled as links, the creases as hinge joints, and origami vertices as spherical mechanisms [15], as shown in Fig. 1. Thinking of origami as a kinematic linkage allows designers to apply engineering concepts (such as mechanical advantage [16], motion prediction [17], and stability [18]) to origami design.

2.2 Visualizing Stability. A *stable equilibrium* is a state of a system in which the sum of the net forces is zero and the energy stored in the system is at a local minima. This is often visualized using a “ball on a hill” analogy, where all energy in the system is shown as potential energy [19]. The stable equilibria are the local minima, as illustrated in Fig. 2. This method has been used in origami research to depict the energy storage behaviors of multi-stable vertices [20], bistable leaf-out origami [21], and a bistable waterbomb pattern [22]. The OSIM also uses the “ball on a hill” analogy for conceptualizing stable equilibria. The examples in this paper use approximated energy storage behaviors (including the assumption that origami facets are mass-less and have zero-inertia). However, higher fidelity models that include components such as damping, dynamics, and force-deflection interactions could also be used.

2.3 Origami Stability Techniques. A myriad of stability techniques are seen in origami-based products and origami literature [1,2,5,15,20,23–30], as well as in a number of reviews [30,31].

Many techniques are incorporated as part of the creases or facets. Some examples include strain energy stored in the joints during folding [20–22,32–35] and introducing bias at the joints using other stimuli, such as heat, magnetic fields, or surface tension [1,2,28–30,36].

Other techniques not incorporated as part of the creases or facets include clasps, magnets, actuators, and other constraints [37,38]. For example, the origami solar array [12] uses an expanding external frame (fitted with torsional springs) to pull the array into the unfolded fold state, as shown in Fig. 3.

3 Origami Stability Integration Method

The OSIM is a design tool for visualizing and planning stable equilibria in origami-based devices. This section addresses

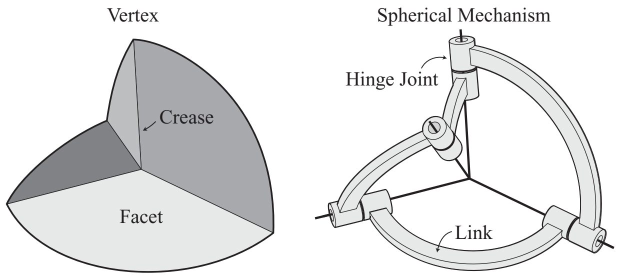

Fig. 1 An origami vertex can be modeled as a spherical mechanism

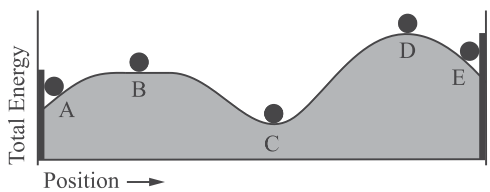

Fig. 2 A "ball on a hill" diagram for visualizing stability through potential energy. Positions A, C, and E are stable equilibrium positions. Position D is an unstable equilibrium position. Position B is a neutrally stable position.

Fig. 3 An expanding external frame (shown in white) is the stability technique used to pull open the origami solar array [12], creating a stable unfolded fold state. This image shows the array mid-deployment.

preliminary concepts, describes the steps of the OSIM, and provides an example.

3.1 Preliminaries. To facilitate the discussion of origami in the OSIM, two concepts, *origami linkage* and *fold-state continuum*, are introduced. The term *origami linkage* provides a way to discuss the facets and creases of a fold state without referring to the fold angles. The *fold-state continuum* provides a simple way to represent a range of fold states.

3.1.1 Origami Linkage. Most origami-based devices utilize the movement of multiple vertices. However, the common terms used to discuss origami as a kinematic linkage (such as *origami vertex*, *origami figure*, *configuration*, *fold state*, and *crease pattern*) either do not refer to a generalized set of facets and creases or they imply a static position. To address these limitations, a new term, *origami linkage*, is presented.

An *origami linkage* is a set of planar facets and their interconnecting creases. This term builds on the traditional use of the

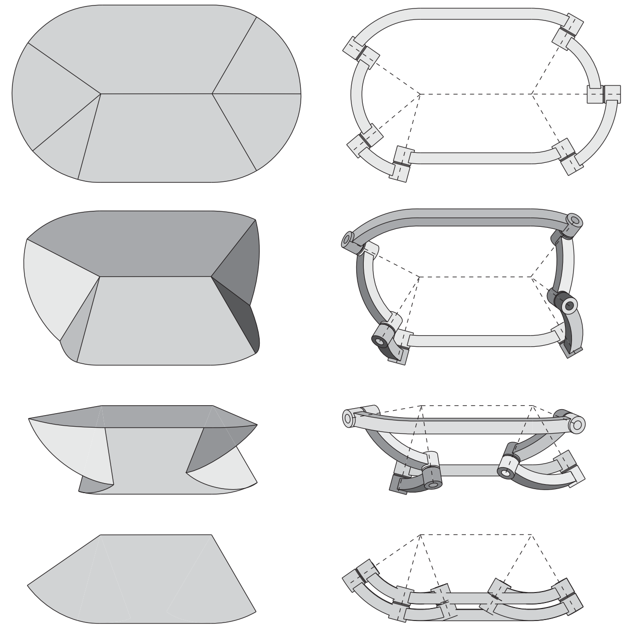

Fig. 4 A two-vertex origami linkage represented as an origami linkage (left) and a kinematically equivalent spherical mechanism (right), shown in four corresponding fold states

term *linkage* in engineering to emphasize that the members exist independent of their position and to strengthen the concept that origami can be an engineering tool. The term origami linkage will be used in the OSIM because the set of facets and creases is represented for a range of its fold angles (not just one fold state).

An origami linkage can be as simple as a single vertex or multiple vertices, as shown in Fig. 4. Typically, an origami linkage has an equivalent spherical mechanism. A *fold state* is an origami linkage in a defined position.

Selecting an origami linkage is an important part of the OSIM. Because a variety of resources are available on this topic [39–44], this paper does not give specific guidance on this step. Furthermore, each of the included examples of the OSIM assume that an origami linkage had already been defined by the designers.

