

Article

Antimatter Free-Fall Experiments and Charge Asymmetry

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Abstract: We propose a method by which one could use modified antimatter gravity experiments in order to perform a high-precision test of antimatter charge neutrality. The proposal is based on the application of a strong, external, vertically oriented electric field during an antimatter free-fall gravity experiment in the gravitational field of the Earth. The proposed experimental setup has the potential to drastically improve the limits on the charge-asymmetry parameter $\bar{\epsilon}_q$ of antimatter. On the theoretical side, we analyze possibilities to describe a putative charge-asymmetry of matter and antimatter, proportional to the parameters ϵ_q and $\bar{\epsilon}_q$, by Lagrangian methods. We found that such an asymmetry could be described by four-dimensional Lorentz-invariant operators that break CPT without destroying the locality of the field theory. The mechanism involves an interaction Lagrangian with field operators decomposed into particle or antiparticle field contributions. Our Lagrangian is otherwise Lorentz, as well as PT invariant. Constraints to be derived on the parameter $\bar{\epsilon}_q$ do not depend on the assumed theoretical model.

Keywords: symmetry and conservation laws; gauge field theories; gauge bosons

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1. Introduction

The CPT theorem [1,2] encompasses large classes of local field theories with Lorentz-invariant, local, and Hermitian Lagrangian terms. All theories investigated in [1,2] are invariant under the combined action of charge conjugation (C), parity inversion (P), and time reversal (T). Due to the (almost) universal character of the theories studied in [1,2], the accepted assumption is that CPT invariance is an inalienable symmetry of nature. Here, we explore the possibility that, by enlarging the class of the possible interactions, it is possible to evade the CPT theorem by adding Hermitian, local, yet CPT-violating interactions that lead to long-range, gravity-mimicking, CPT-violating interactions, and still conserve Lorentz invariance.

Such interactions have been investigated in [3–6] with a separate charge asymmetry parameter for antimatter being introduced on an *ad hoc* basis. We, here, explore if such asymmetry parameters could be introduced via a separation of the quantum-field theoretical fermion operators into positive-energy and negative-energy components, and we discuss a possible related experimental *ansatz*.

Such interactions violate a fundamental principle of the construction of quantum field theories, namely, gauge invariance. This is the primary reason why one can introduce putative charge-asymmetry of matter and antimatter by four-dimensional Lorentz-invariant operators that break CPT without destroying the locality of the field theory. Our interaction terms describe the exchange of virtual bosons but exclude virtual annihilation channels. While, with this approach, several difficulties associated with nonlocal and Lorentz-violating field theories can be avoided, gauge invariance is violated.

The experimental proposal discussed here is independent of the theoretical model introduced; the aim is to set limits for the charge asymmetry parameter for antimatter,

which could otherwise be introduced on an ad hoc basis [3,5,6]. Our approach is based on the explicit assumption that matter–antimatter symmetry perfectly holds for the gravitational interaction. There are strong theoretical arguments to support this assumption [7,8]. This conjecture also is compatible with the experimental results reported in [9], which will hopefully be improved in the near future.

From a phenomenological point of view, the decisive characteristic of our models is the fact that they allow for the presence of a different charge asymmetry parameter for matter versus antimatter. Such effects have been discussed in the literature, independent of a quantum-field theoretical underlying formulation [4–6]. Our proposal for an improvement of the charge asymmetry parameter for antimatter is connected with antimatter gravity experiments with the aim of comparing a residual electrostatic force on an antihydrogen atom with the gravitational force.

Tests of the charge neutrality of matter (and antimatter) have recently attracted considerable attention [4–6]. A conceivable electric charge of the neutron was investigated in [10]. An excellent review on various theoretical models allowing for charge asymmetry is presented in [11]. An essential assumption underlying our investigations is that the gravitational interaction for antimatter in the gravitational field of the Earth is the same as for matter. There are strong theoretical arguments to support this initial assumption [7,8,12].

Past efforts [13–15] to measure the gravitational interaction of electrons and positrons have suffered from the problem of eliminating electric-field effects. This has been a tremendous problem that has never been solved convincingly in experiments [14,15]. It is connected with the elimination of the influence of the electric field generated by the electrons in the metal or other material (drift tube) surrounding the fall line of the charged leptons.

