A Hölder-continuous Extended State Observer for Model-free Position Tracking Control

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***Abstract*— Position tracking control in three spatial dimensions in the presence of unknown or uncertain dynamics, is applicable to unmanned aerial, ground, (under)water and space vehicles. This work gives a new approach to model-free position tracking control by designing an extended state observer to estimate the states and the uncertain dynamics, with guaranteed accuracy of estimates. The estimated states and uncertainties can be used in a control scheme in real-time for position tracking control. The uncertainty (disturbance input) estimate is provided by an extended state observer (ESO) that is finite-time stable (FTS), to provide accuracy and robustness. The ideas of homogeneous vector fields and real-valued functions are utilized for the ESO design and to prove FTS. The estimated disturbance is then utilized for compensation of this uncertainty in real-time, and to enhance the stability and robustness of the feedback tracking control scheme.**

I. Introduction

The majority of linear and nonlinear control approaches are model-based, for which a model of the dynamics of the system being controlled is necessary. However, as the complexities of control systems continue to increase, uncertainties and difficulties in modeling these systems become increasingly important. This has led to increased research interest and advancement in data-driven control schemes that do not require accurate dynamics models for the systems being controlled. But guaranteed stability and robustness of data-driven control schemes require stable and robust state and uncertainty observers. This work develops an extended state observer for estimating the states and uncertainties of a system that can be used for data-driven control of the system.

The idea of obtaining an estimate of an unknown disturbance, and then utilizing it in the control scheme to compensate such a disturbance, can be traced back to the work [1] in 1956. In the last 15 years, data-driven control for uncertain systems has been used in different senses and settings in the published literature following this paradigm of "learning" the unknown dynamics. These settings are quite varied, and range from "classic" PIDs to feedback control using techniques from, e.g., neural nets, fuzzy logic, and soft computing to learn the uncertainties in the dynamics [2]–[6]. A data-driven output tracking control framework based on classical control techniques, termed the "intelligent PID" (or "iPID") scheme, was proposed by Fliess and Join in [7], [8]. This "iPID" scheme has already been successfully implemented onto different control practical scenarios, such as exoskeletons and micro-aircraft [9], [10]. In addition, if the output (measurement) model is known and the system is differentially flat [11], then a state trajectory can also be tracked and uncertainties in the input-state dynamics can be estimated over time using classical filtering techniques [7], [12]. Some applications where similar model-free control techniques have been considered are given in, e.g., [13]–[16]. More recently, a data-enabled predictive control ("DeePC") method was formulated for data-driven control of unmodeled/uncertain systems that is analogous to the classical model-predictive control (MPC) technique for model-based control of linear systems in [17]. Data-driven control algorithms that use disturbance or uncertainty observers and maintain input constraints have also been treated, e.g., in [18], [19]. Other recent data-driven control schemes based on linear systems theory include [20], [21]. However, the weak link in most data-driven control schemes is lack of guaranteed stability of the overall feedback compensation loop for the unknown/uncertain dynamics. Note that ensuring stability of this compensation, is critical to ensuring the stability and reliability of the entire data-driven control system.

To address this shortfall, a set of prior work focuses on applying the "disturbance/uncertainty observer" idea to ensure the stability and robustness of disturbance estimation, and then utilize the estimated disturbance in the control law. In recent years, the methodology of Extended State Observers (ESO) has been used to estimate disturbance inputs or uncertainties in a dynamical system. In an ESO, the state estimation process is augmented so that the disturbances or uncertainties that cannot be measured directly, are made part of the estimation scheme. Furthermore, the convergence rate of estimation error of the measured states can contribute to the convergence of the disturbance estimation errors for inputs that cannot be measured directly. Prior literature on this topic has obtained disturbance observers and ESO that are finite-time stable or exponentially stable [22]–[25], by applying sliding mode techniques and the signum function $\mathrm{sgn}(\cdot)$ to the estimation process. However, if such estimated disturbances are directly utilized as the term for compensation of the external disturbance, there might arise performance issues due to the slide-mode structure, which can cause harmful chattering (oscillations) in the feedback loop.

