Proceedings of the ASME 2020
International Design Engineering Technical Conferences
and Computers and Information in Engineering Conference
IDETC/CIE2020
August 17-19, 2020, Virtual, Online

DETC2020-22445

DESIGNING DEVELOPABLE MECHANISMS FROM FLAT PATTERNS

Lance P. Hyatt
Department of Mechanical Engineering
Brigham Young University
Provo, Utah 84602
Email: lance_hyatt@byu.edu

Amanda Lytle
Department of Mechanical Engineering
Brigham Young University
Provo, Utah 84602
Email: mandymarielytle@gmail.com

Spencer P. Magleby
Department of Mechanical Engineering
Brigham Young University
Provo, Utah 84602
Email: magleby@byu.edu

Larry L. Howell
Department of Mechanical Engineering
Brigham Young University
Provo, Utah 84602
Email: lhowell@byu.edu

ABSTRACT

This paper presents tools and methods to design cylindrical and conical developable mechanisms from flat, planar patterns. Equations are presented that relate the link lengths and link angles of planar and spherical mechanisms to the dimensions in a flat configuration. These flat patterns can then be formed into curved, developable mechanisms. Guidelines are established to determine if a mechanism described by a flat pattern can exhibit intramobile or extramobile behavior. A developable mechanism can only potentially exhibit intramobile or extramobile behavior if none of the links extend beyond half of the flat pattern. The behavior of a mechanism can change depending on the location of the cut of the flat pattern. Different joint designs are discussed including lamina emergent torsional (LET) joints. Physical examples are presented.

1 INTRODUCTION

Developable surfaces are used in a variety of engineering applications. Several unique properties make them ideal for use in architecture [1, 2], aerospace [3], medical devices [4, 5], and industrial design [6]. When limited to three dimensions, developable surfaces are ruled surfaces that can be formed by transforming a plane without stretching or tearing the surface [7]. In engineering applications, a flat, planar pattern can be created and then formed to achieve the final developable surface. There are many different planar manufacturing methods to cut out complex patterns in a variety of materials that would be difficult on a curved surface. These methods have allowed designers to build three dimensional structures, mechanisms, and robotics from planar patterns [8, 9, 10, 11, 12].

Developable mechanisms are mechanisms that can conform to or are embedded in developable surfaces [13]. By conforming to a surface, a mechanism can be stored in a compact configuration and be deployed when needed. Developable mechanisms offer significant advantages in applications where volume and size constraints are important such as space travel and medical applications. Previous work has outlined general design guidelines for mechanisms on cylindrical and conical surfaces [14, 15]. They outline how to map different mechanisms to a developable surface and characterize the potential motion of the mechanism with respect to the surface. However, most of these methods and tools are independent of any manufacturing considerations.

If a mechanism is embedded into a planar material, it is referred to as a lamina emergent mechanism (LEM). Several authors have explored both planar and spherical LEMs [16, 17].

While these mechanisms have many uses and applications, only certain types of mechanisms can be mapped onto a plane, and if the plane is transformed in any way, the mechanism behavior changes as well.

This paper presents several methods and tools to design developable mechanisms in a planar, undeveloped surface that can be formed into cylindrical and conical surfaces. First, background information is given on developable mechanisms, along with lamina emergent mechanisms. Tools are then presented to create flat patterns of both cylindrical and conical developable mechanisms. A method for classifying flat developable mechanism patterns is introduced to help identify the potential behavior of a mechanism. Several options for different joint designs are discussed. Finally, physical examples are shown in different materials.

2 BACKGROUND

2.1 Developable Mechanisms

By aligning the joint axes of a mechanism with the ruling lines of a developable surface, the link shapes can be chosen to match the rest of the surface. While embedded in the surface, the mechanism is said to be in its conformed position.

The three non-trivial types of developable surfaces in three dimensions are generalized cylinders whose ruling lines are parallel, generalized cones whose ruling lines converge to a point, and tangent developed surfaces whose ruling lines are tangent to a spacial curve. By aligning revolute joints with the ruling lines, different types of mechanisms can be mapped to the different types of surfaces. Planar mechanisms can be mapped to cylindrical surfaces, spherical mechanisms to conical surfaces, and some spatial mechanisms to tangent developed surfaces.

As developable mechanisms are contained within or on a reference surface, it is useful to be able to analyze the motion of the mechanism with respect to the reference surface. Greenwood [14] introduced the terms extramobile, intramobile, and transmobile to describe the behavior of developable mechanisms based on the position of the links and joints on the surface. If a mechanism exhibits extramobile behavior, all of the links can move exclusively outside of the reference surface. Intramobile behavior means that all parts can move into the surface, and transmobile behavior means that parts of the mechanism will be outside and inside the reference surface during motion. He provided two methods (Instant Center Reference Line (ICRL) method and shadow method) to determine what behaviors are possible for a given cylindrical developable mechanism. These methods have also been applied to conical developable mechanisms [15].