Theoretical arguments [13] suggested that, under the presence of the electric field generated by the electrons, the total gravitational acceleration of electrons converges to zero, while that of positrons would be twice the acceleration due to gravity. Indeed, experiments with positrons were not successfully reported despite considerable invested effort [13–15]. We argue the other way and show that, in a gravitational experiment carried out with (supposedly) electrically neutral particles, the influence of any deliberately introduced, strong, external, electric field allows one to display the effect of a residual charge excess, leading to a test of charge neutrality.

The influence of a hypothetical charge excess in antimatter on the dynamics can be explored effectively in a strong, uniform, vertical external electric field. This is because any potential charge asymmetry is compared to the extremely weak gravitational force. Our estimates, reported in this article, suggest that limits on the charge asymmetry of antimatter could be improved by many orders of magnitude if an antihydrogen gravity experiment is done in the presence of a strong external electric field. SI mksA units are used in this article unless stated otherwise.

2. Charge Symmetry and Gravity

Let us assume that electron and proton charges do not quite add up to zero so that there is an ever so slight residual charge to be associated with a hydrogen atom, an idea originally formulated by Einstein at the 1924 Lucerne Meeting of the Swiss Physical Society as was explicitly mentioned in [16,17]. In accordance with [4], we parameterize a putative charge excess as follows,

$$q_e = -|e|, \quad q_p = |e|(1 + \epsilon_{p-e}), \quad (1)$$

$$\epsilon_{p-e} = \frac{q_e + q_p}{|e|}, \quad \epsilon_n = \frac{q_n}{|e|}. \quad (2)$$

Here, $q_e = e$ is the electron charge, and $|e|$ is its modulus, while q_p and q_n are the proton and neutron charges, respectively. If we assume, with [4], charge conservation in the β decay of the neutron, then the charge-neutrality violating parameter ϵ_n for the

neutron (let us be clear that the neutron acquires an infinitesimal electric charge under the assumptions made in [4]) becomes

$$\epsilon_n = \epsilon_{p-e} \equiv \epsilon_q. \quad (3)$$

If a body containing Z protons and electrons, as well as N neutrons is measured as being neutral with sensitivity δq , the one can obtain a limit on $|\epsilon_q|$, which is on the order of

$$|Z\epsilon_{p-e} + N\epsilon_n| = (Z + N)|\epsilon_q| \leq \delta q, \quad (4a)$$

$$|\epsilon_q| \leq \frac{\delta q}{(Z + N)|e|}. \quad (4b)$$

The essential idea of the acoustic method used in [4] is that, under the assumption of a nonvanishing charge asymmetry in matter, electromagnetic waves incident on an electrically neutral gas would set the gas atoms in motion, inducing sound waves. However, this method of determining limits on $|\epsilon_q|$ is not free from pitfalls and requires a considerable additional mathematical formalism in the evaluation of the experiment. For example, according to a note to Table I of [18], data published in the previous work [19] may exhibit inconsistencies.

A paper [20] that initially claimed an accuracy on the level of 10^{-23} for $|\epsilon_q|$ has recently been questioned in [4], with the claim that their result on $|\epsilon_q|$ could not be considered to be better than 10^{-19} if all inaccuracies and neglected systematic effects in the paper [20] are properly taken into account. The paper [4] also indicates additional rectification of the analysis of the resonant modes in the gas-filled capacitor used in the previous experiment [20]. Table I of [4] contains a comprehensive compilation of previous measurements of ϵ_q . We will use their result, given in an unnumbered equation on the second-to-last page of [4],

$$\epsilon_q = (-0.1 \pm 1.1) \times 10^{-21}, \quad (5)$$

for matter particles (both a hydrogen atom as well as the constituent atoms of the Earth). Limits on the charge asymmetry of matter have also been derived on the basis of model-dependent astrophysical methods [21,22]. Separate investigations put limits on the neutrality of neutrinos on the basis of astrophysical observations [23–27].