Inspired by the work [24], this research gives an FTS ESO design to estimate the disturbance during flight of a multirotor aircraft in translational dimensions. The idea of homogeneous function is utilized in the ESO design. The signum function is not used in the ESO design to avoid harm-

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ful chattering and oscillations due to measurement noise, which is always present. Moreover, a control scheme using the estimated disturbance for compensation, is proposed here. A simulation work is carried out to show validity of the ESO design and the proposed control scheme.

The remainder of this paper is organized as follows. Section II outlines the problem formulation. The position kinematics and dynamics model of a vehicle moving in three spatial dimensions is provided. The rigid body tracking control and disturbance observer problem on $\mathbb{R}^3$ are set up separately based on the kinematics and dynamics. Section III finds the observer law for the disturbance observer. Lyapunov stability analysis for the disturbance observer is carried out in Section III. A tracking control scheme based on this observer is presented in Section IV. Numerical simulation results of this Hölder-continuous control law obtained by using Matlab/Simulink, are presented in Section V. The concluding section VI provides a summary of results presented, and mentions related research directions to be pursued in the near future.

II. Preliminaries and Problem Formulation

A. Preliminaries

We start with a few preliminary results and definitions given in this subsection.

Definition 1: [26] [27] A real-valued function $V:\mathbb{R}^n\mapsto\mathbb{R}$ is called homogeneous of degree d with respect to weights $\{r_i{>}0\}_{i=1}^n$, if for all $\lambda>0$ and for all $(x_1,x_2,...,x_n)^{\mathrm{T}}\in\mathbb{R}^n$, there is :

$$V(\lambda^{r_1}x_1,\lambda^{r_2}x_2,...,\lambda^{r_n}x_n)=\lambda^d V(x_1,x_2,...,x_n) \quad (1)$$

Definition 2: [26] [27] A vector $F:\mathbb{R}^n\mapsto\mathbb{R}^n$ is called homogeneous of degree d with respect to weights $\{r_i{>}0\}_{i=1}^n$, if for all $\lambda>0$ and for all $(x_1,x_2,...,x_n)^{\mathrm{T}}\in\mathbb{R}^n$, there is :

$$\begin{aligned}&F_i(\lambda^{r_1}x_1,\lambda^{r_2}x_2,...,\lambda^{r_i}x_i,...,\lambda^{r_n}x_n)\\&\quad=\lambda^{d+r_i}F_i(x_1,x_2,...,x_i,...,x_n)\end{aligned} \quad (2)$$

If $V:\mathbb{R}^n\mapsto\mathbb{R}$ satisfies (1) and is differentiable with respect to x_i, then the partial derivative of V in x_i satisfies

$$\begin{aligned}&\lambda^{r_i}\frac{\partial}{\partial x_i}V(\lambda^{r_1}x_1,\lambda^{r_2}x_2,...,\lambda^{r_i}x_i,...,\lambda^{r_n}x_n)\\&\quad=\lambda^d\frac{\partial}{\partial x_i}V(x_1,x_2,...,x_i,...,x_n)\end{aligned} \quad (3)$$

With the knowledge that V is homogeneous, eq. (3) can be conveniently used to check the homogeneity of $\frac{\partial V}{\partial x_i}$.

Theorem 1: [27] *Suppose V_1 and V_2 are continuous real-valued functions on $\mathbb{R}^n$, homogeneous with respect to ν of degrees $l_1>0$ and $l_2>0$, respectively, and V_1 is positive definite. Then for every $x\in\mathbb{R}^n$, it holds that:*

$$\begin{aligned}&[\min_{z:V_1(z)=1}V_2(z)][V_1(x)]^{\frac{l_2}{l_1}}\le V_2(x)\\&\quad\le[\max_{z:V_1(z)=1}V_2(z)][V_1(x)]^{\frac{l_2}{l_1}}.\end{aligned} \quad (4)$$

