

5 Perspective on the Development and 6 Application of Light-Field Cameras in Flow 7 Diagnostics 8

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23 Abstract 24

25 Multi-camera flow diagnostics have made large gains in recent years in the field of three-dimensional and
26 multi-physics measurements. However, cost, complexity and optical access pose challenges that place multi-
27 camera techniques out of reach for many labs. In that context, light-field (LF) imaging represents an
28 alternative approach that can potentially alleviate some of these challenges. LF flow diagnostics is a branch
29 of measurement techniques introduced within the last decade that are based on a plenoptic camera's unique
30 ability to capture three-dimensional and multi-spectral data via a single objective lens and image sensor. Thus
31 far, LF flow diagnostics have successfully achieved significant camera-reduction alongside other
32 performance improvements in 3D flow velocimetry, 3D particle tracking, 3D scalar-field tomography, micro-
33 fluidic velocimetry and multi-spectral imaging, as well as early demonstrations of single-camera multi-
34 physics measurements for applications such as 3D fluid-structure interactions. Here, we discuss the state of
35 development in LF flow diagnostics, highlight on-going challenges, and project potential advancements in
36 the near future.
37

38 Keywords: light-field, plenoptic, microlens array, 3D flow diagnostics, particle image velocimetry,
39 tomographic PIV, plenoptic PIV, particle tracking velocimetry, 3D-PTV, light-field microscopy, scalar-field
40 tomography, 3D background-oriented schlieren, multi-spectral imaging
41

42 1. Introduction 43

44 Modern advances in fluid dynamics require
45 understanding of increasingly three-dimensional
46 (3D), unsteady and multi-physics phenomena,
47 ranging from 3D fluid-structure interactions (FSI) in
48 soft robotics to the aerothermodynamics of
49 hypersonic vehicles. Image-based flow diagnostics
50 continue to be indispensable, as more complex
51

52 computational models in increasingly extreme
53 environments require on-going empirical
54 validations. However, the progress and application
55 of advanced image-based flow diagnostics are
56 currently impeded by the paradigm of increasing
57 camera-count, where capturing more complex flows
58 require increasingly more costly scientific-grade
59 cameras per experiment.
60

3 For example, as Fig. 1 illustrates, early particle
 4 image velocimetry (PIV) experiments employed
 5 only one camera for 2D two-component (2C)
 6 measurements. Later, stereo-PIV extended the
 7 technique to 2D-3C by adding a second camera,
 8 while tomographic PIV (tomo-PIV) and 3D particle
 9 tracking velocimetry (3D-PTV) used three to four
 10 *high-speed cameras* to obtain 3D-3C time-resolved
 11 velocity-fields [1]. Numerous contemporary
 12 research have also begun to combine tomo-PIV/3D-
 13 PTV with simultaneous measurements of additional
 14 physics using setups with five or more cameras; e.g.
 15 FSI [2,3] and simultaneous velocimetry with flame-
 16 front visualization [4,5]. This paradigm can present
 17 significant challenges for many applications due to:
 18

- Inflating costs of experiments.
- Increasing alignment complexity and sensitivity.
- Expanding footprint and optical access requirement that are incompatible with facilities such as combustion rigs, hypersonic wind tunnels or small biological samples.
- And finally, the more technical disadvantage of depth-of-field (DOF) versus light-sensitivity tradeoff, where reduction of lens aperture in already light-starved volumetric measurement is often necessary to gain sufficiently large DOF to encompass the volume.

32 Breaking the existing paradigm is therefore critical
 33 in order for modern image-based flow diagnostics to
 34 be applied to an even broader set of problems that
 35 might benefit from these advanced measurement
 36 techniques.

37 Attempts to break the paradigm of increasing
 38 camera-count thus far include the view-splitter (“quadscope”) [6,7] and fiber-optic techniques [8]
 39 that combine multiple perspective-views onto a
 40 single sensor; the MiniShaker [9] and co-axial
 41 volumetric PIV [10] that pack multiple small
 42 cameras into a compact camera head; as well as
 43 defocusing PIV [11], digital holographic PIV [12]
 44 and color-coded PIV [13] that leverage aperture-
 45 related or non-perspective-views physics to generate
 46 3D data. In addition to these techniques, a unique
 47 branch of camera-reduction strategies has emerged
 48 in recent years based on the novel plenoptic (aka.
 49 light-field, LF) imaging principle. Unlike
 50 conventional cameras, plenoptic imagers employ a
 51 microlens array (MLA) to capture 4D light-ray data
 52 within a single shot via a single objective lens,
 53 following which, a multiplicity of images with
 54 varying perspectives, focal points and extended
 55

56 DOF can be rendered. These powerful capabilities
 57 have been successfully leveraged to achieve 3D flow
 58 velocimetry, fragment/particle-tracking,
 59 microscopy, scalar-field tomography, and
 60 hyperspectral measurements. Here, we review
 61 progress in the nascent field of plenoptic flow
 62 diagnostics and provide perspectives on likely future
 63 developments.