In contrast, the constraints on charge neutrality for antimatter are looser by many orders of magnitude. Tests on the electric charges of positrons and antiprotons can be derived from measurements of their cyclotron resonance frequencies and from spectroscopic data [3]. The most recent direct tests [5,6] revealed a 1σ limit 68.3% confidence level)

$$|\bar{\epsilon}_q| \leq 7.1 \times 10^{-10} \quad (6)$$

for antimatter. Here, the parameters for antiparticles are given by

$$q_{\bar{e}} = |e|, \quad q_{\bar{p}} = -|e| (1 + \epsilon_{\bar{p}-\bar{e}}), \quad (7)$$

$$\epsilon_{\bar{p}-\bar{e}} = \frac{q_{\bar{e}} + q_{\bar{p}}}{-|e|}, \quad \epsilon_{\bar{n}} = \frac{q_{\bar{n}}}{-|e|}. \quad (8)$$

The parameters $\bar{e}$, $\bar{p}$, and $\bar{n}$ stand for the positron, antiproton, and antineutron, respectively. We shall also make the assumption that charge is conserved in the β decay of the antineutron and write

$$\epsilon_{\bar{n}} = \epsilon_{\bar{p}-\bar{e}} \equiv \bar{\epsilon}_q. \quad (9)$$

Furthermore, we shall assume, as demonstrated in a number of experiments [28–30], that gravitational and gravity-like interactions are equivalently realized on the microscopic (atomic) and macroscopic level.

The electric charge asymmetry of antihydrogen, if it exists, is not necessarily opposite to that found in hydrogen. However, there might be good arguments to support this

conjecture. Namely, electrons and positrons constitute a particle–antiparticle pair and, therefore, are described by the same Dirac equation, which predicts that the electric charges of the positron and electron add up to zero. The same applies to protons and antiprotons. However, one observes that leptons and hadrons are still two completely different particle species. Therefore, it could appear easier to speculate about the broken electric neutrality of a hydrogen atom rather than the broken electric neutrality of, say, positronium.

3. Charge Asymmetry and CPT Violation

Typically (see [4,16,17]), the charge-asymmetry parameters ϵ_q and $\bar{\epsilon}_q$ are used as *ad hoc* parameters without any attempt being made to formulate an underlying quantum field theory that could describe the charge-symmetry violation. Let us attempt to realize somewhat higher ambitions and explore candidate models. In the field-theoretical sense, let us explore if a slight breaking of the charge symmetry could be formulated in terms of a gauge-symmetry breaking (GB) modification of the quantum electrodynamics (QED) interaction Lagrangian. Denoting field operators by a hat, we write the Lagrangian (we temporarily switch to the natural unit system with $\hbar = c = \epsilon_0 = 1$, as is customary in particle physics)

$$\begin{aligned}\hat{\mathcal{L}}_{\text{GB}} = & \sum_{f=e,p,n} \hat{\bar{\psi}}_f (i\gamma^\mu \partial_\mu - m) \hat{\psi}_f - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & - e \left(\hat{\bar{\psi}}_e \gamma^\mu \hat{\psi}_e - \hat{\bar{\psi}}_p \gamma^\mu \hat{\psi}_p \right) \hat{A}_\mu \\ & - \epsilon_q |e| \left(\hat{\bar{\psi}}_p \gamma^\mu \hat{\psi}_p + \hat{\bar{\psi}}_n \gamma^\mu \hat{\psi}_n \right) \hat{A}_\mu.\end{aligned}\quad (10)$$

Here, the electron-positron field operator is ψ_e , while the composite spin-1/2 operator for the proton–antiproton field is ψ_p , and the neutron–antineutron field could be described by a spin-1 generalization of the Dirac equation (see [31]). The quantized photon field is described by the four-vector field operator $\hat{A}_\mu$, which enters the field-strength tensor operator $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu$, where $\partial_\mu \equiv \partial/\partial x^\mu$ is the partial derivative with respect to the space-time coordinate x^μ . In the form of $\hat{\mathcal{L}}_{\text{GB}}$ given in Equation (10), we use the assumption (9). As a side remark, we note that the current conservation implies that $\epsilon_{p-e} = 2\epsilon_u + \epsilon_d$, where the valence quark couplings for the up and the down quark are ϵ_u and ϵ_d , respectively.

From the form of the Lagrangian (10), it follows that electrons (and consequently positrons) carry a charge $\pm e$, while protons (and antiprotons) carry a charge $\pm(1 + \epsilon_{p-e})|e|$. This results in a hydrogen atom having a charge $\epsilon_{p-e}|e|$, while antihydrogen atoms carry a charge $(-\epsilon_{p-e}|e|)$. One might, therefore, assume that $\epsilon_q = -\bar{\epsilon}_q$.

