Neutron stars in scalar-tensor theories: Analytic scalar charges and universal relations

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Neutron stars are ideal astrophysical sources to probe general relativity due to their large compactnesses and strong gravitational fields. For example, binary pulsar and gravitational wave observations have placed stringent bounds on certain scalar-tensor theories in which a massless scalar field is coupled to the metric through matter. A remarkable phenomenon of neutron stars in such scalar-tensor theories is spontaneous scalarization, where a normalized scalar charge remains of order unity even if the matter-scalar coupling vanishes asymptotically far from the neutron star. While most works on scalarization of neutron stars focus on numerical analysis, this paper aims to derive accurate scalar charges analytically. To achieve this, we consider a simple energy density profile of the Tolman VII form and work in a weak-field expansion. We solve the modified Tolman-Oppenheimer-Volkoff equations order by order and apply a Padé resummation to account for higher order effects. We find that our analytic scalar charges (in terms of the stellar compactness) beautifully model those computed numerically. We also find a quasiuniversal relation between the scalar charge and stellar binding energy that is insensitive to the underlying equations of state. A comparison of analytic scalar charges for Tolman VII and constant density stars mathematically supports this quasiuniversal relation. The analytic results found here provide physically motivated, ready to use accurate expressions for scalar charges.

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I. INTRODUCTION

Neutron stars (NSs) are ideal compact astrophysical objects to probe fundamental physics. Due to their high central density that exceeds the saturation density of nuclear matter by several fold, NSs can efficiently test nuclear physics. For example, recent observations of 2 $M_\odot$ pulsars [1], x-ray emissions from hot spots on a rotating NS surface by NICER [2,3], and gravitational waves from colliding NSs by LIGO and Virgo [4] have constrained properties of nuclear/quark matter and certain nuclear parameters (see e.g., [5–21]). Due to their large compactnesses, NSs are also perfect sources to probe strong-field gravity. Indeed, observations of binary pulsars [22–30] and the binary neutron star merger GW170817 [31–36] have constrained various modifications to general relativity (GR).

Some of the most well-studied modified theories of gravity are scalar-tensor theories in which scalar fields are introduced (either minimally or nonminimally coupled to the metric) to the action. A simple scalar-tensor theory proposed by Damour and Esposito-Farèse has two theoretical parameters (α_0, β_0) and allows NSs to scalarize spontaneously [37–39]. Namely, when β_0 is sufficiently negative, NSs can have scalar charges of order unity even when $\alpha_0 \ll 1$ (or equivalently, when the value of the scalar field at infinity is small). Such a phenomenon can be understood as NSs undergoing a tachyonic instability, where the effective mass of the scalar field becomes imaginary [40]. A catalog of NS scalar charges is provided in [41] while a surrogate model has recently been constructed in [35] based on numerical calculations. The parameters (α_0, β_0) have been constrained with solar system experiments, various binary pulsars [22,26,27,29] and the binary NS merger event GW170817 [35] to be $|\alpha_0| \lesssim 3 \times 10^{-4}$ and $\beta_0 \gtrsim -4.4$. Future forecasts on probing this scalar-tensor theory with black-hole/NS binaries have been made with pulsar [25] and gravitational-wave [26,35,42] observations.

The goal of this paper is to provide accurate expressions for scalar charges of NSs in the scalar-tensor theory in [38] by solving the field equations analytically. Such expressions are complementary to e.g., the surrogate model [35] mentioned earlier. To achieve this, instead of using realistic equations of state (EoS) given in tables, we adopt the Tolman VII model [43] that approximates the energy density profile inside a NS to a quadratic function in a radial coordinate. Such a simple profile allows one to solve the Tolman-Oppenheimer-Volkoff (TOV) equations analytically in GR. Unfortunately, due to the complication of the modified TOV equations in scalar-tensor theories including the coupling with the scalar field, it is challenging to solve them analytically. To overcome this, we work in a

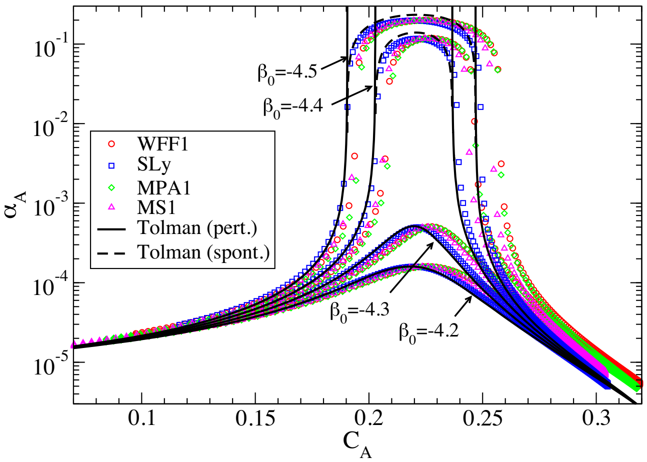

FIG. 1. Comparison of analytic scalar charges with the Tolman VII model against numerical scalar charges with realistic EoSs as a function of the compactness. For the former, we show the results for both perturbative (solid) and spontaneous (dashed) scalarization. We present the results with four different choices of β_0. α_0 is fixed to $\alpha_0 = 10^{-5}$ except for spontaneous scalarization for the Tolman case which assumes $\alpha_0 = 0$. Observe that the analytic Tolman results accurately model numerical results, especially with the SLy EoS.

weak-field approximation and solve the field equations order by order in the expansion.[1] To take in to account higher order contributions in the expansion, we apply a Padé resummation. We consider both spontaneous scalarization (mentioned earlier) and "perturbative" scalarization where the scalar charge is proportional to α_0.

Figure 1 summarizes our findings, which compares the analytic scalar charges with the Tolman VII model in terms of the stellar compactness (the ratio between the mass and radius) against those with realistic EoSs (WFF1 [45], SLy [46], MPA1 [47], MS1 [48]) computed numerically. Notice that the former can accurately model the latter, especially for the SLy EoS, for both the spontaneous and perturbative scalarization cases. We also find that when we plot the scalar charges as a function of the stellar binding energy (the difference between the gravitational and baryonic mass), the relation between these quantities becomes quasiuniversal and is insensitive to EoSs with an error of $\sim 1\%$ (see [49] for other universal relations involving scalar charges in scalar-tensor theories). We estimate analytically the amount of the quasiuniversality by comparing analytic expressions for scalar charges with the Tolman VII model and with constant density stars and find that it is quasiuniversal with an error of 0.3%. The technique developed here should easily be applicable to other theories beyond GR to compute the stellar charges (or sensitivities).

[1]Similar calculations were carried out in quadratic gravity [44] and Einstein-Æ ther theory [30] to respectively find analytic scalar charges and sensitivities, which were then used in tandem with binary pulsar observations to place bounds on these theories.

The rest of the paper is organized as follows. In Sec. II, we review the scalar-tensor theory proposed in [38], explain the modified TOV equations, present the relation between mass and radius, and review spontaneous scalarization following [50]. In Sec. III, we present the formalism for computing scalar charges analytically in the Tolman VII model by combining weak-field expansions and a Padé resummation. We compare such analytic results against numerical ones in Sec. IV and also present the quasiuniversal relation between the scalar charge and the binding energy. We conclude in Sec. V and give several different directions for future work. In Appendix, we repeat the analytic calculation for constant density stars and derive scalar charges similar to the Tolman VII case. We use the geometric units of $c = G = 1$. The main expressions for scalar charges are summarized in a Supplemental Mathematica notebook [51].

II. SCALAR-TENSOR THEORIES AND NEUTRON STARS

We begin by reviewing scalar-tensor theories and spontaneous scalarization of NSs.

A. Theory

Presented here are the action and field equations for scalar-tensor theories in the Einstein frame. The former is given by [37–39]

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi}(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi) + S_{\rm mat}[\psi, A^2(\varphi)g_{\mu\nu}], \tag{2.1}$$

where R is the Ricci scalar for the metric $g_{\mu\nu}$ in this frame, g is its determinant, φ is the scalar field and ψ is the matter field. A is the conformal factor that relates $g_{\mu\nu}$ and the metric $\tilde{g}_{\mu\nu}$ in the (physical) Jordan frame as $\tilde{g}_{\mu\nu} = A(\varphi)g_{\mu\nu}$. Notice that we have set $G = 1$ for the bare gravitational constant G. Varying the above action with respect to $g_{\mu\nu}$ and φ, the field equations are given by [38,39]

$$R_{\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi + 8\pi\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right), \tag{2.2}$$

$$\Box\varphi = -4\pi\alpha(\varphi)T, \tag{2.3}$$

where $T_{\mu\nu}$ is the matter stress-energy tensor and

$$\alpha(\varphi) \equiv \frac{\partial \ln A(\varphi)}{\partial\varphi}. \tag{2.4}$$

In this paper, we consider an example scalar-tensor theory first considered by Damour and Esposito-Farèse [37–39] with

$$A(\varphi) = \exp\left(\frac{\beta_0}{2}\varphi^2\right). \tag{2.5}$$

Another parameter of the theory is $\alpha_0 \equiv \alpha(\varphi_0) = \beta_0\varphi_0$ with φ_0 representing the scalar field at infinity.

