Proceedings of the ASME 2020
International Design Engineering Technical Conferences
and Computers and Information in Engineering Conference
IDETC/CIE2020
August 17-19, 2020, Virtual, Online

DETC2020-22544

LIMITS OF EXTRAMOBILE AND INTRAMOBILE MOTION OF CYLINDRICAL DEVELOPABLE MECHANISMS

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ABSTRACT

Mechanisms that can both deploy and perform desired tasks offer a multifunctional advantage over traditional mechanisms. Developable mechanisms (DMs) are devices capable of conforming to a predetermined developable surface and deploying from that surface to achieve specific motions. This paper presents new terminology that defines motion of developable mechanisms while interior and exterior to a developable surface. The limits of motion are identified using defined conditions, and it is shown that the more difficult of these conditions may be treated as a non-factor during the design of cylindrical developable mechanisms given certain assumptions.

1 INTRODUCTION

Progress in mechanical design has led to the development of mechanical systems that achieve increasingly specified tasks. Mechanisms that can create customized paths, positions, and force outputs are important, and combining these behaviors with other functionalities is an area of increasing interest. In particular, mechanisms that can both deploy and perform desired tasks offer a multifunctional advantage over traditional mechanisms. Examples include deployable straight-line linkages [1], deployable mechanisms with intentionally-shaped parts [2], and mechanisms that conform to a predetermined surface, such as a rectangle [3]. Other means of obtaining desired shapes have been shown through the use of compliant parts [4] and harmonic linkages [5].

Developable mechanisms (DMs) are devices capable of conforming to a predetermined developable surface (such as a cylinder or cone [6]) and deploying from that surface to achieve specific motions, as shown in Figure 1. Their ability to lie within a surface makes them compact, and if embedded into or part of the surface (such as if a compliant mechanism made from part of the surface) it possible to be hyper-compact. Because of the prevalence of developable surfaces in many engineering applications, DMs provide a way to integrate multifunctionality into

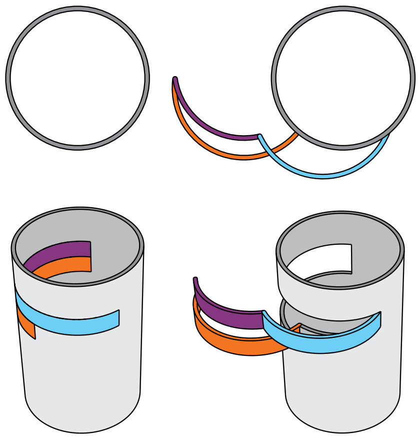


FIGURE 1: This four-bar linkage embedded within a cylinder illustrates a developable mechanism that (a) conforms to and (b) emerges from a cylinder.

previously underutilized surfaces. Foundational work in this field has defined DMs [7], described behaviors unique to cylindrical [8] and conical [9] DMs, and demonstrated their usefulness in certain applications, such as minimally-invasive surgical devices [10].

Because DMs are designed within the context of a developable surface, the movement of the mechanism relative to that surface becomes of particular importance. For example, a device may be made to exist on the interior of a pressurized pipe and requires all parts to remain interior to the pipe during actuation. Another possibility would be mechanisms that lie on the outside of a rocket body where penetrating the pressure vessel would lead to catastrophic failure. Past work investigated whether a given mechanism is capable of moving into or away from a developable surface (referred to as intramobility and extramobility [8]). This behavior allows a DM to (1) lie on a pre-existing surface and (2) exhibit motion without penetrating the surface. Mechanisms that exhibit extramobility and intramobility can move without interfering with existing subsystems, thereby providing multifunctionality in pre-existing systems with minimal impact. However, these terms say little about the motions a mechanism can obtain while interior or exterior to the developable surface. In this paper, we advance the understanding of these mechanisms by identifying the range of motion that can be achieved by a DM while interior or exterior to a developable surface.

Another challenge that exists in the design of mechanisms is the existence of toggle positions and change-points. Early identification of these positions can aid in maintaining a mechanism's desired characteristics throughout its motion [11–13]. Change-points in DMs have not been previously investigated, meaning that the plausibility of extramobile and intramobile motion of a cylindrical DM is unclear when the linkage is a change-point mechanism.

