Circuit-based Inverse Design of Metastructured MIMO Devices

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***Abstract*—In this work, an inverse design method for multi-input multi-output (MIMO) metastructured devices is developed. Large-scale inverse design problems are difficult to solve directly and often require heuristic methods or design optimization to find a solution. Inherent errors introduced by heuristic methods makes design optimization a more promising route to the realization of high performance devices. Here, a fast frequency domain solver for grids of Y-parameter matrices is developed. The solver is used together with an adjoint-based optimization routine to solve inverse metastructured design problems. The design procedure is demonstrated through the realization of a planar beamforming network for a multi-beam antenna.**

***Index Terms*—Adjoint methods, MIMO, metamaterials.**

I. INTRODUCTION

This paper presents an inverse-design method for wideband multi-input multi-output (MIMO) metastructured devices. Previously, wideband MIMO metastructured devices were realized using heuristic methods, [1], and design optimization, [2]. However, the heuristic method is restricted to the design of devices acting as lenses and suffers from phase errors for all but one input/output pair. Using design optimization eliminates these issues but can be computationally expensive especially if the forward problem is solved using full-wave methods as in [2]. Another factor contributing to the computational cost of large-scale optimization problems is the evaluation of the gradient of the objective function at each step. One way to reduce this cost is through the use of adjoint-based methods which, have been shown to greatly reduce computational cost in electromagnetic inverse-design [3]–[5].

Here, a fast frequency domain solver for Y-matrices is developed which allows for the inclusion of a broad range of electromagnetic responses including bianisotropy. The forward problem solver is then used together with a nonlinear constrained optimization routine utilizing the adjoint variable method to compute the gradient. This design platform enables the inverse-design of electrically large devices containing many degrees of freedom. These capabilities are demonstrated through the design of a beamformer for a multi-beam aperture antenna.

II. INVERSE-DESIGN METHOD

The inverse-design method uses a frequency domain solver leveraging the relationship between TE modes in a 2-D region and the modes that exist in a 2-D circuit network, similar to [6]. Properly exploiting this relationship provides reasonable accuracy and significant gains in computational efficiency compared to full-wave methods. The solver is constructed by representing the computational domain as a tiled grid of four-port Y-parameter matrices, representing unit cells, and solving for the currents and voltages in the network. Electromagnetic sources and other boundary conditions are imposed at the edge ports of the computational domain using lumped impedances and voltage sources, shown in Fig. 1. The values of the lumped impedances and voltage sources are determined by the excitation, desired response, and medium that surrounds the structure.

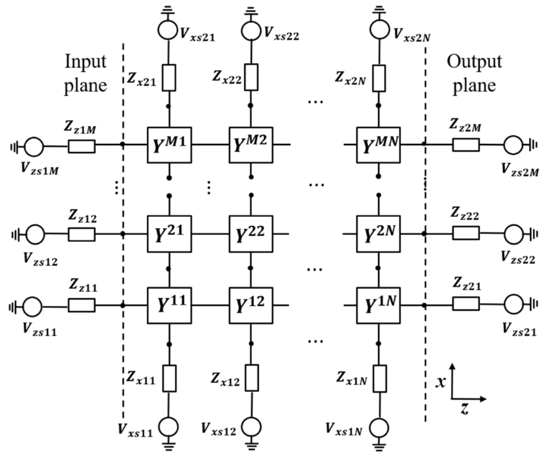

Fig. 1. An MxN grid of Y-parameter matrices representing the solution domain of the frequency domain solver. The voltage response at every node in the network is determined by the elements of the Y-parameter matrices Y^{ij} and the boundary terminations represented by the voltage sources and lumped impedance's in the network. The input and output planes represent the location of the sources and the aperture in the beamformer design.

The frequency-domain solver computes the electromagnetic response, given the Y-parameters of all of the unit cells, subject to a set of boundary terminations and source voltages. However, in a inverse-design problem the electromagnetic response is given and the Y-parameters of the unit cells are unknown. To determine the unit cell Y-parameters that provide the desired MIMO functionality a design optimization problem is formulated as follows,

$$\arg\min_{y} \sum_{k=1}^{K} v_k(y)^T \mathbf{P} v_k(y) \quad (1)$$

for K input/output profiles. Where y is a vector that determines the value of all of the Y-matrices in the design, $v_k(y)$ is a real vector that represents the error in the amplitude and phase of the boundary voltages for the kth input/output profile and **P** is scaling matrix.