B. Neutron stars

To construct a NS solution, we begin with the metric ansatz given by

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{2.6}$$

For the matter sector, we consider a perfect fluid whose stress-energy tensor in the Einstein frame is given by [52]

$$T_{\mu\nu} = A^4(\varphi)[(\tilde{\rho} + \tilde{P})u_\mu u_\nu + g_{\mu\nu}\tilde{P}], \tag{2.7}$$

where u^μ is the four-velocity of the fluid. Notice that $\tilde{\rho}$ and $\tilde{P}$ are the energy density and pressure in the Jordan frame that directly enters in the EoS $\tilde{P}(\tilde{\rho})$. Plugging these into the field equations in Eqs. (2.2) and (2.3), one finds [38,39][2]

$$m' = 4\pi r^2 A^4(\varphi)\tilde{\rho} + \frac{1}{2}r(r-2m)\varphi'^2, \tag{2.8}$$

$$\tilde{P}' = -(\tilde{\rho} + \tilde{P})\left[\frac{m + 4\pi A^4\tilde{P}r^3}{r(r-2m)} + \frac{1}{2}r\varphi'^2 + \alpha(\varphi)\varphi'\right], \tag{2.9}$$

$$\varphi'' = 4\pi\frac{r}{r-2m}A^4(\varphi)[\alpha(\varphi)(\tilde{\rho} - 3\tilde{P}) + r\varphi'(\tilde{\rho} - \tilde{P})] - \frac{2(r-m)}{r(r-2m)}\varphi', \tag{2.10}$$

where a prime represents an r derivative and

$$e^{-\lambda(r)} = 1 - \frac{2m(r)}{r}. \tag{2.11}$$

The asymptotic behavior of the scalar field at infinity is given by

$$\varphi = \varphi_0 - \alpha_A\frac{M_A}{r} + \mathcal{O}\left(\frac{M_A^2}{r^2}\right), \tag{2.12}$$

where

$$\alpha_A = \frac{\partial \ln M_A}{\partial \varphi_0} \tag{2.13}$$

is the scalar charge while M_A is the total gravitational mass that can be read off from the asymptotic behavior of $m(r)$ at infinity as

$$m(r) = M_A + \mathcal{O}\left(\frac{M_A}{r}\right). \tag{2.14}$$

[2]There is also an equation for ν that we do not present here since it is unnecessary for deriving scalar charges.

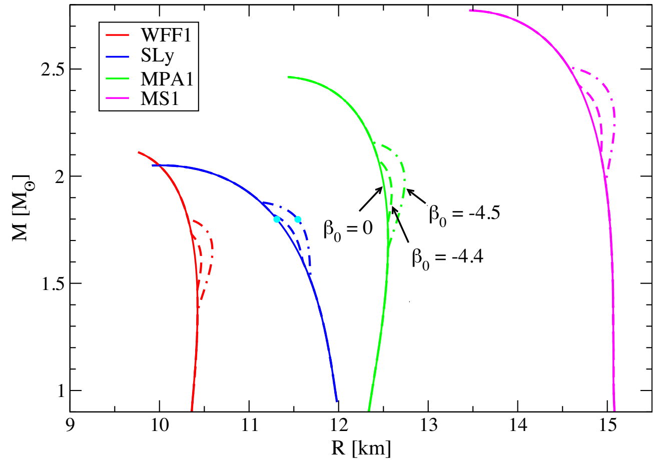

FIG. 2. Mass-radius relation for NSs with four representative EoSs in GR ($\alpha_0 = \beta_0 = 0$) and the scalar-tensor theory ($\alpha_0 = 10^{-5}$ and $\beta_0 = -4.4, -4.5$). The latter deviates from the former due to spontaneous scalarization. Two cyan dots correspond to the NSs of 1.8 $M_\odot$ with SLy EoS whose energy density profiles are shown in Fig. 4.

We can construct NS solutions as follows. First, we impose the initial conditions of

$$m(0) = 0, \quad \tilde{P}(0) = \tilde{P}_c, \quad \varphi(0) = \varphi_c, \quad \varphi'(0) = 0, \tag{2.15}$$

where $\tilde{P}_c$ is the central pressure while φ_c is the central value for the scalar field. Under these initial conditions together with a choice of (α_0, β_0) and an EoS, we solve Eqs. (2.8)–(2.10) in the interior region numerically and the stellar radius R is determined by $\tilde{P}(R) = 0$. We then solve the equations in the exterior region with $\tilde{P} = \tilde{\rho} = 0$. Finally, we read off M_A, φ_0 and α_A by comparing the asymptotic behavior of the numerically-computed φ and m with Eqs. (2.12) and (2.14). Figure 2 presents the mass-radius relation of NSs in GR and the scalar-tensor theory for four representative EoSs (WFF1 [45], SLy [46], MPA1 [47], MS1 [48]) with different stiffness.[3] "Humps" in the relation for the scalar-tensor theory correspond to NSs with spontaneous scalarization.

C. Spontaneous scalarization

An analytic attempt of computing scalar charges with spontaneous scalarization was taken in [50]. First, when α at the center of a star, α_c, is small, one can expand both α_0 and α_A in terms of α_c as[4]

[3]These numerical results are computed with a Mathematica notebook developed in [52].

[4]The scalar field (and its derivative) enter in even powers in Eqs. (2.8) and (2.9), and in odd powers in Eq. (2.10). This means that α_0 enters in odd powers in α_c and α_A. Inverting the former order by order (and substituting it to the latter), one finds that α_0 and α_A enter in odd powers in α_c.

$$\alpha_0 = d_1\alpha_c + d_2\alpha_c^3 + \mathcal{O}(\alpha_c^5), \tag{2.16}$$

$$\alpha_A = e_1\alpha_c + e_2\alpha_c^3 + \mathcal{O}(\alpha_c^5). \tag{2.17}$$

Next, one can solve Eq. (2.16) for α_c as

$$\alpha_c = \omega C_+ + \bar{\omega} C_-, \tag{2.18}$$

with

$$\omega = 1, \quad -e^{\pm i\pi/3}, \tag{2.19}$$

and

$$C_\pm = \left(\frac{\alpha_0}{2d_2} \pm \sqrt{D}\right)^{1/3}, \quad D = \left(\frac{\alpha_0}{2d_2}\right)^2 + \left(\frac{d_1}{3d_2}\right)^3. \tag{2.20}$$

From Eqs. (2.17) and (2.18), the scalar charge is given by

$$\alpha_A = (\omega C_+ + \bar{\omega}C_-)e_1 + (\omega C_+ + \bar{\omega}C_-)^3 e_2 + \mathcal{O}(\alpha_c^5). \tag{2.21}$$

For example, when $\alpha_0 = 0$,[5] α_c can take nonvanishing values as $\alpha_c = \pm\sqrt{-d_1/d_2}$ and

$$\alpha_A = \pm\left[\left(-\frac{d_1}{d_2}\right)^{1/2} e_1 + \left(-\frac{d_1}{d_2}\right)^{3/2} e_2\right]. \tag{2.22}$$

Thus, although $\alpha_0 = 0$, the scalar charge becomes nonvanishing when $d_1/d_2 < 0$.

For constant density stars, d_1 and e_1 are given in a closed analytic form as [50]

$$d_1 = \mathrm{HeunG}\left(\tilde{a}, \tilde{q}; \tilde{\alpha}_-, \tilde{\alpha}_+, \frac{3}{2}, \frac{3}{2}; Z\right) - (\tilde{Z}\log\tilde{Z})\mathrm{HeunG}'\left(\tilde{a}, \tilde{q}; \tilde{\alpha}_-, \tilde{\alpha}_+, \frac{3}{2}, \frac{3}{2}; Z\right), \tag{2.23}$$

$$e_1 = \beta_0 \tilde{Z}\mathrm{HeunG}'\left(\tilde{a}, \tilde{q}; \tilde{\alpha}_-, \tilde{\alpha}_+, \frac{3}{2}, \frac{3}{2}; Z\right), \tag{2.24}$$

with

$$\tilde{a} = -\frac{1}{1+3\tilde{p}_c}, \tag{2.25}$$

$$\tilde{q} = \frac{3\beta_0}{2}\left(\frac{3\tilde{p}_c - 1}{3\tilde{p}_c + 1}\right), \tag{2.26}$$

$$\tilde{\alpha}_\pm = \frac{3}{2}\left(1 \pm \sqrt{1 - \frac{8\beta_0}{3}}\right), \tag{2.27}$$

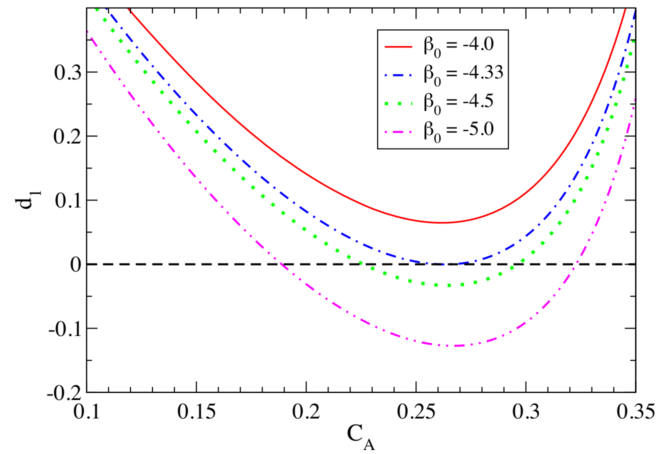

FIG. 3. The linear coefficient d_1 in Eq. (2.16) as a function of the stellar compactness C_A for constant density stars with $\alpha_0 = 0$ and various choices of β_0. Spontaneous scalarization is realized when $d_1 < 0$.

$$Z = \frac{\tilde{p}_c}{1+3\tilde{p}_c}, \tag{2.28}$$

$$\tilde{Z} = \frac{1+\tilde{p}_c}{1+3\tilde{p}_c}, \tag{2.29}$$

and $\tilde{p}_c = \tilde{P}_c/\tilde{\rho}_c$ with $\tilde{P}_c$ and $\tilde{\rho}_c$ being the central pressure and energy density. HeunG is the general Heun function (a generalization of the hypergeometric function) while its prime refers to a derivative with respect to Z.

