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Planning a research lesson online: pre-service teachers' documentation work

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Abstract

Purpose – The authors developed a lesson study innovation for bridging pre-service teachers' experiences in an early methods course and clinical experiences focusing on the development of technological pedagogical content knowledge (TPACK). The authors analyze one planning meeting by a lesson study team comprised of four pre-service teachers and one cooperating teacher. The purpose of this research was to determine the nature of documentation during the online planning meeting and how the cooperating teacher facilitated the documentation process.

Design/methodology/approach – The authors used Gueudet and Trouche's (2009) documentation framework to determine the way the lesson study team in our study used all the resources available to plan a lesson. They analyzed the video recordings of the meeting to examine the interplay between material, didactical and mathematical components during the discussions. The material components included the Teacher Desmos Activity Builder and the eTextbook. The didactical components included assessment, scaffolding, multiple representations and problem-solving activities. The mathematical components pertained to systems of linear equations and inequalities with two variables.

Findings – The authors' findings show that the cooperating teacher performed an invariant set of actions for improving the research lesson and, also, gave recommendations about how to implement the lesson. In facilitating the planning discussions, the cooperating teacher made explicit the relationship between material, didactical and mathematical components. The authors' work has implications for supporting the preparation of facilitators of online planning sessions during lesson study.

Research limitations/implications – The authors did not have access to the planning meeting where the PSTs created the draft of the research lesson. In addition, they are reporting the observations of only one online meeting.

Originality/value – The authors' work has implications for supporting the preparation of facilitators of online planning sessions during lesson study.

Keywords Online lesson study, Planning in lesson study, Research lesson, Documentation, Mathematics pre-service teachers, Cooperating teachers

Paper type Research paper

The process of planning a research lesson is a core lesson study (LS) activity that anchors future activities in the cycle. Teachers' busy schedules can make it difficult to participate in the planning process during the school day. Online environments offer opportunities to

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overcome the logistical challenges of scheduling face-to-face planning meetings. However, issues about how to develop shared instructional materials and how to facilitate LS online are open questions. A careful examination of how the planning process happens online can illuminate an understanding of supports needed for conducting LS online.

Our work is situated in the context of mathematics pre-service teacher education in Puerto Rico as a part of a funded project aiming at improving mathematics teacher education. We developed a LS innovation for bridging pre-service teachers' [1] (PSTs) experiences in an early methods course and clinical experiences focusing on the development of *technological pedagogical content knowledge* (TPACK). In mathematics education, TPACK refers to the intersection of three types of knowledge: mathematical, technological and pedagogical. Researchers have provided a detailed model of TPACK including its components (Rosenberg and Koehler, 2015). Harris *et al.* (2009) recommend "using the technology, pedagogy and content knowledge (TPACK) framework as a way to think about effective technology integration, recognizing technology, pedagogy, content and context as interdependent aspects of teachers' knowledge necessary to teach content-based curricula effectively with educational technologies" (p. 393). Clinical experiences are common in teacher education programs and consist of PSTs' school-based visits under the supervision of an experienced teacher, called cooperating teacher, with the purpose of apprenticing into the work of teaching. We focus on one LS team led by one cooperating teacher (CT) with PSTs. The team planned in-person meetings, however, due to scheduling conflicts, the team held one online planning meeting through Microsoft Teams. We ask: (1) *What was the nature of documentation during the online planning meeting?* (2) *How did the cooperating teacher facilitate the documentation work?* By documentation, we refer to the framework proposed by Gueudet and Trouche (2009) about using various resources to develop a scheme for planning a lesson.

Our questions address the problem of facilitating lesson study in an online environment since this information could help to design professional development initiatives for qualified LS facilitators as recommended by Huang *et al.* (2020). Although we did not anticipate conducting lesson study online when designing the intervention, the online planning meeting provided us with the opportunity to examine how the traditional LS model translates to an online environment. Recent work has focused on identifying affordances of LS online and our work intends to contribute to this line of research (e.g., Huang *et al.*, 2020; Nickerson *et al.*, 2014; Soto *et al.*, 2019). We are particularly interested in the process of planning a research lesson with PSTs who have limited or no prior experiences with lesson planning and participants with no prior experiences with LS. The examination of interactions between a CT who is also facilitating LS online and PSTs participating in a LS team can be useful for teacher education for future implementation of LS during clinical experiences.

Planning in lesson study

LS is a professional development strategy based on teachers' cooperation that has been traditionally used in China and Japan (Huang *et al.*, 2017; Lewis, 2000; Lewis and Tsuchida, 1998). It was introduced in the US, at the end of the twentieth century (Lewis, 2000; Lewis and Tsuchida, 1998; Murata, 2011). Lewis *et al.* (2009) identified four phases for LS: investigation, planning, research lesson and reflection. The team establishes the focus of the LS and examines current knowledge about the topic they want to study; identifies the learning goals and develops a lesson plan; teaches the lesson developed; collects information about students' thinking; and, reflects and integrates new knowledge from LS into their own practice (Lewis *et al.*, 2019). Optional steps are to revise and re-teach the research lesson to a new group of students (Lewis *et al.*, 2009). LS teams often include the participation of a "knowledgeable other," a more experienced teacher or an expert in the field who provides insights about the research lesson (Takahashi, 2014). Math education researchers started using LS soon after it was introduced in the US and the results of early LS investigations were promising. Fernandez (2005) reported the benefits of LS, along with micro-teaching, on the understanding and development of subject matter knowledge. Since then, there has been a steady growth in research on implementation of LS in teacher education programs (Amador and Carter, 2016; Bieda *et al.*, 2015; da Ponte, 2017; Fernandez and Zilliox, 2011; Matthews *et al.*, 2009). Most of the research about implementations of LS in teacher education programs is motivated by investigating how adaptations to the traditional LS model promote PSTs' learning.