Theorem 2: [28] *Let f be a vector field satisfying $f\in C(\mathbb{R}^n,\mathbb{R}^n)$, $f(0)=0$, f is homogeneous:*

$$\begin{aligned}&\forall\lambda>0,\ f_i(\lambda^{r_1}x_1,\lambda^{r_2}x_2,...,\lambda^{r_i}x_i,...,\lambda^{r_n}x_n)\\&\quad=\lambda^{\tau+r_i}f_i(x_1,x_2,...x_i,...,x_n)\end{aligned} \quad (5)$$

and the trivial solution $x=0$ of system $\dot{x}=f(x)$ is locally asymptotically stable. Then let p be positive integer, and k be a real number larger than $p\cdot\max_{1\le i\le n}r_i$. There exists a function $V:\mathbb{R}^n\mapsto\mathbb{R}$ such that:
(i) $V\in C^p(\mathbb{R}^n,\mathbb{R})\cap C^\infty(\mathbb{R}^n\backslash\{0\},\mathbb{R})$;
(ii) $V(0)=0, V(x)>0$ for all $x\neq 0$ and $V(x)\mapsto+\infty$ as $||x||\mapsto+\infty$;
(iii) V is homogeneous: $\forall\lambda>0$,

$$V(\lambda^{r_1}x_1,\lambda^{r_2}x_2,...,\lambda^{r_n}x_n)=V^k f_i(x_1,x_2,...,x_n)$$

(iv) $\forall x\neq 0, \nabla V(x)\cdot f(x)<0$.
These results are used in the FTS ESO design in Section III.

B. System Kinematics and Dynamics

The dynamics model that we use is that of a multirotor UAV in translational motion represented in the inertial frame, and is based on eq. (1) in [29]. In this model, actuator dynamics (such as rotor dynamics and blade flapping), and ground effect, are accounted for as unknown dynamics as follows:

$$\begin{cases}\dot{\eta}=\mu,\\ M\dot{\mu}=-C\mu+Mg+\tau_d+\tau,\end{cases} \quad (6)$$

where $\eta\in\mathbb{R}^3$ and $\mu\in\mathbb{R}^3$ denote the position and velocity of the vehicle in inertial frame $\mathcal{I}$, M is the mass of the UAV, C is the damping (drag) coefficient which is unknown, $g\in\mathbb{R}^3$ is the gravitational acceleration vector, τ_d is the unmodeled and unknown (disturbance) dynamics, and τ is the control force acting on the UAV.

For the disturbance observer design and the control scheme design based on this observer, the dynamics model is simplified from (6) as follows:

$$\begin{cases}\dot{\eta}=\mu,\\ \dot{\mu}=\sigma+g+M^{-1}\tau,\ \text{and}\\ \dot{\sigma}=\delta.\end{cases} \quad (7)$$

The term $\tau_d-C\mu$ is replaced by the term σ, which contains the resultant of the unknown dynamics involved in the flight dynamics. The following assumption is made for this unknown dynamics acting on the vehicle.

Assumption 1: The rate of σ, $\dot{\sigma}$, is unknown but bounded, and satisfies the inequality: $\|\dot{\sigma}\|\le\delta$. Further, $\bar{\delta}$ denotes a known upperbound on δ.

With the dynamics model (11), the disturbance estimation problem for the term σ and a tracking control problem for η and μ are formulated. These are described next in Sections III and IV respectively.

III. Finite-time Stable Extended State Observer (FTSESO)

The disturbance observer design with finite-time stability is described in details in this section, along with a stability analysis. The disturbance observer is formed as an ESO and the estimation errors are defined as follows:

$$\tilde{\eta}=\eta-\hat{\eta},\ \tilde{\mu}=\mu-\hat{\mu},\ \tilde{\sigma}=\sigma-\hat{\sigma},$$

which are the position, velocity, and unknown disturbance estimation errors respectively.

