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## Deep learning for 2D passive source detection in presence of complex cargo

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8 **Deep learning for 2D passive source detection in**  
9 **presence of complex cargo**

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14 4 **W. Baines<sup>1</sup>, P. Kuchment<sup>2</sup>, and J. Ragusa<sup>3</sup>**  
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<sup>1</sup> Mathematics Department, Texas A&M University, College Station, TX, USA

<sup>2</sup> Mathematics Department, Texas A&M University, College Station, TX, USA

<sup>3</sup> Nuclear Engineering Department, Texas A&M University, College Station, TX, USA

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20 **Abstract.** Methods for source detection in high noise environments are important for  
21 single-photon emission computed tomography (SPECT) medical imaging and especially  
22 crucial for homeland security applications, which is our main interest. In the latter case, one  
23 deals with passively detecting the presence of low emission nuclear sources with significant  
24 background noise (with Signal To Noise Ratio (*SNR*) 1% or less). In passive emission  
25 problems, direction sensitive detectors are needed, to match the dimensionalities of the  
26 image and the data. Collimation, used for that purpose in standard Anger  $\gamma$ -cameras, is  
27 not an option. Instead, Compton  $\gamma$ -cameras (and their analogs for other types of radiation)  
28 can be utilized. Backprojection methods suggested before by two of the authors and their  
29 collaborators enable detection in the presence of a random uniform background. In most  
30 practical applications, however, cargo packing in shipping containers and trucks creates  
31 regions of strong absorption and scattering, while leaving some streaming gaps open. In such  
32 cases backprojection methods prove ineffective and lose their detection ability. Nonetheless,  
33 visual perception of the backprojection pictures suggested that some indications of presence  
34 of a source might still be in the data. To learn such features (if they do exist), a deep  
35 neural network approach is implemented in 2D, which indeed exhibits higher sensitivity and  
36 specificity than the backprojection techniques in a low scattering case and works well when  
37 presence of complex cargo makes backprojection fail completely.  
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41 26 **Keywords:** source detection, Compton camera, illicit nuclear material  
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45 27 Submitted to: *Inverse Problems*  
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3      28 **1. Introduction**

4      29 Checking for presence of illicit nuclear materials (most probably in small quantities and  
5      30 shielded by cargo) at border crossings and shipping cargo containers in harbors is an  
6      31 important homeland security task. Ideally, one would try to reconstruct from the detected  
7      32 signals the source distribution inside the cargo. When the data is sufficiently well behaved  
8      33 (e.g., in SPECT), analytic reconstruction is often possible [30]. However, in a very low SNR  
9      34 environment, as in the case of illicit nuclear source detection, this is impossible. Indeed, the  
10     35 forward analytic (integral transform type) models are not applicable. Moreover, even if they  
11     36 were, attempts of any filtration in FBP-type techniques lead to reconstruction deterioration.  
12     37 The saving grace is that in this case practitioners are mostly interested in getting reliable  
13     38 (i.e., with low rates of false positives and false negatives) information about the presence of  
14     39 a source, rather than its exact location.

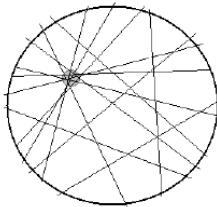
20     40 In passive emission imaging, detectors must be direction sensitive. Indeed, otherwise the  
21     41 data measured has insufficient dimension for recovery of an image. Directional information  
22     42 is especially critical when SNR is too low for the intensity fluctuations that arise due to  
23     43 the presence of a source to be statistically significant. The following options for obtaining  
24     44 directional sensitivity are available:

- 27     45 • *Mechanical collimation*, when only rays incident along (or close to) a certain line are  
28     46 allowed to reach the detector (see Section 2). This, while determining the incoming  
29     47 photon's direction, significantly reduces the signal strength and thus becomes unsuitable  
30     48 for low SNR.
- 33     49 • *Compton  $\gamma$ -cameras* represent a more recent, and gaining its appreciation, type of  $\gamma$   
34     50 radiation detectors that determine a surface cone of possible incident trajectories, rather  
35     51 than the exact directions.
- 37     52 • Neutron detectors are being developed that (albeit based on different physics principles)  
38     53 produce similar cone information and lead to similar mathematical analysis.

40     54 Backprojection detection technique introduced in [5, 33] relied upon finding suspicious  
41     55 **locations**. It utilized the following three assumptions:

- 43     56 (i) geometric smallness of the source (usually of linear dimension on the order of 1% of the  
44     57 linear cargo size);
- 46     58 (ii) existence of a sufficient number of particles from the source reaching the detector being  
47     59 **ballistic** (non-scattered);
- 49     60 (iii) unstructured strong random background.

51     61 The idea is rather simple: backprojecting the incoming trajectories (or, in the Compton case,  
52     62 the whole surface cones of possible trajectories) of particles, one hopes that maybe, due to  
53     63 sufficient presence of ballistic particles detected from the source, one can see a statistically  
54     64 significant accumulation at the geometrically small source's location (see Fig. 1)

12 **Figure 1.** An idea of the backprojection method.  
13  
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15 Analysis done in [5] provided a crude formula for the total number  $N$  of particles (and  
16 thus observation time) needed to make detection with high (on the order of 99%) sensitivity  
17 and specificity (i.e., with low levels of false negatives and false positives).  
18

$$19 \quad 20 \quad 21 \quad 22 \quad 23 \quad 24 \quad 25 \quad 26 \quad 27 \quad 28 \quad 29 \quad 30 \quad 31 \quad 32 \quad 33 \quad 34 \quad 35 \quad 36 \quad 37 \quad 38 \quad 39 \quad 40 \quad 41 \quad 42 \quad 43 \quad 44 \quad 45 \quad 46 \quad 47 \quad 48 \quad 49 \quad 50 \quad 51 \quad 52 \quad 53 \quad 54 \quad 55 \quad 56 \quad 57 \quad 58 \quad 59 \quad 60$$

$$N \gtrsim \left(\frac{8}{S}\right) p(1 - p). \quad (1)$$

65 Here  $p$  is the ratio of the linear dimension of the source relative to the dimension of the cargo  
66 and  $S$  is the SNR, defined as the proportion of the ballistic particles from the source versus  
67 the total number of source and background particles. In the cases considered in [5]  $N$  had  
68 to be on the order of 600000, which is not unrealistic for  $\gamma$  photons **not** screened by heavily  
69 shielding cargo. High specificity has been hardwired into the method, so satisfying (1) was  
70 only needed in [5, 6] to ensure high sensitivity.

71 The implementation of the technique worked as follows [5, 6]: the data was  
72 backprojected, which resulted in a large background level throughout the volume. When  
73 the object was completely surrounded by detectors, this level was essentially constant and  
74 the mean was removed. When the detectors did not surround the object completely (e.g.,  
75 no detector below the object), the global mean is irrelevant, and at each location the mean  
76 over a smaller patch was removed. After this clean-up the locations with an intensity less  
77 than five standard deviations above the mean suggested by the Central Limit Theorem were  
78 cut off. The results were interpreted as indications of a source being present. Thousands  
79 of Monte Carlo simulations showed that the inequality (1) performs well and if  $N$  is at or  
80 above this threshold, detection occurs with high sensitivity and specificity ‡.

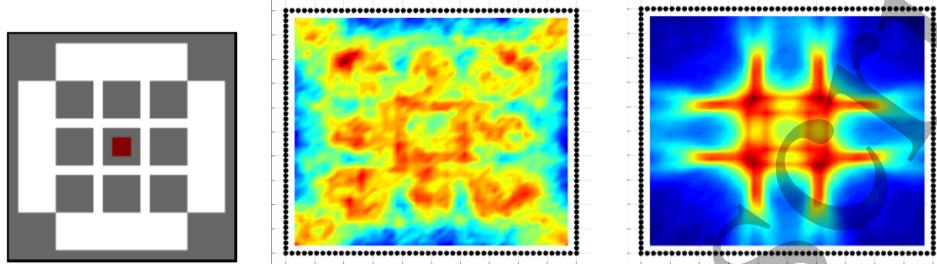
81 This technique works reasonably well in the absence of complex cargo, but starts failing  
82 if such cargo is present [6], due to the second and third assumptions being inapplicable§.  
83 However, visual inspection of the backprojected data (see [6]) seems to indicate that the  
84 data **might** still contain a signature of the source presence. Indeed, when the method of [5]  
85 was applied to some cases of complex cargo in [6], despite its failure to detect presence  
86 of the source, such signatures (e.g., different highlighting of the pathways between cargo  
87 boxes) seemed to appear only when a source was present (see Figure 2). The reader should

‡ An alternative Bayesian approach was implemented in [33].

§ This cargo problem is mostly non-existent when detecting neutrons coming from the source. However, some other (non-mathematical) issues arise, such as for instance lower number of particles detected.