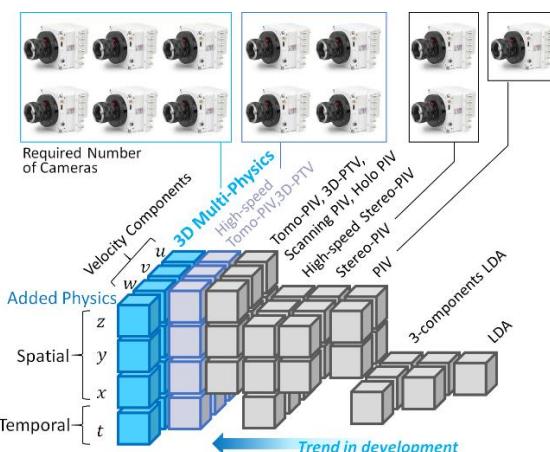


Fig. 1 The curse of camera-count. Based on: [1]

2. Principles and Hardware of Light-Field Imaging

LF imaging begins with the treatment of light as 4D rays in 3D space, each ray having a 2D spatial coordinate (s, t) on a datum plane, as well as 2D directional coordinate (u, v) on a parallel plane at another depth z . In the reference frame of a camera (Fig. 2), (u, v) is mapped on the main-lens aperture while (s, t) is mapped to the sensor-plane. Thus the (x, y, z) position of a light-source relative to the camera can be computed if the LF coordinates of its rays are known. A regular camera does not capture LF data because the act of focusing collapses (u, v) information to a point, as illustrated in Fig. 2. Plenoptic cameras preserve (u, v) by imaging the aperture plane (u, v information) at discrete locations along the sensor using the MLA.

Two distinct architectures of plenoptic cameras exist: the original plenoptic 1.0 camera (aka. unfocused plenoptic) [14,15] and the later plenoptic 2.0 (aka. focused plenoptic) camera [16–21]. Plenoptic 1.0 and 2.0 differ in the positioning of their MLA (Fig. 2) which subsequently affects the sampling of LF information (Fig. 3). The plenoptic 1.0’s MLA coincides with the main-lens’ nominal

image-plane, while the sensor is one microlens focal-length behind the MLA. As such, rays focused on the nominal image-plane are neatly re-expanded into unfocused circular sub-images on the sensor occupying the same footprint as a microlens diameter. In contrast, plenoptic 2.0 has an MLA that is focused on the nominal image-plane, such that each microlens acts as a mini relay-lens conveying cropped, overlapping versions of the full image onto the sensor. Example of plenoptic 1.0 and 2.0's raw images are given in [16]. Consequently, as Fig. 3

shows, a given microlens discretely samples multiple (u, v) at a fixed (s, t) coordinate in plenoptic 1.0, whereas in plenoptic 2.0 a microlens samples a range of (s, t) and (u, v) whose values are coupled. Typically, plenoptic 2.0's pixels are distributed more densely in (s, t) and less in (u, v) relative to 1.0. Notably, the pixel distribution is fixed to hardware for 1.0, but can be easily re-optimized via shifting the MLA in 2.0- a key advantage of the latter.