3.1.2 Fold-State Continuum of Origami Linkages. As an origami linkage folds, it progresses through a continuous set of fold states that can form a continuum. A fold-state continuum is a useful tool in analyzing and understanding the range of motion of an origami linkage and the properties of the linkage throughout the folding process.

The fold-state continuum for a single DOF origami linkage can be plotted along a single line, as seen in Fig. 5. This 1D continuum is used as the x-axis in the OSIM. Linkages with multiple DOF can also be represented using a 1D fold-state continuum using symmetry or other constraints (such as constraints applied by a user during manipulation) [21,22,45]. Care should be taken to ensure that origami linkages deploy correctly from any change-point position (such as the fully unfolded state).

Joints in spherical mechanisms can undergo continuous rotation. However, origami linkages typically interfere at ± 180 deg due to adjacent facet interference. Other constraints, such as limited facet extension, may further limit the motion. Because of this, 1D fold-state continuums are usually limited to a certain range bounded by the possible motion of the origami linkage. This is represented as vertical lines on either side of the continuum, as shown in Fig. 5.

3.2 Steps of the OSIM. The OSIM is intended as a conceptual design tool. The steps are arranged to be a systematic method to think through the problem, to understand how all the components

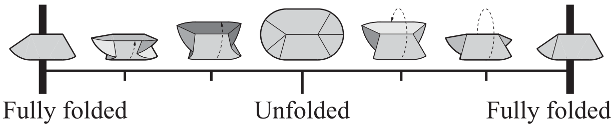

Fig. 5 A 1D fold-state continuum of a single DOF origami linkage in Fig. 4. Seven fold states along the continuum are shown and marked on the continuum (three are labeled). Dashed arrows are also included on some fold states to help visualize the position.

interact, and to get a good idea of which stability techniques would be most beneficial for the application.

The steps of the method are as follows:

(1) Select an origami linkage (see Sec. 3.1.1).
(2) Outline a 1D fold-state continuum for the linkage (see Sec. 3.1.2.).
(3) Designate zones critical to the function of the origami-based device, as shown in Fig. 6.
  (a) *Desired stability zones*: Stable fold states are desired in the final device. Ideally, origami linkages are stable in a single, predetermined fold state (i.e., one position along the continuum). However, some margin of error is often acceptable. The width of the desired stability zone corresponds with this acceptable stability tolerance.
  (b) *Unacceptable motion zones*: Fold states that the origami linkage should not reach (such as zones that contain bifurcation points). This zone should be "blocked," meaning that it should have a high energy storage behavior.
(4) Determine (approximate or calculate) the energy components inherent in the system (such as gravity, strain in the hinges, and other loading conditions). Graph the energy stored by each component along the fold-state continuum (see Sec. 2.2.).
(5) Sum the energy components from step 4.
(6) *Superimpose a "desired energy curve"*: A curve that achieves an energy storage that satisfies the desired behaviors outlined in step 3—stable equilibrium in the desired stability zones and high energy storage in the unacceptable motion zones, as shown in Fig. 6.
(7) Select stability techniques to obtain this desired energy curve and determine (approximate or calculate) their energy behavior along the continuum. (Resources for selecting techniques are given in Secs. 4 and 5.)
(8) Sum each energy component from steps 4 and 7. Compare the result of the sum to the desired energy behavior. Prototype and revisit steps as necessary.

Different combinations of techniques and linkages can produce favorable results. As such, the steps of the OSIM are meant to be

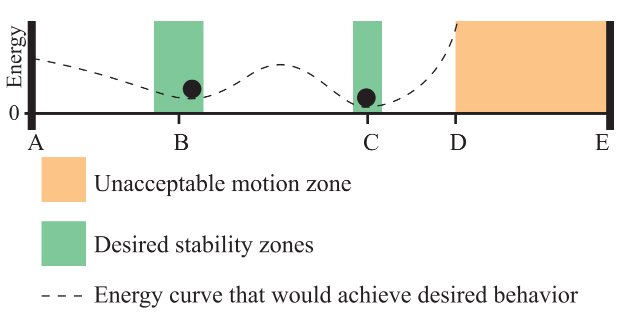

Fig. 6 An example of how to indicate "desired stability zones" and "unacceptable motion zones," with an energy curve that achieves the desired behavior. Five example fold states are labeled (A)–(E). Fold states B and C are desired stable fold states. The motion of the origami linkage should not progress past fold state D.

Fig. 7 An origami-based colander that uses the limited facet extension technique in combination with the loading condition (gravity) to achieve stability in its unfolded state (image used with permission from B&R Plastics, Inc®)

an iterative process. Throughout the process, the origami linkage could be modified or replaced. A well-designed origami linkage and loading conditions can reduce the number of techniques necessary for achieving the desired energy conditions. For example, while most loading conditions create instability in the unfolded state, the linkage used for the origami colander [46] (shown in Fig. 7)[1] effectively uses the loading condition to bias toward a stable equilibrium. Gravity pulls the side facets into extension and this creates stability.

3.3 Example of the OSIM: Origami Baby Tub.

The PUJ tub® is an origami-based product that is flat for storage and can be folded into a seat shape and placed in a sink to be used as a baby bathtub.[2] While the OSIM was not used to design stability in this product, it is used here to illustrate the steps of the OSIM (see Fig. 8).

Step 1: A 2-DOF origami linkage is selected. This origami linkage is not rigidly foldable, so a flexible foam is used.

Step 2: Applying symmetry constraints (simulating the constraints applied by a user), the 2-DOF motion is simplified to be represented along a 1D fold-state continuum. Adjacent facet interferences are the end boundaries of the continuum (shown by thicker lines on each side of the fold-state continuum as described in Sec. 3.1.2).

Step 3: There are two desired stability zones: when the tub is in the unfolded fold state (for storage) and when fully folded (tub shape). The design should also be blocked from folding the wrong direction (shown by the unacceptable motion zone).

Step 4: Strain of the largest panel is the only significant energy component. Its energy behavior is graphed along the fold-state continuum.

Step 5: No summing necessary because step 4 had only one energy component.

Step 6: A desired energy curve is superimposed.

Step 7: Two additional techniques are used to provide stability. Hinge interference prevents the tub from being folded in the wrong direction (shown in Fig. 11)[3] and magnets help keep it in the tub shape.