The process of planning the research lesson is fundamental in LS (Fujii, 2016; Lewis *et al.*, 2006). LS contributes to the improvement of instruction since it proposes the enhancement of learning resources such as study plans (Lewis *et al.*, 2009). Fernandez (2002) states, "Work on every study lesson begins by teachers coming together to meticulously plan the lesson as a group. The actual product of this collaborative planning is a written lesson plan that describes in detail the design of the lesson" (p. 394). During the process of planning a lesson, the team engages in various activities including "specifying learning goals, aligning the instructional activities with the learning goals, and anticipating student responses that show achievement of the goals, and then evaluating lessons by examining evidence of students' thinking and learning" (Morris *et al.*, 2009). There is evidence that some LS teams value the process of consulting and examining different resources to plan the research lesson. Takahashi *et al.* (2005) describe the importance of consulting various textbooks to determine different ways to teach a topic and of reading research findings to establish the effect of the diverse teaching methods. Lewis *et al.* (2019) state the importance of continuing to study the standards, the curriculum and relevant research about how students learn a topic when planning a research lesson. Through the collaborative elaboration of the plan for the research lesson, the LS team develops an understanding of how to promote student learning and how to capture evidence of student thinking while observing the implementation of the lesson (Wake *et al.*, 2016). Moreover, in the process of planning the research lesson, the team determines the tasks that will make student thinking evident to the LS team and allow for post-lesson reflections about whether or not the students met the established learning goals.

Previous studies have shown that mathematics PSTs that participated in LS demonstrated significant improvement in the lesson planning process (Cheng and Zhang, 2019). Nevertheless, LS implementations with PSTs require scaffolds to support the planning process. For example, Matthews *et al.* (2009) developed a four-column table for planning the research lesson, including LS aspects within the traditional way of planning in the U.S. The table included columns to describe the learning activities and key questions, the expected students' reactions and responses, the teacher's response to student reactions and the goals and methods of evaluation. Arranging information in this way helped the PSTs to improve their planning skills and student-centered observations.

The LS facilitation is another way to scaffold PSTs' engagement in lesson planning. Some researchers have investigated the role of the knowledgeable other with PSTs. In teacher education programs that rely on LS, a teacher educator, a cooperating teacher, or a university collaborator assumes the role of the knowledgeable other and, also, the facilitator of LS, providing guidance for going through the lesson study cycle effectively (Takahashi, 2014). Fernandez (2010) reports the importance of the knowledgeable other in posing questions about how students will meet the learning goals, offering suggestions for teaching the lesson, and supporting in the reflection and collaborative deliberations. The role of the knowledgeable other has been less documented in the US context and our examination of the work of a cooperating teacher who is both the facilitator of lesson study and the knowledgeable other intends to contribute to this research.

Other researchers have stressed the importance of allowing PSTs to work with curricular materials. Nicol and Crespo (2006) consider that curricular materials help PSTs prepare for planning, stating that, "the textbook offered them a place to begin in their planning of mathematics lessons" (p. 336). Fernandez (2010) reported that PSTs pay attention to hands-on activities and real-world examples from textbooks, Internet resources and references books in the planning process. Overall, providing opportunities for PSTs to rely on curricular materials in the planning process grounds their lessons in authentic school-based contexts.

Recent work has broadened traditional LS implementation by engaging teachers in modified versions of LS that take advantage of online environments. For example, Nickerson *et al.* (2014) report that their development of a hybrid LS model extended face-to-face meetings with teachers' engagement in a web site. The web site discussions enabled the participation in LS of teachers who were in distant locations by discussing examples of student thinking observed in the research lesson. The instructional coach who facilitated LS initiated many of the substantial interactions by focusing the teachers' attention on students' thinking and consolidating the teachers' learning in relation to their participation in the LS cycle. Another example is a group of early career mathematics teacher educators who used LS for their professional development (Soto *et al.*, 2019). Working at different states of the US they created a virtual community of practice to develop an activity in which the PST would analyze samples of students' work and decide the following steps in instruction for an individual student or the whole class. The benefits of the virtual resources included the possibility of joining the class and having a real time discussion, using text messaging, while another team member was teaching. Overall, these LS innovations allow overcoming the logistical challenge of holding face-to-face meetings for planning or observing the research lesson implementation while another team member is teaching, thus broadening opportunities to participate in LS.

Theoretical framework

We use Gueudet and Trouche's (2009) documentation framework to determine the way the LS team in our study used the resources available. According to the framework, there is a difference between resources and documents. A resource is a material or virtual "thing" that is available for teaching. A document is the result of the interplay between the resources available and their scheme of use. By scheme of use, they mean the set of operational invariants executed by the person when performing a process. A scheme is the set of regularities appearing in the teachers' activity across different contexts. Resources evolve to documents due to the changes that the teacher introduces to the resources (Trouche *et al.*, 2020). As a result, the framework helps to describe the way teachers select and use resources during the planning process.

In the analysis of how teachers create a document, the focus is on the material, didactical and mathematical components and the connections between these components. The material components refer to the resources available to teachers when planning such as national standards, state standards, curricular guidelines, textbooks, reference books, students' notebooks, manipulatives, worksheets, computer software and websites. The didactical components refer to teaching strategies and the pedagogical justifications for the strategies or resources used in relation to student learning goals. The mathematical component refers to the mathematical ideas in the lesson, including notions, tasks and procedures (Gueudet and Trouche, 2009).