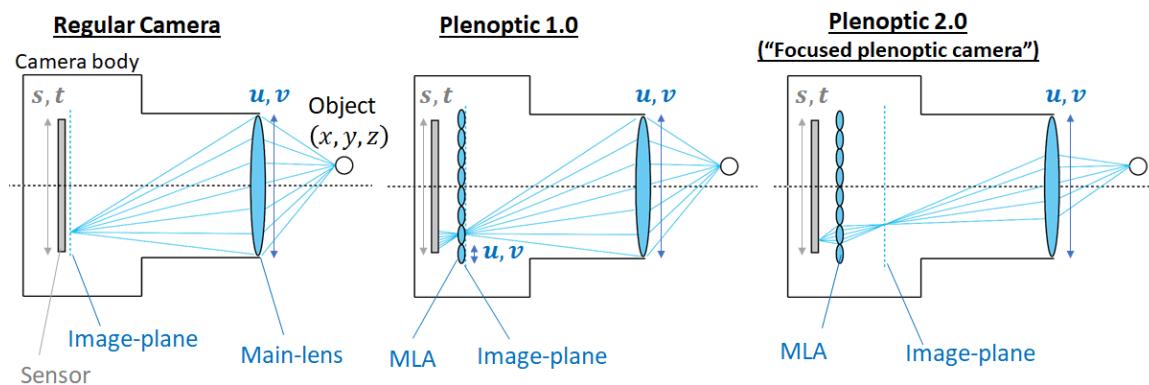


Fig. 2 Comparison of a conventional versus plenoptic 1.0 and 2.0 cameras.

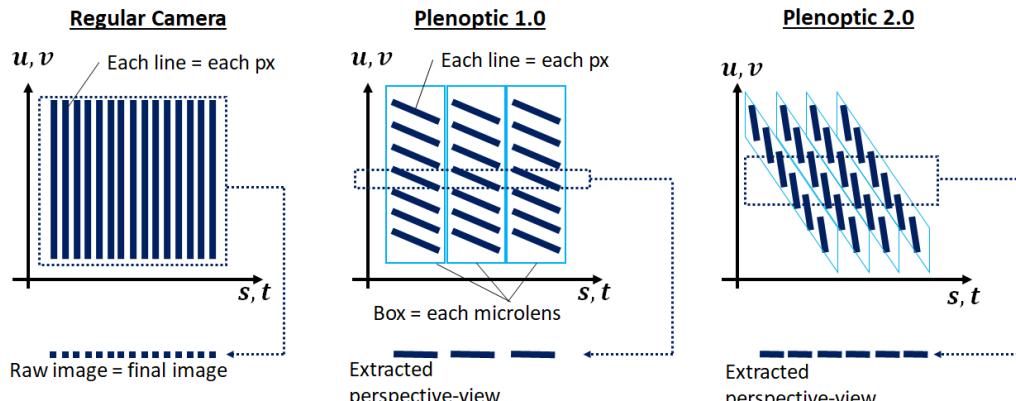


Fig. 3 Comparison of recorded information between conventional versus plenoptic 1.0 and 2.0 cameras. The schematic illustrates sampling for focal-plane objects, and is sheared when rays originate from off-focal planes.

The raw images of both plenoptic architectures must be decoded to provide meaningful viewing. Two commonly used decoding methods are as follows:

1. *Perspective-view generation*: a slice of pixels is extracted out of Fig. 3's diagram at the desired perspective angle (u, v) . Physically this represents extracting a single pixel from behind each microlens at a constant location and

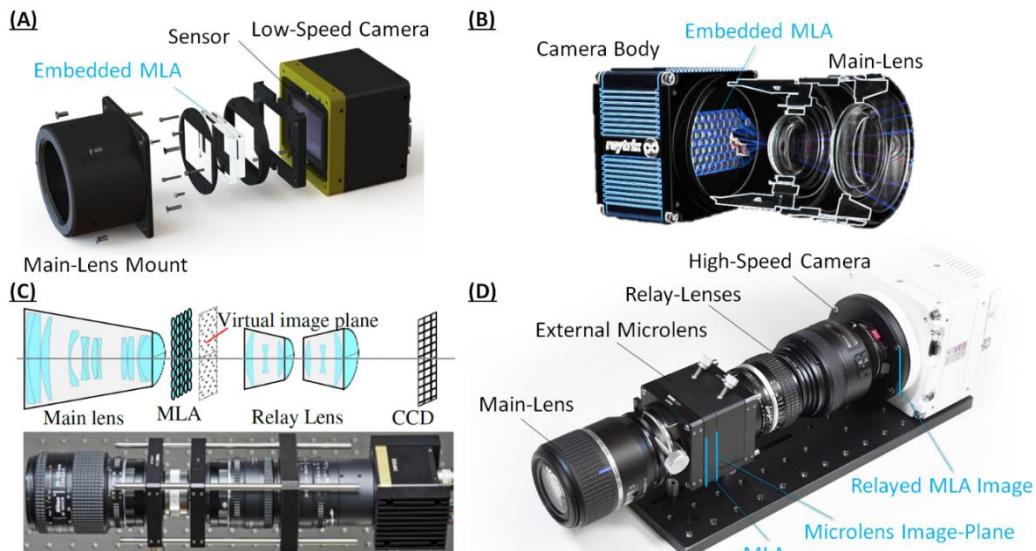
assembling them to form an image. A perspective-view has unusually large DOF for the given main-lens setup (more details later), while changing the extraction (u, v) shifts the viewer's perspective.