Developable mechanisms on both regular cylinders and circular cones can be represented by a circular cross-section as seen in Figs. 1 and 2. The mechanism can then be described by the angles between the joints on that cross-section. For planar mechanisms, traditional kinematic analysis is done using the distance between joints [18]. The relationship between the link angle of the cylindrical developable mechanism ϕ_i and the link length l_i is

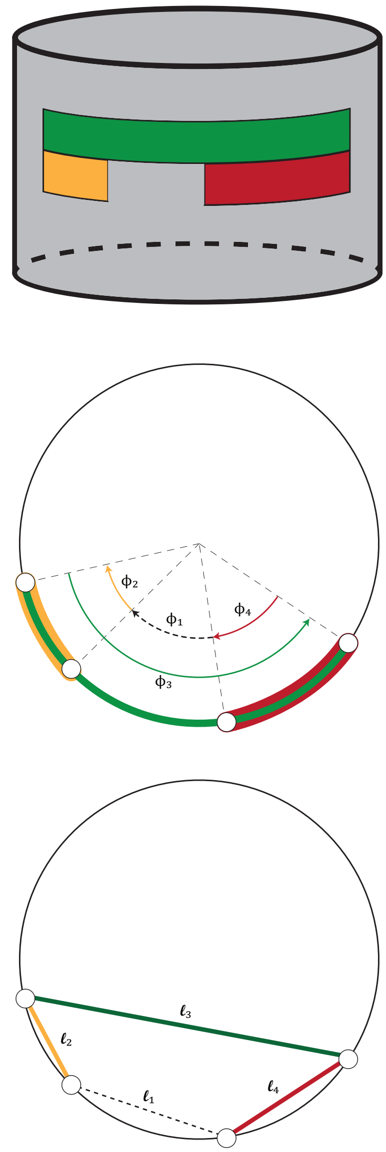

FIGURE 1: A cylindrical developable mechanism can be represented with a circular cross-section. The link angles (ϕ_i) and the skeleton diagram with the link lengths (l_i) are labelled.

$$l_1 = 2R\sin\frac{\phi_i}{2} \tag{1}$$

where R is the radius of the reference circle.

While the links of a developable mechanism conform to the reference surface it is often useful to view the skeleton diagram

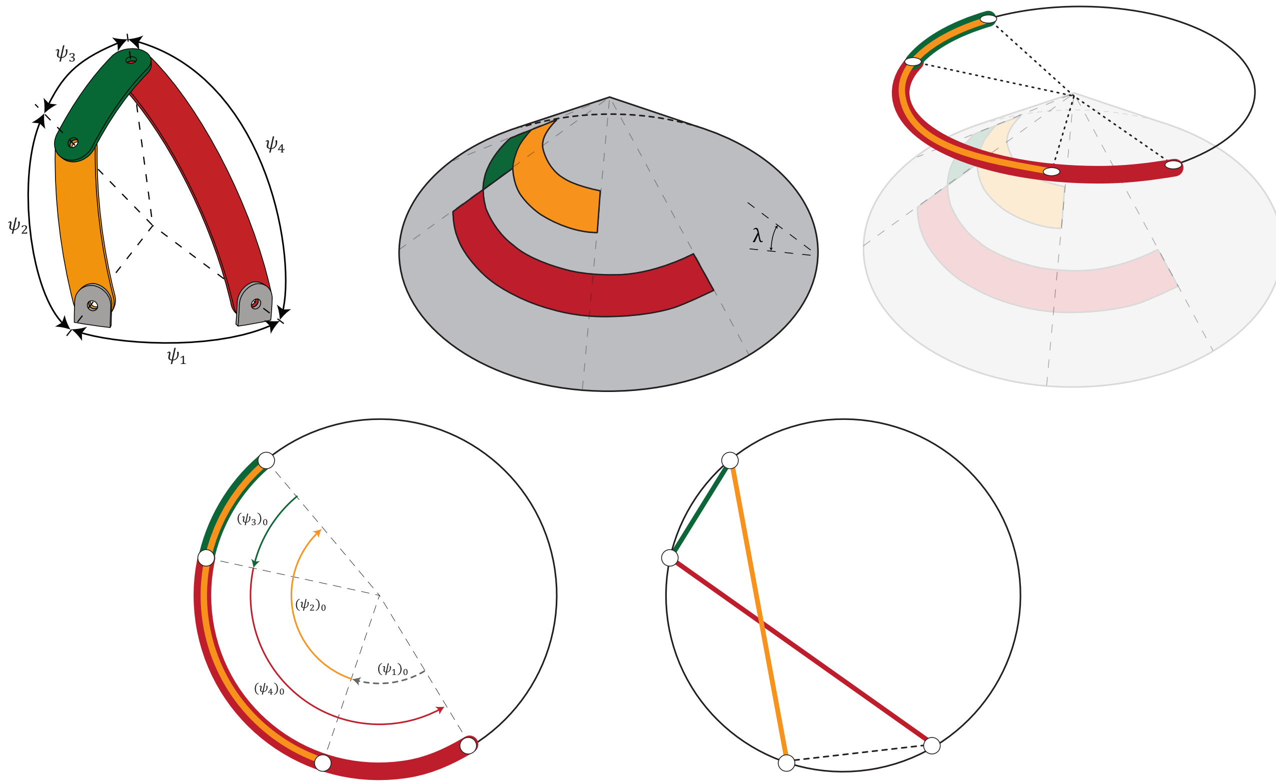

FIGURE 2: A spherical mechanism can be mapped onto a conical surface by aligning the joint axes and ruling lines. The spherical link angles (ψ_i) and projected angles ($(\psi_i)_0$) are labelled along with the cone angle (λ).

to understand the basic kinematic motion of the joints. A mechanism can exist in either an open or crossed configuration, and the skeleton diagram helps identify the configuration of a developable mechanism.