According to the framework, the identification of the documentation scheme requires observers to describe the process of creating the documents; that is, the way teachers create, select, and modify tasks for the lesson. Teachers' documentation schemes are evident in their decisions when designing new tasks, when stating the reasons for sequencing tasks, and when determining the time spent on tasks. In the analysis of the documentation scheme, observers must attend to the way the teachers use the material, didactical and mathematical components during the creation of a lesson. In our study, the framework allowed us to determine how the CT and the PSTs used the resources available to create collaboratively a document. Additionally, we applied the framework to identify the CT's contributions to the documentation process and how the documentation process contributed to the PSTs' professional learning.

Teacher Desmos Activity Builder

The Teacher Desmos Activity Builder (TDAB) is a web-based program to create learning environments to learn mathematics beyond the traditional graphing calculator capabilities. TDAB (https://teacher.desmos.com) was developed by Desmos, widely known in the mathematics teacher community for its free online graphing calculator. Instructional activities created with the TDAB can include a combination of graphs and animations. Teachers can also use TDAB to design slideshows with online graphic calculators' capabilities. Using TDAB, teachers can design interactive lessons including features such as the use multiple representations (i.e., algebraic, tabular and graphic), various types of assessments (e.g., multiple choice questions, open-ended questions, card sort activities), and options for students to see their peers' answers. Lessons created with the TDAB can be organized as a slideshow, providing students a sequence of tasks where they can graph a function on the Cartesian plane, draw a figure, answer a question or interact with a video or other multimedia resource. One of the most useful features of TDAB is its capability to facilitate online interactions among students and among the teacher and the students. During the lesson, teachers can observe and monitor students' work on a given task, select students' answers and copy these answers to a clipboard for sharing them with the class. This feature can support teachers in the process of selecting and sequencing students' answers in preparation for a whole class discussion where they can display students' answers (Smith and Stein, 2018). For example, teachers can use the TDAB to aggregate students' answers by presenting them simultaneously in a graph. In this way teachers can engage students in discussions that reveal their mathematical understandings. All students' work with the lesson is stored in the company's servers and students and teachers can have access to the lesson later on for further review. Teachers can use TDAB to create activities that promote productive interactions in their classrooms. Ideally, teachers can use TDAB to develop activities that engage students in discussions for analyzing a problem, formulating conjectures, testing the conjectures and building mathematical conclusions.

Research context

This study is situated in Puerto Rico as part of an innovation intended to connect methods courses and clinical experiences through LS. Participants were PSTs enrolled in an early methods course for using technology in mathematics teaching that precedes the traditional methods course for teaching mathematics. The PSTs were in their third or fourth year of a secondary mathematics teacher education program in a large public institution. At that stage in the program, the PSTs have completed most of the required mathematics content courses including Calculus I and II, College Geometry, Statistics and Discrete Mathematics. As a part of the course, the PSTs learned how to use TDAB and a modified version of the Matthews *et al.* (2009) four columns table for planning a lesson [2]. The PSTs were divided in three teams each with the leadership of an experienced CT. Each team engaged in LS to develop and teach a research lesson using interactive technology. LS is not well known in Puerto Rico and interactive online calculators are not used often in classrooms.

We focus on one LS team led by a CT, Mr. Martínez, and four PSTs, Alice, Bruce, Yenny and Yesenia [3]. At the moment of the intervention, Mr. Martínez had ten years of teaching experience and almost six years as CT. Although Mr. Martínez started using TDAB as part of his involvement in this study, he is highly proficient using technology for mathematics instruction. We selected Mr. Martínez to participate in the research for his experience as a math teacher and as a CT.

The team had an initial meeting to review instructional materials where Mr. Martínez notified them of the focus on systems of linear equations as the topic of the research lesson for an Algebra I class with mostly eighth graders. After this initial meeting, the team held two other meetings where they discussed ideas for the research lesson. This study focuses on the fourth meeting where Mr. Martínez reviewed a draft of the research lesson prepared by the PSTs. Since the team had difficulties scheduling a face-to-face meeting, they scheduled an online meeting. The meeting had a duration of 1 h and 36 min. The PSTs had prepared a slide show with the tasks for the lesson using the TDAB. The TDAB platform allowed them to create interactive slides for the lesson that the Algebra students would individually work on during the research lesson. In the meeting, Mr. Martínez led the discussion, assuming the role of LS facilitator, and also editing and vetting the slides. While the plan was for Mr. Martínez to teach the first version of the research lesson, the COVID-19 pandemic affected the implementation of the successive LS steps [4]. In this study, our focus is on the planning process and the interactions between the CT and the PSTs.

The online session was video-recorded using Microsoft Teams. We had access to the audio- and video-recordings of each team member. Additionally, the video showed the slides prepared with the Teacher Desmos Activity Builder (TDAB) and the electronic textbook (eTextbook) for the Algebra I class (Larson *et al.*, 2012), for which Mr. Martínez had provided access as a source for planning the lesson. The video-recording showed how the team members actively edited the slides and how they used the textbook. We also had access to the plan, and the PSTs' version of the research lesson prepared with the TDAB and the reviewed version after the fourth planning meeting. Nevertheless, the video from the online planning meeting was the main data source for this study.