2. *Refocusing*: an image with regular (thin) DOF is generated by integrating across (u, v) in Fig. 3's diagram. Direct integration leads to an image focused on the main-lens' nominal focal-plane.

5 Conversely, if the sample points in Fig. 3 are
 6 sheared prior to integration (representing
 7 propagation of rays in z), the image can be
 8 synthetically refocused to different depths.

9 Due to the different pixel distributions,
 10 perspective-views with more precise (u, v) can be
 11 obtained in plenoptic 1.0, albeit at lower spatial
 12 (s, t) resolution than 2.0. Similarly, refocused
 13 images with higher spatial resolutions can be
 14 obtained from 2.0. However, artifact-free images are
 15 difficult to obtain from 2.0 as a raw pixel straddles a
 16 wider range of (u, v) . Additional procedures to
 17 reduce artifacts often must be implemented and
 18 typically include either an explicit or implicit

19 determination of an object's depth within the scene,
 20 which can present challenges in more complex 3D
 21 scenes. Aside from resolution trade-offs, plenoptic
 22 1.0 and 2.0 also offer different conveniences from a
 23 workflow viewpoint: the decoupled (s, t) and (u, v)
 24 in 1.0 is suitable to applications requiring high
 25 (u, v) resolution such as hyperspectral imaging
 26 (discussed later), while the ability of 2.0 to re-
 27 optimize (s, t) versus (u, v) precisions by shifting
 28 the MLA is highly desirable for applications such as
 29 PIV albeit with a significant increase in complexity
 30 of the associated image processing scheme. For
 31 brevity, further details on architectures and decoding
 32 are left to [20,22,23].



38 Fig. 4 Examples of plenoptic hardware used in fluid dynamics research: (A) a modified low-speed camera
 39 with embedded MLA for plenoptic 1.0 imaging. (B) Raytrix's commercial plenoptic 2.0 camera. [24] (C)
 40 Relayed MLA to support rapid prototyping. [25] (D) Relayed MLA used on a high-speed camera. [26,27]

41 Finally, two approaches are used to physically
 42 implement plenoptic imaging, neither of which is
 43 trivial as the MLA's minuscule focal length requires
 44 tenths of a millimeter or smaller tolerances. In
 45 matured research cameras such as Fig. 4A or
 46 commercial cameras such as Lytro and Raytrix
 47 devices (Fig. 4B), the camera sensor is removed of
 48 its glass cover and the MLA embedded directly in a
 49 factory clean room. Embedded MLA provides
 50 superior optical performance and compactness, but is
 51 less feasible for rapid prototyping or flexibly
 52 retrofitting high-cost systems like high-speed
 53 cameras and intensifiers. The latter requirement
 54 prompted a second type of design where the MLA is
 55 located externally and has its image relayed via
 56 lenses onto the sensor, thereby requiring no
 57 modification to the sensor body (see Fig. 4C-D)
 58 [14,25–30]. This design is especially demanding of
 59 the relay lenses' field flatness and aperture, which
 60 must accommodate the MLA's focal length tolerance
 and divergent ray angles.

61 In the next few years, we expect improvements
 62 in sensors and MLA fabrication to gradually enable
 63 higher resolution and cheaper plenoptic cameras.
 64 Additionally, several innovations on the horizon are
 65 also expected to reform plenoptic imaging. These
 66 include but are not limited to: (i) Actively-driven
 67 MLA and associated algorithms that seamlessly

5 transition between plenoptic 1.0 and 2.0 to provide
 6 on-the-fly optimization for varying experimental
 7 needs. (ii) Better image decoding to improve
 8 resolutions and reduce image artifacts of rendered
 9 plenoptic images for a given hardware design; e.g.
 10 super-resolution by conventional [31] or deep-
 11 learning methods. And (iii) LF imaging based on
 12 camera array instead of MLA such as the synthetic
 13 aperture PIV technique [32], especially via the use of
 14 cheap but increasingly capable smartphone camera
 15 sensors- most notably in the footsteps of Pelican [33]
 16 and Light's multi-camera LF-imaging phones [34].