Conical developable mechanisms are spherical mechanisms, and analysis is typically done using the angle between joint axes [19]. When these joint axes are projected onto a circular cross-section of the conical reference surface, they resemble a cylindrical developable mechanism. This paper will use the convention used in [15] where a projected angle is indicated by a subscript of zero. An example of a spherical mechanism mapped onto a conical surface is shown in Fig. 2. The joint axes are aligned with the ruling lines of the conical surface and the links are shaped to conform with the surface. To find the projected angles, the axes are projected onto a plane perpendicular to the cone axis. The relationship between a spherical link angle ψ_i and the projected angle $(\psi_i)_0$ is

$$\cos\psi_i = \sin^2(\lambda) + \cos^2(\lambda)\cos((\psi_i)_0) \tag{2}$$

where λ is the cone angle measured from the base of the cone up to the surface.

Both cylindrical link angles and conical projected angles can have a positive or negative value depending on the direction of rotation about the center axis. The values of these angles are defined as the minimum angle between the joint locations. Therefore, both link angles and projected angles are between -180° and 180° and non-zero. This paper will use the convention that a counter-clockwise rotation is positive. The sum of these link angles or projected angles will always equal an integer multiple of 360° shown below for an n-bar mechanism

$$\phi_1 + \phi_2 + ... + \phi_n = 360k \tag{3}$$

where k is an integer.

It is important to note that while the link angles and projected angles are defined as the minimum angle between joint locations, the physical link may extend past the joint axes on the surface. This will not change the motion of the mechanism, as kinematic behavior is only determined by the relative location of the joints. However, this can affect the mobility of a mechanism with respect to the reference surface. A general rule for a mechanism to exhibit intramobile or extramobile behavior is that a link cannot wrap around more than half of the reference surface. Examples of cylindrical developable mechanisms whose links extend past the joint locations are found in [14].

2.2 Lamina Emergent Mechanisms

Lamina emergent mechanisms (LEMs) are mechanisms that can be made into a planar material [20]. When actuated, the mechanisms is deployed from the surface, but can conform when not in use. One of the biggest advantages of LEMs is being able to use planar manufacturing methods such as laser cutting, water jet cutting, stamping, along with single-layer fabrication methods like those used in microelectromechanical systems (MEMS) [21, 22, 23]. Depending on the orientation of the joint axes, both planar and spherical LEMs can be created [17, 24]. As all of the links and joints of a LEM are co-planar, the mechanism is considered a change-point or folding mechanism [25].

Lamina emergent mechanisms are compliant mechanisms and therefore achieve some or all of their motion through the deflection of flexible members [26]. Several compliant joints have been presented as options to allow for approximations for the motion different rigid joints such as revolute, spherical, spatial, and prismatic joints [27, 28, 29]. A common joint used in LEMs are lamina emergent torsional (LET) joints [30, 31, 32, 33]. LET joints can be manufactured in a plane and allow for large out-of-plane motion and angular deflection. Several different designs have been introduced to mitigate adverse effects of tension or compression during motion [16]. While these investigations focused primarily on LEMs made from plastics, work has been done to design LEMs in other materials like sheet metal [34].

Nelson [35] presented a method to use arrays of LET joints to enable the formation of deployable mechanisms and structures. This was done by aligning the LET joints with the ruling lines of a surface to get an approximation of a curved surface when deflected. Zimmerman and Seymour presented some models and discussion of LET joints on a cylindrical surface [36, 37]

3 FLAT PATTERNS OF DEVELOPABLE MECHANISMS

To create developable mechanisms from flat sheets of material, a two dimensional pattern can be created in a plane, and then the sheet can be formed into a curved developable surface without stretching or tearing. This work outlines how to design cylindrical and conical developable mechanisms using an initially flat pattern. In both cases the process is the same. If a developable surface is “cut” along one of the ruling lines it can be flattened onto a plane. For a cylinder, this results in a rectangle, and a cone becomes a segment of a circle as shown in Fig. 3. The ruling lines on the flat cylinder are all parallel, and the ruling lines on the flat cone converge at the center of the circle. Compliant joints are aligned with the appropriate ruling lines, and links are designed to connect the joint axes. Finally, the sheet can be formed into a developable surface to create the developable mechanism with the desired behavior.

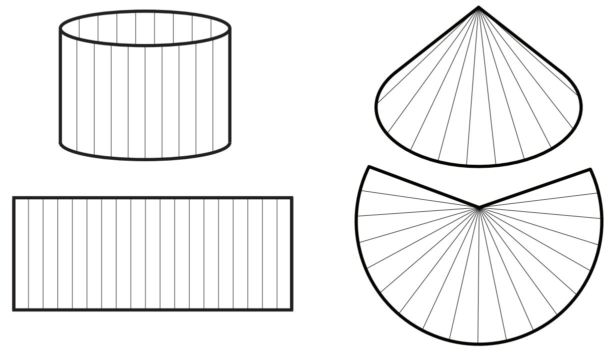

FIGURE 3: An example of a cylindrical and conical surface in their flat and formed states.

The location of the cut does not affect the kinematic behavior of the final mechanism, so the location can be arbitrary. However, it is typically chosen at a location that does not cut a developable link. This will minimize the size of the link in the flat pattern. For certain developable mechanisms this is inevitable, and the following sections introduce a way to design for those cases.

3.1 Cylindrical Developable Mechanisms

As previously stated, the flat pattern of a cylindrical surface can be represented by a rectangle. The length of the rectangle L is the circumference of the cylindrical surface with radius, R.