Our analysis of the planning meeting started by preparing a timeline of the meeting to segment the video into intervals of approximately 4 min, noting changes in the activity structure of the session (see Herbst *et al.*, 2011). There were 26 intervals in the meeting. In our second viewing of the video, we focused on identifying components that supported the process of documentation in relation to the theoretical framework by Gueudet and Trouche (2009), namely, materials, didactical and mathematical components and their scheme of use. To record our observations, we used a table divided in three sections. In the first section we described the material component in four columns for (1) a brief description of the material component (i.e., textbook, TDAB feature, notebook, or other resource mentioned), (2) how the material component was used, (3) the purpose of using the material component and (4) the CT's observations or reactions. The second section included descriptions of the didactical component with two columns to denote (1) the pedagogical moves and (2) a description of how the move was used. The third section was used to describe the mathematical component in three columns for (1) the type of math component (i.e., mathematical concept or procedure), (2) a brief description of the math component and (3) how the component was represented. We watched the video independently and produced a list of the components used in each interval and the CT's and PSTs' ways of use. Each observer coded two of components in each interval. The first author compiled a list of the components noted, aggregating the results from each author's list (Table 1).

In a subsequent research meeting, we discussed the components, clarifying the identification of didactical components in relation to the discussions during the planning session in the video. For example, we had questions as to whether the initial slides were meant

Table 1.
Coding for interval No. 9

Documentation component	Observations
Material component	TDAB; e-Textbook
Resource	TDAB slide show card and vocab cards
How was the resource used?	The team is creating an activity for students to match a term with its definition
Purpose	To review earlier concepts
CT's reactions or observations	CT makes references to the definitions provided in the textbook CT writes definitions on cards CT controls the discourse
Didactical component	CT says that the definitions must be simple
Pedagogical move	CT provides expectations for the activity and makes sure that all of the relevant terms are included The CT checks if there is redundant information and tells PSTs which concepts are important
How was the move used?	Stating important concepts is key because the CT helps PSTs to make instructional decisions according to terms most relevant to the topic CT shows that it is possible to match more than two cards in the sort card activity
Mathematical Component	System of equations and systems of inequalities
Description	Mathematical terms related to the mathematical content (linear equation, solution of an equation, ordered pair, slope, inequality, symbols ($<, \leq, =, \geq, >$)
Representation	They started with a verbal representation CT shows how to include mathematical representations

to scaffold students' work on the task or review prior knowledge about terms pertaining to linear functions such as graphic representations, slope and intercepts. We decided to label them as "scaffolding" since the main purpose was to establish the vocabulary to be used in the lesson. We also produced detailed annotations with descriptions about how the team members were using the various components for crafting the research lesson. We paid careful attention to the CT's comments and his editions. We recorded the CT's interactions in a separate column, describing Mr. Martínez's actions for making explicit the documentation process during the planning meeting.

The next layer of analysis pertained to the connections between the various components to identify the scheme of utilization. Looking across the table that we generated, we identified whether the team's audible discussions included explicit connections between the materials, the mathematical and didactical components. We produced a visual representation with nodes for specific components and links for explicit connections. The visual representation helped us to examine the process of documentation in relation to the purposeful use of various components for planning the research lesson.

Findings

We start by answering the first research question regarding the nature of documentation during the online planning meeting of the research lesson for the LS cycle. Prior to the online planning meeting, the team members had agreed that the PSTs would develop a draft of a lesson with tasks for reviewing math concepts related to systems of linear equations and inequalities with two variables. At the beginning of the planning meeting, the PSTs shared a lesson created with the TDAB that included a sorting card review activity and fourteen tasks. The tasks included open-ended questions (11), closed questions (1), and multiple-choice exercises (2). The purpose of the tasks was to assess students' [5] knowledge of definitions, algebraic procedures and strategies for solving word problems. Table 2 includes the description of the tasks that the PSTs brought to the meeting.

(1)	Card sorting
(2)	Write a standard linear equation in slope-intercept form
(3)	Graph a system of linear equations and find the solution
(4)	Name the solution of a system of linear equations given its graph
(5)	Name the strategies to solve a system of linear equations
(6)	Solve a system of linear equations given in standard form (multiple choice)
(7)	Solve a system of linear equations by elimination (two exercises, one of them multiple choice)
(8)	Solve a system of linear equations by substitution
(9)	Solve a verbal problem using a system of linear equations
(10)	Estimate the solution of a system of linear equations given its graph
(11)	Write the definition of system of linear inequalities
(12)	Graph a system of linear inequalities to find some solutions
(13)	Solve a verbal problem using a system of linear inequalities
(14)	Determine if a point is solution of a system of linear inequalities

Table 2.
Tasks that the PSTs brought to the planning meeting

The PSTs used various TDAB features to develop each task, such as the inclusion of graphs, interaction screens and the mathematical text editor. The tasks were composed of two screens: one where the problem was stated, in the upper part of the screen, and the other, in the lower part, for the students to write the answer. The PSTs wrote the problem statement on a note or on a graph page. The PSTs considered different formats for students to record their responses such as entering normal or math text, selecting an option from the multiple-choice options, using the graphic feature so they could enter the coordinates of a point, writing an equation, graphing a system of equations, or entering a definition.

At the beginning of the online meeting, the CT verified that all PSTs had simultaneous access to the lesson, so they could edit the TDAB document. Then, the CT started to review all the slides elaborated by the PSTs. Table 3 shows a description of all the components used in each interval, including the CT's actions for facilitating the LS planning meeting.