This paper introduces new terminology that defines motion of DMs while interior and exterior to a developable surface. The limits of motion are identified using defined conditions, and it is shown that the more difficult of these conditions may be treated as a non-factor during the design of cylindrical DMs given certain assumptions. Possibilities of change-point mechanisms existing in intramobile and extramobile DMs are investigated. Discussion is provided on the implications of intramobile and extramobile motion.

2 DEVELOPABLE MECHANISM BACKGROUND

When modeled with zero thickness, DMs are constrained to have at least one position in which their joints are all coincident to and aligned with the ruling lines of a developable surface [7]. (This position is referred to as the conformed position.) This constraint creates unique conditions that influence the possible outcomes of mechanism synthesis. Constraining kinematic linkages to conform to predetermined developable surfaces can result in the identification of mechanism behaviors that are relevant in reference to those surfaces, such as the ability to be extramobile, intramobile, and transmobile [8].

In a zero-thickness model, the surface to which the mechanism conforms is called the "reference" surface. When conformed, all of the mechanism's joint axes must intersect and be aligned to the ruling lines on the reference surface. It should be noted that the reference surface does not need to represent a physical surface. As such, the reference surface is merely a representation of where the joint axes must align in space. A cylindrical DM is a mechanism that has at least one position where all parts of the mechanism conform to a cylindrical reference surface.

3 INTRAMOBILE AND EXTRAMOBILE MOTION

We define *intramobile motion* as motion where all moving parts of a mechanism remain interior to the reference surface. Similarly, *extramobile motion* is motion where all moving parts of a mechanism remain exterior to the reference surface. This paper investigates the limits of intramobile and extramobile motion for regular cylindrical developable mechanisms.

Because DMs on cylinders are planar mechanisms, it is often convenient to model them when viewed along the cylinder centerline, as shown in Figure 2. While cylindrical DMs physically are created using curved links, it can be advantageous in the kinematic modeling of DMs to view each linkage as a straight line. Figure 2 shows a developable mechanism (a) and its straight-linkage equivalent (b). The two mechanisms are kinematically equivalent since the distance between pivots is identical. This paper uses both methods to represent the links within a cylindrical DM.

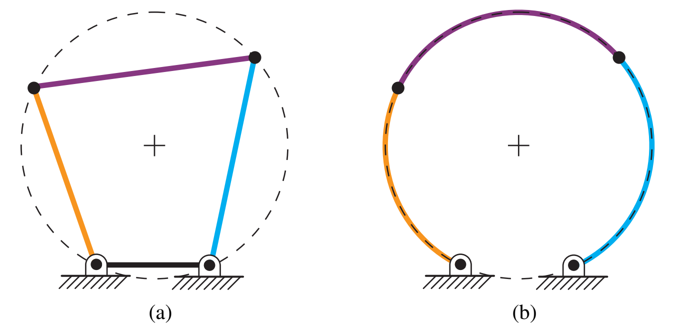


FIGURE 2: When modeling the kinematics of a developable mechanism, only the minimum distance between pivots (a) needs to be considered. However, when identifying potential contact between the links and the reference surface, the actual curved links (b) must be used. Note that for all figures in this paper, the black link is ground, the purple link is the coupler link, and the orange and blue links are links 2 and 4, respectively.

3.1 Conditions for Intramobile and Extramobile motion

The requirement that all parts of a mechanism must remain interior or exterior to the reference surface can be decomposed into the following conditions:

Condition 1: No grounded link may rotate from the conformed position far enough to again intersect the reference surface.
Condition 2: No grounded link may rotate interior to (exterior to) the reference surface for extramobile (intramobile) motion.
Condition 3: No portion of the coupler may cross the reference surface.

The motion of a cylindrical DM will remain extramobile or intramobile if none of these three conditions are violated. It is therefore to the designer's benefit to accurately identify the limits of each of these conditions. Predicting the motion limits of grounded links (Conditions 1 and 2) is straightforward. In contrast, predicting the location of a coupler relative to the reference surface (Condition 3) can be much more complex. However, if the first two conditions were to always occur prior to Condition 3, a designer would be free to ignore Condition 3, making its complexity a non-factor when designing for extramobile and intramobile motion. Sections 3 and 4 demonstrate that Conditions 1 and 2 will always be violated prior to Condition 3 for any mechanism exhibiting intramobile or extramobile motion given the following assumptions:

A. All links have an arc length $\leq \pi R$.
B. All links have the same curvature as the reference surface.
C. All links are modeled with zero thickness.