Step 8: The combined energy components result in the desired energy behavior.

[1]https://www.brplastics.com
[2]https://www.puj.com
[3]See Note 2

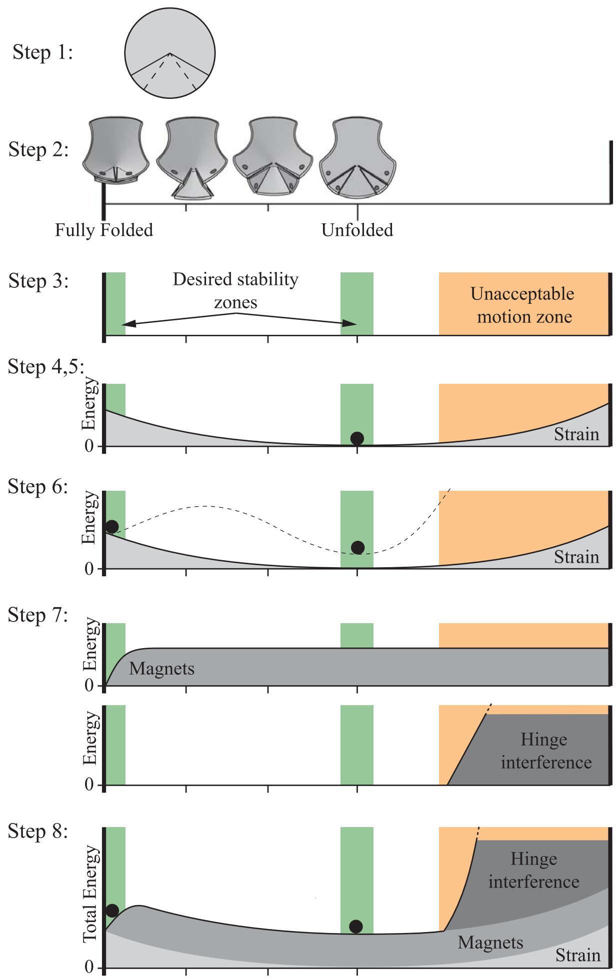

Fig. 8 The steps of the OSIM for the origami baby tub, as described in Sec. 3.3

Two additional examples of this process are given in Sec. 6 as case studies.

4 Stability Technique Classification

Selecting stability techniques is a critical part of the OSIM (step 7). Therefore, this section presents criteria to help designers select appropriate techniques, provides some benefits and drawbacks of each criteria, and reviews how often techniques have been used in existing products. The proposed classification helps designers choose techniques that (1) can be integrated into their device and (2) can achieve the desired energy storage behavior.

4.1 Classification Criteria. Two criteria are introduced: intrinsic versus extrinsic and gradual versus non-gradual. This results in four groups of techniques: intrinsic gradual, intrinsic non-gradual, extrinsic gradual, and extrinsic non-gradual. Details about each group and its techniques are given in Sec. 5. Common example techniques for each type are given in Table 1.

4.1.1 Intrinsic Versus Extrinsic. This criterion classifies how the technique is applied to an origami linkage.

An *intrinsic stability technique* assists in realizing stable equilibria using only the creases and facets of an origami linkage. Examples include hinge interference, compliant joints, and non-rigidly foldable linkages [5]. In this paper, stimuli-actuated techniques are also considered intrinsic despite the need for outside influence.

Table 1 Examples of stability techniques within each of the four technique groups

	Gradual	Non-gradual
Intrinsic	Non-rigid origami Compliant joints Stimuli-actuated joints	Hinge interference Adjacent facet interference Global facet interference Sequential flap interference Limited facet extension
Extrinsic	Actuators Torsional springs Linear springs Gravity	Offsets Snaps, clasps, buckles Telescoping poles Hook-and-loop

An *extrinsic stability technique* uses external hardware (not just the facets and creases) to help realize stable equilibria. Examples include clasps, magnets, straps, and actuators.

Decisions on whether to use intrinsic or extrinsic techniques are often about appearance and manufacturability rather than energy storage behavior. Referencing the baby tub example (Sec. 3.3), because the appearance of the final product is a top priority, intrinsic techniques would be more preferred than extrinsic techniques.

4.1.2 Gradual Versus Non-Gradual. This criterion is meant to classify the energy storage behavior of the technique. Decisions on whether to choose gradual or non-gradual techniques are often based on the shape of the desired energy curve (step 6 of the OSIM).

A *gradual stability technique* creates a gradual (i.e., smooth) change in the energy of the linkage along the fold-state continuum. These techniques may affect the energy along the entire continuum or certain ranges. If the desired energy curve includes bistability or long smooth sections, a gradual stability technique may be useful. Examples include springs and non-rigidly foldable linkages.

Non-gradual stability techniques have a sharp change in energy along the fold-state continuum. If the desired energy curve includes areas of high energy behavior (such as an unacceptable motion zone), a non-gradual stability technique may be useful. Examples include clasps, telescoping poles, and facet interference.

4.2 Benefits and Drawbacks. This section provides some benefits and drawbacks that pertain to the entire criteria. For example, the benefits of intrinsic techniques apply to both intrinsic gradual and intrinsic non-gradual techniques. A summary of benefits and drawbacks is given in Table 2.

Table 2 Typical benefits (+) and drawbacks (−) of the four types of stability techniques

	Gradual	Non-gradual
Intrinsic	+ No added parts + Helps actuation − Hinders actuation − Limited by material − Complex to design − Low-energy threshold	+ No added parts + Easy to design + Distinct stable points + High-energy threshold + Versatile energy storage
Extrinsic	+ Typically passive + Helps actuation − Hinders actuation − Added parts − Low-energy threshold	+ High-energy threshold + Easiest to implement + Directional effect − Added parts − User input may be required

Note: Explanations are given in their respective subsections.

4.2.1 Intrinsic Techniques. Benefits: The primary advantage of intrinsic techniques is that they only involve the members of the origami linkage, meaning they do not require additional parts. For applications that are concerned with appearance, these techniques can highlight the simple, elegant nature of origami.