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4      88 take into account that the color scales are different in the three pictures there and assigned  
5      89 automatically by the visualization software. This is of no importance, since it is not the  
6      90 intensity, but rather the patterns of highlighted pathways between boxes seem different.



18      **Figure 2.** (Left): Example of complex cargo configuration for which backprojection  
19      methods fail (i.e., no statistically suspicious locations are found). The red spot denotes the  
20      91 source location, the grey area represents iron and the white area represents air. (Middle):  
21      92 Backprojection results in absence of source. (Right): Backprojection results in presence of  
22      93 source.

24      94 No model of this effect has been developed, no telling features have been learned, and  
25      95 thus no detection algorithm came out of such observations.

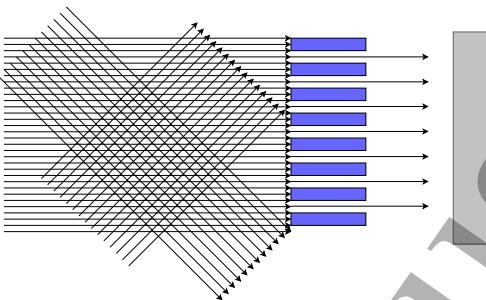
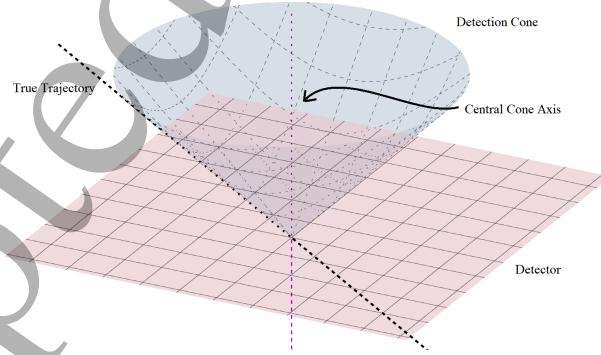
26      96 This has led the authors to attempt deep learning for the source inference in the  
27      97 hope that a network could learn what we could not. Our main goal is to detect the  
28      98 presence/absence of a source, not necessarily its location. If there is high probability of  
29      99 presence of the source, in practice one would check the cargo with other (hand-held) devices.  
30      100 However, one also needs to achieve high specificity, to avoid large numbers of false positives.

31      101 One should note that quality of tomographic image reconstructions using neural  
32      102 networks has been questioned recently, see e.g. [7,8]. This critique, however, does not apply  
33      103 to the problem at hand, where we only look for a binary output rather than an image.

34      104 We describe now the structure of the article. Section 2 contains a brief description  
35      105 of the Compton type cameras and references to the known analytic approaches. Success  
36      106 of deploying neural networks is predicated upon our access to sufficient data for neural  
37      107 network training. Thus, the first step - generating various complex cargo scenarios is  
38      108 described in Section 3. To avoid the inverse crime (overfitting), different processes of  
39      109 generating cargos are used for creating training and testing samples. Then, in absence  
40      110 of real data (which would require having weapons grade nuclear materials and physically  
41      111 creating thousands of different cargoes), we use (Section 4) the technique of forward radiation  
42      112 transport simulations customarily used in nuclear engineering. As has been mentioned, the  
43      113 actual type of radiation is mathematically irrelevant, but to be close to real world scenarios  
44      114 and numerical parameter values, the case of  $\gamma$ -photons coming from an U-238 source and  
45      115 real world material parameters for cargo are used. The design of the network is described  
46      116 in section 5. The results are presented in Section 6. Additional remarks can be found in  
47      117 section 7. Acknowledgements are provided in section 8. The algorithm description is located  
48      118 in the Appendix.

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3      **2. Collimated and Compton  $\gamma$ -Cameras**4      116 Mechanical collimators (see Figure 3) can be installed in front of a direction insensitive  
5      117  $\gamma$ -camera to block all particles but those incident along (or close to) a desired trajectory.  
6      1187      Mechanical collimators are widely used in medical imaging. They, however, significantly  
8      11921      **Figure 3.** Light collimation diagram22      119 attenuate the signal and require rotating the detector (or the object). In the applications  
23      120 with sufficiently high *SNR*, this additional data loss is not such a problem. In dealing with  
24      121 low *SNR* signals however, this renders recovery of weak signals impossible. For this reason  
25      122 one can consider Compton type cameras instead.26      123 The **Compton camera** is a type of  $\gamma$ -particle detector|| that does not attenuate the  
27      124 incident particles. The price to pay is that it provides less precise direction information  
28      125 than collimation would give. Namely, only a surface cone of possible incoming directions  
29      126 is measured rather than a precise trajectory (see Fig 4). In the absence of mechanical  
30      12747      **Figure 4.** Surface cone produced by Compton camera from particle detection48      127 collimation, signal strength is effectively maintained, although the directional information  
49      128 is less precise and thus data analysis becomes more complex. On the other hand, the data  
50      129 provided is significantly over-determined (e.g., the space of cones in 3D is five-dimensional,  
51      13052      || As we have mentioned before, novel neutron detectors (albeit based upon different physics rather than  
53      127 Compton scattering) that provide mostly similar cone information are currently being developed.

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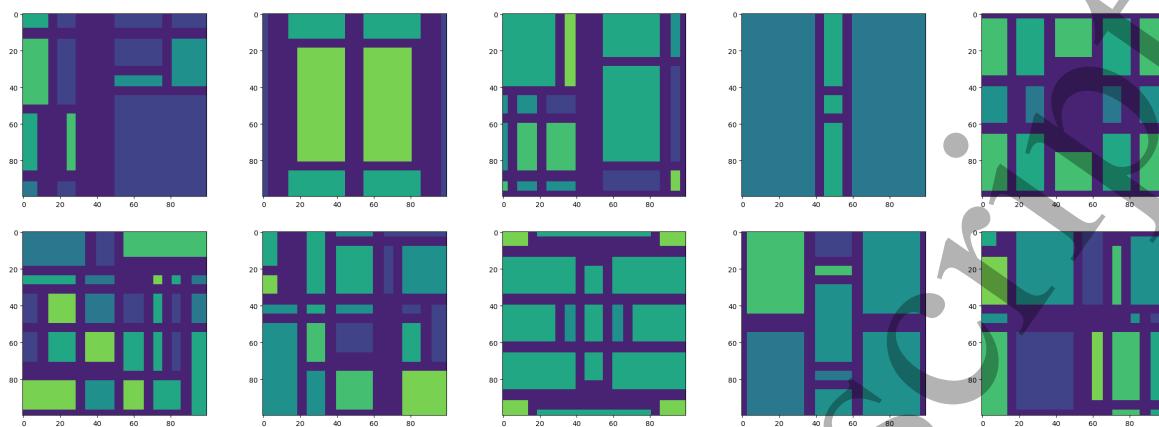
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3      131 versus the unknown distribution being three-dimensional). This turns out not to be a  
4      132 bad thing at all, but rather a blessing for stable inversion (see [30] for details and further  
5      133 references).6      134 A variety of exact inversion formulas from Compton data of filtered-backprojection and  
7      135 other types have been developed and implemented (see [30] and references therein). The  
8      136 choices are much more diverse than for the usual Radon transform inversions (see [24]). The  
9      137 reason is that the Compton data is highly overdetermined. It was shown that this feature  
10     138 can be used to get high quality reconstructions in SPECT in presence of 50% noise and  
11     139 higher. However, this is a far cry from the low SNRs encountered in the homeland security  
12     140 problems described above.13     **3. Simulating Cargo Scenarios**14     141 If one intends to tackle a problem using deep learning, it is natural to start by acquiring  
15     142 large amounts of training and testing data.16     143 In order to obtain rich training data for a neural network, at least thousands (better  
17     144 hundreds of thousands or millions) of cargo configurations are needed. Due to the sensitive  
18     145 nature of the materials involved in this work, we are unable to procure real-world data, so we  
19     146 resort to synthetic simulation. The high computation costs of these simulations restricted  
20     147 us to several thousands of samples, reaching up to  $4 \times 10^5$ . However, our results (see Section  
21     148 6) already show a success in detection.22     149 To start, we randomly produce several thousand cargo configurations and compute  
23     150 forward radiation data simulations with up to four randomly placed sources and without  
24     151 them for each one. In order to avoid overfitting (and an inverse crime), different cargo  
25     152 generation procedures are used for producing the training and testing data.26     *3.1. Procedural Generation of Training Cargo Configurations*27     153 A square cargo hold of size of  $2.4m \times 2.4m$  is assumed and partitioned into  $2.4cm \times 2.4cm$   
28     154 cells (the possible source would occupy one of them). Each cell can be indexed via a pair of  
29     155 row and column indices,  $(i, j)$ , with  $1 \leq i, j \leq 100$  and is assigned a material identification  
30     156 number  $ID_{i,j}$ . These numbers correspond to a variety of materials, including Air, concrete,  
31     157 highly enriched uranium, iron, cotton, wood, plastic, and fertilized (their detailed chemical  
32     158 content described in [6]).33     159 Real cargo typically consists of several boxes with small spaces in between. In order  
34     160 to emulate this, an algorithm is implemented to generate different cargo configurations. It  
35     161 consists of three main steps:36     162     • A network of several horizontal and vertical “corridors” between boxes with random  
37     163 widths and locations is generated. The number of corridors  $c$  is selected randomly in a  
38     164 desired range  $c_{min} \leq c \leq c_{max}$ .