As there exist very stringent limits on ϵ_q (for particles, see [4]), it is interesting to explore the possibility of different charge-asymmetry parameters for particles and antiparticles, i.e., a Lagrangian with two different parameters ϵ_q and $\bar{\epsilon}_q$. One may, thus, explore the phenomenological consequences of the Lagrangian

$$\begin{aligned}\hat{\mathcal{L}}_{\text{SYM}} = & \sum_{f=e,p,n} \hat{\bar{\psi}}_f (i\gamma^\mu \partial_\mu - m) \hat{\psi}_f - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \\ & - e \left(\hat{\bar{\psi}}_e \gamma^\mu \hat{\psi}_e - \hat{\bar{\psi}}_p \gamma^\mu \hat{\psi}_p \right) \hat{A}_\mu \\ & - \epsilon_q |e| \left(\hat{\bar{\psi}}_p^{(+)} \gamma^\mu \hat{\psi}_p^{(+)} + \hat{\bar{\psi}}_n^{(+)} \gamma^\mu \hat{\psi}_n^{(+)} \right) \hat{A}_\mu \\ & + \bar{\epsilon}_q |e| \left(\hat{\bar{\psi}}_p^{(-)} \gamma^\mu \hat{\psi}_p^{(-)} + \hat{\bar{\psi}}_n^{(-)} \gamma^\mu \hat{\psi}_n^{(-)} \right) \hat{A}_\mu.\end{aligned}\quad (11)$$

Here, we have induced CPT violation by decomposing a general fermionic field operator $\hat{\psi}(x)$ into a positive-frequency matter contribution $\psi^{(+)}(x)$ and a negative-frequency antimatter contribution $\psi^{(-)}(x)$, as follows,

$$\hat{\psi}(x) = \psi^{(-)}(x) + \psi^{(+)}(x), \quad (12)$$

$$\hat{\psi}^{(+)}(x) = \sum_s \int \frac{d^3p}{(2\pi)^3} \frac{m}{E} a_s(\vec{p}) u_s(\vec{p}) e^{-ip \cdot x}, \quad (13)$$

$$\hat{\psi}^{(-)}(x) = \sum_s \int \frac{d^3p}{(2\pi)^3} \frac{m}{E} b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ip \cdot x}. \quad (14)$$

In view of the impossibility to transform positive-energy states into negative-energy states via Lorentz transformations (due the mass gap for Dirac particles), this decomposition is Lorentz invariant (for details, see [32]). It is useful to remark that the decomposition is also PT invariant. The decomposition of the effective proton and neutron field operators into positive-frequency and negative-frequency contributions has led to CPT violation and allows for different charge-asymmetry parameters ϵ_q (for particles, i.e., for hydrogen) and $\bar{\epsilon}_q$ (for antiparticles, i.e., for antihydrogen).

A closer inspection reveals that the CPT-violating terms in Equation (11) (those proportional to ϵ_q and $\bar{\epsilon}_q$) only modify the exchange of virtual photons between protons (and neutrons, due to the infinitesimal neutron charge); however, they do not lead to any annihilation of a proton–antiproton pair into a virtual photon. In that sense, the physics deduced from the interaction (11) implies a slight modification of the electromagnetic interaction between protons as compared to electrons, affecting the charge neutrality of the hydrogen and antihydrogen atoms.

This leads to a violation of the electromagnetic gauge invariance (both because of the slightly different electron and proton charge and because of the separation of the field operator in the interaction Hamiltonian into positive-energy and negative-energy components). The consequences of this assumption of astrophysical scales need to be explored. Two aspects can be mentioned here.

First, let us remember that a conceivable charge asymmetry of matter has been discussed as a possible (partial) explanation for the expansion of the Universe [33,34]. Even if the commonly accepted explanation involves a nonvanishing cosmological constant [35,36], it would be interesting to explore conceivable additional contributions to the expansion of the Universe due to antimatter and matter charge asymmetries.

There is a second aspect that might be even more interesting. Namely, a closer inspection reveals that the infinitesimal charge excess of the proton against the electron mimics, in the nonrelativistic limit, a gravitational interaction (see also Equation (16) below). However, the relativistic corrections to the gravitational interaction are known to be different for gravity as compared to electromagnetism [12,37].

If a nonvanishing charge asymmetry parameter were found for antimatter or matter, then one might need to investigate the effective modifications of the gravitational interaction (now adjusted for the retardation corrections to the electromagnetic admixtures due to the charge asymmetry). These could potentially have interesting connections to the observed discrepancies between astrophysical observations and the accepted theory of gravitation and, thus, to the dark matter problem.