Proposition 1: The Hölder-continuous extended state observer equations for a point mass with the kinematics and dynamics (7) and with position and velocity measurements, is of the following form:

$$\begin{cases} \dot{\hat{\eta}}=\hat{\mu}+k_1(\psi^T\psi)^{\frac{\alpha-1}{2}}\psi, \\ \dot{\hat{\mu}}=\hat{\sigma}+g+M^{-1}\tau+\frac{1}{\kappa}\tilde{\mu}+\frac{k_2}{\kappa}(\psi^T\psi)^{\frac{\alpha-1}{2}}\psi, \\ \dot{\hat{\sigma}}=k_3(\psi^T\psi)^{\alpha-1}\psi, \end{cases} \tag{8}$$

where $\frac{1}{2}<\alpha<1$.

Note that unlike most of the FTSESO designs that are FTS because they use the signum function and variable structure design, the proposed FTSESO (8) is Hölder-continuous near the origin.

With (7), (8) and the estimation errors defined earlier, the error dynamics of the FTSESO are given by:

$$\begin{cases} \dot{\tilde{\eta}}=\tilde{\mu}-k_1(\psi^{\mathrm{T}}\psi)^{\frac{\alpha-1}{2}}\psi, \\ \dot{\tilde{\mu}}=\tilde{\sigma}-\frac{1}{\kappa}\tilde{\mu}-\frac{k_2}{\kappa}(\psi^{\mathrm{T}}\psi)^{\frac{\alpha-1}{2}}\psi, \\ \dot{\tilde{\sigma}}=-k_3(\psi^{\mathrm{T}}\psi)^{\alpha-1}\psi+\delta. \end{cases} \tag{9}$$

To reduce the difficulty of proving the stability of (9), term δ in (9) is neglected temporarily. The following theorem considers the stability of the auxiliary system obtained from the above error dynamics without the term δ.

Theorem 3: The auxiliary system given below by (10) is globally finite time stable (FTS) at $(\tilde{\eta}^T,\tilde{\mu}^T,\tilde{\sigma}^T)^T=0\in\mathbb{R}^9$*:*

$$\begin{cases} \dot{\tilde{\eta}}=\tilde{\mu}-k_1(\psi^T\psi)^{\frac{\alpha-1}{2}}\psi, \\ \dot{\tilde{\mu}}=\tilde{\sigma}-\frac{1}{\kappa}\tilde{\mu}-\frac{k_2}{\kappa}(\psi^T\psi)^{\frac{\alpha-1}{2}}\psi, \\ \dot{\tilde{\sigma}}=-k_3(\psi^T\psi)^{\alpha-1}\psi. \end{cases} \tag{10}$$

Proof: The proof of this result is based on the main results of [28] (given by Theorem 2) and [27] (given by Theorem 1), both stated in Section II. Define the vector: $z=(\psi^{\mathrm{T}},\tilde{\sigma}^{\mathrm{T}})^{\mathrm{T}}\in\mathbb{R}^6$ and $f(\psi,\tilde{\sigma})\in\mathbb{R}^6$, such that the stability proof of (10) can be reduced to the stability proof of:

$$\dot{z}=(\dot{\psi}^{\mathrm{T}},\dot{\tilde{\sigma}}^{\mathrm{T}})^{\mathrm{T}}=f(\psi,\tilde{\sigma}) \tag{11}$$

Carry out the linear combination of the first and second equation in (10) with coefficients $\{1,\kappa\}$. Then the auxiliary system $(\dot{\psi}^{\mathrm{T}},\dot{\tilde{\sigma}}^{\mathrm{T}})^{\mathrm{T}}=f(\psi,\tilde{\sigma})$ can be expressed in the following form:

$$\begin{cases} \dot{\psi}=\kappa\tilde{\sigma}-(k_1+k_2)(\psi^{\mathrm{T}}\psi)^{\frac{\alpha-1}{2}}\psi \\ \dot{\tilde{\sigma}}=-k_3(\psi^{\mathrm{T}}\psi)^{\alpha-1}\psi \end{cases} \tag{12}$$