Figure 3 shows d_1 for constant density stars as a function of the stellar compactness C_A defined by

$$C_A = \frac{M_A}{R}. \tag{2.30}$$

One can check numerically that spontaneous scalarization occurs when $d_1 < 0$ [50]. This means that $d_2 > 0$ in the relevant parameter region, which we checked with the analytic expression for d_2 in Appendix. Notice that $\beta_0 = -4.33$[6] is the critical value below which the spontaneous scalarization is realized for a certain range of C_A. In [50], closed analytic expressions are not found for d_2 and e_2.

III. ANALYTIC SCALAR CHARGES

We now explain how we can use the weak-field expansion and a Padé resummation to analytically calculate NS scalar charges with the Tolman VII model.

[5]Given current stringent bounds on α_0, $\alpha_0 \approx 0$ is a valid approximation when discussing spontaneous scalarization.

[6]See e.g., [53] for a related, analytic estimate of the critical β_0 with constant density stars. The critical value may vary slightly for neutron stars with other equations of state.

A. Formalism

To construct approximate, analytic stellar solutions in the scalar-tensor theory, we work under a weak-field expansion. Namely, we decompose each unknown function $f(r)$ as

$$f(r) = \sum_{k=0} f_k(r)\epsilon^k, \tag{3.1}$$

where ϵ is a book-keeping parameter that counts the order of the GR compactness $C_0 = M_0/R$ where M_0 is the GR mass. We have

$$m_0(r) = \tilde{\rho}_0(r) = \tilde{p}_0(r) = \tilde{p}_1(r) = 0, \tag{3.2}$$

and thus to leading order, $m(r) = \mathcal{O}(\epsilon)$, $\tilde{\rho}(r) = \mathcal{O}(\epsilon)$ and $\tilde{p}(r) = \mathcal{O}(\epsilon^2)$. Notice that $\tilde{\rho} \gg \tilde{p}$ in the weak-field limit. For the scalar field, we find it convenient to introduce

$$\bar{\varphi}(r) \equiv \frac{\varphi(r)}{\alpha_0}, \tag{3.3}$$

so that $\bar{\varphi}(r) = \mathcal{O}(\alpha_0^0)$ when $\alpha_0 \ll 1$. We decompose $\bar{\varphi}(r)$ as in Eq. (3.1) and we have

$$\bar{\varphi}_0(r) = \frac{1}{\beta_0}. \tag{3.4}$$

To see the effect of spontaneous scalarization mentioned in Sec. II C, we need to derive the scalar charge α_A valid up to $\mathcal{O}(\alpha_0^3)$ [which, in turn, means that it is valid to $\mathcal{O}(\alpha_c^3)$ as in Eq. (2.16)]. Namely, we need to find solutions up to to $\mathcal{O}(\alpha_0^2)$ higher than the leading. For this, we seek for a solution $\bar{\varphi}$ to $\mathcal{O}(\alpha_0^2)$. For $m(r)$, since $m_k = \mathcal{O}(\alpha_0^2)$ when $k \geq 2$ [namely, $m(r) = m_1(r)$ in GR] we need to find a solution for $m_k(r)$ valid to $\mathcal{O}(\alpha_0^4)$ such that α_A is valid up to $\mathcal{O}(\alpha_0^3)$. Since $\tilde{p}$ does not enter in the $m'(r)$ equation in Eq. (2.8), we only need a solution for $\tilde{p}$ to $\mathcal{O}(\alpha_0^2)$. Next, we substitute the weak-field expansion of each unknown function in Eq. (3.1) to the differential equations in Eqs. (2.8)–(2.10) and solve order by order in ϵ. Below, we present the equations and solutions mainly to leading order in ϵ for an illustration purpose. In the actual calculation, we derived higher-order contributions and applied a Padé resummation that we explain later.

B. Interior solutions

Since realistic EoSs are given in tables, one can only construct stellar solutions numerically. To overcome this, we use the Tolman VII model [43,54] that is known to accurately model realistic NSs in GR.

The density profile is given by a simple quadratic form as

$$\tilde{\rho}_1 = \frac{15M_0}{8\pi R^3}\left(1 - \frac{r^2}{R^2}\right), \qquad \tilde{\rho}_i(r) = 0 \quad (i \geq 2). \tag{3.5}$$

Thus, these stars are parametrized by M_0 and R, where the former is the stellar mass in GR. To check the validity of the above profile, we present in Fig. 4 the normalized energy density profile for NSs with 1.8 $M_\odot$ and the SLy EoS found numerically for both GR and scalar-tensor theory. For the latter, the star is spontaneously scalarized (see Fig. 2). Notice that the energy density profiles in both theories are almost identical and can be approximated by Eq. (3.5) shown by the black solid curve. This justifies the use of the Tolman VII model even for NSs in scalar-tensor theory.

We next derive the field equations and solutions in the interior region. The leading differential equations are given by

$$m_1'(r) = \frac{15}{2}\left(1 + 4\frac{\alpha_0^2}{\beta_0} + 8\frac{\alpha_0^4}{\beta_0^2}\right)\frac{M_0}{R^5} r^2(R^2 - r^2) + \mathcal{O}(\alpha_0^6), \tag{3.6}$$

$$\bar{\varphi}_1''(r) = -\frac{2\bar{\varphi}_1'}{r} + \frac{15}{2}\left(1 + 4\frac{\alpha_0^2}{\beta_0}\right)\frac{M_0}{R^5}(R^2 - r^2) + \mathcal{O}(\alpha_0^4), \tag{3.7}$$

$$\tilde{P}_2'(r) = -\frac{15M_0}{8\pi R^5}(R^2 - r^2)\left(\frac{m_1}{r^2} + \alpha_0^2\bar{\varphi}_1'\right). \tag{3.8}$$

Imposing the boundary conditions

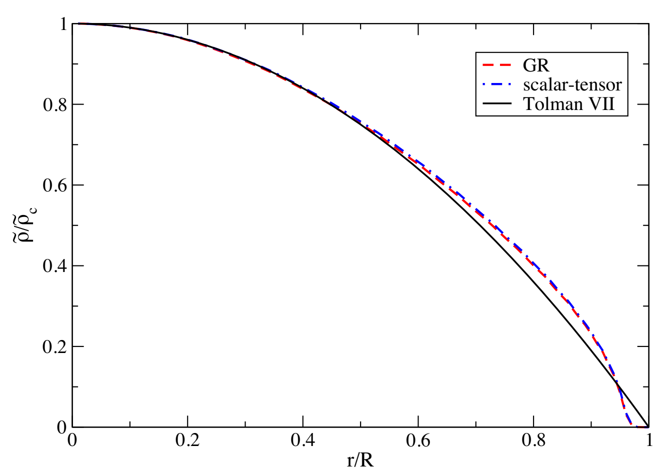

FIG. 4. Energy density profiles (normalized by the central value) for NSs with 1.8 $M_\odot$ in GR and in the scalar-tensor theory with $\alpha_0 = 10^{-5}$ and $\beta_0 = -4.5$ (corresponding to cyan dots in Fig. 2). We use the SLy EoS. For comparison, we also show the profile for the Tolman VII model given in Eq. (3.5). Observe that such a model can accurately describe the profile for realistic NSs.

$$m_k(0) = 0, \qquad \bar{\varphi}_k(0) = \bar{\varphi}_{1c}, \qquad \tilde{P}_k(R) = 0, \tag{3.9}$$

we find

$$m_1^{(\mathrm{int})}(r) = \frac{5}{2}\left(1 + 4\frac{\alpha_0^2}{\beta_0} + 8\frac{\alpha_0^4}{\beta_0^2}\right)\frac{M_0}{R^3}r^3\left(1 - \frac{3}{5}\frac{r^2}{R^2}\right) + \mathcal{O}(\alpha_0^6), \tag{3.10}$$

$$\bar{\varphi}_1^{(\mathrm{int})}(r) = \bar{\varphi}_{1c} + \frac{5}{4}\left(1 + 4\frac{\alpha_0^2}{\beta_0}\right)\frac{M_0}{R^3}r^2\left(1 - \frac{3}{10}\frac{r^2}{R^2}\right) + \mathcal{O}(\alpha_0^4), \tag{3.11}$$

$$\tilde{P}_2^{(\mathrm{int})}(r) = \frac{15}{16\pi}\left(1 + \frac{4+\beta_0}{\beta_0}\alpha_0^2\right)\frac{M_0^2}{R^4} \times \left(1 - \frac{r^2}{R^2}\right)^2\left(1 - \frac{1}{2}\frac{r^2}{R^2}\right) + \mathcal{O}(\alpha_0^4). \tag{3.12}$$