Following Gueudet and Trouche's (2009) framework, an analysis of the columns helped us to determine the material, didactical and mathematical components in the planning meeting. The TDAB features used were the card sorting, the tools to create open questions and closed questions. In the writing of the questions, text, graphs, tables, sketches, [6] mathematical symbols and mathematics editors were included. Similarly, students could include graphical mathematical objects such as points, lines and textual mathematical objects such as equations of lines, coordinates, inequalities, in the responses. Although some of these features were included by the PSTs, the CT had a critical role showing the PSTs different ways to improve the tasks.

We identified various didactical components in the planning meeting: scaffolding, assessment, use of multiple representations and emphasis on problem solving. The team discussed strategies for scaffolding on several occasions by the emphasizing on reviewing the terms that are used, organizing the tasks in a certain sequence, translating the textbook definitions from English to Spanish, and instructing students to write the procedures for solving the problems in their notebooks [7]. The CT stated the significance of well-defined terms and of proving students' opportunities to justify their answers. In their lesson draft, the PSTs had included graphical, algebraic and verbal representations of the concepts related to linear systems of equations and inequalities. The CT emphasized the importance of supporting students in the transition from one representation to another. For example, he added a table to one of the tasks that the PSTs included in the lesson for students to find the solution of a system of linear equations by listing solutions of each equation and noticing a common solution. With this table, the CT expected students to better understand that the solution of the given system of linear equations can be found with the aid of a table, a graph or algebraically. Either way, students will arrive to the same answer.

Table 3.
Components used for documentation during online lesson planning

Interval No	Material	Didactical	Mathematical	CT's actions
8	(1) Teacher Desmos Activity Builder (2) Note slide (3) Card sort (4) eTextbook	(1) Stating definitions at the beginning of the lesson (2) Translating the textbook definitions from English to Spanish	(1) Definitions of systems of equations and inequalities	(1) Asks PSTS the purpose of the task (2) Opens online eTextbook (3) Suggests including an introduction to the topic (4) CT includes eTextbook definitions
9	(1) Teacher Desmos Activity Builder (2) Answer key for card sorting	(1) Creating a task for reviewing math terms	(1) Inequality (2) Linear equation (3) Ordered pair (4) Slope (5) Solution of an equation (6) Symbols ($<, \leq, =, \geq, >$)	(1) CT includes card sorting answer key (2) Indicates how to add math symbols
10	(1) Teacher Desmos Activity Builder (2) Card sorting	(1) Establishing learning objectives for the task (Review)	(1) Graphic representation of a linear equation of two variables as a line	(1) Offers ideas of how to edit the cards by rephrasing
11	(1) Teacher Desmos Activity Builder (2) Card sorting	(1) Stating the importance of well-defined terms	(1) Solutions of a system of linear equations (2) Inconsistent, dependent, and independent systems of equations	(1) Shows how to match more than two cards (2) Offers further editing
12	(1) Teacher Desmos Activity Builder (2) Card sorting	(1) Including different representation for math terms	(1) Graphic representation of a system of two linear equations and its relationship with the solution	(1) Continues adding cards (2) Asks PSTs to review the tasks
13	(1) Teacher Desmos Activity Builder (2) Open questions: Note Page, Math Input (3) Features: Math text editor, randomize option, see classmates' responses, anonymizing answers	(1) Providing scaffolds for the task	(1) Changing a linear equation in standard form to slope- intercept form	(1) Asks the purpose of the tasks (2) Shows how to use TDAB features (3) Discusses how to promote students' interactions

(continued)

Interval No	Material	Didactical	Mathematical	CT's actions
14	(1) Teacher Desmos Activity Builder (2) Open question: Graphs, Math Input (3) Features: rearranging the window, adjusting brightness, combining windows, include justifications for answers	(1) Justifying answers	(1) Graphing a system of linear equations (2) Connecting an algebraic and graphic representation of a linear equation (3) Identifying the intersection point of two lines as the solution of the linear system	(1) Offers suggestions about sequencing (2) Shows how to use TDAB features (3) Discusses the importance of requiring students' justification to their answers
15	(1) Teacher Desmos Activity Builder (2) Open question: Graphs, Math Input (3) Features: Include and view all students' responses on the same cartesian plane	(1) Multiple representations (2) Assessing students' answers	(1) Graphic solution of a system of linear equations (2) Notation for the axes in the Cartesian plane	(1) Asks the purpose of the tasks (2) Asks for PSTs' justification of solution (3) Shows how to use the overlay feature
16	(1) Teacher Desmos Activity Builder (2) Open question: Note page, Text Input	(1) Assessing students' knowledge	(1) Strategies for solving a system of linear equations	(1) Asks the purpose of the tasks (2) Suggests changing the instructions
17	(1) Teacher Desmos Activity Builder (2) Note page, Multiple choice (3) Features: Randomize choices	(1) Assessing students' procedural fluency	(1) Solving a system of linear equations by substitution	(1) Asks PSTs how they can be sure students use the required method (2) Shows how to randomize alternatives
18	(1) Teacher Desmos Activity Builder (2) Note page, Multiple choice (3) Features: Randomize choices	(1) Assessing students' procedural fluency	(1) Solving a system of linear equations by elimination	(1) Adds instruction to write procedure on the notebooks

(*continued*)

Table 3.

Table 3.