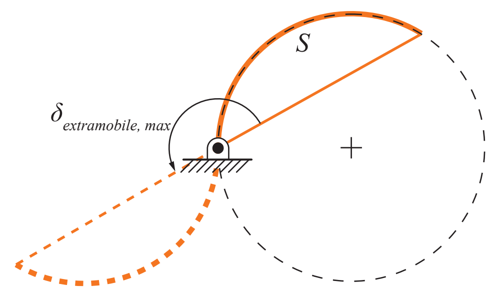


FIGURE 3: The maximum amount of rotation outside the reference surface for a grounded link.

D. All grounded links only extend in one direction past their grounded pivot.
E. The coupler does not extend beyond either of the moving pivots.

Assumptions A and D are necessary for mechanisms to exhibit intramobility or extramobility [8]. Assumption B is a requirement for cylindrical developable mechanisms. Assumptions C and E build on previous work in this area [7, 8] and provide a foundation for mechanisms with thickness and more complex geometries.

3.2 Conditions for Extramobile Motion

This section will detail Conditions 1-3 for both Grashof and non-Grashof cylindrical developable mechanisms. Note that special-case Grashof mechanisms (change-point mechanisms) will be discussed in a later section.

3.2.1 Conditions 1 and 2

For grounded links (i.e. links 2 and 4 in traditional four-bar mechanisms), the maximum exterior rotation for the link (the point at which Condition 1 is violated) can be calculated as described in the equation below and shown in Figure 3, where S is the arc length of the link.

$$\delta_{extramobile,max} = \pi \text{ for } (0 < S \leq \pi R) \tag{1}$$

To violate Condition 2, a grounded link would need to move exterior to the reference surface, then return to its initial position on the reference surface. At this point, a continuation of motion would move the link interior to the reference surface. Hence, the limits of extramobile motion for grounded links are represented by the conformed position and Eq. 1.

3.2.2 Condition 3

To violate Condition 3 prior to Conditions 1 or 2, the convex side of the curved coupler would need to intersect the reference surface prior to the endpoints crossing

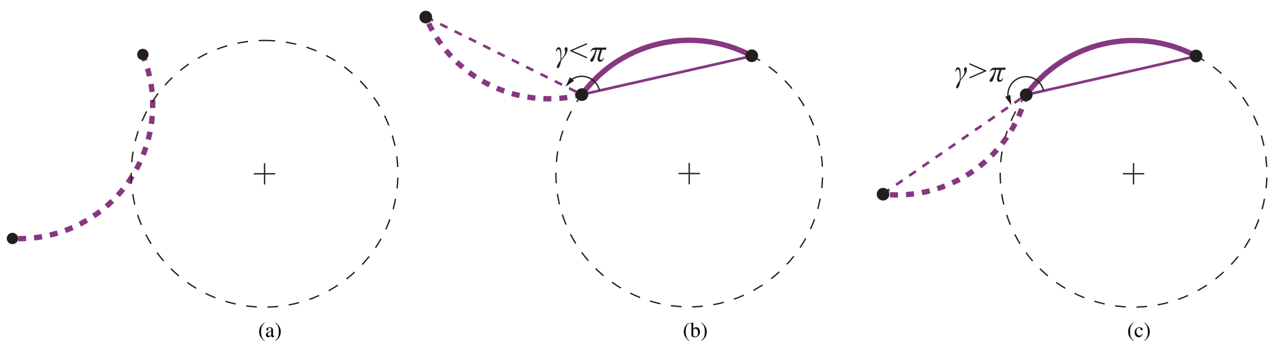


FIGURE 4: (a) The convex side of a coupler, shown in purple, may intersect the reference surface before the endpoints intersect the surface. The rotation of the coupler, γ, can help determine if Condition 3 is violated before Condition 1 or 2, as shown in (b) and (c).

the surface, as shown in Figure 4a. Hence, there are two scenarios that must be necessary for this to happen.