The size of a link l_i on a cylindrical surface is the arc length between the joint axes a_i given by Eqn. 4.

$$a_i = \phi_i R \tag{4}$$

where ϕ_i is in radians

As the link angles can be either positive or negative, the value a_i can be represented by a vector pointing either right (positive) or left (negative). The arc length calculated will be referred to as the link vectors and will always be perpendicular to the ruling lines of the reference surface. This leads to a similar result as Eqn. 3, where the sum of all of the link vectors will be an integer multiple of L shown here

$$a_1 + a_2 + \ldots + a_n = Lk \tag{5}$$

where k is an integer.

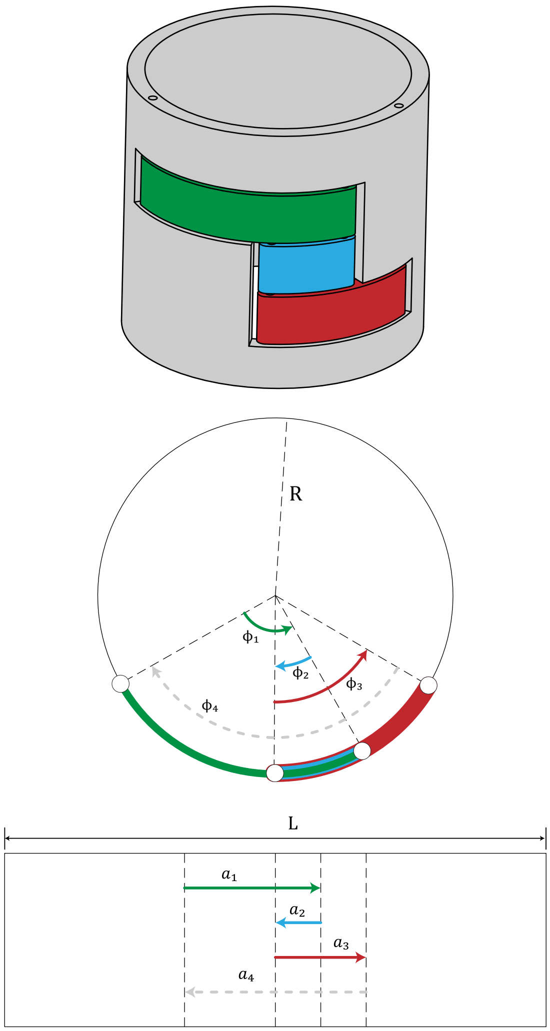

FIGURE 4: A four-bar cylindrical developable mechanism is shown with its corresponding skeleton diagram and flat pattern with link vectors labelled.

Figure 4 shows an example of a four-bar cylindrical developable mechanism and the corresponding flat pattern. In this case, the sum of the link vectors equals zero.

If the sum of the link vectors in not zero, another step must be taken to create a flat pattern. When viewing the circular cross-section, the link angles can rotate completely about the center and still create a closed loop. However, when a surface is cut along a ruling line, both edges of the flat pattern correspond to the same ruling line of the formed surface. Any links that cross or meet at this ruling line must exit one edge of the flat pattern and continue on the other side. One way to account for this discontinuity is by joining the link segments once the surface is formed. However, another option is to reverse the vector direction. The new magnitude of the link vector is $L-|a_i|$. This ensures that the

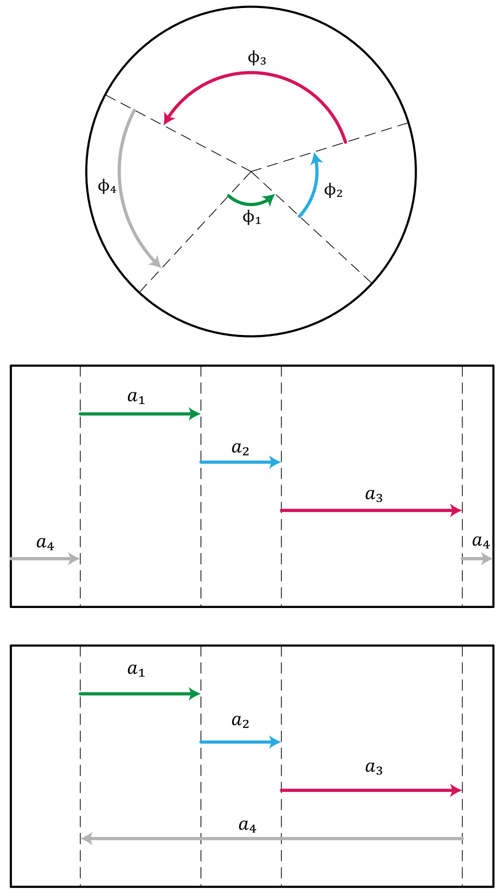

FIGURE 5: The links in the skeleton diagram would wrap around the surface which is not possible in the flat pattern. The link vector a_4 is reversed to connect the appropriate ruling lines.

sum of the link vectors of all developable mechanism flat patterns will equal zero. This allows the mechanism to be complete when the developable surface is formed without any additional assembly. An example of this process is shown by Fig. 5.

When a link vector reversal occurs, the joint axes do not move, so the kinematic behavior of the mechanism remains the same. However, depending on which link vector is reversed, the motion of the mechanism with respect to the reference surface can change. The description of these effects is discussed in Section 4.1.