17 3. Application to Flow Velocimetry

18 “Plenoptic-PIV” forms the core of LF flow
 19 diagnostics with successful applications in the
 20 studies of small marine animals [35] (Fig. 5A, Fig.
 21 6A), compressor linear cascade [36] (Fig. 5B, Fig.
 22 6B), birds and maneuvering wings [37], riverbed
 23 boundary-layer [38], shock-boundary layer
 24 interactions [39,40] (Fig. 5C, Fig. 6C), thin liquid
 25 film [41], rotating helicopter blade [42] (Fig. 5D, Fig.
 26 6D), transcatheter heart valves [43] and numerous
 27 others. In Fig. 5-Fig. 6’s examples, constraints on
 28 optical access and depth of volume would have made
 29 a multi-camera system very challenging to
 30 implement.

31 As with many other plenoptic flow diagnostic
 32 techniques introduced below, the earliest 3D
 33 velocimetry via plenoptic-PIV was achieved by
 34 leveraging the camera’s refocusing capability. Image
 35 of a particle-field was refocused to planes at discrete
 36 depths (called a “focal stack”), after which
 37 sharpness-detection or intensity-based segmentation
 38 attempts to localize particles to their corresponding
 39 depth based on defocus blurring. This technique is
 40 not robust and has relatively low depth resolution.

41 The current realization of plenoptic-PIV with
 42 improved robustness and resolution adopts a similar
 43 workflow as tomo-PIV (see Fig. 7). The raw
 44 plenoptic image of the particle field is first decoded
 45 into a stack of perspective-views at discrete (u, v) . A

4 virtual particle volume is then reconstructed by
 5 operating on the perspective-views with a
 6 tomographic algorithm such as the standard
 7 Multiplicative Algebraic Reconstruction Technique
 8 (MART):

$$9 E(x_j, y_j, z_j)^{k+1} \\ 10 = E(x_j, y_j, z_j)^k \left[\frac{I(s_i, t_i)}{\sum_{j \in N_i} w_{i,j} E(x_j, y_j, z_j)^k} \right]^{\mu w_{i,j}}$$

11 where E denotes voxel intensity at (x_j, y_j, z_j) on the
 12 k^{th} iteration; I denotes intensity of pixel at (s_i, t_i) ;
 13 and summation in the denominator is carried out for
 14 the set of voxels N_i in the line-of-sight of pixel
 15 $I(s_i, t_i)$; while, μ is the iteration’s relaxation factor.
 16 Critically, $w_{i,j}$ is a weighting factor that relates the
 17 projection of a 2D pixel in a perspective-view to 3D
 18 voxels, and is closely related to 3D calibration of the
 19 camera system. After reconstruction, two sequential
 20 volumes are cross-correlated to produce a 3D
 21 velocity-field. In this respect, the only key distinction
 22 between plenoptic-PIV and tomo-PIV is the nature of
 23 the perspective-views. In plenoptic-PIV, upwards to
 24 100 perspective-views can be obtained from a single
 25 raw image, but only with a parallax baseline as wide
 26 as the main-lens’ aperture, which fundamentally
 27 limits depth resolution.

28 For more details, early development of the
 29 plenoptic-PIV technique is described by Lynch et al.
 30 [44], while subsequent improvements are covered in
 31 [45–47]. Like tomo-PIV, numerous alternatives to
 32 MART were proposed to improve accuracy or
 33 expedite convergence, including dense ray-tracing
 34 reconstruction [48], filtered refocusing [49],
 35 deconvolution [50] and expectation-maximization
 36 with summed line-of-sight estimation [51].
 37 Expediting is significant for plenoptic-PIV due to the
 38 computational cost of iterating through a large
 39 number of perspective-views compared to just four
 40 in tomo-PIV. Concurrent studies are also exploring
 41 whether under-sampling the available perspective-
 42 views but maintaining total parallax baseline will
 43 reduce computation without adverse effects on
 44 reconstruction.