Drawbacks: Intrinsic techniques achieve stability using the geometry and material of the origami linkage. This creates two disadvantages. First, these techniques are limited by the material of the origami linkage. For example, if the device must be made from a stiff or brittle material, it could be difficult to create good torsional springs for large deflection. However, advances in materials and methods for increasing material compliance are making intrinsic techniques more accessible.

Another challenge is that these techniques usually place constraints on which origami linkages can be used because the two affect each other. For example, if the facet interference technique is selected to help maintain a non-planar fold state, the origami linkage must either be non-flat foldable or have more than one degree-of-freedom [47,48].

4.2.2 Extrinsic Techniques. Benefits: Extrinsic techniques are useful for applications where origami linkage dimensions are constrained. This may be the case for applications with required kinematic properties.

Extrinsic techniques are not limited to the materials used for the origami linkage. For example, a designer could build the origami linkage from a rigid material and add compliant members, such as torsional springs, to provide stability [49].

Drawbacks: These techniques usually require added parts. Some extrinsic techniques, such as clips or bolts, need additional user input to create stability.

4.2.3 Gradual Techniques. Benefits: Gradual techniques are useful for assisting actuation because their influence typically extends over a range of folds states.

Gradual techniques can be advantageous because they can achieve multiple stable locations with one implementation. This is possible because the mechanical advantage of origami can change as it folds [16].

Drawbacks: Gradual techniques may not be able to guarantee reliable stability in the stability zone if the desired stability zones are very small (low margin of error).

4.2.4 Non-Gradual Techniques. Benefits: When an application needs a static equilibrium position to fall on a specific fold state, or in a narrow range of fold states, non-gradual techniques are valuable because they can create distinct, stable equilibria. These techniques can also be useful in creating locations with high energy behavior because they usually have high rates of energy storage.

Drawbacks: Non-gradual techniques generally do not assist with actuation.

4.3 Investigating Techniques Used in Products. Additional insights can be gained by evaluating the stability techniques currently used in origami-based products. A collection of 69 origami-based products was evaluated to determine which techniques are being used in products and how often they are used. (These were the same 69 products analyzed by Avila et al. [44].) The results are shown in Fig. 9. Some of the products are used in Sec. 5 as examples. A summary of the results, separated by the classification criteria, is included below.

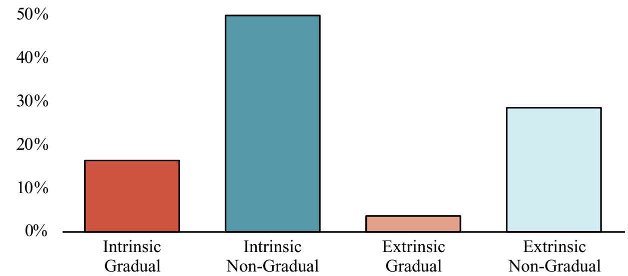

Fig. 9 Comparison of technique type and how often they were implemented in a sample of 69 origami-based products. Most of the products used several stability techniques; the techniques were counted once for each stable equilibrium to which they contributed.

4.3.1 Results for Intrinsic Techniques. The most common intrinsic technique used in products was adjacent facet interference, accounting for 65% of the intrinsic non-gradual techniques used. This technique is often used to achieve stability when the product is in its compact state (59% of the cases).

None of the products in the study used stimuli-actuated techniques. This may be because stimuli-actuated techniques require very specific environmental conditions (i.e., heat, magnetic fields) that are not common in everyday use cases.

Many products used thick materials, but very few used thickness-based stability techniques (such as hinge interference). The field may benefit from further research on intrinsic techniques that use thickness to achieve stability. This could be done by using thickness accommodation methods. For example, a vertex that is monostable when modeled as zero-thickness can achieve bistability when thickness is applied and a compliant material is used for the joints [33].

For more in-depth analysis on specific intrinsic techniques used in origami-based products, see Sec. 5.

4.3.2 Results for Extrinsic Techniques. Extrinsic techniques are being used in many products but are not very prevalent in the literature. This may be because many extrinsic techniques (such as springs and clips) are not mathematically complex or unique to origami. Of the 69 devices analyzed, five devices (7%) used at least one extrinsic gradual technique and 34 devices (49%) used at least one extrinsic non-gradual technique.

Extrinsic non-gradual techniques come in a wide variety (such as snaps, clasps, fasteners, and restraints) and are easy to add to the origami linkage at any fold state. Many of these techniques require user input, which is valuable for affecting energy storage only when the linkage is in a specific fold state. This also gives these techniques versatility; they can have a directional effect on energy storage, where the energy threshold for entering a fold state is low, but exiting is high.

While extrinsic techniques are relatively simple to add to products, they were used less often than intrinsic techniques. One reason may be that extrinsic techniques are not as visually appealing as intrinsic techniques. Future research could include how to subtly incorporate extrinsic techniques without having them distract from the origami.

4.3.3 Results for Gradual Techniques. Gradual techniques have received notable attention in academia [20,27]. In products, however, gradual techniques are used much less often than non-gradual techniques, as shown in Fig. 9. Below are four possible reasons for this.

(1) It can be more difficult to achieve a precise stable location with gradual techniques than with non-gradual techniques. This may dissuade product designers from choosing gradual techniques, especially if they have a narrow desired stability zone.
(2) Gradual techniques generally cannot provide as high of energy barriers as non-gradual techniques. Many products are designed to function (and exhibit loads) in one of their stable fold states. A small energy well may not be enough to ensure stability, especially if there are large loads in one of the desired stable fold states.
(3) Gradual techniques, especially intrinsic gradual techniques, require more analysis than non-gradual techniques.

Academia is more prone to tackle problems with heavy analysis than those designing products.

(4) Many non-gradual techniques are not specific to origami. Academic seeks to publish novel methods and tools (such as calculating the stability behavior of non-rigidly foldable origami), while product designers are content using existing technology (such as traditional snaps, clips, and buckles).

4.3.4 Results for Non-Gradual Techniques. Non-gradual techniques were used more often in the 69 devices than gradual techniques. Four possible reasons for this are discussed in Sec. 4.3.3.