3.2 Conical Developable Mechanisms

Flat patterns for conical developable mechanisms can be created in a similar manner as flat patterns for cylindrical developable mechanisms. When a circular conical surface with a cone angle λ is cut along a ruling line and flattened, the resulting pattern is a segment of a circle with an inner angle Θ. The relationship between these values is

$$\Theta = 2\pi * \cos(\lambda) \tag{6}$$

While a link on a cylindrical flat pattern is represented by a straight vector, the link on a conical flat pattern is represented by an arc rotated about the center of the flat circle. The links on a flat conical pattern will be referred to as link arc angles. The relationship between a projected angle $(\psi_i)_0$ and the arc angle α_i is

$$\alpha_i = (\psi_i)_0 * \cos(\lambda) \tag{7}$$

Just as link vectors have a direction based on the sign of the cylindrical link angles, arc angles have a direction based on the sign of the conical projected angles. A positive value indicates a counter-clockwise rotation about the center of the circle segment. Figure 6 shows an example of a four-bar conical developable mechanism and its corresponding flat pattern. It is important to note that while the distance from the center of the circle segment can change, the link arc angles will remain constant meaning that the kinematic behavior of the mechanism will stay the same.

Arc angles of flat conical patterns are directly analogous to the link vectors of flat cylindrical patterns. The sum of the arc angles of any developable mechanism will equal an integer multiple of Θ. If this sum is non-zero, a reversal of direction can be applied to an arc angle. The magnitude of the reversed arc angle will be $\Theta - |\alpha_i|$. This process is identical to the link vector reversal of cylindrical flat patterns.

4 CLASSIFICATION OF FLAT PATTERNS

This section will present a classification scheme to describe flat patterns created from a developable mechanism. The terms and definitions apply to both cylindrical and conical developable mechanisms. For simplicity, this section will present the classification in terms of cylindrical flat patterns. However, all of the statements can be applied to conical flat patterns by substituting any reference to the pattern length L with the angle of the circular segment Θ, and link vectors a_i with arc angles α_i.

For a four-bar cylindrical mechanism with link angles ϕ_1, ϕ_2, ϕ_3, and ϕ_4, the corresponding link vectors are a_1, a_2, a_3, and a_4, where a_4 is the ground link. As these angles can be positive or negative, there are 16 possible combinations of link vectors, eight of which are opposites of the others. For the following description, only the eight cases where a_1 is positive will be examined. In every case, the sum of the four link vectors is equal

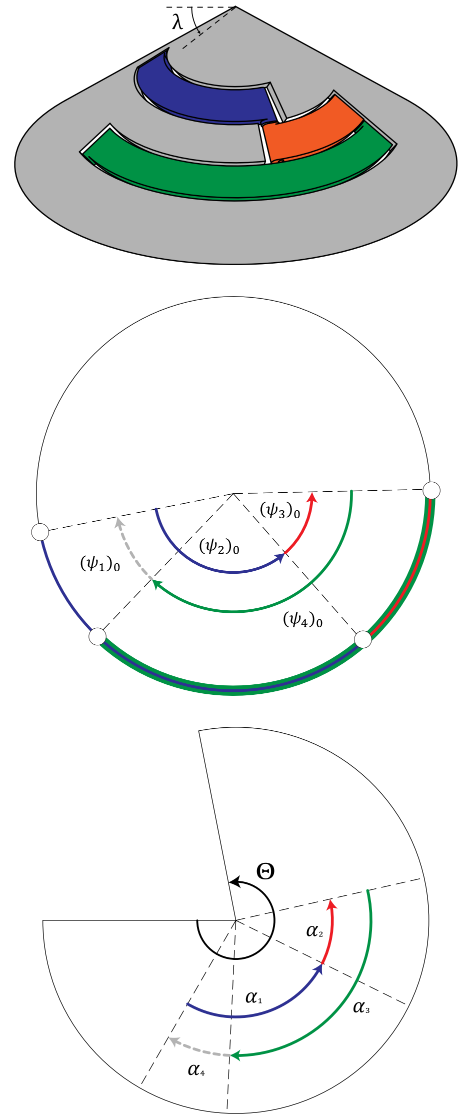

FIGURE 6: A conical developable mechanism with its circular diagram with projected angles ($(\psi_1)_0, (\psi_2)_0, (\psi_3)_0, (\psi_4)_0$) and flat pattern with the arc angles $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ labelled.

to a multiple of the length of the flat pattern as outlined in Eqn. 5. By using the link vector reversal process described in section 3.1, every cylindrical developable mechanism can be modeled by a flat pattern where the sum of the link vectors equals zero. If the sum of the vectors is equal to zero, it means that it is impossible to have all positive link angles $(++++)$, so the total number of possible unique cases is seven. The seven mechanisms are outlined in Table 1. These seven cases can be divided into two classes based on the sign of the product of the four link vectors. The product of the signs can determine if the formed mechanism will be in a crossed or open configuration. If the product is negative, the formed mechanism will be in an open circuit, and a positive product indicates a crossed circuit. Previous work has shown that if a developable mechanism in its conformed position is crossed, it must be a Grashof mechanism [38]. This means that if the product of the link vectors is positive, the formed developable mechanism is guaranteed to be a Grashof mechanism. If the product is negative, further calculation is needed to determine the Grashof condition of the mechanism.