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**Figure 5.** A selection of cargo configurations procedurally generated via Algorithm 1

- 167 The resulting configurations are unlikely to be symmetric, while real cargo might happen  
168 to be symmetric. To check whether symmetry plays any role in detectability ¶, a portion  
169 of the samples produced are “symmetrized” by enforcing various (rotation and mirror)  
170 symmetry rules.
- 171 • Connected components of the rest of the space are identified as distinct “cargo boxes.”  
172 Then material contents are assigned to all boxes. In a subset of (rather than all)  
173 symmetric cargo configurations, material contents are also “symmetrized” according to  
174 the corresponding rule.

32 Generating the corridors between pieces of cargo is performed using a modification of  
33 the procedure outlined in [11] for generating road networks. For the training set, we only  
34 use networks consisting of horizontal and vertical segments, while for the testing set tilted  
35 and non-orthogonal pathways are allowed.

37 *Remark.* Instead of selecting corridor locations uniformly randomly, their locations for  
38 training are selected according to a probability distribution generated from a type of gradient  
39 noise developed in [25] in order to automate the production of realistic looking textures in  
40 computer graphics. A different algorithm is used for testing samples.

43 Identification of connected components (“boxes”) is performed using SciPy’s (Scientific  
44 Python, a popular Python package for scientific computing [17]) implementation of the  
45 algorithms outlined in [32].

47 The entire generation procedure is summarized in Algorithm 1 in the Appendix (Section  
48 9).

51 *3.2. Procedural Generation of Testing Cargo Configurations*

53 To avoid the inverse crime of overfitting, testing configurations are produced by a somewhat  
54 similar, but independent algorithm. Namely, the middle points, the lengths and width of the

56 ¶ Disclosure: Our results show that symmetries do not influence detectability.

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3      191 corridors are selected randomly and independently. Moreover, the corridors are not required  
4      192 to be vertical or horizontal, or even orthogonal at their intersections anymore. Finding the  
5      193 boxes (connected components of the complement) and filling them with materials is also  
6      194 done randomly, similarly to the training case.

7      195 *3.3. Source placement*

8      196 A source of a (randomized) strength corresponding to approximately 1% SNR is placed  
9      197 randomly into the cargo.

10     198 Multiple sources (0, 1, 2, 3, or 4) are also modeled to see the effect on detection. Two  
11     199 scenarios are used:

12     200 (i) when all the sources have the same strength  $\approx 1\%$  SNR  
13     201 and

14     202 (ii) when the strength of the source is diluted between several locations.

15     203 One naturally expects deterioration of the detection in the 2nd case, while *a priori* it would  
16     204 not be surprising if it happened in the 1st as well (although our results will demonstrate  
17     205 that this does not happen). Indeed, the backprojection detection, as well most probably  
18     206 the one by deep networks, if successful, should use some geometric assumptions (e.g.,  
19     207 geometric smallness of the source), since the source's strength alone would not be statistically  
20     208 significant. Thus, multiplying the number of sources in principle might degrade the geometric  
21     209 features of importance (albeit one does not know what these are).

22     210 **4. Forward radiation simulations**

23     211 As previously mentioned, the nature of particles is irrelevant, but in order to be in realistic  
24     212 situations, the  $\gamma$  particle detection is considered, where the material parameters and emission  
25     213 and background rates that are used assume realistic values.

26     214 After the cargo scenario has been created, one needs to simulate training and testing  
27     215 data by solving a massive forward radiation transport computation. Fortunately, reliable  
28     216 simulation tools have been developed by nuclear engineering researchers.

29     217 *4.1. Physics Preliminary*

30     218 U-238 (Uranium-238) photons from the 1.001 MeV emission line have mean-free-path in  
31     219 high-Z materials sufficiently high to be detected outside the container (13.3mm mean-free-  
32     220 paths) [28]. In our application, sources of background radiation include a concrete base  
33     221 located some distance below the container. (Cosmic rays and other natural sources can be  
34     222 easily included and do not influence the results much.) These background sources radiate at  
35     223 much higher energies than 1.001 MeV, including 1.461 MeV from Potassium-40, 1.12 MeV  
36     224 and 1.76 MeV from Bismuth-214, and 2.61 MeV from Thallium-208 (Bismuth and Thallium  
37     225 are products of the decay of Uranium-238 and Thorium 232 respectively, and are present in

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3      226 trace amounts in concrete). Gamma photons which downscatter from these sources into the  
 4      227 energy group surrounding the 1.001 MeV line account for the noise in our signal. Gamma  
 5      228 photons from the source will also undergo scattering and absorption within the volume of the  
 6      229 container, which will reduce the number of ballistic source particles reaching the detectors  
 7      230 placed around the container, thus weakening the signal.

10     *4.2. Mathematics of the forward radiation data simulation*

11     The radiation transport within the cargo container is modeled by the linear Boltzmann  
 12     equation, given below using the multigroup approximation:

$$13 \quad \vec{\Omega} \cdot \vec{\nabla} + \Sigma_t^g(\vec{r})\Psi^g(\vec{r}, \vec{\Omega}) = \sum_{g'=1}^G \sum_{l=0}^L \Sigma_{s,l}^{g' \rightarrow g}(\vec{r}) \sum_{m=-l}^l \Phi_{l,m}^{g'}(\vec{r}) + Q^g(\vec{r}, \vec{\Omega}) \quad (2)$$

14     where  $\vec{r} \in \mathcal{D}$  is the position,  $\vec{\Omega} \in \mathbb{S}^2$  the set of discrete directions and  $g \in [1, G]$  the energy  
 15     group.  $\mathcal{D}$  is the volume of the cargo container,  $\mathbb{S}^2$  is the unit sphere,  $G$  is the total number of  
 16     energy groups,  $\Psi^g$  is the photon angular flux in the energy group  $g$ ,  $\Sigma_t^g$  is the total interaction  
 17     cross section in group  $g$ ,  $\Sigma_{s,l}^{g' \rightarrow g}$  is the  $l^{th}$ -Legendre moment of the scattering cross section  
 18     from group  $g'$  to group  $g$ ,  $L$  is the maximum anisotropy expansion order, and  $Q^g$  is the  
 19     volumetric source of photons in group  $g$  (stemming from the U-238 source). The moments  
 20     of the angular flux are given by

$$21 \quad \Phi_{l,m}^g(\vec{r}) = \int_{4\pi} Y_{l,m}(\vec{\Omega}) \Psi^g(\vec{r}, \vec{\Omega}) d\Omega \quad (3)$$

22     where  $Y_{l,m}$  is the spherical harmonic of order of  $l$  and degree  $m$ . Eq. (2) is supplied with  
 23     boundary conditions:

$$24 \quad \Psi^g(\vec{r}, \vec{\Omega}) = h^g(\vec{r}, \vec{\Omega}) \quad \forall \vec{r} \in \partial\mathcal{D}^- \quad (4)$$

25     where  $\partial\mathcal{D}^-$  is the incoming boundary defined as  $\partial\mathcal{D}^- = \{\vec{r} \in \partial\mathcal{D} \text{ such that } \vec{\Omega} \cdot \vec{n}(\vec{r}) < 0\}$  with  
 26      $\vec{n}(\vec{r})$  the outward unit normal vector at position  $\vec{r}$ . The function  $h^g$  describes the background  
 27     radiation due to a large concrete slab underneath the container, as previously described.  
 28     Cross sections for various materials were generated using NJOY-99 [23]. The multigroup  
 29     structure employed ranges from 1.00099 MeV to 2.61449 MeV with narrow bands centered  
 30     at the radiation lines of the background and U-238.

31     For the purposes of this paper, calculations are carried out in two-dimensional space  
 32     and only the energy group corresponding to the 1.001 MeV line is considered after solving  
 33     Eq. (2). The photon transport equation, Eq. (2), is discretized using standard techniques:

34     (i)  $S_n$  product Gauss-Legendre-Tchebychev angular quadrature [27] is employed (only a  
 35     small number of polar angles are needed, but a very high number of azimuthal angles  
 36     are needed to resolve properly the angular distribution in the 2D domain.)