First, the coupler must invert orientation (convex side of arc facing the reference surface), as shown in Figure 4a. This is only possible if the mechanism can reach both its open and crossed configurations in the same circuit (as is the case with double rockers and all non-Grashof mechanisms).

Second, the convex side of the coupler must intersect the reference surface prior to Conditions 1 or 2 being violated. This can be evaluated by analyzing the rotation of the coupler, γ, when Condition 2 is violated. If $\gamma < \pi$, the coupler has not already crossed into the reference surface when Condition 2 is met, as shown in Figure 4b. If $\gamma > \pi$, the coupler has crossed into the reference surface prior to Condition 2 being met, as shown in Figure 4c.

Greenwood [8] classified extramobile and intramobile mechanisms into three classes and demonstrated how only specific types of four-bar mechanisms [14, 15] can be created within each class. These classifications are shown in Figure 5. (While there are several subclasses identified by Greenwood, there is no need to discuss these due to symmetry.) To demonstrate that Condition 3 is not violated prior to Conditions 1 and 2, we will look at the possible motions of the coupler in each of these three classes. Without loss of generality, we will assign θ_1 (angle of the ground link) in each class to equal 0.

Class 1 Under Class 1, and using Barker's classification for planar four-bar linkages [16], it is possible to obtain a GCCC (double crank), GCRR/GRRC (crank rocker), GRCR (double rocker), and RRR2/RRR4 (triple rocker) (excluding change-point mechanisms) [8]. Note that change-points will be discussed in a later section. Only GRCR and RRR2/RRR4 are capable of reaching both their open and crossed configurations, resulting in the convex side of the coupler facing the reference surface as the coupler moves toward the surface. The other mechanism types (GCCC and GCRR/GRRC) cannot invert the coupler and the convex side of the coupler can therefore not contact the reference surface prior to Conditions 1 and 2.

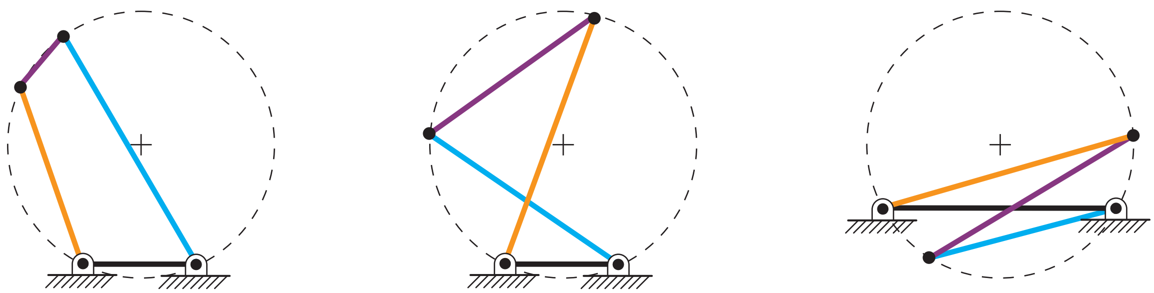
For mechanism types GRCR and RRR2/RRR4, the mechanism can deploy off the surface, toggle to its crossed configuration, and then link 2 can re-conform to the surface, as shown in Figure 6. We will show that when link 2 comes back to the conformed position, the convex side of the coupler has not penetrated the reference surface.

Under Class 1, all joints must reside on the same half of a circle while in the conformed position. This constrains the longest link l to be the link closest to the center of the circle, as shown in Figure 6. It can be seen that the angles adjacent to l, α and ρ, must be less than $\pi/2$. Furthermore, when link 2 re-conforms to the surface (reaches the crossed configuration for the same value of θ_2 at the conformed position) a symmetric polygon is formed by links 3 and 4 in the open and crossed positions.

Because ρ must always be less than $\pi/2$, and because the mirrored polygon is symmetric, the angle opposite ρ is equivalent to ρ and must always be less that $\pi/2$. The angle γ must therefore always be less than π, preventing the coupler curve from moving past its tangent position and into the reference surface. It is then concluded that the defining limits to extramobility for Class 1, given the assumptions above, are set by Conditions 1 and 2.