The free photon field term in the Lagrangian density (11) involves the field-strength tensor

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu. \quad (15)$$

A closer inspection reveals that, in the nonrelativistic limit, the Lagrangian (11) leads to an effective interaction between hydrogen (H) and antihydrogen atoms ($\bar{H}$) of (we now switch back to SI mksA units for the remainder of this article)

$$V_{HH}(R) = \frac{\epsilon_q^2 e^2}{4\pi\epsilon_0 R}, \quad (16a)$$

$$V_{\overline{H}H}(R) = \frac{\epsilon_q \bar{\epsilon}_q e^2}{4\pi\epsilon_0 R}, \quad (16b)$$

$$V_{\overline{H}\overline{H}}(R) = \frac{\bar{\epsilon}_q^2 e^2}{4\pi\epsilon_0 R}, \quad (16c)$$

where R is the interatomic distance. None of the interaction Lagrangians investigated in [1,2] involve the splitting of a field operator into positive-frequency and negative-frequency parts. Our explicit separation of the field operators into positive- and negative-energy components in Equation (11) offers the possibility of introducing different charge asymmetry parameters for matter and antimatter.

4. Possible Implications on Charge Asymmetry of Antimatter

Based on the Lagrangian (11), our goal was to investigate whether or not the limit given in Equation (6), namely, $|\bar{\epsilon}_q| \leq 7.1 \times 10^{-10}$ could be improved by a simple experimental arrangement: Our suggestion was to place freely falling antihydrogen atoms into a uniform, vertically oriented, electric field, which would enable us to compare the gravitational force acting on the (supposedly) neutral antihydrogen atom to the gravitational force. We would, thus, intend to make the background electric field large, not small (as was otherwise attempted in [13–15]).

An estimate of the achievable accuracy of $|\bar{\epsilon}_q|$ can be obtained by a simple calculation (see also Figure 1). The magnitude of the gravitational force on an antihydrogen atom is

$$|\vec{F}_g| = m_{\overline{H}} g, \quad (17)$$

where $m_{\overline{H}}$ is the antihydrogen atom's mass. The magnitude of the residual electric force on the antihydrogen atom is

$$|\vec{F}_e| = \bar{\epsilon}_q |e| |\vec{E}|, \quad (18)$$

where $|\vec{E}|$ is the (possibly strong) external, vertically oriented electric field. Let us assume that we can experimentally establish that

$$|\vec{F}_e| < \chi |\vec{F}_g|, \quad (19)$$

i.e., that the residual putative electric force is less than a fraction χ of the gravitational force. Let us also parameterize the field strength of the vertically oriented field as $|\vec{E}| = |\vec{E}|_{\text{SI}} \frac{\text{V}}{\text{m}}$, where $|\vec{E}|_{\text{SI}}$ is the magnitude of the field, measured in volts per meter. This sets a limit on $|\bar{\epsilon}_q|$, which is of the order of

$$\bar{\epsilon}_q < \frac{\chi m_{\overline{H}} g}{|e| |\vec{E}|} = 1.02 \times 10^{-7} \frac{\chi}{|\vec{E}|_{\text{SI}}}. \quad (20)$$

Typical Cockcroft–Walton voltage multipliers operate at voltages of of 2×10^4 V or more [38–40], and recent developments easily reach the range or 10^5 V (see [41]). Powerful tandem accelerators are known to operate at 2.5×10^7 V (see [42,43]). In view of Paschen's law [44], the electric breakdown strength inside an antihydrogen trap is of no concern in a trap held at a good vacuum (below 10^{-7} atmospheric pressure). Here, we estimate, somewhat conservatively, that an electric field strength of $|\vec{E}| = 10^6 \frac{\text{V}}{\text{m}}$ can be realized in a dedicated experiment, corresponding to a value of $|\vec{E}|_{\text{SI}} = 10^6$.

Under the further conservative assumption that the experiment establishes that the magnitude residual electric force is less than 10% of the gravitational force ($\chi = 0.1$), one could improve the limit on $\bar{\epsilon}_q$ into the range

$$|\bar{\epsilon}_q| \lesssim 10^{-14}, \quad (21)$$

potentially improving the limit (6) by many orders of magnitude, leading to a drastic improvement of the charge neutrality parameter for antimatter. Possible limitations due to systematic effects encountered in the experiment (interatomic interactions and the spin coupling to the electric field) are discussed in the Appendix A.