37     (ii) Spatial discretization based on a standard bilinear discontinuous finite element technique  
 38     with upwinding at cell interfaces. [26, 31]

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23      246 (iii) Transport sweeps and Source Iteration are employed to solve the resulting system. [21]  
45      247 Once the transport equation (2) has been solved, the outgoing angular photon flux at any  
6      248 boundary edge in 2D is recorded, which serves as the input data for use in Deep Learning  
7      249 and Backprojection.8      250 Once configurations have been generated, a radiating source emitting an expected  
9      251 8042.17 photons per second at 1.001 MeV is randomly placed, a forward radiative transfer  
10     252 equation is solved, and from its solution the radiation angular flux distribution on the  
11     253 boundary of the cargo is collected.12     254 Due to linearity of (2), the situations of presence of zero to four randomly placed sources  
13     255 could (and were) easily incorporated.14     256 **5. Convolutional Neural Network**  
1516     257 Using a fully connected network for the problem seems to be hardly feasible even in 2D,  
17     258 less so in 3D, in particular due to high dimensionality of the Compton camera data. The  
18     259 saving grace here is that, as in many imaging problems [22], one expects that the important  
19     260 correlations occur mostly between close pixels, and hence convolutional neural networks,  
20     261 which are much more compact due to weight sharing, offer a hope. We thus construct, train,  
21     262 and test a deep convolutional neural network (CNN). This hand-waving argument for using  
22     263 CNN needs to be confirmed by computations, which is done in this text.23     The suggested CNN architecture is summarized in Figure 6 below. The input data  
24     dimension is  $144 \times 10^3 = 400 \times 360 \times 1$ , as we model 400 equally spaced detectors with  
25     360 equally spaced angular bins and only one energy bin is used. The network is trained  
26     on 1689 unique simulated cargo configurations with varying numbers of sources present. By  
27     exploiting the fact that the Boltzmann equation (2) is linear, we can produce multiple new  
28     samples from each configuration by taking varying combinations of sources and detectors.  
29     We simulate up to four sources per configuration, and four linear arrays of detectors along  
30     each edge of the cargo. This leads to a total of  $1689 \times 15 \times 16 = 405360$  total samples.  
31     The various combinations are summarized in Table 1 Below. The output of the CNN is two  
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Number of Sources	One Detector	Two Adjacent Detectors	Two Opposite Detectors	Three Detectors	Four Detectors	Total
0	6756	6756	3378	6756	1689	25335
1	27024	27024	13512	27024	6756	101340
2	40536	40536	20268	40536	10134	152010
3	27024	27024	13512	27024	6756	101340
4	6756	6756	3378	6756	1689	25335
Total	108096	108096	54048	108096	27024	405360

56     **Table 1.** Number of training samples in each category  
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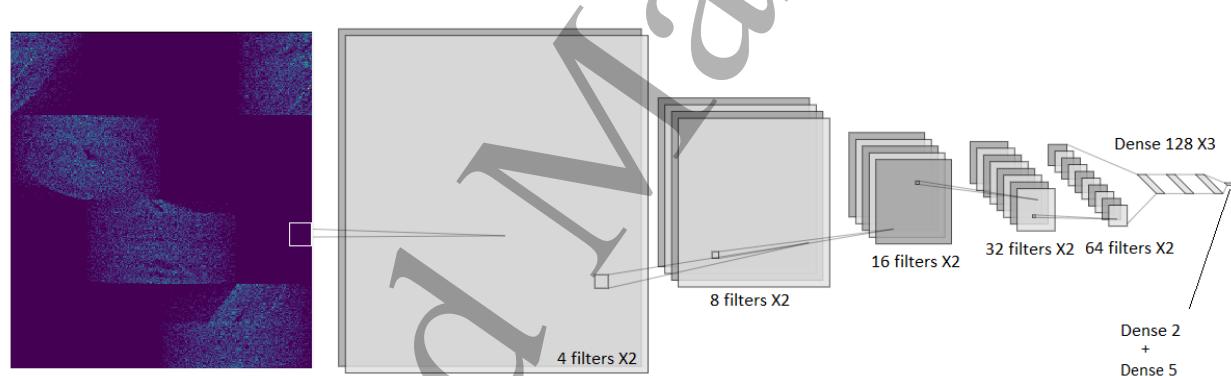
1    *Deep learning for source detection*

2    11

3    probability measures:  $\mathbb{P}_d$  on  $\{0, 1\}$  and  $\mathbb{P}_n$  on  $\{0, 1, 2, 3, 4\}$ . A source is determined to be  
 4    present if  $\mathbb{P}(x = 1) > 0.5$ , and absent otherwise.  $\mathbb{P}_n$  predicts the number of sources present,  
 5    which we set to  $k = \text{argmax}_{0 \leq j \leq 4} \mathbb{P}_n(x = j)$ . The loss function used for training is the binary  
 6    cross-entropy loss:  
 7   

$$10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15 \quad 16 \quad 17 \quad 18 \quad 19 \quad 20 \quad 21 \quad 22 \quad 23 \quad 24 \quad 25 \quad 26 \quad 27 \quad 28 \quad 29 \quad 30 \quad 31 \quad 32 \quad 33 \quad 34 \quad 35 \quad 36 \quad 37 \quad 38 \quad 39 \quad 40 \quad 41 \quad 42 \quad 43 \quad 44 \quad 45 \quad 46 \quad 47 \quad 48 \quad 49 \quad 50 \quad 51 \quad 52 \quad 53 \quad 54 \quad 55 \quad 56 \quad 57 \quad 58 \quad 59 \quad 60 \quad \mathcal{L}(y, \hat{y}) = -y \log \hat{y} - (1 - y) \log(1 - \hat{y}), \quad (5)$$

264    where  $y$  is the network prediction and  $\hat{y}$  is the target value (see [15]). The CNN was  
 265    trained on simulations of a localized source in the presence of high background noise  
 266    ( $SNR = 0.01$ ). In all cases, early stopping is used to halt training before over-fitting.  
 267    The various hyper-parameter values used in training are summarized in Table 2 below. The  
 268    CNN is implemented using Keras with Tensorflow as its backend. Keras is a high level API  
 269    (Application Programming Interface) for interfacing with machine learning toolkits such as  
 270    Tensorflow, Theano, and Microsoft Cognitive Toolkit. It helps streamline the construction  
 271    and training of neural networks [12]. Tensorflow is Google's machine learning toolkit and was  
 272    chosen due to its scalability, wide range of features, and the wide range of documentation  
 273    and tutorials available [1]. Any parameters not explicitly mentioned here were set to default  
 274    values.



38    **Figure 6.** CNN architecture used for source detection. The left-most cell shows an example  
 39    of the detector data input to the CNN.  $2 \times 2$  Max pooling layers are placed after every second  
 40    convolutional layer.  
 41   

274    42    43    44    45    46    47    48    49    50    51    52    53    54    55    56    57    58    59    60    **6. Results**

275    After training the CNN, we considered a large variety of cargo scenarios to test and to  
 276    compare and contrast the performance of the CNN against the backprojection method  
 277    of [5, 6]. We detail some interesting specific example scenarios in Sections 6.1 and 6.2. We  
 278    then investigate the statistical performance of the CNN on large scale data sets to evaluate  
 279    the sensitivity and specificity of the CNN in Section 6.3, and to assess its performance  
 280    with different numbers of sources and detectors in Section 6.4. Finally, in Section 6.5 we  
 281    discuss the relation between cargo configuration and exposure time and how this affects the  
 282    practicality of our technique.  
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Deep learning for source detection

12

Optimization Method	Adam (See [18])
Activation	RELU (Softmax at output)
Bias	True
Convolution Window Size	3x3
Learning Rate	$2.0 \times 10^{-5}$
Learning Rate Decay Rate	0
Batch Size	4
Early Stopping Patience	3 epochs
Loss	Binary Cross-Entropy

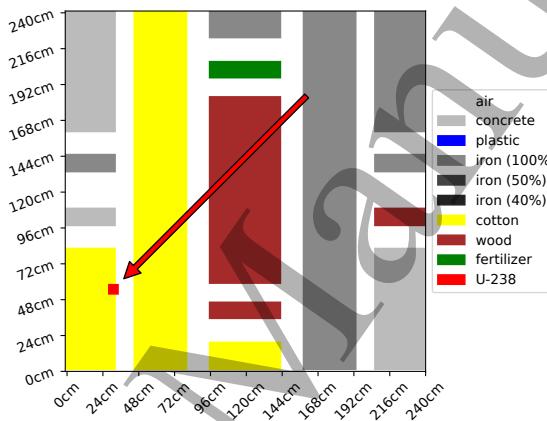
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**Table 2.** Hyper-parameters used during training284  
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6.1. Example Scenarios287  
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We describe now several (out of many, see later on in this text) sample results of testing the trained network on various scenarios not included in the training set.294  
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1      *Deep learning for source detection*

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3      295    *6.1.2. Example #2*

4      296    Next we consider the scenario shown in Figure 8, where backprojection fails to detect  
 5      297    the source (and thus is not shown), but the network succeeds. Here the exposure time  
 6      298    needed for the detection is significantly longer. In this configuration a long thick iron slab  
 7      299    effectively blocks one side of the detectors. Smaller chunks of iron spread throughout the  
 8      300    container further attenuate the signal along certain trajectories. As a result, it would take  
 9      301    9 hours and 26 minutes to detect the needed 101,092 particles. Unless one is talking about  
 10     302    a shipping container, this is practically unfeasible. As the results in Section 6.3 show, twice  
 11     303    shorter time would still do decently, and even five times shorter time might sometimes be  
 12     304    used, although at the expense of higher false positive rate.