Interval No	Material	Didactical	Mathematical	CT's actions
19	(1) Teacher Desmos Activity Builder (2) Open question: Note page, Text Input, fill-in tables	(1) Problem solving (2) Using multiple representations (3) Anticipating students' solutions (4) Creating tasks that go beyond identifying the intersection point of a system of linear equations	(1) Solving a system of linear equations with various methods including by using a table	(1) Asks PSTs to anticipate students' responses (2) States knowledge that the task promotes (3) Adds a Fill-in table with multiple columns
20	(1) Teacher Desmos Activity Builder (2) Graph, Math Input, Sketch page (3) Features: Adjust window, overlay students' answers	(1) Using multiple representations (2) Problem solving	(1) Solving a system of linear equations (2) Estimate the solution of a system of linear equations given its graph	(1) Edits to include that students graph the system (2) Requests to PSTS to go over the problem to see how it works
21	(1) Teacher Desmos Activity Builder (2) Note page, Math Input	(1) Holding students accountable for showing procedures in their notebooks (2) Assessing students' procedural fluency	(1) Solving a system of linear equations by elimination	(1) Adds instruction to write the solution on the notebook
22	(1) Teacher Desmos Activity Builder (2) Open question	(1) Lesson sequence (2) Redundancy (The exercise was eliminated)	(1) System of linear inequalities	(1) Deletes the slide (2) Changes the sequence of the slides
23	(1) Teacher Desmos Activity Builder (2) Open question. Graph, Note page, Math Input, Sketch page (3) eTextbook: definition	(1) Scaffolding for linear inequalities (2) Sequencing	(1) Solutions of a system of linear inequalities	(1) Adds Sketch page to the task (2) Changes instructions to include more than one solution

(*continued*)

Interval No	Material	Didactical	Mathematical	CT's actions
24	(1) Teacher Desmos Activity Builder (2) Open question. Note, Math Input, Sketch page (3) overlay students' answers (4) eTextbook: math problem	(1) Considering a problem's context (2) Anticipating students' solutions	(1) Applying systems of linear inequalities to solving problems	(1) Asks for the purpose of the task (2) Adds Sketch page (3) Edits the instructions (4) Adds questions
25	(1) Teacher Desmos Activity Builder (2) Open question, Note page, Text Input, Sketch page	(1) Multiple representations	(1) Graph methods to solve a system of linear inequalities	(1) Changes the order of instructions to graph first and give the solution later
26	(1) Teacher Desmos Activity Builder (2) Open question. Note, Math Input, Sketch (3) eTextbook: math problem	(1) Considering a problem's context (2) Anticipating students' solutions	(1) Applying systems of linear inequalities to solving problems	(1) Asks PSTs to anticipate students' responses and the way they will analyze solutions

Table 3.

We provide an example of the interaction between Mr. Martínez and the PSTs when preparing the lesson in order to illustrate the documentation process (Figure 1). The team was adding a word problem to the TDAB lesson that they were working on. Bruce (a PST) was editing the text of the word problem while other PSTs provided suggestions. The problem, taken from the e-Textbook and translated to Spanish by the PSTs, was about a store manager that sells DVD and CD players, each one with a given different earning margin. The goal of the problem was to write a linear inequality that considers the number of DVD and CD players that the manager needed to sell to reach an established amount of earnings. All the members of the team read the problem and proposed to change the problem's context from DVDs and CDs, outdated technologies, to smartphones and smartwatches, which are more familiar to students. Mr. Martínez requested the PSTs to anticipate students' strategies for solving the problem. They discussed the possibility of asking for a graphic solution and overlaying the students' solutions. We coded the e-Textbook and the TDAB features used as the material components, considering the problem's context and anticipating the students' solutions as the didactical components, and applying the system of linear inequalities as the mathematical component. We also coded Mr. Martínez's actions as a knowledgeable other and as a LS facilitator. Through the team's interactions we could see the collaborative nature of the discussion during the planning meeting. Mr. Martínez's actions prompted the PSTs' engagement in creating the research lesson (Table 3, Interval No. 24).

During the planning meeting, the mathematical components were evident in discussions about the main concepts in the lesson. The team mentioned terms such as linear equation, linear inequality, systems of linear equations and inequalities, solution of an equation, and solution of a system of linear equations. The math-specific features of the TDAB supported the team in considering mathematical components. For example, the availability of mathematical symbols (i.e., $<, \leq, =, \geq, >$) was useful for writing instructions. Additionally, the accessibility of features for incorporating multiple representations in the slides (i.e., graphs, tables, formulas) helped the team to consider how to strengthen students' understanding of mathematical concepts.

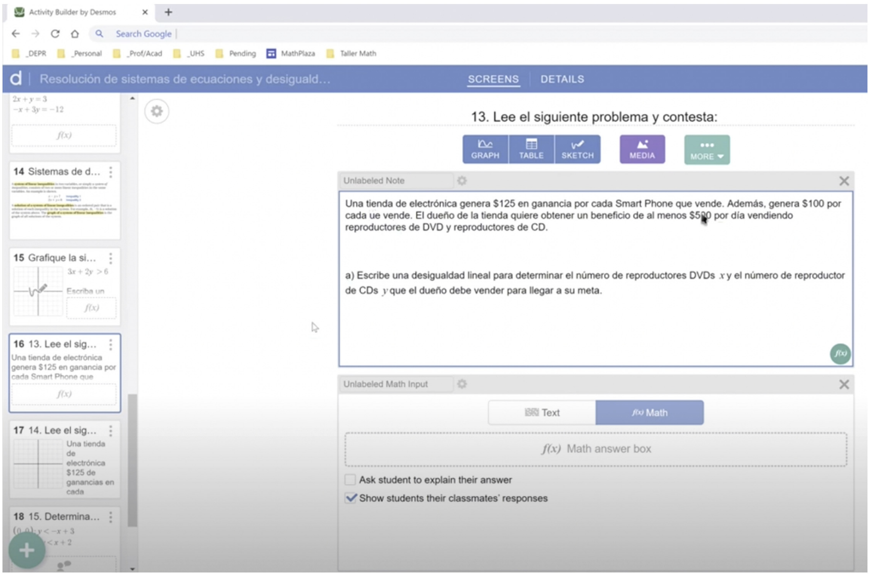

Figure 1.
Example of interaction between Mr. Martínez and the PSTs when preparing the lesson