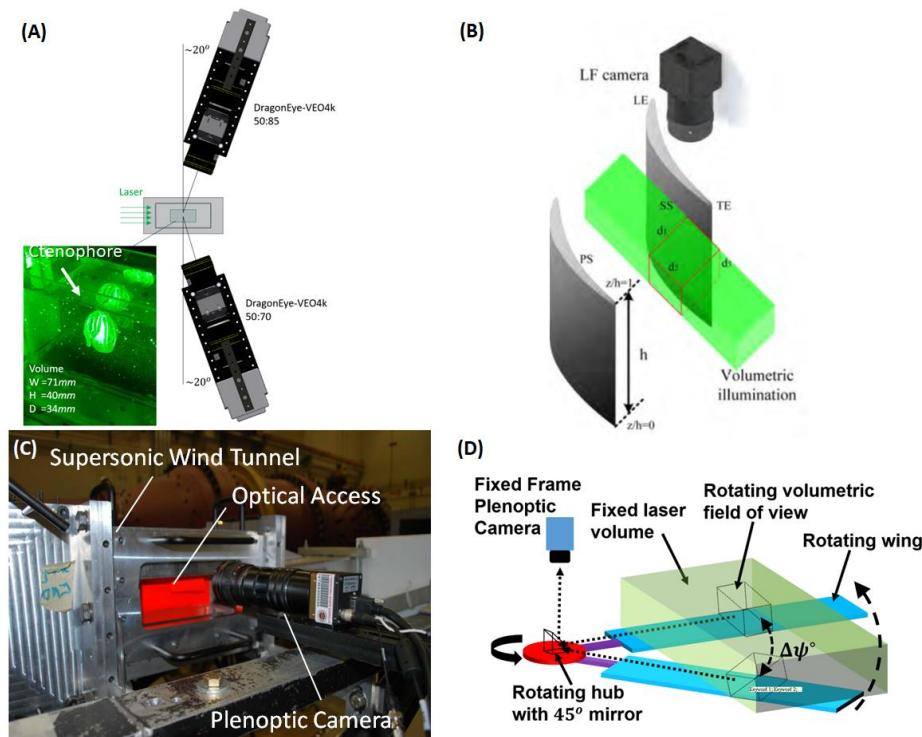


Fig. 5 Application of plenoptic-PIV in the measurements of (A) ctenophore hydrodynamic [35], (B) flow within a compressor linear cascade [36], (C) supersonic swept-fin [40], and (D) leading-edge vortex dynamic about a rotating frame-of-reference [42].

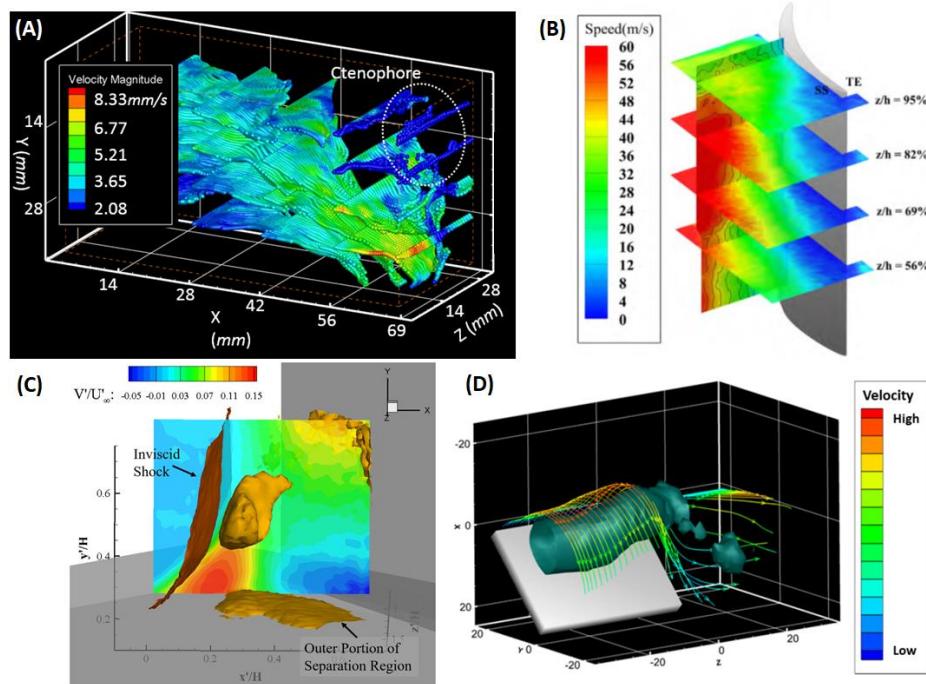
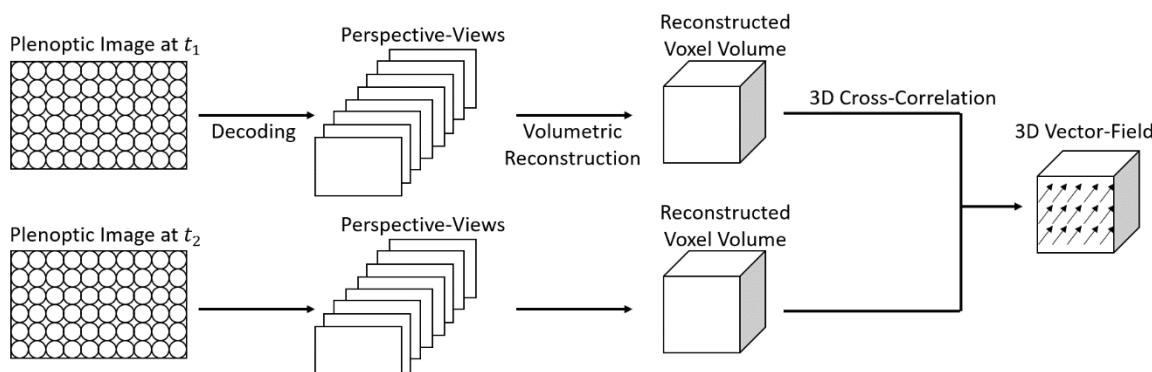