Define the following Lyapunov candidate $V_a(\psi,\tilde{\sigma})$ for the stability analysis of (11):

$$V_a(\psi,\tilde{\sigma})=\frac{1}{2}[\frac{k_3}{\alpha\kappa}(\psi^{\mathrm{T}}\psi)^{\alpha}+\tilde{\sigma}^{\mathrm{T}}\tilde{\sigma}] \tag{13}$$

The Lie derivative of $V_a(\psi,\tilde{\sigma})$ along the vector field $f(\psi,\tilde{\sigma})$, L_fV_a, is obtained as follows:

$$\begin{aligned} \dot{V}_a(\psi,\tilde{\sigma})=L_fV_a&=\frac{k_3}{\alpha\kappa}\alpha(\psi^{\mathrm{T}}\psi)^{\alpha-1}\psi^{\mathrm{T}}\dot{\psi}+\tilde{\sigma}^{\mathrm{T}}\dot{\tilde{\sigma}} \\ &=\frac{k_3}{\alpha\kappa}\alpha\kappa(\psi^{\mathrm{T}}\psi)^{\alpha-1}\psi^{\mathrm{T}}\tilde{\sigma}-\frac{k_3}{\kappa}(k_1+k_2)(\psi^{\mathrm{T}}\psi)^{\frac{\alpha+1}{2}} \\ &-k_3(\psi^{\mathrm{T}}\psi)^{\alpha-1}\tilde{\sigma}^{\mathrm{T}}\psi \\ &=-\frac{k_3}{\kappa}(k_1+k_2)(\psi^{\mathrm{T}}\psi)^{\frac{\alpha+1}{2}}\leq 0. \end{aligned} \tag{14}$$

From eq. (14) and the invariance principle, we see that the auxiliary system (11) is asymptotically stable. According to Definition 2, it can also be verified that $f(\psi,\tilde{\sigma})$ is a homogeneous vector field of degree $\alpha-1$ with respect to the vector of weights $\{1,\alpha\}$. According to Theorem 2, there exists a homogeneous Lyapunov function $V_b(\psi,\tilde{\sigma})$ of degree $\gamma\in\mathbb{N}$ such that $L_fV_b=\nabla V_b(\psi,\tilde{\sigma})\cdot f(\psi,\tilde{\sigma})<0$. Although V_b is not given explicitly by this theorem, through Proposition 2 in [28], V_b can be obtained from V_a. According to Definition 1, the real-valued function $L_fV_b=\nabla V_b(\psi,\tilde{\sigma})\cdot f(\psi,\tilde{\sigma})$ is homogeneous of degree $\gamma+\alpha-1$.

$$\begin{aligned} \dot{V}_b(\psi,\tilde{\sigma})&=\frac{\partial V_b}{\partial\psi}[\kappa\tilde{\sigma}-(k_1+k_2)(\psi^{\mathrm{T}}\psi)^{\frac{\alpha-1}{2}}\psi] \\ &+\frac{\partial V_b}{\partial\tilde{\sigma}}[-k_3(\psi^{\mathrm{T}}\psi)^{\alpha-1}\psi] \\ &<-c_1V_b^{\frac{\gamma+\alpha-1}{\gamma}}. \end{aligned} \tag{15}$$

Further, according to Theorem 1, we get:

$$c_1=-\max_{\{\psi,\tilde{\sigma}:V_b(\psi,\tilde{\sigma})=1\}}\nabla V_b(\psi,\tilde{\sigma})\cdot f(\psi,\tilde{\sigma}). \tag{16}$$

Note that $\nabla V_b(\psi,\tilde{\sigma})\cdot f(\psi,\tilde{\sigma})$ is negative definite, and therefore $c_1>0$. Since $\frac{1}{2}<\alpha<1$, the auxiliary system (11) is proved to be FTS at the origin $(\psi,\tilde{\sigma})=(0,0)$. ■