C. Exterior solutions and perturbative scalar charges

Next, we study the exterior region. By setting $\tilde{P} = \tilde{\rho} = 0$ in Eqs. (2.8)–(2.10), we find

$$m_1'(r) = 0, \qquad \bar{\varphi}_1''(r) = -\frac{2\bar{\varphi}_1'}{r}. \tag{3.13}$$

Imposing regularity at infinity, we can solve the above equations to find the exterior solution as[7]

$$m_1^{(\mathrm{ext})}(r) = M_1, \qquad \bar{\varphi}_1^{(\mathrm{ext})}(r) = \frac{\bar{\omega}_1}{r}, \tag{3.14}$$

where M_1 and $\bar{\omega}_1$ are integration constants. We can determine the integration constants both in the interior and exterior solutions by imposing the boundary conditions at the surface:

$$m_k^{(\mathrm{int})}(R) = m_k^{(\mathrm{ext})}(R), \tag{3.15}$$

$$\bar{\varphi}_k^{(\mathrm{int})}(R) = \bar{\varphi}_k^{(\mathrm{ext})}(R), \tag{3.16}$$

$$\bar{\varphi}_k^{\prime(\mathrm{int})}(R) = \bar{\varphi}_k^{\prime(\mathrm{ext})}(R). \tag{3.17}$$

Similar to Eq. (3.14), we can introduce integration constants M_k and $\bar{\omega}_k$ at order ϵ^k as

$$m_k^{(\mathrm{ext})}(r) = M_k\left[1 + \mathcal{O}\left(\frac{M_0}{r}\right)\right], \tag{3.18}$$

$$\bar{\varphi}_k^{(\mathrm{ext})}(r) = \frac{\bar{\omega}_k}{r} + \mathcal{O}\left(\frac{M_0^2}{r^2}\right). \tag{3.19}$$

Once the integration constants are determined, we can compute the perturbative scalar charge from Eq. (2.12) as

$$\alpha_A = -\alpha_0\frac{\sum_{k=1}^N \bar{\omega}_k\epsilon^k}{\sum_{k=1}^N M_k\epsilon^k}. \tag{3.20}$$

In this paper, we computed up to $N = 10$. Equation (3.20) allows us to express α_A in a series of the GR compactness C_0. It would be more useful to express the scalar charges in terms of the physical compactness in the scalar-tensor theory defined by

$$C_A = \frac{M_A}{R}, \qquad M_A = \sum_{k=1} M_k\epsilon^k. \tag{3.21}$$

We can solve order by order to find C_0 in terms of C_A and plug this into Eq. (3.20). To have the results consistent up to the order analyzed, we expand Eq. (3.20) about $\alpha_0 = 0$ and $C_A = 0$ and keep only to $\mathcal{O}(\alpha_0^3, C_A^N)$. To have the series converge, we then construct the Padé approximant of order $N/2$ (when N is an even number) in C_A.

At leading order, the integration constants are derived as

$$\bar{\varphi}_{1c} = -\frac{15}{8}\left(1 + 4\frac{\alpha_0^2}{\beta_0}\right)\frac{M_0}{R} + \mathcal{O}(\alpha_0^4), \tag{3.22}$$

$$M_1 = \left(1 + 4\frac{\alpha_0^2}{\beta_0} + 8\frac{\alpha_0^4}{\beta_0^2}\right)M_0 + \mathcal{O}(\alpha_0^6), \tag{3.23}$$

$$\bar{\omega}_1 = -\left(1 + 4\frac{\alpha_0^2}{\beta_0}\right)M_0 + \mathcal{O}(\alpha_0^4). \tag{3.24}$$

Using Eq. (3.20), the scalar charge to this order is given by

$$\alpha_A = \alpha_0\frac{(1 + 4\frac{\alpha_0^2}{\beta_0})}{(1 + 4\frac{\alpha_0^2}{\beta_0} + 8\frac{\alpha_0^4}{\beta_0^2})} = \alpha_0 + \mathcal{O}(\alpha_0^5, C_A). \tag{3.25}$$

We have derived α_A valid to $\mathcal{O}(C_A^{10})$. The first few terms are given by

$$\alpha_A = \alpha_0 - \frac{10}{7}\alpha_0(\beta_0+1)C_A + \frac{5\alpha_0}{3003} \times (1253\beta_0^2 + 1514\beta_0 - 1126)C_A^2 + \mathcal{O}(\alpha_0^3, C_A^3). \tag{3.26}$$

[7] An analytic solution for a NS exterior in scalar-tensor theories without the weak-field expansion has been found in a different coordinate system [37,39]. We found that working in the same coordinate system as the interior case is easier due to some nonlinearity in the coordinate transformation.

We then construct the Padé approximant to fifth order in C_A, which we provide in a Supplemental Mathematica notebook [51].

D. Spontaneous scalarization

So far, we have constructed α_A in a series of α_0 and C_A in a form

$$\alpha_A = \bar{e}_1(C_A)\alpha_0 + \bar{e}_3(C_A)\alpha_0^3 + \mathcal{O}(\alpha_0^5), \tag{3.27}$$

where $\bar{e}_1$ and $\bar{e}_3$ are functions of C_A valid to $\mathcal{O}(C_A^N)$. We also have

$$\begin{aligned}\alpha_c &= \alpha_0\beta_0 \sum_{k=1}^{N} \bar{\varphi}_{kc}\epsilon^k \\ &= \bar{d}_1(C_A)\alpha_0 + \bar{d}_3(C_A)\alpha_0^3 + \mathcal{O}(\alpha_0^5).\end{aligned} \tag{3.28}$$

We can invert Eq. (3.28) order by order in α_c to find $\alpha_0(\alpha_c)$ in a form given by Eq. (2.16). Then, we substitute this into Eq. (3.27) and expand about $\alpha_c = 0$ to find $\alpha_A(\alpha_c)$ in the form in Eq. (2.17). Following Sec. II C, we can then find α_A for NSs under spontaneous scalarization.

In fact, it turns out that the contribution for the second term in Eq. (2.22) is negligible and we focus on deriving d_1, d_2, and e_1. We first find these coefficients in terms of a series expansion of C_A valid to $\mathcal{O}(C_A^{10})$. The first few terms of these functions are given by

$$\begin{aligned} d_1 = 1 + \frac{15\beta_0}{8}C_A + \frac{5\beta_0}{128}(17\beta_0 - 14)C_A^2 \\ + \frac{\beta_0}{7168}(695\beta_0^2 - 2294\beta_0 - 27720)C_A^3 + \mathcal{O}(C_A^4),\end{aligned} \tag{3.29}$$

$$\begin{aligned} d_2 = -\frac{1955\beta_0}{448}C_A^2 + \frac{(1630328 - 3417469\beta_0)\beta_0}{512512}C_A^3 \\ + \mathcal{O}(C_A^4),\end{aligned} \tag{3.30}$$

$$\begin{aligned} e_1 &= 1 + \frac{5}{56}(5\beta_0 - 16)C_A \\ &+ \frac{5}{384384}(5515\beta_0^2 - 54170\beta_0 - 144128)C_A^2 \\ &+ \frac{1}{156828672}(941985\beta_0^3 - 19472682\beta_0^2 \\ &- 258360664\beta_0 - 441114624)C_A^3 + \mathcal{O}(C_A^4).\end{aligned} \tag{3.31}$$

Next, we construct the Padé approximants on these. We first tried fifth order Padé approximants but found that there are some unphysical divergences (Fig. 5). Instead, we use fourth order Padé approximants. We substituted these into the following expression to find scalar charges for NSs under spontaneous scalarization:

$$\alpha_A = \pm\left(-\frac{d_1}{d_2}\right)^{1/2} e_1. \tag{3.32}$$

The final expression is available in the Supplemental Mathematica notebook [51].

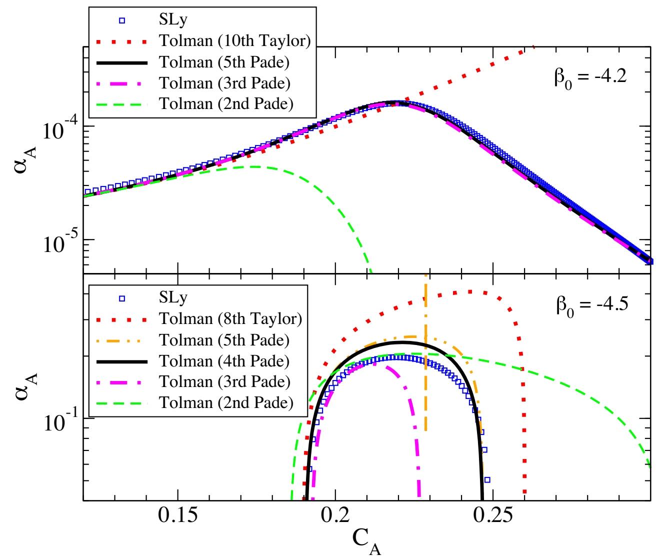

FIG. 5. Convergence of scalar charge calculations with the Tolman model for $(\alpha_0, \beta_0) = (10^{-5}, -4.2)$ (top) and $(\alpha_0, \beta_0) = (0, -4.5)$ (bottom) under various approximations. We compare the Taylor series results and various Padé resummation results against numerical results for the SLy EoS. Observe that a Padé resummation is crucial to accurately model the numerical result. The expressions for the black solid curves are the ones that we will use in the remaining part of this paper.