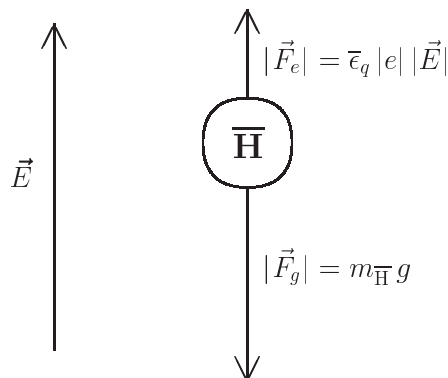


Figure 1. The schematic of the proposed experiment involves a freely falling antihydrogen atom $\bar{H}$ in a strong external, static, vertically oriented electric field $\vec{E}$. If the antihydrogen atom fails to be electrically neutral, then the gravitational force of magnitude $|\vec{F}_g| = m_{\bar{H}}g$ competes with the residual electrostatic force $|\vec{F}_e| = \bar{e}_q |e| |\vec{E}|$, thus, leading to a corresponding reduction of the resultant force (or, to an enhancement, upon flipping the sign of the electric field). In proposing the experiment, we assumed that the gravitational interaction for antimatter in the gravitational field of the Earth is the same as for matter [7,8,12].

5. Conclusions

In this work, we discussed possible antimatter charge asymmetry characterized by an antimatter charge asymmetry parameter $\bar{e}_q$. Our considerations were motivated by the fact that the charge-asymmetry parameter $\bar{e}_q$ for antimatter might be different from the charge asymmetry parameter e_q for particles (see Equation (11)). We, thus, advocate an electrically counterbalanced antimatter gravity for the potential determination of an improved bound on the charge-asymmetry parameter $\bar{e}_q$ for antimatter.

In the proposed experimental setup, a strong, vertically oriented, electric field is placed in the line of free fall of an antihydrogen atom. Any putative charge asymmetry of antimatter, under the influence of the additional external field, would modify the resultant acceleration of the antihydrogen atom, which, in the absence of charge asymmetry, would exclusively be due to gravity. Even under pessimistic parametric estimates, the bounds on $\bar{e}_q$ could potentially be improved by many orders of magnitude when compared to the current limits (see Equations (6) and (21) and [5,6]).

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Appendix A. Interatomic Interactions and Spin-Orbit Force

A potential issue with any experiment is systematic effects. While details of the realization of the current proposal are beyond the scope of the current paper, some remarks on the role of non-gravitational matter–antimatter interactions and spin couplings to the electric field are in order.

The nonretarded van-der-Waals interaction energy among hydrogen atoms, in the leading approximation, is [45–47]

$$E_{\text{HH}}(R) \approx -\frac{D_6}{R^6}, \quad D_6 = 6.499 E_h a_0^6, \quad R \ll \frac{a_0}{\alpha}, \quad (\text{A1})$$

where E_h is the Hartree energy and a_0 is the Bohr radius (the fine-structure constant is α). In the long-range limit, the result changes due to retardation to [47]

$$E_{\text{HH}}(R) \approx -\frac{23}{4\pi} \frac{\hbar c}{(4\pi\epsilon_0)^2} \frac{1}{R^7} \alpha_{1s}(0) \alpha_{1s}(0), \quad R \gg \frac{a_0}{\alpha}, \quad (\text{A2})$$

where

$$\alpha_{1s}(0) = \frac{9 e^2 a_0^2}{2 E_h} \quad (\text{A3})$$

is the static polarizability of hydrogen. In the long-range limit, the interatomic interaction is also referred to as the Casimir–Polder interaction [48]. Due to the $R^{-6} \dots R^{-7}$ dependence, the van-der-Waals (viz., Casimir–Polder) interaction can typically be considered as negligible at large interatomic separations. However, we remember that, here, we compare it to gravitational interactions, and thus a closer look is required.

Of interest in our context are matter–antimatter interatomic interactions, i.e., the van-der-Waals and Casimir–Polder interactions between hydrogen and antihydrogen atoms. A closer inspection reveals that the both the van-der-Waals as well as the Casimir–Polder interaction between hydrogen and antihydrogen has exactly the same coefficients and the same attractive sign when compared to the interaction between two hydrogen atoms, i.e., one has

$$E_{\overline{\text{HH}}}(R) = E_{\text{HH}}(R). \quad (\text{A4})$$

This conclusion is not completely trivial. The (first-order) van-der-Waals Hamiltonian, given in Equation (2c) of [49], actually changes sign in the transition from the hydrogen–hydrogen to the hydrogen–antihydrogen system. This is evident because the nonretarded van-der-Waals Hamiltonian counts the electrostatic interactions between the constituent particles of both atoms. For the hydrogen–hydrogen system, one exemplary term in the van-der-Waals Hamiltonian is due to the interaction of the orbiting particle of atom A (the electron) with the nucleus of atom B (the proton).