Now consider the disturbance term δ, and the disturbance observer error dynamics (9) expressed as follows:

$$\begin{cases} \dot{\psi}=\kappa\tilde{\sigma}-(k_1+k_2)(\psi^{\mathrm{T}}\psi)^{\frac{\alpha-1}{2}}\psi, \\ \dot{\tilde{\sigma}}=-k_3(\psi^{\mathrm{T}}\psi)^{\alpha-1}\psi+\delta. \end{cases} \tag{17}$$

Theorem 4: When $V_b>(\frac{c_2\bar{\delta}}{c_1})^{\frac{\gamma}{2\alpha-1}}$*, where* c_1 *is defined previously in (16) and* c_2 *is defined as:*

$$c_2=\max_{\{\psi,\tilde{\sigma}:V_b(\psi,\tilde{\sigma})=1\}}|\frac{\partial V_b}{\partial\tilde{\sigma}}|, \tag{18}$$

the error dynamics (17) is FTS.

Proof: Use the Lyapunov function V_b in Theorem 3 for the observer error dynamics. Note that V_b is homogeneous of degree γ with respect to $\{1,\alpha\}$. According to Definition 1 and 2, it can be further concluded that $|\frac{\partial V_b}{\partial\tilde{\sigma}}|$ is homogeneous of degree $\gamma-\alpha$ with respect to $\{1,\alpha\}$. Applying Theorem 1, the following inequality holds:

$$|\frac{\partial V_b}{\partial \tilde{\sigma}}| < c_2 V_b^{\frac{\gamma-\alpha}{\gamma}}, \quad c_2 > 0. \tag{19}$$

Consider again the disturbance observer error dynamics (17). The time derivative of V_b satisfies:

$$\begin{aligned}\dot{V}_b(\psi,\tilde{\sigma}) &= \frac{\partial V_b}{\partial \psi}[\kappa\tilde{\sigma} - (k_1+k_2)(\psi^{\mathrm{T}}\psi)^{\frac{\alpha-1}{2}}\psi] \\ &+ \frac{\partial V_b}{\partial \tilde{\sigma}}[-k_3(\psi^{\mathrm{T}}\psi)^{\alpha-1}\psi] + \frac{\partial V_b}{\partial \tilde{\sigma}}\delta \\ &< -c_1 V_b^{\frac{\gamma+\alpha-1}{\gamma}} + |\frac{\partial V_b}{\partial \tilde{\sigma}}||\delta| \\ &< -c_1 V_b^{\frac{\gamma+\alpha-1}{\gamma}} + c_2 V_b^{\frac{\gamma-\alpha}{\gamma}}\bar{\delta} \\ &= (-c_1 V_b^{\frac{2\alpha-1}{\gamma}} + c_2\bar{\delta}) V_b^{\frac{\gamma-\alpha}{\gamma}}\end{aligned} \tag{20}$$

From (20), when the Lyapunov function V_b satisfies:

$$\begin{aligned}-c_1 V_b^{\frac{2\alpha-1}{\gamma}} + c_2\bar{\delta} < 0 \text{ or} \\ V_b > (\frac{c_2\bar{\delta}}{c_1})^{\frac{\gamma}{2\alpha-1}},\end{aligned} \tag{21}$$

we conclude that the error dynamics (17) is FTS. This concludes our proof. ■

It can be seen from this result that outside a neighbourhood of the origin, the error dynamics (17) is FTS.