IV. COMPARISON WITH NUMERICAL RESULTS

Let us now compare the analytic scalar charge expression for the Tolman VII model with scalar charges for realistic NSs found numerically. Scalar charges can be both positive and negative but we focus on the former.

A. Scalar charges vs compactness

First, let us check the convergence of the analytic result for scalar charges in terms of the order of the Padé resummation. Figure 5 compares the analytic scalar charges at various orders of the Padé approximants. We show the results for the perturbative scalarization case $(\alpha_0, \beta_0) = (10^{-5}, -4.2)$ and the spontaneous scalarization case $(\alpha_0, \beta_0) = (0, -4.5)$. We also present Taylor series (without the Padé resummation) up to $\mathcal{O}(C_A^{10})$ for perturbative scalarization and to $\mathcal{O}(C_A^8)$ for spontaneous scalarization, as well as numerical results computed with the SLy EoS. Observe that for the perturbative scalarization case (top panel), the Padé resummation at third order is already a good approximation and is converging fast to the numerical result.[8] On the other hand, the Taylor series result becomes less accurate when the compactness is relatively high. For the spontaneous scalarization case (bottom panel), there is little difference between the fourth and fifth order Padé results, though the latter has some unphysical divergence. We therefore use the former to avoid this divergence. Additionally, the former has the advantages of a simpler functional expression and better agreement with the numerical SLy data. All in all, these findings suggest that it is crucial to perform the resummation to find an accurate modeling.

Having understood the convergence, let us now carry out the comparison in more detail for various values of β_0 and EoSs. Figure 1 shows such a comparison. Observe that the analytic scalar charges for the Tolman VII case beautifully captures the numerical results for realistic NSs, especially for the SLy EoS. Notice that the agreement between the analytic and numerical results is good even for stars with spontaneous scalarization. This shows that the analytic result serves as an accurate, ready to use expression for the NS scalar charge in this scalar-tensor theory.

B. Scalar charges vs binding energy

We next look at the relation between the scalar charge and the stellar binding energy, where the latter is defined by

$$\Omega_A = -\frac{1}{2}\int d^3x\tilde{\rho}(r)\int d^3x'\,\frac{\tilde{\rho}(r')}{|\boldsymbol{x}-\boldsymbol{x}'|}, \tag{4.1}$$

$$= -16\pi^2\int_0^R dr r^2\tilde{\rho}(r)\left(\int_r^R dr' r'\tilde{\rho}(r')\right), \tag{4.2}$$

where the second equality is valid only for spherically symmetric systems. Physically, this quantity measures the difference between the gravitational and baryonic mass of a star and can be used to e.g., probe certain formation scenario of NSs [55]. For the Tolman VII model, the relation between the binding energy and compactness is given by

$$\frac{\Omega_A}{M_A} = -\frac{5}{7}C_A + \mathcal{O}(\alpha_0^2). \tag{4.3}$$

Such a GR relation is sufficient to find the scalar charge expression in terms of the binding energy for perturbative scalar charges since the latter is already proportional to α_0. We can invert this relation and substitute it to the $\alpha_A(C_A)$ expressions.

[8]The third and fifth order Padé results for the perturbative scalar charge expressions have a noticeable difference for lower β_0 where the expressions diverge due to the onset of spontaneous scalarization (not shown in Fig. 5) and thus we use the fifth order result in the remainder of this section.

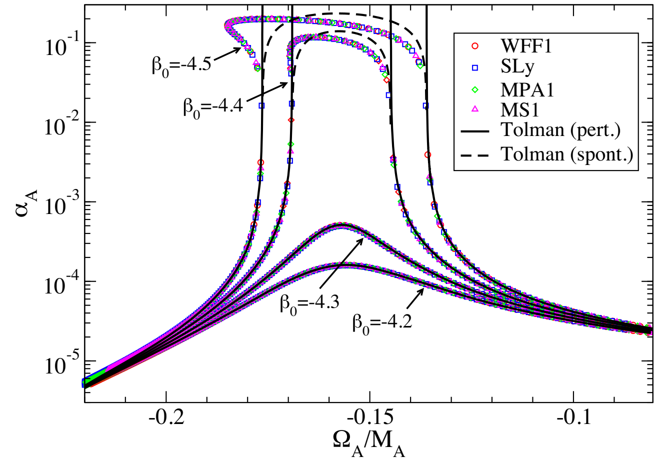

FIG. 6. Similar to Fig. 1 but as a function of the binding energy. Notice that the results are quasiuniversal and insensitive to the EoSs for fixed β_0.

Figure 6 presents the scalar charge as a function of the binding energy (normalized by the stellar mass) for the same choices of α_0 and β_0 and EoSs as in Fig. 1. Observe that numerical results are quasiuniversal among different EoSs for a fixed β_0. Observe also that the analytic result accurately describes the numerical ones for perturbative scalarization. By taking the fractional difference between the numerical and analytic results for $\beta_0 = -4.2$ or $\beta_0 = -4.3$, we found that the quasiuniversality holds to $\sim 1\%$ for $\Omega_A/M_A > -0.2$.

Analytic expressions help us to study the quasiuniversal relation in more detail. When we expand the perturbative scalar charge expression for the Tolman VII model for small binding energy, we find

$$\begin{aligned}\alpha_A^{(\mathrm{Tol})} &= \alpha_0 + 2\alpha_0(\beta_0+1)\frac{\Omega_A}{M_A} \\ &\quad + \frac{7\alpha_0}{2145}(1253\beta_0^2 + 1514\beta_0 - 1126)\frac{\Omega_A^2}{M_A^2} \\ &\quad + \mathcal{O}\left(\alpha_0^3, \frac{\Omega_A^3}{M_A^3}\right).\end{aligned} \tag{4.4}$$

In Appendix, we derive analytic scalar charges for constant density stars. We show that although $\alpha_A(C_A)$ is quite different from those for Tolman VII and realistic EoS cases, $\alpha_A(\Omega_A/M_A)$ is similar to the latter two cases, supporting the quasiuniversal relation. For the perturbative scalar charge, the Taylor-series expansion in small binding energy is given by

$$\begin{aligned}\alpha_A^{(\mathrm{CD})} &= \alpha_0 + 2\alpha_0(\beta_0+1)\frac{\Omega_A}{M_A} \\ &\quad + \frac{5\alpha_0}{21}(17\beta_0^2 + 20\beta_0 - 16)\frac{\Omega_A^2}{M_A^2} + \mathcal{O}\left(\alpha_0^3, \frac{\Omega_A^3}{M_A^3}\right).\end{aligned} \tag{4.5}$$

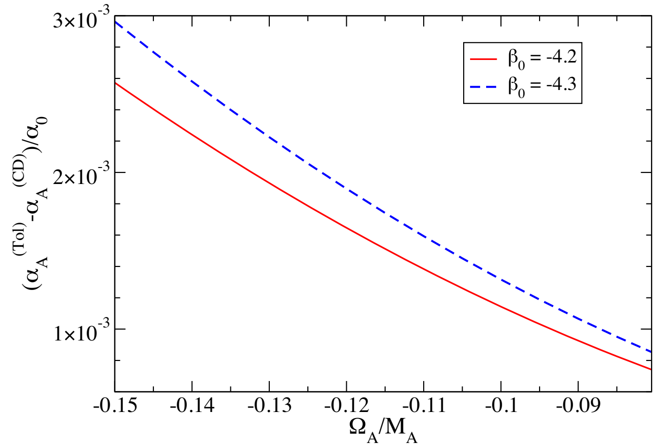

FIG. 7. Fractional difference in the scalar charge between Tolman VII and constant density cases in terms of the binding energy [Eq. (4.6)] for two choices of β_0. Notice that the two agree within an error of $\mathcal{O}(10^{-3})$, indicating the quasiuniversality of the relation between the scalar charge and binding energy.

Comparing this with Eq. (4.4) we see that the two expressions are identical up to $\mathcal{O}(\Omega_A/M_A)$ and the difference only appears at $\mathcal{O}(\Omega_A^2/M_A^2)$. When taking the fractional difference between $\alpha_A^{(\mathrm{Tol})}$ and $\alpha_A^{(\mathrm{CD})}$, we find

$$\frac{\alpha_A^{(\mathrm{Tol})}-\alpha_A^{(\mathrm{CD})}}{\alpha_0}=\frac{2(311\beta_0^2+1343\beta_0+1013)}{15015}\frac{\Omega_A^2}{M_A^2}+\mathcal{O}\left(\alpha_0^3,\frac{\Omega_A^3}{M_A^3}\right). \tag{4.6}$$

Figure 7 presents this fractional difference for $\beta_0=-4.2$ and -4.3. Observe that the Tolman and constant density cases agree within an error of $\mathcal{O}(10^{-3})$. This provides strong analytic support for the quasiuniversality of the relation between α_A and Ω_A/M_A.

For spontaneous scalarization, although the analytic expressions are valid as an order-of-magnitude estimate, the agreement with numerical results are not as good as the perturbative scalarization case. This is partially because we have used the relation between the binding energy and compactness in GR in Eq. (4.3). Though, due to multi-valued scalar charges for some fixed binding energy with small β_0 (e.g., $\beta_0=-4.5$), it would be difficult to find a closed, analytic expression for $\alpha_A(\Omega_A/M_A)$.