For the hydrogen–antihydrogen system, the same term is replaced by the interaction of the orbiting particle of atom A (the electron) with the nucleus of atom B (the antiproton). An obvious generalization of this consideration explains the overall sign change of the van-der-Waals Hamiltonian. However, the $1/R^6$ van-der-Waals interaction is given by a *second-order* perturbation theory result (see [49–51]). Thus, the sign change of the van-der-Waals Hamiltonian leads to an invariant expression for the interaction energy, in second-order perturbation theory. Analogous considerations apply in the long-range limit, where the virtual photon exchange between the two atoms has to be formulated with the full energy dependence of the photon propagator, and two mutually compensating sign changes occur [52]. This justifies Equation (A4).

To analyze the role of interatomic interactions in our proposed test of antimatter charge neutrality, one should compare the interatomic interactions with the gravitational force acting on antihydrogen in the gravitational field of the Earth. This calculation is easily done and reveals that the gravitational force on the Earth’s surface dominates over the van-der-Waals interaction between two hydrogen or antihydrogen atoms for all relevant interatomic distances greater than a Bohr radius. This observation, together with the functional form of

the interatomic interactions, clarifies that interatomic interactions do not affect the viability of our proposal.

Another consideration is the role of the spin coupling of the electric field, especially in view of the fact that antihydrogen (just like hydrogen) has a magnetic moment largely determined by the positron spin orientation (the antiproton spin contributes only a fraction to the total magnetic moment of the antihydrogen ground state). Oscillating electric fields can induce the spin flip of an electron [53], a fact that is also used in spintronics (for a review, see [54]). The dominant term responsible for the spin coupling to the electric field is given by the spin-orbit (SO) coupling Hamiltonian [37,55], which, in SI mksA units, takes the form

$$H_{\text{SO}} = -\frac{\hbar q}{4m^2c^2} \vec{\sigma} \cdot (\vec{E} \times \vec{p}). \quad (\text{A5})$$

Here, q is the charge of the particle, m is its mass, $\vec{\sigma}$ denotes the vector of the Pauli spin matrices, $\vec{E}$ is the electric field, and $\vec{p}$ is the particle momentum operator. For a bound electron in a hydrogen-like ion of charge number Z , one replaces $q\vec{E} \rightarrow -Ze^2\vec{r}/(4\pi\epsilon_0 r^3)$. In this case, the spin-orbit coupling reduces to the familiar Russell–Saunders coupling Hamiltonian, and H_{SO} is replaced as follows,

$$H_{\text{SO}} \rightarrow \frac{\hbar^2 Z\alpha \vec{\sigma} \cdot \vec{L}}{4m^2c r^3}. \quad (\text{A6})$$

As is well known [55,56], the Russell–Saunders coupling determines the fine-structure of hydrogen in leading order. For our proposed experiment, the dominant electric field in Equation (A5) is the external, strong, vertical electric field, and $\vec{p}$ is the electron momentum operator. The momenta of the positron and of the antiproton in the freely falling antihydrogen atom can be transformed as follows into center-of-mass coordinates,

$$p_{e^+} = \frac{m_{e^+}}{m_{\text{H}}} \vec{P} + \vec{p}_r, \quad p_{\bar{p}} = \frac{m_{\bar{p}}}{m_{\text{H}}} \vec{P} - \vec{p}_r, \quad m_{\text{H}} = m_{e^+} + m_{\bar{p}}. \quad (\text{A7})$$