IV. Finite-time Tracking Controller Based on Disturbance Observer

The FTS position tracking control scheme described in [30] is reproduced here with the estimated disturbance implemented inside the control scheme design for disturbance compensation. Define η_d and μ_d as the desired position and velocity and it can be derived that $\dot{\eta}_d = \mu_d$. Define the position tracking control error as $e_\eta = \eta - \eta_d$ and $e_\mu = \mu - \mu_d$. Based on the defined tracking error, the tracking error dynamics generated based on (7) is shown as follows:

$$\begin{aligned}\dot{e}_\eta &= e_\mu \\ \dot{e}_\mu &= \sigma + g + M^{-1}\tau - \dot{\mu}_d\end{aligned} \tag{22}$$

Proposition 2: [30] *Define* $E_\eta = \frac{e_\eta}{(e_\eta^T e_\eta)^{1-\frac{1}{p}}}$ *and consider the feedback control law as follows:*

$$\begin{aligned}\tau &= -M(\hat{\sigma} + g - \dot{\mu}_d + \lambda\dot{E}_\eta + \lambda e_\eta) \\ &- \frac{\lambda M P(e_\mu + \lambda E_\eta)}{((e_\mu + \lambda E_\eta)^T P(e_\mu + \lambda E_\eta))^{1-\frac{1}{p}}}.\end{aligned} \tag{23}$$

This stabilizes the error dynamics (22) *with FTS.*

The proof of FTS of the control scheme without the term $-\hat{\sigma}$ has already been given in [30]. Note that $\tilde{\sigma} = \sigma - \hat{\sigma}$ is proved to be FTS outside a given neighbourhood of the origin by Theorem 4 in the previous section. Therefore, it leads to FTS of the control scheme using the disturbance observer for compensation of the unknown (disturbance) dynamics.

V. Numerical Simulations

This section presents two simulated multirotor flights for two position tracking control schemes. One is the FTS position tracking control scheme that is presented in Theorem 1 in [30]. The other is the control scheme of Proposition 2, which is the FTS position tracking control scheme with the estimated disturbance obtained from the FTSESO. Given exactly the same dynamics model, control parameters, disturbance and desired trajectories, other than the term containing the estimated disturbance for compensation, the latter control scheme is expected to show better performance compared with the former one. This comparison is intended to show the validity of the proposed ESO and control scheme. The simulation is carried out using MATLAB/Simulink. The model parameters and the external disturbance force τ_d are selected as follows:

$$M = 2, C = \text{diag}[0.01, 0.01, 0.01], \ g = [0.0, 0.0, 9.8]^{\mathrm{T}},$$
$$\text{and } \tau_d = [6\sin(t); \ 4\cos(t); \ 8\sin(t)\cos(t)],$$

where t denotes time. Control system gain parameters are selected as follows:

$$p = 1.2, \ P = \text{diag}[12, \ 12, \ 12], \ \text{and } \lambda = 1.8.$$

Finally, the FTSESO gain parameters are selected as follows:

$$\alpha = 0.8, \kappa = 5.0, k_1 = 10, k_2 = 30, \text{ and } k_3 = 100.$$

The desired trajectory is generated as

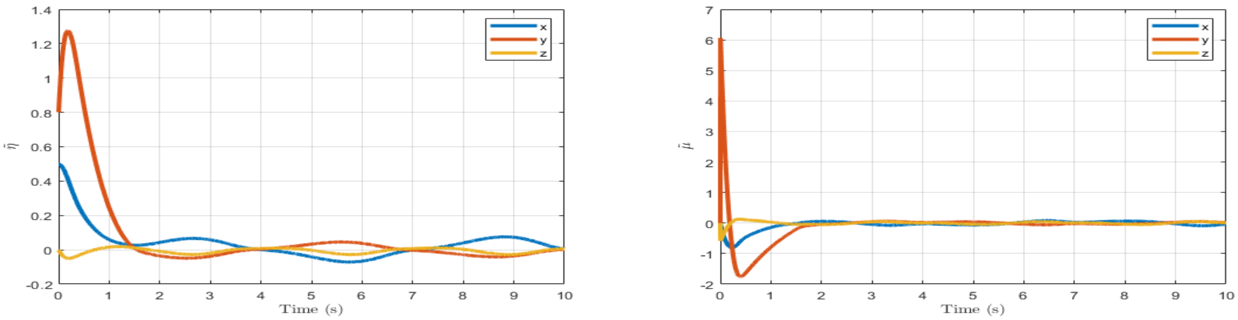
$$\eta_d = [5\cos(t); 5\sin(t); -0.5\,t]. \tag{24}$$

From Fig. 3, it can be seen that the proposed estimation scheme can successfully estimate the unmodeled disturbance input with high accuracy. The disturbance estimation error converges to a small neighbourhood near the origin.