4.1 Intramobility and Extramobility

To determine if a developable mechanism can exhibit intramobile or extramobile behavior, Greenwood introduced the Instant Center Reference Line method and the shadow method using the circular cross-section of a cylindrical developable mechanism [14]. These same methods can be applied to conical developable mechanisms [15]. Both of these methods are useful for analyzing the behavior of a mechanism when in the final, formed position. This section discusses ways to determine the possible behavior of the mechanisms defined by flat patterns.

If all of the links of a flat pattern do not extend past the joint axes, the general rule for designing intramobile and extramobile mechanisms is that no link should have a link angle or projected angle greater than 180°. If the link wraps around more than half of a cylindrical or conical surface, it can only exhibit transmobile behavior. This rule can be extended to the link vectors and arc angles on a flat pattern. If a link vector has a magnitude greater than $L/2$ or an arc angle has a magnitude greater than $\Theta/2$ as can be the case after a link vector reversal, the mechanism will exhibit exclusively transmobile behavior.

Knowing the configuration of the circuit of a mechanism can also help determine if a mechanism can have intramobile or extramobile behavior. Considering four-bar mechanisms, if the joint axes all lie on one half of the flat pattern, all seven mechanism types listed in Table 1 can potentially exhibit intramobile or extramobile behavior. To determine the mobility of a specific mechanism that lies completely on one side, further analysis is necessary, and depends on the position of the ground link. Previous work has explored the potential mobility of a mechanism based on the joint locations on the developable surface [38]. When considering flat patterns, if the joint axes cannot fit on one half of the flat pattern, the four open mechanism types can only exhibit transmobile behavior. As only one of the link vectors or arc angles goes in the opposite direction as the other three, it will always be greater than half the flat pattern. Only the crossed mechanisms have the potential to exhibit intramobile and extramobile behavior if the joint axes cannot fit on one half of the surface.

TABLE 1: Determination of the circuit configuration based on the projected angles.

Product	Circuit	Grashof Condition	Combination
$a_1a_2a_3a_4 < 0$	*Open*	*Undetermined*	+ + + -
			+ + - +
			+ - + +
			+ - - -
$a_1a_2a_3a_4 > 0$	*Crossed*	*Grashof*	+ - + -
			+ - - +
			+ + - -

5 FABRICATION AND PHYSICAL EXAMPLES

The principles presented in this paper were applied in the creation of several examples of developable mechanisms manufactured from a flat sheet and formed into a developable surface. In each case, LET joints or LET arrays were used as compliant joints to achieve motion. While other options exist such as small-length flexural pivots, curved beams, and membranes [37], LET joints can be made with the same planar processes used to fabricate the flat pattern reducing part count and eliminating the need for assembly. The examples shown here all use LET joints.

One challenge of forming developable mechanisms from a flat pattern is ensuring that the surface and links all have the same curvature. Once cut, the links are only connected to the rest of the surface by the grounded joints. During any forming process, considerations must be made to create a constant curvature throughout the mechanism. The methods presented here include adhering to a sacrificial layer during the forming process, designing small tabs to keep the links connected which can be cut once formed, and using a mold to maintain shape during forming.

Once the flat pattern is created, the planar undeveloped pattern must be formed into the developable surface. For the aluminum prototype shown in Fig. 7, the flat pattern was cut with a water jet, and was bent into shape by a slip roll [39].

The remaining examples of hardware shown in Figs. 8, 9, and 10 were made from different types of plastic. The cylindrical and conical developable mechanisms in Figs. 8 and 9 were made from a polypropylene (PP) Formex sheet, and the conical developable mechanisms in Fig. 10 were made from a polyethylene terephthalate (PET) film.

The flat plastic patterns were cut using a laser cutter. Both materials were formed into their developable state then heat set to keep their shape [40]. The PET patterns with designed with small tabs that could be connected during the forming process and cut once heat set as seen in Fig. 10. The PET was heat set in an oven at 150$°C$ for five minutes. Both the cylindrical and conical flat PP patterns were sandwiched between two molds that

FIGURE 7: The flat pattern of a cylindrical developable mechanism was cut with a water jet in aluminum and formed with a slip roll [39]. During the forming process, the mechanism was attached to a sacrificial layer to ensure the links and surface had the same curvature.

FIGURE 8: A polypropylene cylindrical developable mechanism in its flat and formed states. The pattern was cut with a laser cutter and formed with a cylindrical mold.

had the final desired shape, then set in an oven at 160°C for 30 minutes. An example of a pattern in a mold is shown in Fig. 11.

6 CONCLUSION

This work presented equations and design methods to create developable mechanisms from flat, undeveloped patterns. As developable surfaces can be formed from a plane without tear-

FIGURE 9: Various polypropylene conical developable mechanisms in their flat and formed states. The two dimensional patterns were cut with a laser cutter, and formed in a conical mold.

FIGURE 10: A conical developable mechanism made from PET. The pattern was cut on a laser cutter and formed by heat setting.

ing or stretching, developable mechanisms can be designed in a planar state and then formed to achieve a desired behavior. This allows engineers to use a variety of simple planar manufacturing methods to design these complex curved planar and spherical mechanisms.

Cylindrical developable mechanisms can be represented with a rectangular flat pattern with parallel ruling lines. The links are represented by link vectors that travel perpendicular to the ruling lines. Conical developable mechanisms are represented by a segment of a circle with the ruling lines converging at the center of the circle. The spherical links are represented by arc angles measured about the center. While a developable mechanism may have links that loop completely around a developable surface, when designing them on a flat surface, certain considerations must be made to account for the discontinuity. This paper presents a way to modify any set of link vectors or arc angles to be able to fit on the flat pattern.