Figure 2 shows the material, didactical and mathematical components during the planning discussions and the relationships between them. The most frequently used TDAB features were the card sorting and the open questions. Card sorting is the TDAB feature with most connections with the mathematics component but less connections with the didactical components. Card sorting was used at the beginning of the lesson to review mathematical concepts to ensure that students have the prior knowledge necessary to proceed with the other tasks. Open questions are the TDAB feature with most connections with didactical and mathematical components (i.e., definition of a linear system of equation, changing equation in standard form to slope-intercept form, recognizing the graph of a system of linear equations, and solving linear systems of equations and inequalities). The open questions included in the research lesson intended to help the team in gathering evidence about various solution strategies that students could apply. Moreover, the CT recommended to the PSTs adding instructions to the open-ended tasks so that students show their procedures in their notebooks. In that way, the students' notebooks became a new "document" in the planning process because the CT instilled in the PSTs the importance of using the students' responses in their notebook to collect evidence of student thinking.

In answer to the second research question about the way in which the CT facilitated the documentation work, we found that the CT assumed two roles: facilitator of the planning meeting and knowledgeable other. As the facilitator of LS, he asked the PSTs to provide a rationale for the selection of tasks, prompted them to anticipate possible responses, and connected the research lesson with learning standards and curricular goals. As knowledgeable other, the CT reviewed and made edits to improve the research lesson. The CT modeled how to improve the tasks. For example, the CT modified some of the tasks to include more versatile interactions and to seize on TDAB features. The CT stressed the importance of sequencing the tasks by considering the connections between tasks in the lesson. The CT also prompted the PSTs to avoid repeating tasks that targeted the same objectives of tasks included earlier in the lesson. The CT told the PSTs that students should justify their answers and, in doing so, the LS team could gather information about student

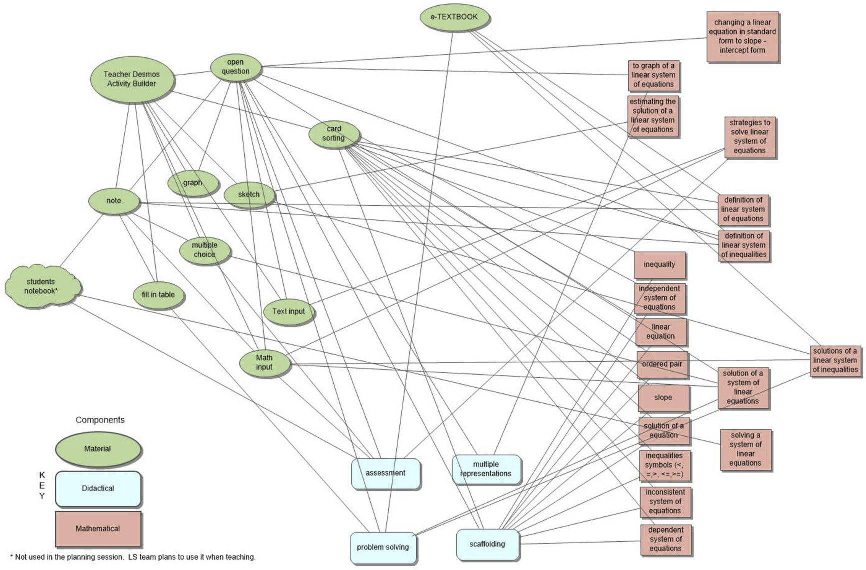

Figure 2.
Material, didactical and mathematical components during the planning discussions

thinking. The CT also emphasized allowing students to see their classmates' answers while working on a task to promote collaborative knowledge-building. The CT held individual students accountable by requesting them to write on their notebooks all the procedure to solve a problem. The students' notebooks became a new document for the teacher to assess students' individual needs and use of mathematical language.

The CT modeled to the PSTs how to plan the lesson through documentation. Specifically, the CT's actions showcased how to integrate the material, mathematical, and didactical components in the research lesson. The CT demonstrated to the PSTs how to use several TDAB features. For example, the CT showed the PSTs how to build the answer key to the card sorting tasks and how to match two or more cards. Figure 3 shows the answer key to the sorting card where multiple cards match one mathematical concept. The CT also asked questions to the PSTs for them to make explicit the didactical components in the research lesson. For example, the CT frequently asked the PSTs to state the purpose of a task and justify their didactical decisions. At the same, time the CT led the planning meeting by checking that all of the PSTs were actively involved in editing the slides. Specifically, the CT gave the PSTs control of the TDAB platform so that they could edit the slides and contribute to the elaboration of the research lesson. Altogether, the CT's actions allowed the team to craft a lesson that would allow them to observe student thinking by considering various components of mathematical proficiency.

Conclusion

We applied Gueudet and Trouche's (2009) framework to examine the documentation process of the LS team led by a CT and including four PSTs. We found evidence of collaboration during the online planning meeting. All team members had access to the material components, including the e-Textbook and the TDAB slides. Moreover, the TDAB platform allowed all team members to simultaneously edit the slides and engage in the documentation process. The CT's actions were fundamental in promoting a collaborative environment during the planning meeting by assuming a dual role of facilitator and knowledgeable other.