For the comparison between the two tracking control schemes, it can be seen that compared with the tracking control error in Fig. 1 the tracking control error in Fig. 2 converges to a smaller neighbourhood of the origin, both for velocity and position tracking. This confirms our expectations of the greater robustness of the model-free (data-driven) control scheme using the FTSESO, when compared to a control scheme that does not use such a ESO for disturbance compensation.

VI. Conclusion and Future Works

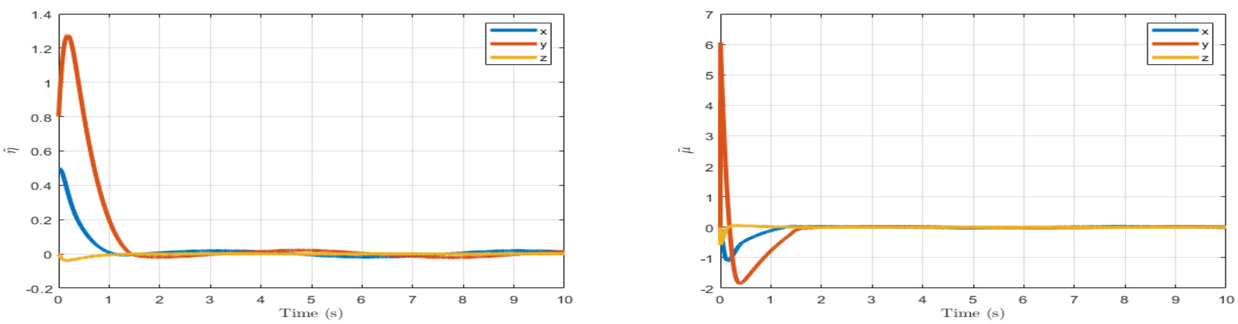
This research proposes a new type of extended state observer with finite time stability, to estimate unknown (disturbance) forces affecting the translational dynamics of a rotorcraft aircraft during the flight. The finite-time stability of the proposed ESO is established using Theorems 2 and 1 based on the use of geometric homogeneity. With the proposed ESO, the estimated disturbance is implemented as a term in the control scheme to compensate for this disturbance force during the flight. The simulation results show the validity of the proposed ESO and its efficiency in disturbance compensation when combined with the control scheme design. In the near future, a finite-time stable ESO



(a) position tracking error

(b) velocity tracking error

Fig. 1: Tracking control error without disturbance compensation from ESO.



(a) position tracking error

(b) velocity tracking error

Fig. 2: Tracking control error with disturbance compensation from ESO.

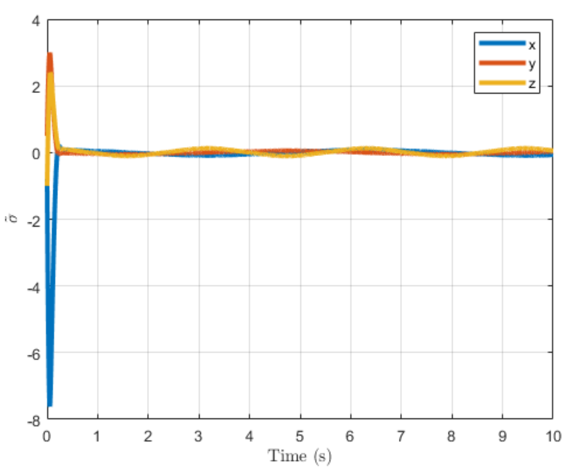


Fig. 3: FTSESO estimation error.

for attitude kinematics and dynamics will be investigated and used for attitude compensation. Furthermore, methods for tuning this ESO and improving its stability abd robustness properties will appear in future publications.

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