This paper presents a way to classify four-bar developable mechanisms based on the direction of the link vectors and arc angles. Seven unique types of mechanisms are outlined, and the criteria for mechanisms in open or crossed configurations. Flat

FIGURE 11: An example of a PP conical developable mechanism in a metal mold used in the forming process.

patterns that indicate a crossed configuration are guaranteed to be Grashof mechanisms. This work also presents rules to determine if a mechanism formed from a flat pattern can have intramobile or extramobile behavior.

Several physical examples are presented. The examples use compliant joints similar to those used in planar LEMs such as LET joints to achieve their motion. Rolling and heat setting processes were used to form the developable mechanisms from their planar pattern. During the formation, some deformation occurs due to a non-zero thickness of the material. While some investigations have been made into the behavior of curved LET joints on a cylindrical surface, future work may include investigations into the conical LET joints and their accuracy compared to planar joints. If LET joints on conical surfaces are designed following the ruling lines, the cross-section of the torsional beams will have a curved, annular shape like the cylindrical LET joints, but also a non-constant cross-section along the ruling line.

ACKNOWLEDGMENT

This material is based on work supported by the National Science Foundation under NSF Grant No. 1663345.

REFERENCES

[1] Gavriil, K., Schiftner, A., and Pottmann, H., 2019. "Optimizing B-spline surfaces for developability and paneling architectural freeform surfaces". *CAD Computer Aided Design, **111***, 6, pp. 29–43.

[2] Glaeser, G., and Gruber, F., 2007. "Developable surfaces in contemporary architecture". *Journal of Mathematics and the Arts, **1***(1), 3, pp. 59–71.

[3] Jha, R., Bodhari, S., Sudhakar, V., and Mahapatra, P., 1989. "A surface modeling paradigm for electromagnetic applications in aerospace structures". In Digest on Antennas and Propagation Society International Symposium, IEEE, pp. 227–230.

[4] Seymour, K., Sheffield, J., Magleby, S. P., and Howell, L. L., 2019. "Cylindrical developable mechanisms for minimally invasive surgical instruments". In Proceedings of the ASME Design Engineering Technical Conference, Vol. 5B-2019, American Society of Mechanical Engineers (ASME).

[5] Lim, J. J., and Erdman, A. G., 2003. "A review of mechanism used in laparoscopic surgical instruments". *Mechanism and Machine Theory, **38***(11), 11, pp. 1133–1147.

[6] Liu, Y. J., Tang, K., Gong, W. Y., and Wu, T. R., 2011. "Industrial design using interpolatory discrete developable surfaces". *CAD Computer Aided Design, **43***(9), 9, pp. 1089–1098.

[7] Ushakov, V., 1999. "Developable surfaces in Euclidean space". *Journal of the Australian Mathematical Society, **66***(03), 6, p. 388.

[8] Sung, C., Demaine, E. D., Demaine, M. L., and Rus, D., 2013. "Joining unfoldings of 3-D surfaces". In Proceedings of the ASME Design Engineering Technical Conference, Vol. 6 B, American Society of Mechanical Engineers.

[9] Sung, C., and Rus, D., 2015. "Foldable joints for foldable robots". *Journal of Mechanisms and Robotics, **7***(2), 5.

[10] Aukes, D. M., Ozcan, , and Wood, R. J., 2014. "Monolithic design and fabrication of a 2-DOF bio-inspired leg transmission". In Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), Vol. 8608 LNAI, Springer Verlag, pp. 1–10.

[11] Koh, J.-s., Aukes, D. M., Araki, B., Pohorecky, S., Mulgaonkar, Y., Tolley, M. T., Kumar, V., Rus, D., and Wood, R. J., 2017. "A Modular Folded Laminate Robot Capable of Multi Modal Locomotion". Springer, Cham, pp. 59–70.

[12] Alfattani, R., and Lusk, C., 2018. "A lamina-emergent frustum using a bistable collapsible compliant mechanism". *Journal of Mechanical Design, Transactions of the ASME, **140***(12), 12.

[13] Nelson, T. G., Zimmerman, T. K., Magleby, S. P., Lang, R. J., and Howell, L. L., 2019. "Developable mechanisms on developable surfaces". *Science Robotics, **4***(27), 2, p. eaau5171.

[14] Greenwood, J. R., Magleby, S. P., and Howell, L. L., 2019. "Developable mechanisms on regular cylindrical surfaces". *Mechanism and Machine Theory, **142***, 12, p. 103584.

[15] Hyatt, L. P., Magleby, S. P., and Howell, L. L., 2020. "Developable mechanisms on right conical surfaces". *Mechanism and Machine Theory, **149***, 7, p. 103813.

[16] Wilding, S. E., Howell, L. L., and Magleby, S. P., 2012. "Introduction of planar compliant joints designed for combined bending and axial loading conditions in lamina emergent mechanisms". *Mechanism and Machine Theory, **56***, 10, pp. 1–15.