Class 2 Under Class 2, it is possible to obtain GCCC and GRCR mechanisms (excluding change-point mechanisms). GCCC is unable to change configurations within the same circuit, which means that the convex side of the coupler cannot penetrate the surface before Conditions 1 or 2 occur.

GRCR can reach both open and crossed configurations (and invert the coupler). In this case, the mechanism can deploy off the surface, toggle to its open configuration, and then link 2 can re-conform to the surface, as shown in Figure 7. We will show that



(a) Class 1 mechanism. Possible configurations include GCCC, GRCC/GCCR, GRCR, and RRR2/RRR4 (excluding change-point mechanisms).

(b) Class 2 mechanism. are conformed when in their crossed configuration. Possible configurations include GCCC and GRCR (excluding change-point mechanisms).

(c) Class 3 mechanism. Possible configurations include GRCC/GCCR (excluding change-point mechanisms).

FIGURE 5: The three classes of extra/intramobile mechanisms. The black line represents the ground link and is constrained to have $\theta_1 = 0$. Class 1 mechanisms are conformed in their open configuration while Class 2 and 3 mechanisms are conformed in their crossed configuration. Note that there are 2 subclasses for each class but only 1 is shown here and discussed due to symmetry.

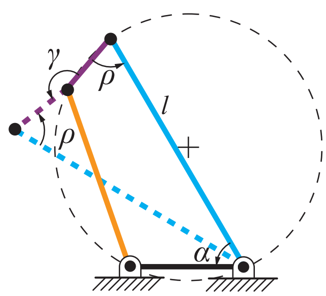


FIGURE 6: Class 1 mechanism in its open (solid, conformed) and crossed (dashed) configuration.

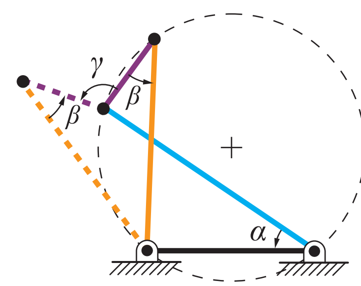


FIGURE 7: Class 2 mechanism in its crossed (solid, conformed) and open (dashed) configuration.

when link 2 comes back to the conformed position, the convex side of the coupler has not penetrated the reference surface.

In Class 2, $\alpha < \pi/2$ (the angle between links 1 and 4) because link 4 must not cross over the center of the circle to maintain extramobility. The angle α subtends the same arc as the angle between links 1 and 3 (β). Therefore $\alpha = \beta$ due to the inscribed angle theorem, which states that any two angles that subtend the same arc on a circle will have the same value. Hence, $\beta < \pi/2$.

When link 2 re-conforms to the surface a symmetric polygon is formed by links 3 and 4 in their open and crossed positions. Following the same logic as Class 1, $\gamma < \pi$. It is then concluded for Class 2 that the defining limits to extramobility, given the assumptions above, are set by Conditions 1 and 2.

Class 3 The only possible mechanism under Class 3 is GCRR (excluding change-point mechanisms). This mechanism type cannot reach both the open and crossed circuits, meaning it cannot toggle the coupler to place the convex side adjacent to the reference surface. Therefore, for Class 3, Conditions 1 and 2 will always occur before the convex side of the coupler comes into contact with the reference surface (Condition 3).

3.3 Conditions for Intramobile Motion

For grounded links, (usually links 2 and 4 in traditional four-bar mechanisms), the maximum interior rotation for the link (the maximum rotation before violating Condition 1) can be calculated as described in the equation below and shown in Figure 8, where S is the arc length of the link.

$$\delta_{intramobile,max} = \pi - \frac{S}{R} \text{ for } (0 < S \leq \pi R) \tag{2}$$

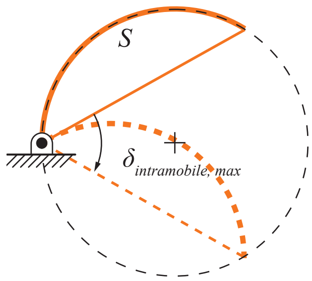


FIGURE 8: The maximum amount of rotation inside the reference surface for a grounded link.