The study found that the majority of non-gradual techniques used in products are hard stops: offsets, strings, clasps, retaining channels, detents, and facet interference. A *hard stop* inhibits the origami's motion due to interference [23]. Hard stops can be compression hard stops or tension hard stops. Figure 10 illustrates examples of both types as intrinsic and extrinsic techniques in an origami linkage.

5 Intrinsic Stability Techniques Design Resource

This section contains design considerations and a detailed discussion of a number of intrinsic techniques. (It should be noted that this is not an exhaustive list of all possible intrinsic techniques.)

The discussions consist of (1) a brief description, (2) design considerations, (3) resources, and (4) examples. Many of the examples come from the study of 69 origami-based products (see Sec. 4.3).

5.1 Intrinsic Gradual Techniques. Origami linkages have several gradual ways of storing energy in their facets and creases. This section will discuss three types of intrinsic gradual techniques: non-rigid foldability, compliant joints, and stimuli-actuated joints.

5.1.1 Non-Rigid Origami. Description: Non-rigidly foldable origami linkages require deformation of the facets or creases in order to fold. The deformed members allow motion and store energy.

Design Considerations: This technique is generally useful for creating monostable energy storage behaviors. However, some linkages, such as the flasher, triangulated tube, and square twist, can be bistable [6,50].

One variation on this technique is to make the origami non-rigidly foldable by offsetting hinges in thick origami. This was demonstrated by creating bistability in an origami antenna [33].

Some materials may not be well suited for this technique because it requires deformation of the creases and/or facets. Analysis and/or testing should be done to ensure that the deformation will not cause yielding or failure.

Resources: A useful method for calculating energy storage in non-rigidly foldable origami is given by Saito et al. [26].

A non-rigidly foldable origami linkage behaves similar to a traditional over-constrained, compliant mechanism [51,52]. Also see the following resources for creating bistability in traditional compliant mechanisms [18,51,53,54].

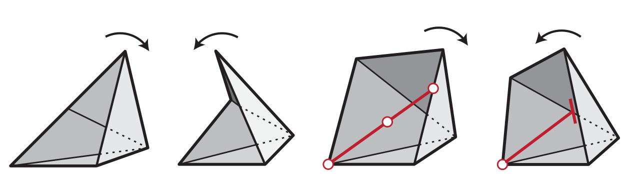

Fig. 10 Origami linkages (in gray) with (*a*) and (*b*) intrinsic and (c) and (*d*) extrinsic hard stops in (*a*) and (c) tension and (*b*) and (*d*) compression. The added links shown in (c) and (*d*) represent members that are not part of the original origami linkage.

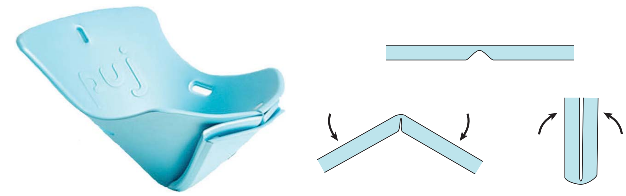

Fig. 11 The baby bathtub, discussed in Sec. 3.3, uses several stability techniques. (*a*) The fully folded state is maintained using magnets and adjacent facet interference. (*b*) Hinge interference prevents the linkage from folding the wrong way (used with permission from PUJ®.).

Examples: The foam origami tub in Fig. 11 is based on a non-rigidly foldable origami linkage. The largest facet bends as it folds, storing energy which biases the tub toward the unfolded state.

5.1.2 Compliant Joints. Description: The creases themselves are compliant joints that store strain energy like a torsional spring.

Design Considerations: Material is a major consideration in this technique. The materials must be compliant enough to handle maximum deflections, while remaining stiff enough to provide the desired energy storage.

Like the non-rigidly foldable technique, one of the lowest energy states occurs at the manufactured fold state, unless the creases have been modified through plastic deformation or stimuli (See Sec. 5.1.3.).

Resources: Research has shown that the stiffness and low-energy state of a crease in a degree-4 vertex can be tailored to create up to six stable states [20].

Francis et al. [55] provide a resource for evaluation of various creased materials.

The geometry of the material can be altered to facilitate folding, for example, using a LET array [25].

Examples: Stable behavior outside of the unfolded state can be seen even in paper origami, such as the waterbomb base and kaleidocycle [15,22,56].

A number of the origami-based products (colander, bathtub, Kayak, bowl, and glasses case) use living hinges to fold and store energy [38,46,57,58].[4]

Because of its limited current use but high potential for application, this is a technique that would benefit from further research into potential applications and design methods.

5.1.3 Stimuli-Actuated Joints. Description: Stimuli-actuated joints are different from other intrinsic gradual techniques because often the energy storage is not due to elastic deformation. The energy storage is more akin to plastic deformation, where the lowest energy state of the hinges is redefined completely.

Design Considerations: On the microscopic level, these techniques have a distinct advantage because the materials are often stimulated using a field rather than tactile manipulation [59].

Resources: See the resources discussed in Sec. 2, including Refs. [1,2,28–30,60].

Examples: The origami robot is one of the few macroscopic examples of stimuli-actuated joints. It uses heat to actuate the joints [1].

5.2 Intrinsic Non-Gradual Techniques. Intrinsic non-gradual techniques create a sharp change of energy using the facets and creases of the origami linkage. The techniques discussed below fall into two main categories: interference (between creases with thickness, between adjacent facets, between non-adjacent

[4] See Note 1.

facets, and from sequential folding) and extension (from facets in tension).

5.2.1 Hinge Interference. Description: Hinge interference occurs when the thickness of facets obstructs the folding of the crease.

Design Considerations: This technique is a low-profile intrinsic technique that is beneficial because it only affects the kinematics of the linkage once the thickness of the facets interfere. Facet chamfer angles determine when the facet interferes. Applications that use thin material are not conducive to this technique because the technique requires out-of-plane thickness.

Resources: Lang et al. [6] and Huffman [61] provide fold angle equations. Tachi shows an implementation of this technique in the tapered panel technique (a thickness accommodation technique) [5]. Ku gives a novel variation for creating creases with interference [62]. Directional hinge techniques that bridge the gap between extrinsic and intrinsic techniques are provided by Shemenski and Trease [23].