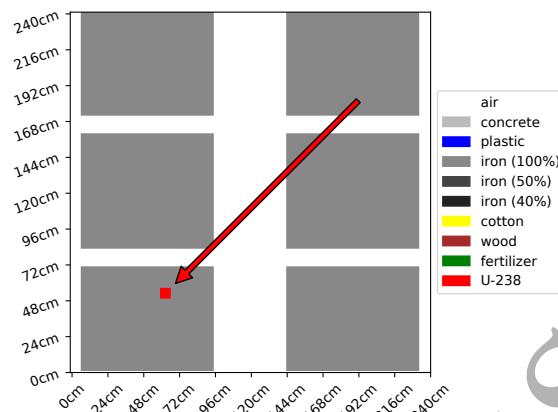
33      **Figure 8.** Cargo configuration with source location indicated by arrow. 101,092 particles  
 34      detected, 100,095 background particles and 997 source particles. Exposure time is 9 hours  
 35      and 26 minutes.

36      305    *6.1.3. Example #3*

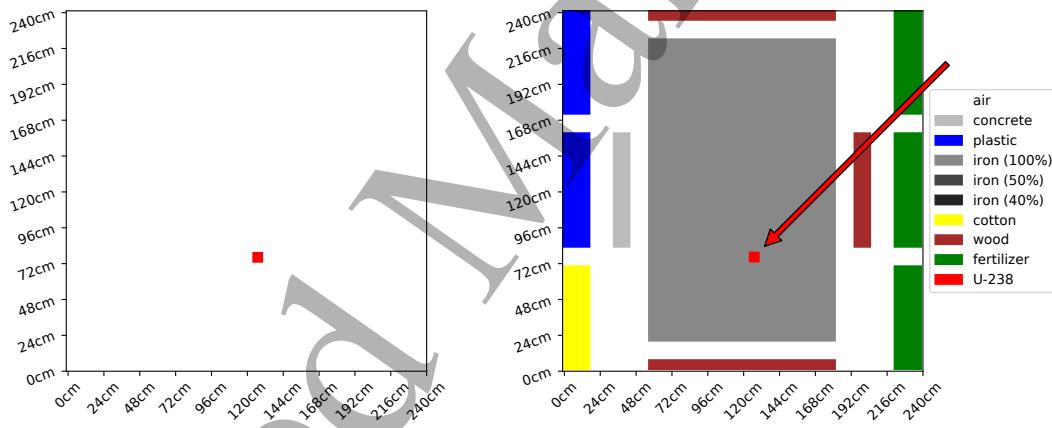
37      306    Here we consider a somewhat more tenable scenario shown in Figure 9, where  
 38      307    backprojection fails to detect the source, yet the network succeeds. In this case the exposure  
 39      308    time is 50 minutes and 17 seconds for 100,866 particles. In this configuration several large  
 40      309    blocks of iron are periodically tiled in the container, with the source located within one of  
 41      310    the blocks.

42      311    *6.1.4. Example #4*

43      312    Now we consider a somewhat extreme scenario (Figure 10), where both approaches  
 44      313    succeed in detecting the source. In this case the exposure time is 3 days and 12 hours  
 45      314    for collecting 101,272 particles. In this configuration one very large block of iron in the  
 46      315    center of the container surrounds the source. The source is still localized relatively well by  
 47      316    backprojection for this scenario. Most of the cargo is filled with a homogeneous material,  
 48      317    which might explain why backprojection did not fail.



**Figure 9.** Cargo configuration with source location indicated by arrow. 100,866 particles detected, 99,867 background particles and 999 source particles. Exposure time is 50 minutes and 17 seconds.



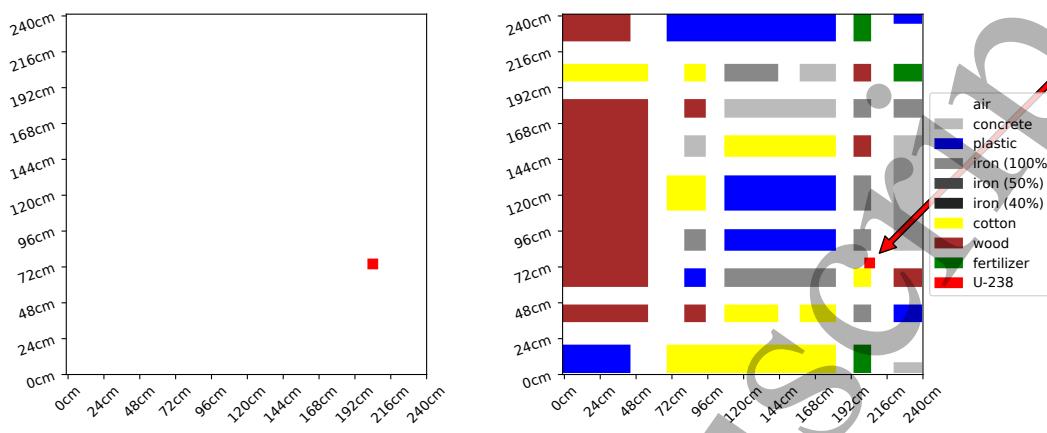
**Figure 10.** Left: Backprojection with source detected. Right: Cargo configuration with source location indicated by arrow. 101,272 particles detected, 100,328 background particles and 944 source particles. Exposure time is 3 days and 12 hours.

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318 6.1.5. *Example #5*

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319 Next, we consider a rather easy scenario (Figure 11), where both backprojection and  
46 the network succeed. In this case the exposure time is 276 milliseconds for 100,898 particles.  
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320 In this configuration several small blocks of different materials are spread throughout the  
48 container. Only an insignificant amount of particles are scattered, so backprojection recovers  
49 the source distribution extremely well.  
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20 **Figure 11.** Left: Backprojection with source detected. Right: Cargo configuration with  
21 source configuration indicated by arrow. 100,898 particles detected, 99,911 background  
22 particles and 987 source particles. Exposure time is 276milliseconds.  
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24 324 *6.2. Generalization to more complex scenarios*  
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26 Now we will include more complex situations, considerably different from the ones used for  
27 training. Namely, the corridors are not necessarily aligned vertically and horizontally, nor are  
28 intersecting corridors orthogonal. The algorithm of producing configurations was different  
29 from the one used in training. Additionally, we allow multiple sources to be present. The  
30 results show that the network passes well this generalization test.  
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33 330 *6.2.1. Example #6*  
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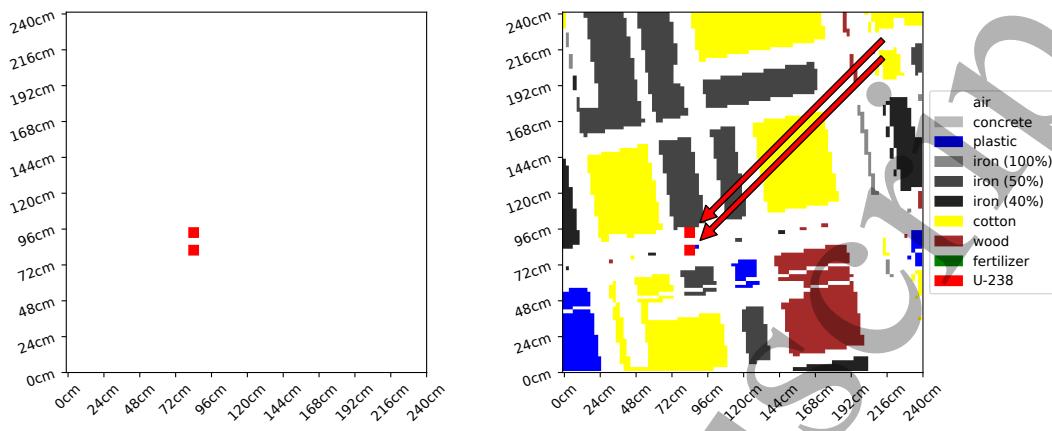
35 In this configuration (see Figure 12) several iron blocks are spread throughout the  
36 container, but a sufficient amount of low attenuating paths exist between the sources and  
37 detector arrays for backprojection to recover the sources well. There are two sources present  
38 very near to each other. This clearly aids the backprojection method in successfully detecting  
39 the sources. The CNN also succeeds in detecting presence of both of the sources. Here the  
40 exposure time is 649 milliseconds for 101,497 particles.  
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43 337 *6.2.2. Example #7*  
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45 In this configuration (Figure 13) several heavy iron blocks cut diagonally through the  
46 container slightly off-center. Three sources are present in this scenario, the two sources  
47 around the middle are localized well with backprojection, since most of the materials only  
48 weakly attenuate the signal, but the source on the other side of the heavy iron has several  
49 attenuating materials to contend with, so the backprojection smears its signature throughout  
50 the diagonal corridor it's in. Both backprojection and the CNN successfully predict that there  
51 is a source, although backprojection fails to locate the third source. This third source may  
52 prove difficult for the CNN to contend with as well, as the CNN predicts there are only two  
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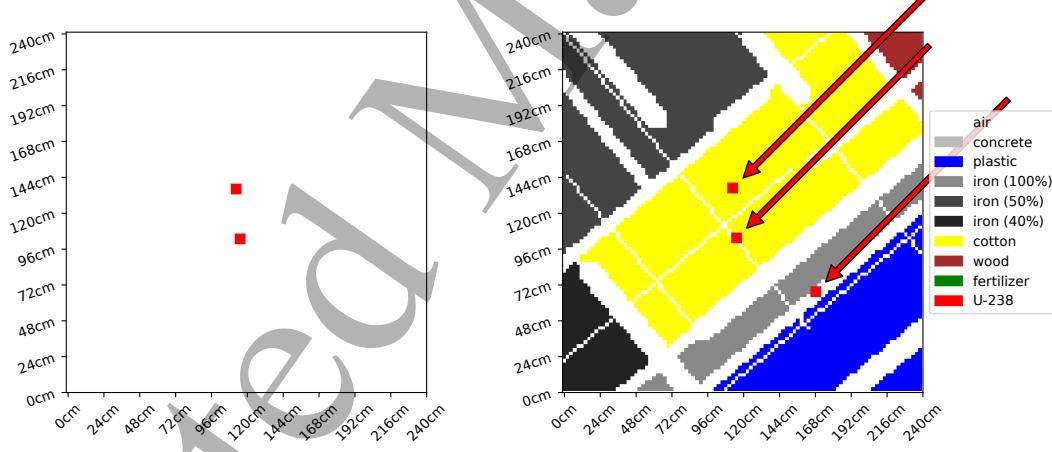
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5 *Deep learning for source detection*