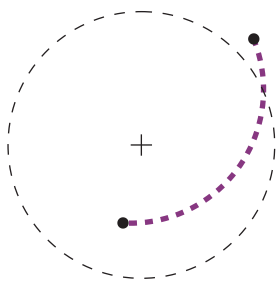


FIGURE 9: The coupler link has the same curvature as the reference surface. The only way for the coupler link to cross from the inside to the outside of the reference surface is if one of the coupler endpoints crosses first.

Condition 2 can be violated if any grounded link moves away from its initial position on the reference surface then moves back to its initial position. At this point, a continuation of motion will move that link exterior to the reference surface.

To violate Condition 3, the convex side of the coupler would need to cross the reference surface. Because each link is shaped to the reference surface (see Assumption B in Section 3), the only way that any point on the coupler link (link 3 in traditional four-bar mechanisms) can cross the reference surface is if one or more of the endpoints has already crossed, as shown in Figure 9. Therefore, the intramobile motion for a regular cylindrical DM is bounded by the motion of links 2 and 4 (Conditions 1 and 2).

4 CHANGE-POINT MECHANISMS

This section is modeled after Section 3, but is specific to change-point mechanisms. Change-point mechanisms exist when

$$s + l = p + q \tag{3}$$

where s is the shortest link, l is the longest link, and p and q are the remaining two links. Because of the unique geometry that exists within a change-point mechanism, there is often crossover between where they exist in terms of the three classes of extramobile and intramobile cylindrical DMs discussed above. This section discusses where each type of change-point mechanism is found in the three classes of extramobile and intramobile mechanisms (summarized in Table 1) and logic on why these mechanisms are still unable to violate Condition 3 prior to Conditions 1 and 2.

The considerations described in previous sections apply for change-point mechanisms pertaining to Conditions 1 and 2 and for intramobility.

4.1 Open Change-Points

Change-point mechanisms that exist in Class 1 must, by definition, exist in an open configuration in the conformed position. As was shown in Figure 6, the longest link l in a Class 1 mechanism lies closest to the center of the circle. Hence, no other link may be of equal length to l, making it impossible to create a CP2X (two pairs of equal length links) or CP3X mechanism (all links have equal length) in a Class 1 mechanism. All other change point mechanisms (CPCCC, CPCRR/CPRRC, and CPRCR) can be created depending on the location of the shortest link relative to ground, as shown in Table 1.

4.2 Crossed Change-Points

Mechanisms in Class 2 and 3 must be in their crossed configuration when conformed. Not all change-points are capable of existing in a crossed configuration while mapped to a circle. According to Josefsson [17], the area of a crossed cyclic quadrilateral (a crossed four-bar mapped to a circle) is found by

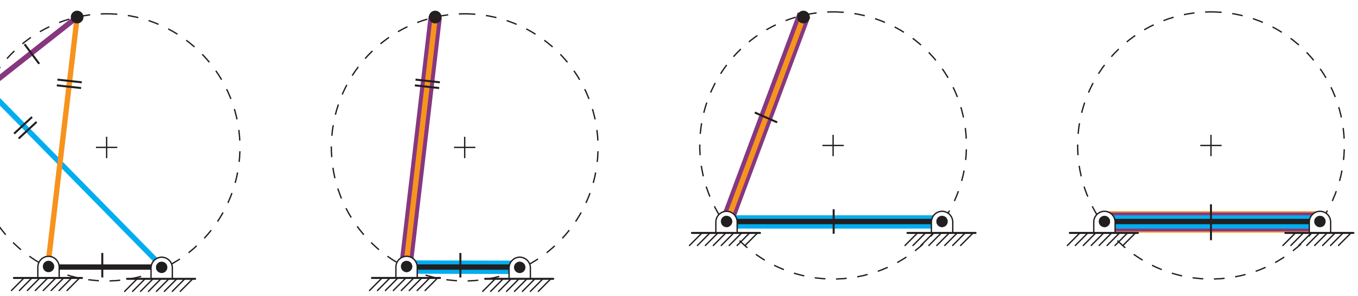
$$K = \frac{1}{4}\sqrt{(P_1)(P_2)(P_3)(P_4)} \tag{4}$$

where

$$P_1 = -a + b + c - d$$
$$P_2 = a - b + c - d$$
$$P_3 = a + b - c - d$$
$$P_4 = a + b + c + d$$

Terms a, b, c, and d represent the four link lengths of a crossed four-bar, in no particular order. Because each link must have a positive, non-zero length, $P_4 > 0$.