Examples: The linkage shown in Fig. 12 is blocked from reaching the unfolded state by hinge interference. Figure 11(*b*) shows this technique used to block the tub from folding in the wrong direction. This example is also used in Sec. 3.3.

5.2.2 Adjacent Facet Interference. Description: Adjacent facets interfere with each other and inhibit further motion, as shown in Fig. 10(*b*). Typically, this interference is at ±180 deg, as discussed in Sec. 3.1.2.

Design Considerations: For applications that need a large exposed surface area, this technique is not ideal because facets double back on one another.

Adjacent facet interference also only blocks one fold direction. Thus, in applications that require fully constrained states, additional techniques will be needed. One benefit is that it is relatively easy to constrain adjacent facets using bolts, magnets, and hook-and-loop fasteners.

This technique is often used in load bearing applications, such as the origami-based chairs and tablet cases [64–67].

Resources: A resource for designing origami linkages to interfere in non-planar fold states is given by Foschi and Tachi [47].

Examples: Technically, the bathtub in Fig. 11 and Sec. 3.3 is fully folded when it functions as a tub, but because it is not rigidly foldable it has a non-planar shape. The adjacent facets are a good technique for this application because the facets are sandwiched between the baby and the sink. The adjacent facets are also constrained by magnets.

5.2.3 Global Facet Interference. Description: Global facet interference is when two or more non-adjacent facets interfere.

Design Considerations: Calculating global interference is more involved than calculating local interference because the position of facets in multiple vertices must be evaluated.

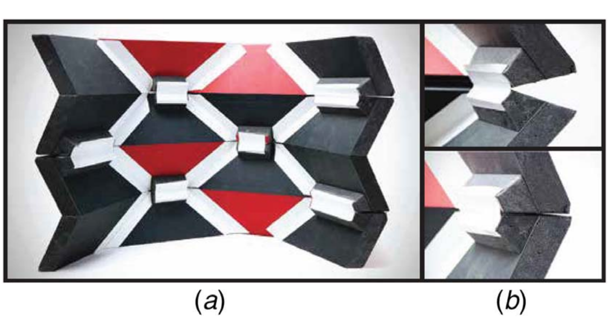

Fig. 12 The facets of the thick origami linkage (*a*) are negatively chamfered to create a stable partially folded state [63]. (*b*) Details of the hinge interference.

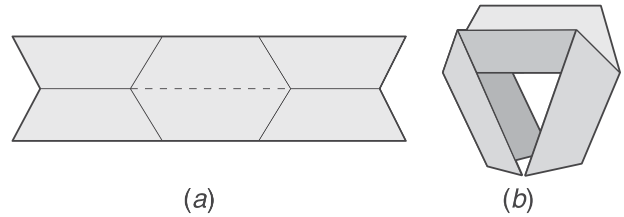

Fig. 13 (*a*) A symmetric crease pattern. (*b*) A fold state with global facet interference where the two wings come into contact.

Resources: One method for simplifying implementation of this technique is the use of symmetry.

Examples: The majority of the products that use this technique use symmetry—the kayak, canoe, forceps, and several of the chairs [68–70].[5] A simple example is also shown in Fig. 13.

5.2.4 Sequential Flap Interference. Description: This technique occurs when two creases are made collinear to form a flap in a fold state that can bifurcate. The flap is then folded, blocking other bifurcation modes.

Design Considerations: This technique can be difficult to implement if an application requires thick materials because creases that form the flap must be nearly collinear. This means that, without modifications, hinge interference will occur in either fold direction (see Sec. 5.2.1).

Resources: See the following resources for methods that allow sequential folding in thick materials [71,72].

Examples: In the sample of 69 products, sequential flap interference only occurs in the battery, ice bucket, and sofa [38,73]. Flaps are formed in a number of other products such as the boat, bowl, and camping pot, but they are a slight variation on the technique where the flaps do not create the need for sequential folding [37,57,74]. This is one of the few intrinsic techniques that can be toggled on and off by a user (only the ice bucket uses the flap in this way).

5.2.5 Limited Facet Extension. Description: Limited facet extension occurs when the crease in between the actuated facet and a grounded facet are unfolded, as seen in Fig. 10(*a*).

Design Considerations: The limited facet extension technique is useful for creating containers or devices that are kept in tension. If used to create stability in a non-planar configuration, the technique usually only works if the origami linkage is non-developable.

Examples: The origami colander (shown in Fig. 7) has a stable state that uses intrinsic hard stops in tension.[6] Since the origami linkage is non-developable, the facets that make up the sides reach full extension when the other facets are non-coplanar.

6 Case Studies

Two case studies are included: an origami anti-buckling guide and an origami ballistic barrier. These case studies use the OSIM to conceptualize the energy behavior of the origami linkage. They also utilize the four technique groups to determine which techniques may help achieve stability and work best with the final device.

6.1 OriGuide Case Study. The OriGuide is an origami support mechanism that encases a catheter and prevents it from buckling during compression [75,76]. The steps of the OSIM for the OriGuide are described below and shown in Fig. 14. This is an example of both choosing techniques and analyzing and modifying the loading conditions based on information gathered during the OSIM.

[5]http://onakcanoes.com/; https://www.youtube.com/watch?v=lgMZsHXJr6w
[6]https://www.brplastics.com/folding-colanders.html