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19 **Figure 12.** Left: Backprojection with source detected. Right: Cargo configuration with  
20 source configuration indicated by arrow. 101,497 particles detected, 99,492 background  
21 particles and 2,005 source particles. Exposure time is 649 milliseconds.

24 sources present. Here the exposure time is 371 milliseconds for 102,790 particles.



42 **Figure 13.** Left: Backprojection with source detected. Right: Cargo configuration with  
43 source configuration indicated by arrow. 102,790 particles detected, 99,846 background  
44 particles and 2,944 source particles. Exposure time is 371 milliseconds.

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48 *6.2.3. Example #8*

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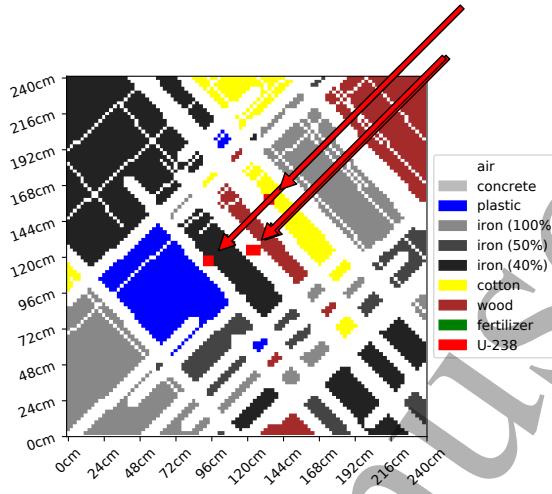
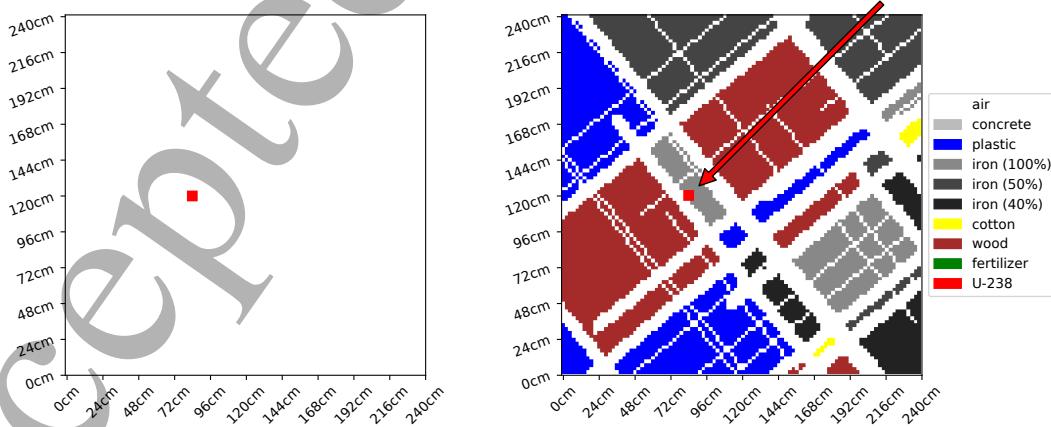
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Here several iron blocks surround the center of the container (Figure 14). Four sources are present in this scenario, two of them directly adjacent (and thus hard to distinguish in the picture) and all four are near the center of the container. In this case backprojection fails to localize any of the sources due the limited angular information in the signal as a result of the attenuating properties of the iron. The CNN, on the other hand succeeds in detecting

1      *Deep learning for source detection*

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3      353 the presence of all four of the sources, even despite the close proximity of two of them. Here  
4      5 the exposure time is 1.97 seconds for 103,789 particles.23      **Figure 14.** Cargo configuration with source configuration indicated by arrow. 103,789  
24      354 particles detected, 99,900 background particles and 3,889 source particles. Exposure time  
25      5 is 1.97 seconds.26      355 6.2.4. *Example #9*27      356 Finally, we consider a simple case where backprojection and the CNN both succeed. In  
28      357 this case there is ample angular information for backprojection to localize the source well  
29      358 and the CNN correctly predicts the presence of a single source. The exposure time is 21.92  
30      5 seconds for 100,672 particles. The configuration can be seen in Figure 15 below.52      359 **Figure 15.** Left: Backprojection with source detected. Right: Cargo configuration with  
53      5 the source configuration indicated by arrow. 100,672 particles detected, 99,683 background  
54      5 particles and 989 source particles. Exposure time is 21.92 seconds.

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34      *6.3. Performance on Large Scale Dataset*  
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6      To test the statistical performance of the CNN on a large scale, 1738 unique cargo  
7      configurations are generated using an alternate (to avoid possible inverse crime) generative  
8      scheme. For each cargo configuration all four linear detector arrays are present, from zero up  
9      to four sources are randomly placed and simulated independently, so that by using linearity  
10     of (2) we can produce  $1738 \times 16 = 27808$  testing samples. Particle detections are simulated  
11     for an exposure time measured by the expected background detection levels of 20000, 50000  
12     and 100000 particles. The data were fed into the trained CNN for source presence detection.  
13     The results for presence detection are summarized in Table 3 below. The results obtained  
14     clearly confirm our expectations (see Section 1).  
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Expected Particle Count	Sensitivity	Specificity
100000	99.90%	99.71%
50000	99.78%	94.59%
20000	99.81%	36.36%

24      **Table 3.** Sensitivity and specificity of the CNN source detection with each source having  
25     1% SNR.  
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28      We remind the reader that sensitivity, or true positive rate, shows the success of  
29     determining the presence of a source (i.e., few false negatives), while specificity reflects  
30     how well the absence of the source is detected (i.e., few false positives). High specificity was  
31     hardwired into the BP techniques [5, 6], it was only the sensitivity that was questionable.  
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34      The accuracy of the prediction generally increases with particle count (and thus  
35     observation time), and sufficient particle counts are required for successful detection. At  
36     the low levels (e.g., of 20000 particles and lower) the network seems biased to think that  
37     a source is always present. This clearly leads to near 100% sensitivity and an extremely  
38     low specificity, which makes the detection practically not feasible, due to high level of false  
39     positives.. An explanation could be that the features that are being detected (albeit we do  
40     not know what they are) are non-smooth, vs. large smooth background. When the total  
41     count is low, the whole dataset becomes non-smooth, which tricks the network.  
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44      For comparison, we show below the analogous backprojection results, which are  
45     significantly worse. This is not surprising, since the basic assumptions for this technique  
46     are not satisfied. For  $10^5$  particles CNN succeeds extremely well and beats hands down the  
47     backprojection technique, which often does not show any statistically significant deviations  
48     and thus does not detect presence of the source. Notice that six times higher number of  
49     detected particles was required in [5, 33] for backprojection stable detection, even without  
50     complex cargo being involved.  
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Expected Particle Count	Sensitivity	Specificity
100000	71.04%	99.31%
50000	65.55%	99.31%
20000	52.13%	98.91%

**Table 4.** Sensitivity and specificity of the backprojection source detection with each source having 1% SNR.

#### 389 6.4. Number of sources and number of detector arrays

390 Here we address the question of whether one has to completely surround the object with four  
 391 detectors, or some results can be achieved with three, two, or one flat detector arrays. We  
 392 thus have simulated each cargo configuration with zero to four independent sources randomly  
 393 placed. Just as with the training data, we take several combinations of which sources and  
 detector arrays are present. The combinations are summarized in Table 5 below.