[17] Wilding, S. E., Howell, L. L., and Magleby, S. P., 2012. "Spherical lamina emergent mechanisms". In Mechanism and Machine Theory, Vol. 49, Pergamon, pp. 187–197.
[18] Norton, R. L., 2004. *Design of Machinery: an introduction to the synthesis and analysis of mechanisms and machines*. McGraw-Hill Professional.
[19] Chiang, C., 1992. "Spherical kinematics in contrast to planar kinematics". *Mechanism and Machine Theory,* ***27***(3), 5, pp. 243–250.
[20] Jacobsen, J. O., Winder, B. G., Howell, L. L., and Magleby, S. P., 2010. "Lamina emergent mechanisms and their basic elements". *Journal of Mechanisms and Robotics,* ***2***(1), 2, pp. 1–9.
[21] Aukes, D. M., Goldberg, B., Cutkosky, M. R., and Wood, R. J., 2014. "An analytic framework for developing inherently-manufacturable pop-up laminate devices". *Smart Materials and Structures,* ***23***(9), 8, p. 094013.
[22] Whitney, J. P., Sreetharan, P. S., Ma, Y., and Wood, R. J., 2011. "Pop-up book MEMS". *Journal of Micromechanics and Microengineering,* ***21***.
[23] Sreetharan, P. S., Whitney, J. P., Strauss, M. D., and Wood, R. J., 2012. "Monolithic fabrication of millimeter-scale machines". *Journal of Micromechanics and Microengineering,* ***22***.
[24] Lusk, C. P., and Howell, L. L., 2008. "Components, building blocks, and demonstrations of spherical mechanisms in microelectromechanical systems". *Journal of Mechanical Design, Transactions of the ASME,* ***130***(3), 3.
[25] McCarthy, J. M., and Soh, G. S., 2011. *Geometric design of linkages*. Springer.
[26] Howell, L., 2001. *Compliant Mechanisms*. John Wiley & Sons.
[27] Winder, B. G., Magleby, S. P., and Howell, L. L., 2008. "A study of joints suitable for lamina emergent mechanisms". In Proceedings of the ASME Design Engineering Technical Conference, Vol. 2, American Society of Mechanical Engineers Digital Collection, pp. 339–349.
[28] Delimont, I. L., Magleby, S. P., and Howell, L. L., 2015. "Evaluating compliant hinge geometries for origami-inspired mechanisms". *Journal of Mechanisms and Robotics,* ***7***(1), 2.
[29] Zhang, H., Zhang, X., and Zhu, B., 2019. "A novel flexural lamina emergent spatial joint". *Mechanism and Machine Theory,* ***142***, 12, p. 103582.
[30] Jacobsen, J. O., Chen, G., Howell, L. L., and Magleby, S. P., 2009. "Lamina Emergent Torsional (LET) Joint". *Mechanism and Machine Theory,* ***44***(11), 11, pp. 2098–2109.
[31] Xie, Z., Qiu, L., and Yang, D., 2017. "Design and analysis of Outside-Deployed Lamina Emergent Joint (OD-LEJ)". *Mechanism and Machine Theory,* ***114***, 8, pp. 111–124.
[32] Xie, Z., Qiu, L., and Yang, D., 2018. "Design and analysis of a variable stiffness Inside-Deployed Lamina Emergent Joint". *Mechanism and Machine Theory,* ***120***, 2, pp. 166–177.
[33] Liu, K., Shan, C. C., and Cao, Y., 2015. "Theoretical modeling and finite element analysis of planar tensile compliant joint". In 2015 IEEE International Conference on Robotics and Biomimetics, IEEE-ROBIO 2015, Institute of Electrical and Electronics Engineers Inc., pp. 1991–1996.
[34] Ferrell, D. B., Isaac, Y. F., Magleby, S. P., and Howell, L. L., 2011. "Development of criteria for lamina emergent mechanism flexures with specific application to metals". *Journal of Mechanical Design, Transactions of the ASME,* ***133***(3), 3.
[35] Nelson, T. G., Lang, R. J., Pehrson, N. A., Magleby, S. P., and Howell, L. L., 2016. "Facilitating Deployable Mechanisms and Structures Via Developable Lamina Emergent Arrays". *Journal of Mechanisms and Robotics,* ***8***(3), p. 031006.
[36] Zimmerman, T., Butler, J., Frandsen, D., Burrow, D., Fullwood, D., Magleby, S., and Howell, L., 2018. "Modified Material Properties in Curved Panels Through Lamina Emergent Torsional Joints". In 2018 International Conference on Reconfigurable Mechanisms and Robots, ReMAR 2018 - Proceedings, Institute of Electrical and Electronics Engineers Inc.
[37] Seymour, K., Bilancia, P., Magleby, S., and Howell, L., 2020. "Hinges and curved lamina emergent torsional joints in cylindrical developable mechanisms". In Proceedings of the ASME Design Engineering Technical Conference, American Society of Mechanical Engineers (ASME). In review.
[38] Hyatt, L. P., Greenwood, J. R., Butler, J. J., Magleby, S. P., and Howell, L. L., 2020. "Using cyclic quadrilaterals to design cylindrical developable mechanisms". In Proceedings of the 2020 USCToMM Symposium on Mechanical Systems and Robotics, P. Larochelle and J. M. McCarthy, eds., Springer International Publishing, pp. 149–159.
[39] Zimmerman, T., 2018. "A Definition and Demonstration of Developable Mechanisms". *Theses and Dissertations*, 4.
[40] Sargent, B., Brown, N., Jensen, B. D., Magleby, S. P., Pitt, W. G., and Howell, L. L., 2019. "Heat set creases in polyethylene terephthalate (PET) sheets to enable origami-based applications". *Smart Materials and Structures,* ***28***(11), 10, p. 115047.