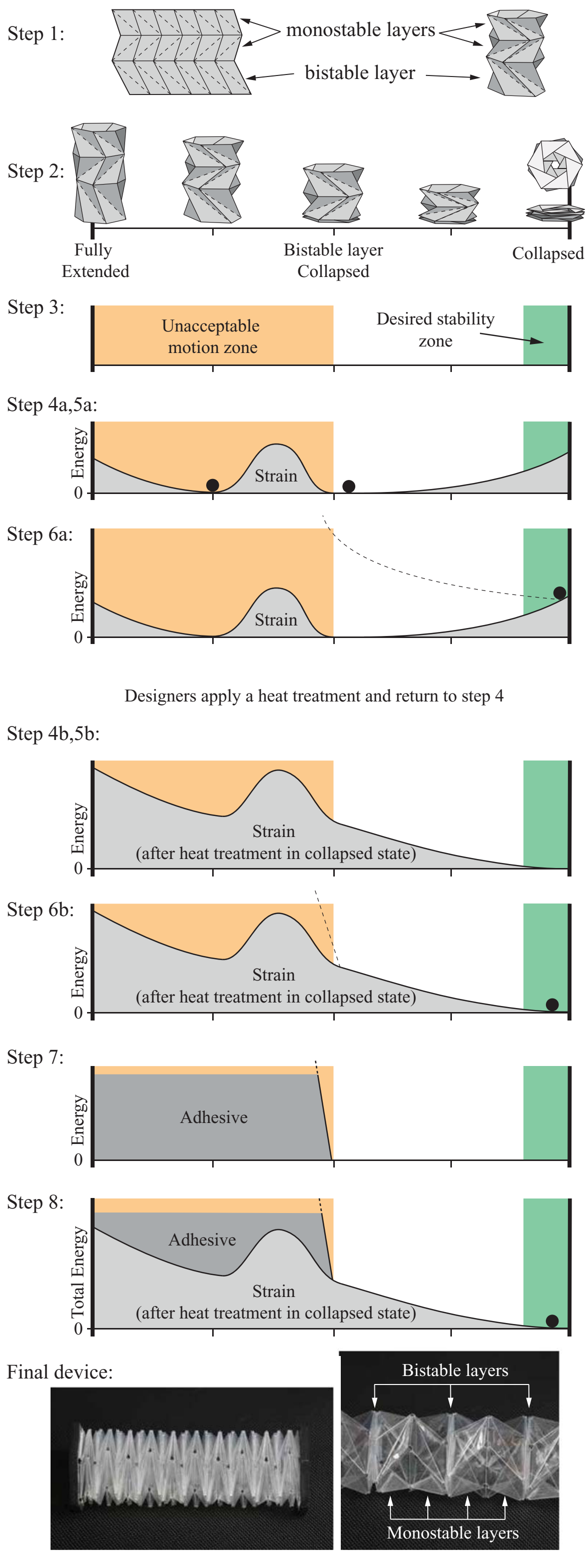

Fig. 14 The steps of the OSIM for the OriGuide, as described in Sec. 6.1. The final device (excluding external hardware) is also shown in its stable state (collapsed) and unstable state (extended).

Step 1: A tube-like origami linkage is selected with a three-story repeating base unit. Note that the OriGuide used many repeating layers but only three layers (the base unit) are shown in the figures. A bistable layer surrounds the catheter and should not actuate during normal motion. Two monostable layers allow the device to collapse uniformly. The geometry of the layers (and the device in general) was chosen to fulfill these requirements. This origami linkage is based on a pattern that has been shown to exhibit monostable and bistable behavior [77], depending on geometry. For this application, the origami linkage was designed to exhibit both monostability and bistability. For a more in-depth discussion on the process of choosing and developing the origami linkage, see Ref. [75].

Step 2: This linkage has multiple DOF. A 1D fold-state continuum was constructed that shows the desired motion path: the bistable layer collapsing first, then the monostable layers collapsing. This motion path is possible by the user constraining the motion to occur in the bistable layer before the monostable layer.

The linkage's motion is limited from further extending due to limited facet extension and from further collapsing due to adjacent facet interference. These are the bounds on the fold-state continuum (marked as thicker lines).

Step 3: In order to not buckle under compression, the OriGuide needs the fully collapsed fold state to have the lowest energy state. The device should not allow the bistable layers to undeploy once fully assembled. The desired stability zone and unacceptable motion zone is placed accordingly.

Step 4a: The only energy component considered is strain: the energy behavior of the linkage due to the flexing of the material. This has an effect near the ends of the fold-state continuum as well as during the collapse of the bistable layer.

Step 5a: No summing necessary.

Step 6a: A desired energy curve is superimposed. The inherent energy conditions (strain) are in opposition to the desired energy conditions so the designers looked at several methods for modifying the strain in the device. The device, made of polyethylene terephthalate (PET), was secured in its collapsed state and given a heat treatment. This effectively changed the inherent energy conditions in the device, so the designers returned to step 4.

Step 4b: The new strain curve (after the heat treatment in the collapsed state) is plotted.

Step 5b: No summing necessary.

Step 6b: A desired energy curve is superimposed. A non-gradual technique might be useful near the unacceptable motion zone for several reasons: the desired energy curve is over a short range, is not bistable, and needs to create a location of high energy storage behavior. The origami linkage should not be modified, so an extrinsic technique is preferred.

Step 7: One additional stability technique is chosen. An adhesive is applied to the bistable layers to prevent them from opening (an extrinsic non-gradual technique).

Step 8: The combined energy components results in the desired energy behavior.

6.2 Ballistic Barrier Case Study. The origami ballistic barrier is a deployable, bullet-resistant shield that stores fully folded state and opens to be a self-standing barrier [63,78]. The steps of the OSIM for the ballistic barrier are described below and shown in Fig. 15.

Step 1: The origami linkage is selected based on its crescent shape in the deployed state and its compact stowed state.

Step 2: The origami linkage has a single degree-of-freedom and is flat foldable which means that the range of the fold-state continuum can be represented as going from one fully folded state to the other. The ends of the continuum are caused by adjacent facet interference.

Step 3: The barrier needs to be stable in the fully folded state and in a partially folded state. The barrier should not reach the unfolded state because it can bifurcate into undesired modes.

Step 4: There are two main energy components: gravity and strain in the hinges. The energy behavior is approximated and graphed along the fold-state continuum.