Fig. 6 Results corresponding to experiments and references (A-D) above, respectively.



17 Fig. 7 Workflow of plenoptic PIV.

18 As noted above, $w_{i,j}$ establishes the relation
 19 between the image and 3D world and is foundational
 20 to 3D reconstruction. The standard practice in four-
 21 camera tomo-PIV involves generating a separate
 22 pinhole or polynomial calibration model for each
 23 camera, from which $w_{i,j}$ is calculated. However, the
 24 availability of ~ 100 perspective-views in plenoptic-
 25 PIV and the possibility of dynamically sampling
 26 *different* sets of perspective-views from the same
 27 image makes tomo-PIV's approach impractical. An
 28 early method of plenoptic calibration developed by
 29 Thomason et al. [52] in 2014 employed a root-mean-
 30 square procedure to estimate the positions of the
 31 sensor, MLA and main-lens, after which a
 32 geometrical model relates image to 3D world. In
 33 2018, Hall et al. [53,54] developed a more robust and
 34 flexible third-order polynomial calibration scheme
 35 for plenoptic 1.0 cameras, where the image-to-world
 36 mapping of all possible perspectives are fitted with
 37 two 3rd-order polynomials, P_s and P_t , for the two
 38 orthogonal sensor directions:

$$41 \quad s = P_s(x, y, z, u, v) \\ 42 \quad t = P_t(x, y, z, u, v)$$

43 Hall et al.'s scheme has the advantage of storing only
 44 a small set of polynomial coefficients, but additional
 45 constraints are required to translate the calibration
 46 scheme into $w_{i,j}$, e.g. the assumption of comparable
 47 size between voxels and microlens pitch. Early
 48 efforts to pre-compute and store the entire $w_{i,j}$
 49 matrix quickly exceeded a typical workstation's
 50 memory. Consequently, a model that translates P_s, P_t
 51 to $w_{i,j}$ is now used to calculate weighting on-the-fly.
 52 Dynamic sampling of (u, v) is allowed in this
 53 method, but the polynomial has the disadvantage of
 54 being uni-directional where solving x, y, z based on
 55 (s, t, u, v) is computationally difficult. Ongoing

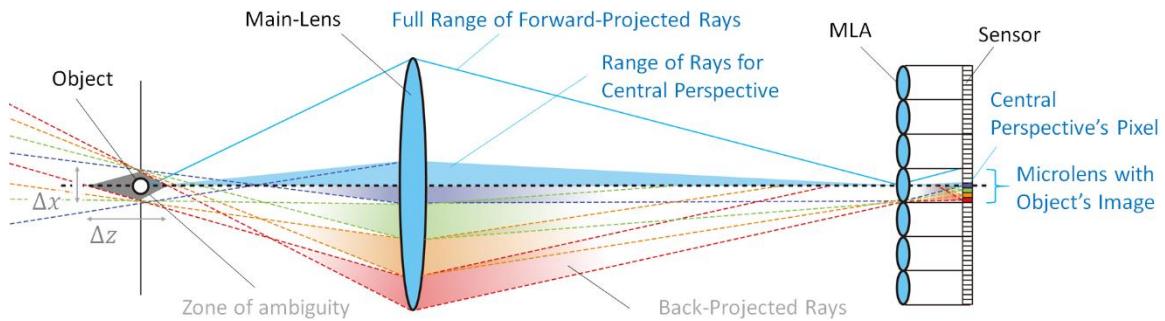
56 work by the authors' groups suggests that re-
 57 fragmenting the polynomial into separate
 58 calibrations for each (u, v) may significantly
 59 improve accuracy, though this once again prevents
 60 dynamic (u, v) sampling without, say, interpolating
 61 coefficients.