V. CONCLUSION

We derived analytically scalar charges for NSs in a scalar-tensor theory proposed in [38]. This was achieved by considering the Tolman VII energy density profile. We first worked within the weak-field approximation and then resummed the series through the Padé approximants, deriving scalar charge expressions for both perturbative and spontaneous scalarization. We found that the analytic scalar charges are in excellent agreement with those computed numerically from realistic EoSs, especially for the SLy EoS. We also found a quasiuniversal relation between the scalar charge and binding energy. The analytic expressions derived here allow one to mathematically support the quasiuniversality by comparing the Tolman VII result with the constant density one. The analytic result provides an accurate, ready to use, and physically-motivated expression for scalar charges.

A similar quasiuniversality between the stellar sensitivities and the binding energy were found recently in Einstein-Æ ther theory [30]. The relation in the weak-field limit was first derived in [56]. This was based on the result for weakly-gravitating stars in [57], where the dipole moment for the vector field (which depends on the sensitivities) was derived within the parametrized post-Newtonian framework and all the internal structure dependence was found to be encoded in the binding energy. It would be interesting to study if a similar analysis can be carried out in scalar-tensor theories to explain the quasiuniversal relation found here.

Various avenues exist for other possible future work. For example, one obvious extension is to apply the analysis presented here to other scalar-tensor theories with spontaneous scalarization, such as the ones with the mass potential [58–61] or quartic interaction [62], scalar-Gauss-Bonnet gravity [63–72], or Horndeski theories [73]. One may also apply the calculation to spontaneous vectorization [40,74,75], tensorization [76], or spinorization [77]. Another possibility might be to consider an analytic representation of the dynamical/induced scalarization [78–83] in compact binary mergers. Finally, the parameter region of the coupling parameter β_0 that gives rise to spontaneous scalarization in the scalar-tensor theories studied here has been shown to be inconsistent with solar system experiments if one includes cosmological evolution of the scalar field [81,84–87]. It would be interesting to consider analytic scalar charges in scalar-tensor theories that has a consistent cosmological evolution of the scalar field [88–90].

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APPENDIX: SCALAR CHARGES FOR CONSTANT DENSITY STARS

In this Appendix we repeat the calculations in the main text to find analytic expressions for scalar charges for constant density stars. For such stars, we give the density profile as

$$\tilde{\rho}_1 = \frac{3M_0}{4\pi R^3}, \qquad \tilde{\rho}_i(r) = 0 \quad (i \geq 2). \tag{A1}$$

Thus, these stars are parametrized by M_0 and R. The leading differential equations are given by

$$m_1'(r) = 3\left(1 + 4\frac{\alpha_0^2}{\beta_0} + 8\frac{\alpha_0^4}{\beta_0^2}\right)\frac{M_0}{R^3}r^2 + \mathcal{O}(\alpha_0^6), \tag{A2}$$

$$\bar{\varphi}_1''(r) = -\frac{2\bar{\varphi}_1'}{r} + 3\left(1 + 4\frac{\alpha_0^2}{\beta_0}\right)\frac{M_0}{R^3} + \mathcal{O}(\alpha_0^4), \tag{A3}$$

$$\tilde{P}_2'(r) = -\frac{3M_0}{4\pi R^3}\left(\frac{m_1}{r^2} + \alpha_0^2\bar{\varphi}_1'\right). \tag{A4}$$

The last equation is valid to full order in α_0. Imposing the boundary condition as in Eq. (3.9), one can solve the above differential equations in the interior region to find

$$m_1^{(\mathrm{int})}(r) = \left(1 + 4\frac{\alpha_0^2}{\beta_0} + 8\frac{\alpha_0^4}{\beta_0^2}\right)\frac{M_0}{R^3}r^3 + \mathcal{O}(\alpha_0^6), \tag{A5}$$

$$\bar{\varphi}_1^{(\mathrm{int})}(r) = \bar{\varphi}_{1c} + \frac{1}{2}\left(1 + 4\frac{\alpha_0^2}{\beta_0}\right)\frac{M_0}{R^3}r^2 + \mathcal{O}(\alpha_0^4), \tag{A6}$$

$$\tilde{P}_2^{(\mathrm{int})}(r) = \frac{3}{8\pi}\left(1 + \frac{4+\beta_0}{\beta_0}\alpha_0^2\right)\frac{M_0^2}{R^4}\left(1 - \frac{r^2}{R^2}\right) + \mathcal{O}(\alpha_0^4). \tag{A7}$$

Next, we study the exterior solution and the scalar charge. At leading order in the weak-field expansion, the integration constants are determined as

$$\bar{\varphi}_{1c} = -\frac{3}{2}\left(1 + 4\frac{\alpha_0^2}{\beta_0}\right)\frac{M_0}{R} + \mathcal{O}(\alpha_0^4), \tag{A8}$$

$$M_1 = \left(1 + 4\frac{\alpha_0^2}{\beta_0} + 8\frac{\alpha_0^4}{\beta_0^2}\right)M_0 + \mathcal{O}(\alpha_0^6), \tag{A9}$$

$$\bar{\omega}_1 = -\left(1 + 4\frac{\alpha_0^2}{\beta_0}\right)M_0 + \mathcal{O}(\alpha_0^4). \tag{A10}$$

Notice that M_1 and $\bar{\omega}_1$ are the same as the Tolman VII case as in Eqs. (3.23) and (3.24). Thus, the scalar charge is also same as in Eq. (3.25). Similar to the Tolman case, we then construct a Padé approximant to fifth order in C_A that we provide in the Supplemental Mathematica notebook [51].

Let us now find the expression for spontaneous scalarization. Although d_1 and e_1 are fully given in Eqs. (2.23) and (2.24), we derived d_1 and e_1 in a fourth order Padé approximant form to make the scalar charge expression similar to that for the Tolman case discussed in the main text. We carried out a similar analysis for d_2 whose full

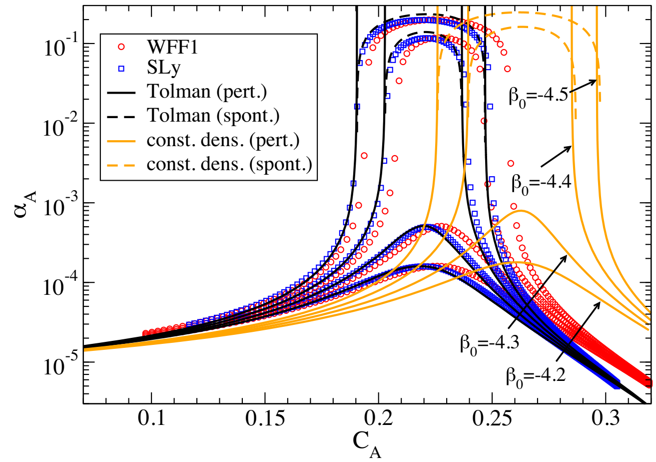

FIG. 8. Similar to Fig. 1 but including analytic results for constant density stars. Notice that scalar charges for such stars are quite different from those for realistic NSs.

expression has not been found yet. The first few terms are given by

$$d_2 = -\frac{21\beta_0}{8}C_A^2 - \frac{3}{560}\beta_0(547\beta_0 - 278)C_A^3 + \mathcal{O}(C_A^4). \tag{A11}$$

We have derived d_2 to $\mathcal{O}(C_A^8)$ and constructed a fourth order Padé approximant. We can substitute these Padé resummed forms for d_1, d_2, and e_1 into Eq. (3.32) to find the scalar charge for constant density stars under spontaneous scalarization when $\alpha_0 \ll 1$. We provide the final expression in the Supplemental Mathematica notebook [51].

Figure 8 compares the analytic scalar charges for constant density stars (as a function of the compactness) with those for the Tolman VII model and numerical charges with two representative EoSs. Notice that the spontaneous scalarization happens for larger compactnesses compared

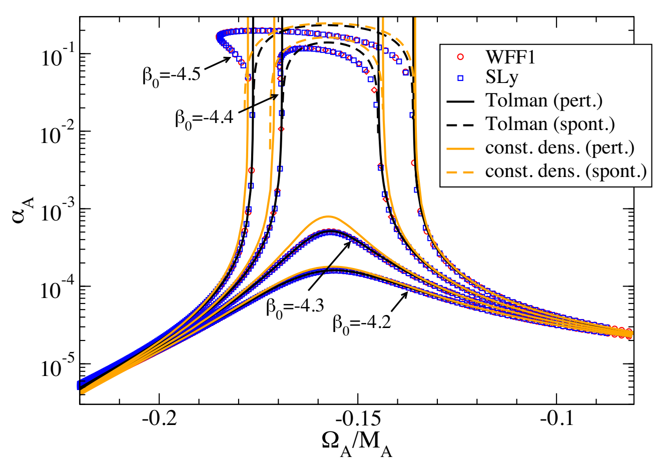

FIG. 9. Similar to Fig. 6 but including analytic results for constant density stars. Unlike in Fig. 8, scalar charges for constant density stars are now similar to realistic NSs.

to the Tolman VII model and realistic NSs. This shows that the former is not an accurate model of the latter.

One can further convert the scalar charge expression in terms of the binding energy. For constant density stars, the leading relation is $\Omega_A/M_A = -(3/5)C_A + \mathcal{O}(\alpha_0^2)$.