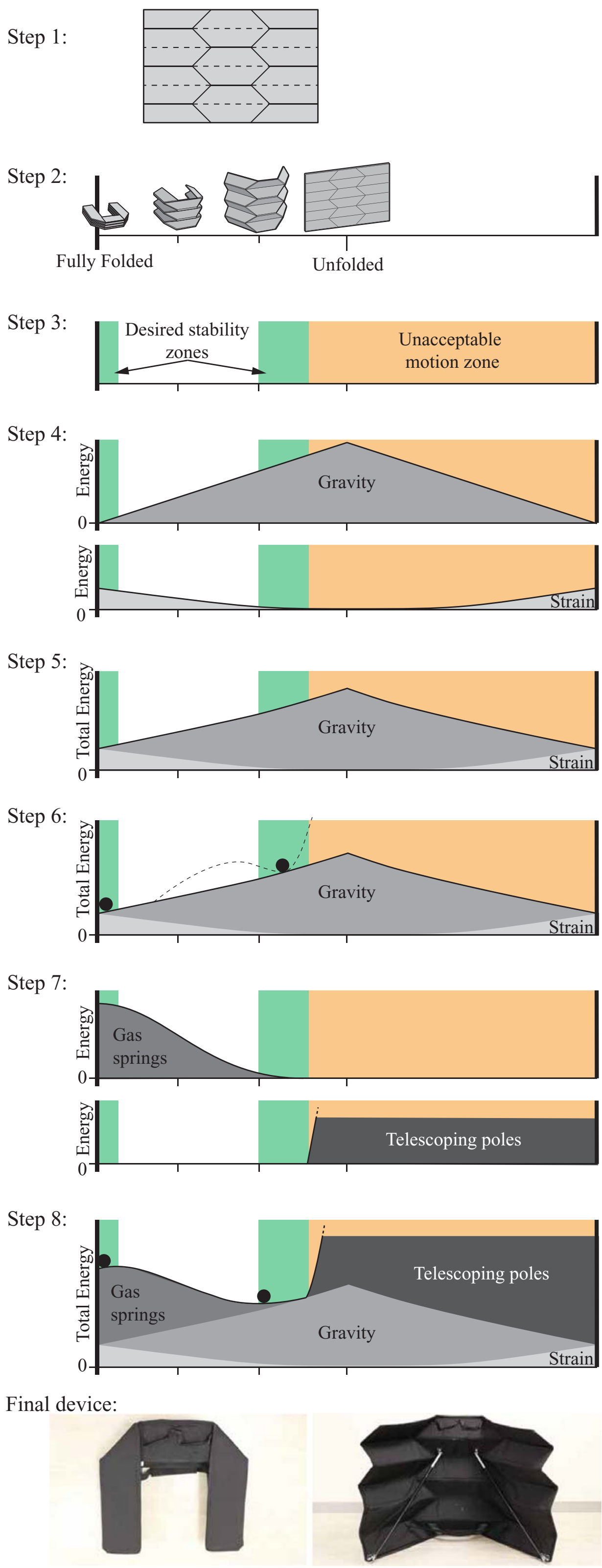

Fig. 15 The steps of the OSIM for the origami ballistic barrier, as described in Sec. 6.2. The final device is also shown, both collapsed and deployed.

Step 5: The two energy components are added together.

Step 6: The desired energy curve is superimposed. Note that the deployed stable condition should have lower energy potential than the fully folded stable configuration to aid in actuation.

Several considerations are made while choosing techniques: (1) the barrier should be kept as light as possible, which indicates that an intrinsic technique could be valuable because no parts are added. (2) The barrier needs to deploy quickly and require little-to-no in-field assembly. This indicates that a gradual technique could be useful. (3) Because the device has an unacceptable motion zone, a non-gradual technique could be needed. (4) The barrier is heavy, so a gradual technique could be useful for assisting actuation.

Because the partially folded state is the more difficult state to maintain, its stability techniques are considered first. Considerations 1, 2, and 4 above indicate that intrinsic gradual techniques should be investigated first.

Several possible intrinsic gradual techniques are evaluated.

Non-rigid origami: Due to the barrier's weight, the amount of energy storage required to keep the barrier in the deployed state would result in excessive facet deformation.

Compliant joints: The aramid fabric does not make a good torsional spring. Compliant joints could only result from added material.

Stimuli-actuated joint: These are often slow and require specific environmental conditions, so it is also not considered for use.

None of the examined intrinsic techniques are very viable, mainly due to the materials used for the barrier. Extrinsic gradual techniques are then evaluated.

Actuators: Actuators that can provide the necessary force would likely be prohibitively heavy.

Springs: Springs are a viable candidate, either torsional springs added at the creases or linear springs attached to facets.

Gravity: Gravity always collapses the chosen linkage so it cannot be used to keep the barrier open.

Springs seem to be the best technique and have few drawbacks. Several embodiments of linear and torsional springs are compared using prototypes. For example, torsional springs in the form of spring-steel sheets are added to the creases and linear springs in the form of gas springs are attached to the panels of a prototype.

Step 7: From the possible candidates, gas springs are selected for several reasons. (1) Their orientation has a low mechanical advantage in the stored state, allowing for bistable behavior. (2) They are attached to non-adjacent facets to counteract parasitic motion with less hardware. (3) The gas springs are inside of telescoping poles, so once the springs reach full extension they act as a hard stop, blocking the barrier from entering the unfolded state. (4) They double as handles.

Step 8: The combined energy components result in a stable equilibrium in both the fully folded and partially folded states. The unacceptable motion zone has high energy storage behavior.

Additional notes: While the OSIM method can help designers choose techniques to provide stability, sometimes the properties of the chosen origami linkage (and corresponding loading conditions) make it difficult to find suitable techniques. In these situations, it may be helpful to reconsider the origami linkage. In this case study, many different techniques were infeasible because of the force necessary to counteract gravity. Choosing a different origami linkage (step 1 of the OSIM) could have reduced the effect of gravity on the linkage and opened up other possibilities for stability techniques.

7 Conclusion

The methods, vocabulary, techniques, and examples presented in this paper are tools for planning stable states in origami. The OSIM is a design tool for conceptualizing how different stability techniques may be used to achieve stability in an origami device, given the inherent energy behavior of the origami linkage. The categorization of stability techniques facilitates their comparison and selection.

The OSIM, as presented in this manuscript, only works for origami linkages that can be represented by a 1D fold-state continuum. Future work in this area could include investigating how to use the OSIM with multidimensional fold-state continuums and how folding along certain branches of an origami linkage can create different energy storage behaviors. For example, an origami linkage may bifurcate or have multiple stable states along one branch and not along another. Future work could also include how to incorporate dynamic loading and friction in the OSIM.

Acknowledgment

This paper is based on work supported by the National Science Foundation and the Air Force Office of Scientific Research through NSF Grant No. EFRI-ODISSEI-1240417 and NSF Grant No. 1663345.

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