This optimization problem is in general non-convex. However, locally optimal solutions that satisfy the design requirements will be considered solutions to the optimization problem. Therefore, gradient-based methods can be used and the problem is solved using MATLAB's nonlinear constrained optimization routine. To enable the optimization of devices with a large number of design variables the adjoint variable method is implemented to calculate the gradient.

III. Design of Multi-beam Antenna Beamformer

In this section, a planar beamforming network for a multi-beam aperture antenna operating at a frequency $f_0 = 10$ GHz is designed. The beamforming network is discretized into 3x3mm ($\lambda_0/10$) unit cells with 61 unit cells in the direction transverse to the aperture and 16 unit cells in the normal direction. There are five input ports separated by 10 unit cells on the input plane and 61 output ports on the output plane, see Fig. 1. The five input profiles to the beamforming network are single port excitations at the five input ports. The output profiles are uniform amplitude with a linear phase gradient corresponding to beam-pointing directions at $\theta_B = 0°, \pm 9.44°, \pm 19.14°$ which, are chosen to maintain orthogonality of the aperture fields. The input port terminations are $Z_{\text{in}} = 50\Omega$ and the terminations at the output ports change for each input to match the scan impedance, $Z_{\text{out}} = 50/\cos\theta_B \Omega$. The extra 56 ports on the input plane and the ports on the transverse boundaries of the beamforming network are terminated in open circuits.

To provide a wideband response a low-pass unit cell was selected for the beamformer. It is an asymmetric microstrip junction patterned on a Rogers RT/Duroid RO5880 substrate with a height of 0.787mm. Each unit cell has six parameters representing lengths and widths of microstrip lines comprising the unit cell. The Y-parameters of the unit cells are calculated by interpolating a database of full-wave simulations performed in Keysight's method of moments solver Momentum.

The design goals for the beamformer are: 1) The desired aperture voltage profile should be produced for each input. 2) The input ports should be impedance matched and isolated from each other. 3) The voltages present at the open-circuited edge terminations should be zero. The third goal ensures that cavity modes are not excited in the beamformer, which could significantly reduce the bandwidth of the device. To find candidate designs, the optimization routine is executed on a desktop computer (CPU: i7-9700 @ 3 GHz, RAM: 64 GB) and is seeded with an initial guess of uniform lengths and widths. After executing for approximately two hours the resulting network has a maximum reflection coefficient, $\Gamma_{\text{max}} = -23$dB, and the lowest isolation between input ports is 35dB. The aperture field has small amplitude and phase errors resulting in minor side-lobe errors, shown in Fig. 2. The radiation pattern is calculated analytically in MATLAB assuming that the aperture

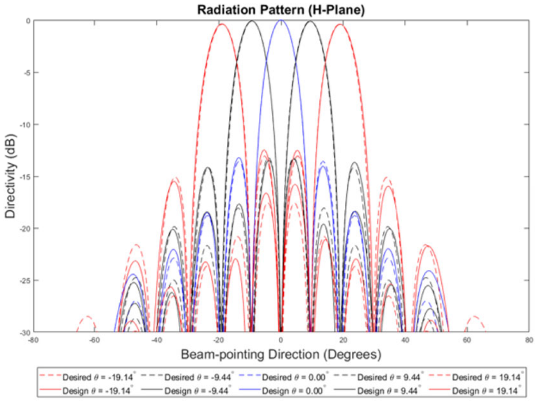

Fig. 2. The ideal H-Plane radiation pattern for a multi-beam aperture antenna using the designed beamformer. The radiation pattern is calculated assuming the aperture is composed of piece-wise uniform amplitude and phase sections that are 3mm in length and modeled as magnetic current sheets. The dashed lines correspond to the radiation pattern resulting from the desired aperture voltage profile and the solid lines correspond to the aperture voltage profile realized in the inverse designed beamforming network.

field has 3mm sections that are piece-wise uniform amplitude and phase. The amplitude and phase at each section is the voltage at the corresponding output port of the beamforming network. Each section is then modeled as a sheet of magnetic current and the total radiation pattern is calculated.

IV. Conclusion

This paper presented an inverse design method for wideband MIMO metastructured devices. The design procedure uses a fast frequency-domain solver for tiled grids of Y-parameter matrices. The fast forward solver is used with an adjoint-based constrained nonlinear optimization routine to synthesize metastructured devices with the desired electromagnetic response. As an example, a planar, antenna beamforming network was designed using the inverse design method.

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