Inverting this and substituting it into the analytic expression for the scalar charge in terms of the compactness, one finds the scalar charge as a function of the binding energy, which is shown in Fig. 9. We compare the constant density result with the Tolman VII result and numerical results with realistic EoSs. Observe that the scalar charges for constant density stars are very similar to other results, supporting the quasiuniversality of the relation between the scalar charge and binding energy.

[1] H. T. Cromartie *et al.* (NANOGrav Collaboration), Relativistic Shapiro delay measurements of an extremely massive millisecond pulsar, Nat. Astron. **4**, 72 (2020).

[2] T. E. Riley *et al.*, A *NICER* View of PSR J0030 + 0451: Millisecond pulsar parameter estimation, Astrophys. J. Lett. **887**, L21 (2019).

[3] M. C. Miller *et al.*, PSR J0030 + 0451 mass and radius from *NICER* data and implications for the properties of neutron star matter, Astrophys. J. Lett. **887**, L24 (2019).

[4] B. P. Abbott *et al.* (Virgo and LIGO Scientific Collaborations), GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral, Phys. Rev. Lett. **119**, 161101 (2017).

[5] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), GW170817: Measurements of Neutron Star Radii and Equation of State, Phys. Rev. Lett. **121**, 161101 (2018).

[6] E. Annala, T. Gorda, A. Kurkela, and A. Vuorinen, Gravitational-Wave Constraints on the Neutron-Star-Matter Equation of State, Phys. Rev. Lett. **120**, 172703 (2018).

[7] C. Raithel, F. Ozel, and D. Psaltis, Tidal deformability from GW170817 as a direct probe of the neutron star radius, Astrophys. J. **857**, L23 (2018).

[8] Y. Lim and J. W. Holt, Neutron Star Tidal Deformabilities Constrained by Nuclear Theory and Experiment, Phys. Rev. Lett. **121**, 062701 (2018).

[9] A. Bauswein, O. Just, H.-T. Janka, and N. Stergioulas, Neutron-star radius constraints from GW170817 and future detections, Astrophys. J. **850**, L34 (2017).

[10] S. De, D. Finstad, J. M. Lattimer, D. A. Brown, E. Berger, and C. M. Biwer, Tidal Deformabilities and Radii of Neutron Stars from the Observation of GW170817, Phys. Rev. Lett. **121**, 091102 (2018); Erratum, Phys. Rev. Lett. **121**, 259902 (2018).

[11] E. R. Most, L. R. Weih, L. Rezzolla, and J. Schaffner-Bielich, New Constraints on Radii and Tidal Deformabilities of Neutron Stars from GW170817, Phys. Rev. Lett. **120**, 261103 (2018).

[12] E. Annala, T. Gorda, A. Kurkela, J. Nättilä, and A. Vuorinen, Quark-matter cores in neutron stars, Nat. Phys. **16**, 907 (2020).

[13] T. Malik, N. Alam, M. Fortin, C. Providência, B. K. Agrawal, T. K. Jha, B. Kumar, and S. K. Patra, GW170817: Constraining the nuclear matter equation of state from the neutron star tidal deformability, Phys. Rev. C **98**, 035804 (2018).

[14] Z. Carson, A. W. Steiner, and K. Yagi, Constraining nuclear matter parameters with GW170817, Phys. Rev. D **99**, 043010 (2019).

[15] Z. Carson, A. W. Steiner, and K. Yagi, Future prospects for constraining nuclear matter parameters with gravitational waves, Phys. Rev. D **100**, 023012 (2019).

[16] C. A. Raithel and F. Ozel, Measurement of the nuclear symmetry energy parameters from gravitational wave events (2019), 10.3847/1538-4357/ab48e6, arXiv:1908.00018.

[17] K. Chatziioannou, Neutron star tidal deformability and equation of state constraints, Gen. Relativ. Gravit. **52**, 109 (2020).

[18] G. Raaijmakers *et al.*, Constraining the dense matter equation of state with joint analysis of NICER and LIGO/Virgo measurements, Astrophys. J. Lett. **893**, L21 (2020).

[19] J. Zimmerman, Z. Carson, K. Schumacher, A. W. Steiner, and K. Yagi, Measuring Nuclear Matter Parameters with NICER and LIGO/Virgo, arXiv:2002.03210.

[20] J.-L. Jiang, S.-P. Tang, Y.-Z. Wang, Y.-Z. Fan, and D.-M. Wei, PSR J0030 + 0451, GW170817 and the nuclear data: Joint constraints on equation of state and bulk properties of neutron stars, Astrophys. J. **892**, 55 (2020).

[21] T. Dietrich, M. W. Coughlin, P. T. H. Pang, M. Bulla, J. Heinzel, L. Issa, I. Tews, and S. Antier, Multimessenger constraints on the neutron-star equation of state and the Hubble constant, Science **370**, 1450 (2020).

[22] P. C. Freire, N. Wex, G. Esposito-Farèse, J. P. W. Verbiest, M. Bailes, B. A. Jacoby, M. Kramer, I. H. Stairs, J. Antoniadis, and G. H. Janssen, The relativistic pulsar-white dwarf binary PSR J1738 + 0333 II. The most stringent test of scalar-tensor gravity, Mon. Not. R. Astron. Soc. **423**, 3328 (2012).

[23] K. Yagi, D. Blas, N. Yunes, and E. Barausse, Strong Binary Pulsar Constraints on Lorentz Violation in Gravity, Phys. Rev. Lett. **112**, 161101 (2014).

[24] K. Yagi, D. Blas, E. Barausse, and N. Yunes, Constraints on Einstein-Æ ther theory and Horava gravity from binary pulsar observations, Phys. Rev. D **89**, 084067 (2014).

[25] E. Berti *et al.*, Testing general relativity with present and future astrophysical observations, Classical Quant. Grav. **32**, 243001 (2015).

[26] L. Shao, N. Sennett, A. Buonanno, M. Kramer, and N. Wex, Constraining Nonperturbative Strong-Field Effects in Scalar-Tensor Gravity by Combining Pulsar Timing and