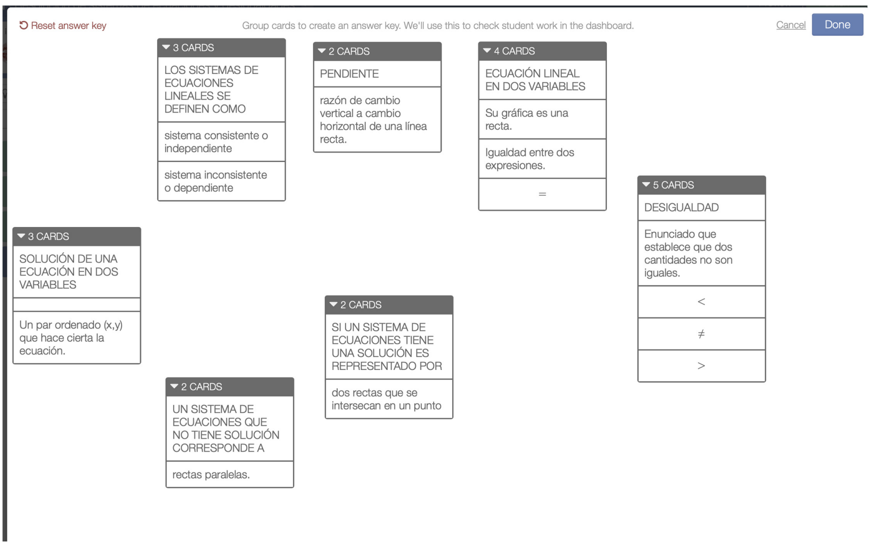

Figure 3.
Answer key to the sorting card where multiple cards match one mathematical concept

The PSTs came to the planning meeting with a draft of the research lesson. We could see, through the documentation process, that the CT helped the team to improve the research lesson by making the didactic decisions explicit, opening opportunities to observe student thinking, and taking advantage of TDAB features.

Some limitations of this study are that we did not have access to the planning meeting where the PSTs created the draft of the research lesson. In addition, we are reporting the observations of only one online meeting. Further research is needed to determine the way PSTs internalize the documentation process.

We found evidence that the CT established a scheme of use of the materials, which is a fundamental activity in the documentation process. The PSTs brought to the online meeting a lesson with 14 tasks. The CT reviewed each task and proposed recommendations for most of the tasks. The recommendations aimed at improving the tasks by incorporating more TDAB features. The CT also identified the didactical value of various TDAB features and asked the PSTs to add instructions for students to reveal their thinking. We conclude that the CT had a scheme since he executed an invariant set of actions on most of the tasks designed by PSTs. At the same time, the CT modeled how to use the material, didactical and mathematical components of the resources. Finally, drawing on his TPACK, the CT took advantage of specific TDAB features, made didactical decisions, and justified those decisions for improving the research lesson. The CT showed his proficiency with TDAB and other technological resources allowing the improvement of the document. The PSTs had the opportunity to see how the CT engaged in documentation for making tasks that revealed students' thinking, thus improving the research lesson. We established that the CT contributed to the improvement of the research lesson, helped to advance PSTs' knowledge, and engaged the LS team in a collaborative process. Our study can help in identifying the knowledge and skills needed for LS facilitators to undertake their roles and, thus, design professional development initiatives for LS facilitators.

We conclude that the typical roles that team members assume during face-to-face LS were translated to the online interactions, with the addition that all of the team members were able to see the changes to the research lesson in real time in their own screens. As one team member made changes to the slides for the research lesson, other team members provided assistance. At times, a team member who was more knowledgeable of TDAB features took over the job of making changes to the research lesson. In a way, various team members became the "knowledgeable other" by proposing changes to the research lesson in real time, while the other team members saw them working on these changes and learning from their expertise and knowledge. In the future, we would like to continue to investigate how online environments provide opportunities for collaboration among team members in the various lesson study steps.

Our study intends to contribute to a better understanding of how LS can be viable online. In our case, since LS has not been implemented in Puerto Rico, we see the potential of using online environments for engaging teachers in LS when they are in various geographical locations. Our finding that the LS facilitator's skills for engaging others in the collaborative process of elaborating the research lesson opens new opportunities for developing leadership among LS facilitators. Teacher education programs can benefit from the knowledge of experienced teachers through lesson study. Moreover, the affordances of the online environment for engaging all in the process of revising and editing the research lesson seems to empower PSTs to assert their knowledge and become agents during LS.

Notes

1. Pre-service teachers are students enrolled in a teacher education program at a higher education institution studying to become teachers.

2. We added a fifth column for PSTs to describe TDAB integration to the lesson.
3. We use pseudonyms for individuals and institutions in compliance with the Institutional Review Board requirements for research.
4. The PSTs had the opportunity to share a mock-up of their research lesson in the technology-based early methods course and get feedback from their peers and the teacher educator teaching the course.
5. We use the term "students" for the Algebra I students in Mr. Martínez class where the research lesson would be implemented.
6. Sketch is a kind of draw board that allows students to draw points, lines, functions, geometrical figures or sketches. Sketches are used for interaction purposes and are very useful to start student discussion because a teacher can aggregate (overlay) all students' answer in the same graph and show it to the whole class.
7. In the school, mathematics instruction is in Spanish but the textbook and other instructional materials are often in English.

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