McCarthy showed that for a change-point mechanism, the product $P_1P_2P_3$ always equals 0 [18]. A combination of Equation 4 and McCarthy's result suggests that a crossed change-point



(a) CP2X where no links lie in the same position.

(b) CP2X where links of the same length lie in the same position.

(c) CP3X where pairs of links lie in the same position and all links are the same length.

(d) CP3X where all links lie in the same position and all links are the same length.

FIGURE 10: Class 2 change-point mechanisms in their conformed positions. Hatch marks indicate equal lengths.

may exist on a circle only if the links are all colinear in the conformed position (the circle has infinite radius). However, Hyatt showed a case where a circle may have a non-infinite radius and still contain a crossed change-point mechanism at the conformed position [19]. This is only possible if at least two of the links are the same length. Therefore, the only way to obtain a change-point mechanism in Class 2 or 3 is through a CP2X or CP3X mechanism, as shown in Table 1.

4.3 Condition 3 for Extramobile Motion

Class 1 CPCCC, CPCRR/CPRRC, and CPRCR change-point mechanisms can be created depending on the location of the shortest link relative to ground. CPCCC and CPRCR mechanisms are unable to invert their coupler and therefore cannot violate Condition 3 before Condition 1 or 2 is violated, as is discussed in Section 3.2.2. CPCRR/CPRRC mechanisms can invert their coupler to place its convex side adjacent to the reference surface. In each case, $\gamma < \pi$, which shows that Condition 3 has not been violated before Condition 1 or 2. Therefore, for Class 1 mechanisms, Condition 1 or 2 will be reached prior to Condition 3.

Class 2 Possible iterations of Class 2 change-point mechanisms are shown in Figure 10. The mechanism in Figure 10a follows the same logic as other crossed mechanisms in Class 2. If the two crossed links cross through the center of the reference surface, the toggled coupler remains tangent to the reference surface through all motion in the open configuration, meaning the coupler at no time crosses the reference surface.

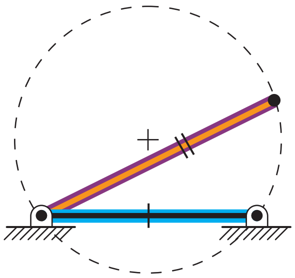
To invert the coupler, the mechanisms in Figures 10b and 10c must rotate links 2 and 3 to be co-linear with links 3 and 4 before it may reach its open configuration. Once all links are co-linear, the mechanism may only move away from the reference surface without violating Condition 1 or 2. Condition 1 is also violated prior to the convex side of the coupler contacting the reference surface.

The mechanism in Figure 10d must rotate a grounded link with the coupler π radians to reach the change-point position. Any further motion would violate Condition 1. If the mechanism moves into its other circuit and away from the reference surface, the convex side of the coupler remains tangent to the reference surface until the grounded links have rotated π radians, at which point Condition 2 would be violated.

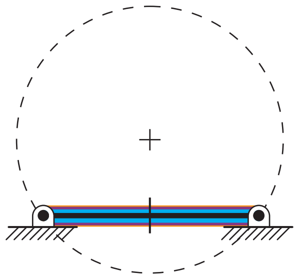
These results lead to the conclusion that all Class 2 change-point mechanisms violate Conditions 1 or 2 prior to Condition 3 given the above assumptions.

TABLE 1: Possible change point mechanisms that exhibit only extramobile and intramobile behaviors.

		CPCRR/CPRRC	CPCCC	CPRCR	CP2X	CP3X
Open change-points	Class 1	✓	✓	✓	✗	✗
Crossed change-points	Class 2	✗	✗	✗	✓	✓
	Class 3	✗	✗	✗	✓	✓



(a) CP2X where links of the same length lie in the same position.



(b) CP3X where all links lie in the same position.

FIGURE 11: Class 3 change-point mechanisms in their conformed positions. Hatch marks indicate equal lengths.