Number of Sources	One Detector	Two Adjacent Detectors	Two Opposite Detectors	Three Detectors	Four Detectors	Total
0	6952	6952	3476	6952	1738	26070
1	27808	27808	13904	27808	6952	104280
2	41712	41712	20856	41712	10428	156420
3	27808	27808	13904	27808	6952	104280
4	6952	6952	3476	6952	1738	26070
Total	111232	111232	55616	111232	27808	417120

**Table 5.** Number of testing samples in each category

Particle detections are simulated for an exposure time measured by the expected background detection levels of 20000, 50000 and 100000 particles. The data were fed into a trained CNN for inference. The results for presence detection are summarized in the bar graphs below, with detailed tables posted in [8].

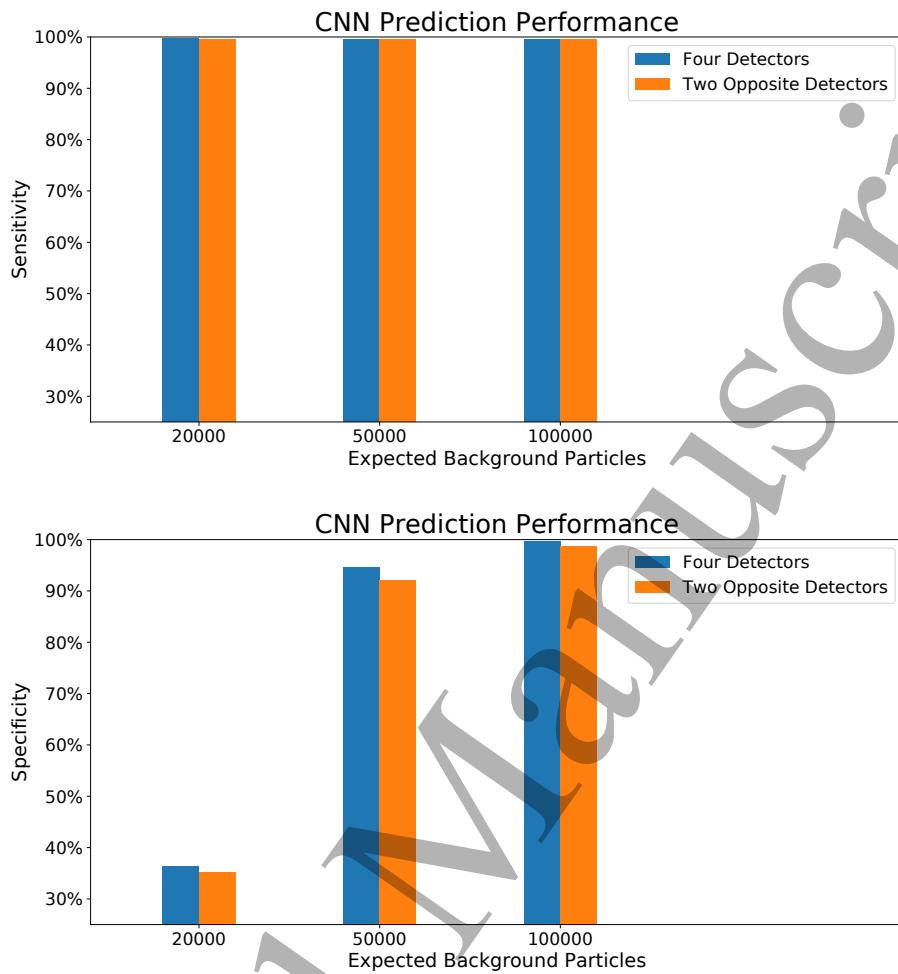
399 Additionally, we investigated the effect of scaling the strength of each source so that  
 400 altogether they had the same strength as a single source, thus effectively diluting the localized  
 401 signature of the source. In the case of backprojection the localized nature of the source is  
 402 the key justification for the method of [5]. This would lead one to believe that splitting the  
 403 source strength will make it more difficult for the CNN to detect any source presence, which  
 404 is indeed confirmed by the results summarized in Table 6 below.

## 405 6.5. Observation time

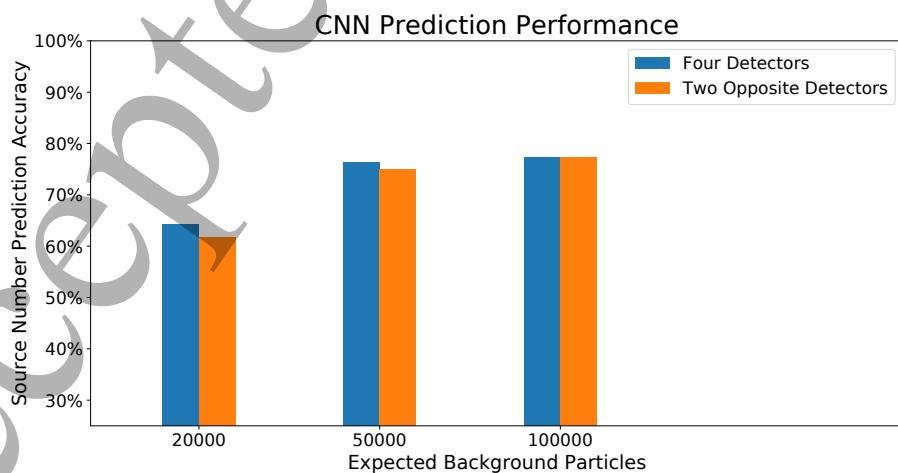
406 The above results are presented in terms of the total number of particles detected. The  
 407 conclusion is natural: the larger - the better. The number of detected particles obviously

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**Figure 16.** Sensitivity and specificity of the source detection.



**Figure 17.** Accuracy of the number of source detection.

Expected Particle Count	Sensitivity one source	Sensitivity two sources	Sensitivity three sources	Sensitivity four sources	Specificity
100000	99.74%	96.57%	89.67%	82.62%	99.71%
50000	99.68%	97.25%	93.51%	89.13%	94.59%
20000	99.99%	99.95%	99.86%	99.77%	36.36%

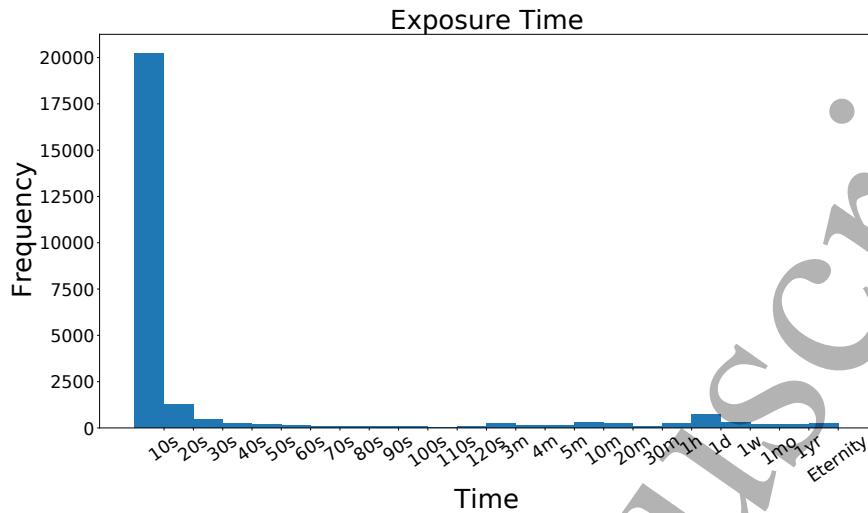
**Table 6.** Sensitivity and specificity of the source detection techniques with split source strength.

408 increases with (essentially proportional to) the time of observation. However, the slope of  
409 this increase clearly depends significantly on the type and configuration of the cargo. Thus,  
410 the exposure time required to reach a certain level of particle detections is a function of  
411 the configuration of the cargo, including source location, material composition, material  
412 placement, and background strength. This makes it difficult to predict boundary flux rates,  
413 even if the configuration is known, without solving the Boltzmann equation (2).

414 To make a fair numerical experiment, many heavily iron (and thus very shielding) cargo  
415 scenarios have been included. Namely, the set of all samples have been divided into 24 sets of  
416 equal size, and the probability of choosing iron as the filling of boxes was increasing linearly  
417 from zero in the first group to almost one in the 24th one. Figure 18 contains the histogram  
418 of the number of runs vs. time required for detection for thousands of configuration runs  
419 for detecting the presence of a source emitting on the order of 1000 particles (assuming four  
420 detectors). The vast majority would require time measured in seconds.