Here, $\vec{p}_r$ is the relative momentum, which, in the ground state of antihydrogen, has the expectation value $\langle \vec{p}_r \rangle = \vec{0}$. We denote the positron mass as m_{e^+} , the antiproton mass as $m_{\bar{p}}$, and the total momentum of the antihydrogen atom by $\vec{P}$. The appropriate spin-orbit coupling Hamiltonian for the coupling of the positron to the vertical electric field is, therefore,

$$H_{\text{SO}} = -\frac{\hbar|e|}{4m_{e^+}m_{\text{H}}c^2} \vec{\sigma}_{e^+} \cdot (\vec{E} \times \vec{P}), \quad (\text{A8})$$

where $\vec{\sigma}_{e^+}$ is the vector of the positron spin matrices. The classical trajectory of a freely falling body in the gravitational field of the Earth implies the relation

$$\vec{P} = -m_{\text{H}} g t \hat{e}_z = -m_{\text{H}} \sqrt{2gh} \hat{e}_z, \quad (\text{A9})$$

where g is the acceleration due to gravity, t is the time, and h is the (downward) distance traveled by the antihydrogen atom in the gravitational field of the Earth. Furthermore, $\hat{e}_z$ is the unit vector in the z direction. If the external electric field is not perfectly aligned with the vertical, i.e., $\vec{E} \times \vec{P} \neq \vec{0}$, then the spin-orbit energy has to be added to the kinetic energy acquired by the antihydrogen atom in free fall. Depending on the spin orientation of the positron, one has

$$E_{\text{SO}} = \langle H_{\text{SO}} \rangle = \pm \frac{\hbar|e||\vec{E}||\vec{P}| \sin \theta}{4m_{e^+}m_{\text{H}}c^2} = \pm \frac{\hbar|e||\vec{E}|\sqrt{gh} \sin \theta}{2\sqrt{2}m_{e^+}c^2}, \quad (\text{A10})$$

where $\theta = \angle(\vec{E}, \vec{P})$. We chose the quantization axis of the positron spin to be parallel to the direction of $\vec{E} \times \vec{P}$.

A force on the freely falling antihydrogen atom is generated when the spin-orbit energy E_{SO} becomes position-dependent, i.e., when there is a field gradient of the electric field $\vec{E}$. Let us estimate that

$$\left| \frac{\partial}{\partial x^i} E^j \right| \sim \frac{|\vec{E}|}{L}, \quad i, j = \{1, 2, 3\}, \quad (\text{A11})$$

where i and j denote Cartesian components, and L is an appropriate length scale for the calculation of the gradient. If we assume that the electric field varies linearly with the coordinates, then L would be the length scale over which the electric field ramps up from zero to its maximum value. The magnitude F_{SO} of the spin-orbit force can, thus, be estimated as

$$F_{\text{SO}} = \frac{\hbar |e| |\vec{E}| \sqrt{g} \hbar \sin \theta}{2\sqrt{2} L m_{e^+} c^2}. \quad (\text{A12})$$

The gravitational force on the antihydrogen atom is $F_g = m_{\bar{\text{H}}} g$, and the ratio is

$$\frac{F_{\text{SO}}}{F_g} = \frac{\hbar |e| |\vec{E}| \sqrt{h} \sin \theta}{2\sqrt{2} \sqrt{g} L m_{e^+} m_{\bar{\text{H}}} c^2} = 1.39 \times 10^{-14} \frac{|\vec{E}|_{\text{SI}} \sqrt{h_{\text{SI}}} \sin \theta}{L_{\text{SI}}}, \quad (\text{A13})$$

where $|\vec{E}|_{\text{SI}}$, h_{SI} , and L_{SI} and the reduced quantities corresponding to the electric field, the fall height h , and the characteristic length scale L , expressed in SI mksA units, i.e.,

$$|\vec{E}|_{\text{SI}} = |E| \frac{\text{m}}{\text{V}}, \quad h_{\text{SI}} = \frac{h}{\text{m}}, \quad L_{\text{SI}} = \frac{L}{\text{m}}. \quad (\text{A14})$$

For $|\vec{E}|_{\text{SI}} \sim 10^6$, $h_{\text{SI}} \sim L_{\text{SI}} \sim 1$, and $\theta \sim 1^\circ$, one has $F_{\text{SO}}/F_g \sim 10^{-10}$. Thus, the spin-orbit force can be neglected for our proposed experiment under realistic assumptions. Intuitively, we could have guessed this result on the basis of the fact that the entire motion is fully nonrelativistic, and the spin-orbit coupling term is a part of the Foldy–Wouthuysen transformed relativistic Dirac Hamiltonian [37,55], which becomes significant only for relativistic systems. In particular, the spin coupling is highly suppressed in comparison to the direct coupling of the external electric field to any residual charge of antihydrogen, which would otherwise result from a putative charge asymmetry of antimatter.

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