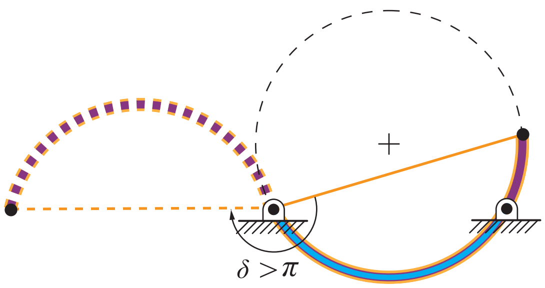


FIGURE 12: Links 2 and 3 of a Class 3 CP2X mechanism must rotate more than π radians to reach the change-point position. Condition 1 is always the limiting case for extramobile motion.

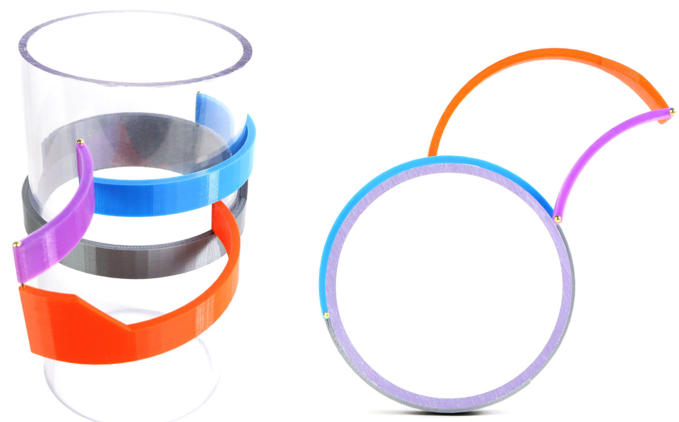
Class 3 Possible iterations of Class 3 change-point mechanisms are shown in Figure 11. For a Class 3 mechanism, one of the grounded links (link 2 in Figure 5c) must be $\geq$ all other links. Therefore, a CP2X mechanism is obtained when link 2 is the same length as link 3, and link 4 is the same length as link 1. Additionally, each pair of links that are of equivalent length must lie at the same location in the conformed position.

To invert the coupler (placing the convex side adjacent to the reference surface), links 2 and 3 must rotate to the change-point position. Figure 12 demonstrates how links 2 and 3 must make a rotation greater than π radians to reach the change-point position, violating Condition 1 prior to the mechanism being able to move back toward the reference surface.

The CP3X mechanism is obtained when all links are the same length. This is the identical case shown in Figure 10d in Class 2. The convex side of the coupler cannot reach the reference surface as the coupler remains parallel with the grounded link throughout its motion.

These results lead to the observation that, for extramobile motion, all Class 3 change-point mechanisms violate Conditions 1 or 2 prior to Condition 3 given the asserted assumptions.

(a) Conformed, crossed position.



(b) Open position (with the blue link returned to the reference surface).

FIGURE 13: A Class 2 mechanism demonstrating the coupler does not cross the reference surface prior to a grounded link moving back to the conformed position.

5 PHYSICAL DEMONSTRATION

Figure 13 shows a sample mechanism conformed to a reference surface. The mechanism moves from a crossed, conformed position (Figure 13a) into an open position (Figure 13b). The blue grounded link in the image is the link that limits the extramobile motion as it moves away from the surface then back to its initial position (Condition 2). While the convex side of the coupler does invert to place it adjacent to the reference surface, it is unable to rotate sufficiently to cross the reference surface prior to the blue grounded link violating Condition 2.

6 CONCLUSION

The ability of a developable mechanism to have large motions exterior or interior to the reference surface provides a powerful way to create multifunctional mechanisms. The observation that Condition 3 is never violated prior to the other two conditions provides great flexibility to designers creating cylindrical DMs. As long as the assumptions above are met, a designer may create a developable mechanism and only take consideration of the grounded links to determine the limits of extramobile and intramobile motion.

A next step to build on this work would be to relax certain constraints, including the constraint that the coupler must remain between the moving pivots. This would increase the functionality of the DM but require further analysis to determine the limits of mobility. Since linkages will need to physically have some appreciable thickness, removing the constraint of zero-thickness would also improve functionality. This will require new terminology and approaches to embed DMs within a material rather than only on a surface.

ACKNOWLEDGMENT

This work was supported by the U.S. National Science Foundation through NSF grant no. 1663345 and the Utah NASA Space Grant Consortium.

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