421 Generally speaking, one would expect the large bin on the left-hand side to correspond  
422 to configurations with less high-Z materials, and the larger bins on the right-hand side  
423 correspond to configurations with more high-Z materials. It can certainly become unrealistic  
424 to detect many source particles in some of the latter cases. Nevertheless, as is evidenced by  
425 some of the examples presented, as well as statistics presented in Section 6.3, quite a few  
426 configurations of high-Z material exist where presence of source(s) source can be detected  
427 in a reasonable amount of time. These lower exposure time scenarios would be the most  
428 appropriate cases for detecting illicit nuclear materials at border crossings. Some of the  
429 longer exposure times (on the order of several minutes to perhaps several days) would be  
430 appropriate for detection of illicit nuclear materials in shipping containers on cargo ships,  
431 where scanning can be done while the container is in transit.

432 Additionally, it is important to note that if one restricts oneself to a smaller number of  
433 detector arrays (incomplete view), it will take longer to reach the same exposure level and  
434 thus would add to the number of undetected cases.



**Figure 18.** Histogram of the number of runs vs. exposure times required for detection for the testing data set. These times are computed in the case that all four linear detector arrays are present and anywhere between one and four sources are present.

## 435 7. Remarks and Conclusions

- 436 • Our work shows that the deep learning approach significantly improves over detection by  
437 backprojection techniques of [6,30,33] and works for complex attenuating and scattering  
438 cargo scenarios, where the latter fails completely. This confirms the opinion expressed  
439 in [6] that some information about source presence was there.
- 440 • This article concentrates on the cases of presence of complex cargo and much (an order  
441 of magnitude) lower number of  $\gamma$ -photon detected than in [5, 33]. This makes the  
442 backprojection detection algorithm of these works not only weak, but also groundless.
- 443 • The network performs well detecting the number of up to four sources (although  
444 naturally somewhat less successfully than detecting mere presence of a source).
- 445 • The authors want to make clear that when producing the results of this paper, no  
446 processing (e.g., backprojecting) of the raw detector data is done before feeding it to the  
447 network. Since the authors do not know what features would be of importance, we have  
448 decided to not impose our prejudices on the data (especially taking into account that  
449 backprojection is a smoothing operator, and the relevant information is most probably  
450 contained in some sharper features).
- 451 • The exposure time required for detection is discussed in Section 6.5. The histogram in  
452 Figure 18 shows detection in a matter of second for a vast majority of configurations.  
453 It is clear that there are some unbeatable shieldings, so one cannot aim for the 100%  
454 success rate. In such cases, other detection techniques could be used: from methods  
455 of detecting presence of significant amounts of (shielding) high-Z materials, to neutron  
456 emission detection, to human intelligence.

3      457     • A strong effort has been made to avoid committing an inverse crime. The testing samples  
4      458     have been produced by an algorithm independent of the one used for the training data.  
5      459     The testing cargo geometries were different from the ones not encountered in the training  
6      460     data, so there was no intersection between the two data-sets.

7      461     • A variety of symmetry rules, including rotational symmetry and mirror symmetry were  
8      462     applied randomly to some of the configurations and their material content, to check  
9      463     whether presence or absence of the symmetry influence the detectability. The network  
10     464     performance does not seem to react to this.

11     465     • The reader should not think that retraining was needed for different tasks and situations,  
12     466     e.g. for heavy iron cargo, or for detecting the number of sources, rather their mere  
13     467     presence. This all was done with a single trained network.

14     468     • Four planar Compton detectors forming a square surrounding the object of interrogation  
15     469     were assumed. It seems that this is the most practical design of such detectors. Effects  
16     470     of removal of some of the detectors have also been studied (Section 6.4).  
17     471     The rectangular shape causes some problems, though, e.g. in backprojection method  
18     472     they create (easily removable) corner artifacts. More importantly, this design lacks full  
19     473     rotational invariance, which could be beneficial for the NN design. On the other hand,  
20     474     the rectangular case is challenged by appearance of tilted cargo structures in the test  
21     475     samples, while they were absent in the training data. The network, however, clearly has  
22     476     overcome this difficulty.

23     477     • Deep learning techniques have been applied for SPECT image reconstruction, but as it  
24     478     has been mentioned in the text the level of SNR we are dealing with in this work makes  
25     479     any attempt to image reconstruction rather than binary detection impossible.

26     480     • There are various further improvements that one should attempt (and are being  
27     481     attempted). Some of them are addressed below.

28     482     (i) It would have been great to figure out what specifically were the signs of presence  
29     483     of the source that the network has learned. This would open a door for developing  
30     484     more analytic methods. However, at this moment the authors do not know what  
31     485     these features are.

32     486     (ii) Producing many more training data is a serious stumbling block in 2D, and  
33     487     especially in 3D case.

34     488     (iii) The CNN architecture should be improved, aiming to reach shorter observation  
35     489     time and even lower SNR levels.

36     490     (iv) We are working on moving to the more realistic 3D situation. The significant  
37     491     difference here is, first, the much higher dimensionality of the data (5D) and  
38     492     corresponding much more massive computations that are needed. Second, in  
39     493     3D, unlike 2D (where a cone consists just of two rays), the Compton data differ  
40     494     significantly from the usual Radon ones. In particular, an issue arises of how to bin  
41     495     the five-dimensional Compton data in such a way, that the use of CNN could be  
42     496     warranted.

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- (v) The neural network (NN) approach should be tested on real data, which the authors clearly do not have. However, the radiative transport forward computations we used are commonly practiced in nuclear engineering, seem to be very realistic, and involve realistic material parameters. There is a chance that when novel neutron detectors that are being developed are deployed, we could get some real data.
- (vi) The approach we describe indicates presence of a source, but not its location (at least in the heavy iron cargo case). One wonders whether location can also be attempted.
- (vii) Although the results presented have been obtained by the same once trained NN, during research various designs of the NN and training sets have been experimented with, all showing consistent ability of detection. It would be still important to study further the model uncertainty (e.g., by using the dropout technique [14]). This will be done in a future work.  
Meanwhile, although the testing samples often deviated from the structures used in the training set, our results have shown that the NN generalized extremely well. High experimental levels of the sensitivity and specificity, as well as more detailed information presented in Section 6.3 about statistical spread of the results instill confidence in the suitability of the network as a detection tool.

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3      604 **9. Appendix: Algorithm for Procedural Generation of Cargo Configurations**

## 4

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7      **Algorithm 1:** Procedural Cargo Configuration

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10     Generate Perlin noise in cargo;

11     Initialize  $n_x$  and  $n_y$  to desired number of vertical and horizontal boundaries

12     (numbers can be chosen randomly);

13     Sum Perlin noise over rows and columns to produce noise function on edge of cargo;

14     Randomly select  $n_x$  distinct  $x$ -coordinates for vertical boundaries and  $n_y$  distinct15      $y$ -coordinates for horizontal boundaries according to edge noise functions. Store16     in  $x$  and  $y$  respectively.;

17

18      $n_{iter} = 0$  ;19     **while**  $n_x > 0$  or  $n_y > 0$  **do**

20

21        **if**  $n_{iter}$  is even and  $n_x > 0$  **then**22            Determine all existing boundary points along the line  $(x[n_{iter}/2], y)$ .23            Randomly select a starting point  $y_s$  and ending point  $y_e$  from among the

24            existing boundary points according to previously generated Perlin noise.

25            Set all points between  $(x[n_{iter}/2], y_s)$  and  $(x[n_{iter}/2], y_e)$  to boundary

26            points.;

27

28             $n_x = n_x - 1$ ;29             $n_{iter} = n_{iter} + 1$ ;

30

31        **else if**  $n_{iter}$  is odd and  $n_y > 0$  **then**32            Determine all existing boundary points along the line  $(x, (y[(n_{iter} - 1)/2]))$ .33            Randomly select a starting point  $x_s$  and ending point  $x_e$  from among the

34            existing boundary points according to previously generated Perlin noise.

35            Set all points between  $(x_s, (y[(n_{iter} - 1)/2]))$  and  $(x_e, (y[(n_{iter} - 1)/2]))$  to

36            boundary points.;

37

38             $n_y = n_y - 1$ ;39             $n_{iter} = n_{iter} + 1$ ;

40

41        **end**

42        Identify connected components (Scipy.Measure.Label);

43        **if** Rotational Symmetry Desired **then**

44            Copy one quadrant of the configuration over all others with appropriate

45            rotation;

46        **if** Mirror Symmetry Desired **then**

47            Copy one side of the configuration over the other with mirroring ;

48            ... Randomly assign material identification to each connected component ;

49        Save configuration to file;

50        **Result:** Single cargo configuration

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