Laser-Interferometer Gravitational-Wave Detectors, Phys. Rev. X **7**, 041025 (2017).
[27] A. M. Archibald, N. V. Gusinskaia, J. W. T. Hessels, A. T. Deller, D. L. Kaplan, D. R. Lorimer, R. S. Lynch, S. M. Ransom, and I. H. Stairs, Universality of free fall from the orbital motion of a pulsar in a stellar triple system, Nature (London) **559**, 73 (2018).
[28] B. C. Seymour and K. Yagi, Probing massive scalar fields from a pulsar in a stellar triple system, Classical Quant. Grav. **37**, 145008 (2020).
[29] D. Anderson, P. Freire, and N. Yunes, Binary pulsar constraints on massless scalar–tensor theories using Bayesian statistics, Classical Quant. Grav. **36**, 225009 (2019).
[30] T. Gupta, M. Herrero-Valea, D. Blas, E. Barausse, N. Cornish, K. Yagi, and N. Yunes, Updated binary pulsar constraints on Einstein-Æther theory in light of gravitational wave constraints on the speed of gravity, arXiv:2104.04596.
[31] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Tests of General Relativity with GW170817, Phys. Rev. Lett. **123**, 011102 (2019).
[32] T. Baker, E. Bellini, P. G. Ferreira, M. Lagos, J. Noller, and I. Sawicki, Strong Constraints on Cosmological Gravity from GW170817 and GRB 170817A, Phys. Rev. Lett. **119**, 251301 (2017).
[33] J. Sakstein and B. Jain, Implications of the Neutron Star Merger GW170817 for Cosmological Scalar-Tensor Theories, Phys. Rev. Lett. **119**, 251303 (2017).
[34] J. M. Ezquiaga and M. Zumalacárregui, Dark Energy After GW170817: Dead Ends and the Road Ahead, Phys. Rev. Lett. **119**, 251304 (2017).
[35] J. Zhao, L. Shao, Z. Cao, and B.-Q. Ma, Reduced-order surrogate models for scalar-tensor gravity in the strong field regime and applications to binary pulsars and GW170817, Phys. Rev. D **100**, 064034 (2019).
[36] H. O. Silva, A. M. Holgado, A. Cárdenas-Avendaño, and N. Yunes, Astrophysical and Theoretical Physics Implications from Multimessenger Neutron Star Observations, Phys. Rev. Lett. **126**, 181101 (2021).
[37] T. Damour and G. Esposito-Farese, Tensor multiscalar theories of gravitation, Classical Quant. Grav. **9**, 2093 (1992).
[38] T. Damour and G. Esposito-Farese, Nonperturbative Strong Field Effects in Tensor-Scalar Theories of Gravitation, Phys. Rev. Lett. **70**, 2220 (1993).
[39] T. Damour and G. Esposito-Farese, Tensor-scalar gravity and binary pulsar experiments, Phys. Rev. D **54**, 1474 (1996).
[40] F. M. Ramazanoğlu, Spontaneous growth of vector fields in gravity, Phys. Rev. D **96**, 064009 (2017).
[41] D. Anderson and N. Yunes, Scalar charges and scaling relations in massless scalar–tensor theories, Classical Quant. Grav. **36**, 165003 (2019).
[42] Z. Carson, B. C. Seymour, and K. Yagi, Future prospects for probing scalar-tensor theories with gravitational waves from mixed binaries, Classical Quant. Grav. **37**, 065008 (2020).
[43] R. C. Tolman, Static solutions of Einstein's field equations for spheres of fluid, Phys. Rev. **55**, 364 (1939).
[44] K. Yagi, L. C. Stein, and N. Yunes, Challenging the Presence of Scalar Charge and Dipolar Radiation in Binary Pulsars, Phys. Rev. D **93**, 024010 (2016).
[45] R. B. Wiringa, V. Fiks, and A. Fabrocini, Equation of state for dense nucleon matter, Phys. Rev. C **38**, 1010 (1988).
[46] F. Douchin and P. Haensel, A unified equation of state of dense matter and neutron star structure, Astron. Astrophys. **380**, 151 (2001).
[47] H. Müther, M. Prakash, and T. L. Ainsworth, The nuclear symmetry energy in relativistic Brueckner-Hartree-Fock calculations, Phys. Lett. B **199**, 469 (1987).
[48] H. Mueller and B. D. Serot, Relativistic mean field theory and the high density nuclear equation of state, Nucl. Phys. **A606**, 508 (1996).
[49] G. Pappas, D. D. Doneva, T. P. Sotiriou, S. S. Yazadjiev, and K. D. Kokkotas, Multipole moments and universal relations for scalarized neutron stars, Phys. Rev. D **99**, 104014 (2019).
[50] M. W. Horbatsch and C. P. Burgess, Semi-analytic stellar structure in scalar-tensor gravity, J. Cosmol. Astropart. Phys.08 (2011) 027.
[51] https://github.com/KentYagi/analytic-scalar-charges.git.
[52] P. Pani and E. Berti, I-Love-Q, Spontaneously: Slowly rotating neutron stars in scalar-tensor theories, Phys. Rev. D **90**, 024025 (2014).
[53] T. Harada, Stability analysis of spherically symmetric star in scalar-tensor theories of gravity, Prog. Theor. Phys. **98**, 359 (1997).
[54] N. Jiang and K. Yagi, Improved Analytic Modeling of Neutron Star Interiors, Phys. Rev. D **99**, 124029 (2019).
[55] W. G. Newton, A. W. Steiner, and K. Yagi, Testing the formation scenarios of binary neutron star systems with measurements of the neutron star moment of inertia, Astrophys. J. **856**, 19 (2018).
[56] B. Z. Foster, Strong field effects on binary systems in Einstein-Æther theory, Phys. Rev. D **76**, 084033 (2007).
[57] B. Z. Foster, Radiation damping in Einstein-Æther theory, Phys. Rev. D **73**, 104012 (2006); Erratum, Phys. Rev. D **75**, 129904 (2007).
[58] P. Chen, T. Suyama, and J. Yokoyama, Spontaneous scalarization: Asymmetron as dark matter, Phys. Rev. D **92**, 124016 (2015).
[59] F. M. Ramazanoğlu and F. Pretorius, Spontaneous scalarization with massive fields, Phys. Rev. D **93**, 064005 (2016).
[60] S. S. Yazadjiev, D. D. Doneva, and D. Popchev, Slowly rotating neutron stars in scalar-tensor theories with a massive scalar field, Phys. Rev. D **93**, 084038 (2016).
[61] D. D. Doneva and S. S. Yazadjiev, Rapidly rotating neutron stars with a massive scalar field—structure and universal relations, J. Cosmol. Astropart. Phys. 11 (2016) 019.
[62] K. V. Staykov, D. Popchev, D. D. Doneva, and S. S. Yazadjiev, Static and slowly rotating neutron stars in scalar–tensor theory with self-interacting massive scalar field, Eur. Phys. J. C **78**, 586 (2018).
[63] D. D. Doneva and S. S. Yazadjiev, New Gauss-Bonnet Black Holes with Curvature-Induced Scalarization in Extended Scalar-Tensor Theories, Phys. Rev. Lett. **120**, 131103 (2018).
[64] H. O. Silva, J. Sakstein, L. Gualtieri, T. P. Sotiriou, and E. Berti, Spontaneous Scalarization of Black Holes and Compact Stars from a Gauss-Bonnet Coupling, Phys. Rev. Lett. **120**, 131104 (2018).

[65] D. D. Doneva and S. S. Yazadjiev, Neutron star solutions with curvature induced scalarization in the extended Gauss-Bonnet scalar-tensor theories, J. Cosmol. Astropart. Phys. 04 (2018) 011.
[66] H. O. Silva, C. F. B. Macedo, T. P. Sotiriou, L. Gualtieri, J. Sakstein, and E. Berti, Stability of scalarized black hole solutions in scalar-Gauss-Bonnet gravity, Phys. Rev. D **99**, 064011 (2019).
[67] M. Minamitsuji and T. Ikeda, Scalarized black holes in the presence of the coupling to Gauss-Bonnet gravity, Phys. Rev. D **99**, 044017 (2019).
[68] C. F. B. Macedo, J. Sakstein, E. Berti, L. Gualtieri, H. O. Silva, and T. P. Sotiriou, Self-interactions and Spontaneous Black Hole Scalarization, Phys. Rev. D **99**, 104041 (2019).
[69] A. Dima, E. Barausse, N. Franchini, and T. P. Sotiriou, Spin-Induced Black Hole Spontaneous Scalarization, Phys. Rev. Lett. **125**, 231101 (2020).
[70] D. D. Doneva, L. G. Collodel, C. J. Krüger, and S. S. Yazadjiev, Black hole scalarization induced by the spin: $2+1$ time evolution, Phys. Rev. D **102**, 104027 (2020).
[71] C. A. R. Herdeiro, E. Radu, H. O. Silva, T. P. Sotiriou, and N. Yunes, Spin-Induced Scalarized Black Holes, Phys. Rev. Lett. **126**, 011103 (2021).
[72] H. O. Silva, H. Witek, M. Elley, and N. Yunes, Dynamical scalarization and descalarization in binary black hole mergers, Phys. Rev. Lett. **127**, 031101 (2021)..
[73] N. Andreou, N. Franchini, G. Ventagli, and T. P. Sotiriou, Spontaneous scalarization in generalised scalar-tensor theory, Phys. Rev. D **99**, 124022 (2019); Erratum, Phys. Rev. D **101**, 109903 (2020).
[74] L. Annulli, V. Cardoso, and L. Gualtieri, Electromagnetism and hidden vector fields in modified gravity theories: Spontaneous and induced vectorization, Phys. Rev. D **99**, 044038 (2019).
[75] R. Kase, M. Minamitsuji, and S. Tsujikawa, Neutron stars with a generalized Proca hair and spontaneous vectorization, Phys. Rev. D **102**, 024067 (2020).
[76] F. M. Ramazanoğlu, Spontaneous tensorization from curvature coupling and beyond, Phys. Rev. D **99**, 084015 (2019).
[77] F. M. Ramazanoğlu, Spontaneous growth of spinor fields in gravity, Phys. Rev. D **98**, 044011 (2018); Erratum, Phys. Rev. D **100**, 029903 (2019).
[78] E. Barausse, C. Palenzuela, M. Ponce, and L. Lehner, Neutron-star mergers in scalar-tensor theories of gravity, Phys. Rev. D **87**, 081506 (2013).
[79] C. Palenzuela, E. Barausse, M. Ponce, and L. Lehner, Dynamical scalarization of neutron stars in scalar-tensor gravity theories, Phys. Rev. D **89**, 044024 (2014).
[80] M. Shibata, K. Taniguchi, H. Okawa, and A. Buonanno, Coalescence of binary neutron stars in a scalar-tensor theory of gravity, Phys. Rev. D **89**, 084005 (2014).
[81] L. Sampson, N. Yunes, N. Cornish, M. Ponce, E. Barausse, A. Klein, C. Palenzuela, and L. Lehner, Projected constraints on scalarization with gravitational waves from neutron star binaries, Phys. Rev. D **90**, 124091 (2014).
[82] N. Sennett and A. Buonanno, Modeling dynamical scalarization with a resummed post-Newtonian expansion, Phys. Rev. D **93**, 124004 (2016).
[83] M. Khalil, N. Sennett, J. Steinhoff, and A. Buonanno, Theory-agnostic framework for dynamical scalarization of compact binaries, Phys. Rev. D **100**, 124013 (2019).
[84] T. Damour and K. Nordtvedt, General Relativity as a Cosmological Attractor of Tensor Scalar Theories, Phys. Rev. Lett. **70**, 2217 (1993).
[85] T. Damour and K. Nordtvedt, Tensor-scalar cosmological models and their relaxation toward general relativity, Phys. Rev. D **48**, 3436 (1993).
[86] D. Anderson, N. Yunes, and E. Barausse, Effect of cosmological evolution on Solar System constraints and on the scalarization of neutron stars in massless scalar-tensor theories, Phys. Rev. D **94**, 104064 (2016).
[87] D. Anderson and N. Yunes, Solar System constraints on massless scalar-tensor gravity with positive coupling constant upon cosmological evolution of the scalar field, Phys. Rev. D **96**, 064037 (2017).
[88] R. F. P. Mendes, Possibility of setting a new constraint to scalar-tensor theories, Phys. Rev. D **91**, 064024 (2015).
[89] R. F. P. Mendes and N. Ortiz, Highly compact neutron stars in scalar-tensor theories of gravity: Spontaneous scalarization versus gravitational collapse, Phys. Rev. D **93**, 124035 (2016).
[90] G. Antoniou, L. Bordin, and T. P. Sotiriou, Compact object scalarization with general relativity as a cosmic attractor, Phys. Rev. D **103**, 024012 (2021).