62 In 2019, Shi et al. [55] proposed an alternative
 63 scheme, which models ray-propagation through the
 64 plenoptic camera using thin-lens model with higher
 65 order corrections for complex lens distortion and
 66 MLA displacement. Notably, the thin-lens
 67 formulation is written in terms of the position and
 68 diameter of the circle-of-confusion that a point-
 69 source produces on a plenoptic image. Calibration
 70 then involves imaging a set of point-sources at
 71 various positions to establish the camera parameters.
 72 The computation of $w_{i,j}$ is subsequently based on
 73 Monte-Carlo tracing of 100 rays through the system
 74 using the camera parameters. The method was
 75 further developed in 2020 [56] to incorporate the
 76 concept of "plenoptic disk" (similar to circle-of-
 77 confusion).

78 While $w_{i,j}$ is often used in the direction of
 79 mapping 3D voxel to 2D pixel, Cao et al. [51]
 80 recently developed a new scheme that employs the
 81 reversed tracing direction for a plenoptic 2.0 camera.
 82 The main motivation being that reversed-tracing
 83 connects one pixel to multiple voxels at once,
 84 thereby reducing to total number of ray-tracing
 85 computations. In another ongoing development, a
 86 unique variant of Hall et al.'s polynomial calibration
 87 was developed by Gururaj et al. [42]. Driven by
 88 engineering needs, their plenoptic camera was
 89 aligned on the hub of a rotating helicopter blade (Fig.
 90 5D), which has a 45° mirror that reflected the view
 91 onto the blade. The setup allows for 3D PIV on a

5 rotating frame-of-reference that was previously
 6 untenable with multi-camera tomo-PIV. A rotating
 7 version of the polynomial calibration was thus
 8 devised for this application. In addition to the
 9 mentioned works, we anticipate that near-future

10 development will likely drive towards increasingly
 11 automatic and physically-informed calibration
 12 schemes, as well as adoption of higher-order
 13 corrections such as tomo-PIV/3D-PTV's volumetric
 14 self-calibration algorithm [57].



20 Fig. 8 Principles behind extended-DOF (top) and particle elongation (bottom) in plenoptic-PIV.

21 It is worth briefly discussing an additional
 22 strength and limitation of plenoptic-PIV: DOF and
 23 particle-elongation, respectively. Multiple cameras
 24 must share a common DOF encompassing the
 25 measured volume in tomo-PIV. As discussed in [58],
 26 DOF for an imaging system is given by:

$$27 \text{DOF} = 2 \frac{d_{aper} c_o}{d_{aper}^2 + c_o^2} z_{nominal}$$

28 Where d_{aper} is the imaging aperture diameter, c_o is
 29 the allowable circle-of-confusion's size in object-
 30 space (e.g. pixel pitch), and $z_{nominal}$ is the nominal
 31 focal-plane's distance from the aperture. For
 32 conventional imaging, d_{aper} corresponds to the
 33 main-lens' aperture diameter. For plenoptic camera,
 34 however, Fig. 8 shows that a perspective-view is
 35 formed by extracting a pixel behind a microlens. The
 36 pixel contains a small range of (u, v) and physically
 37 only gathers rays from a fraction of d_{aper} (see
 38 "Range of Rays for Central Perspective"). This has
 39 the effect of rendering the equivalent aperture as:

$$40 d_{aper, equivalent} = \frac{d_{aper}}{pxpm}$$

41 Where $pxpm$ is the number of pixels per microlens.
 42 Consequently, DOF is significantly increased in
 43 perspective-view. DOF in refocused images remain
 44 unchanged and is dependent on the full aperture,
 45 which is typically quite large in plenoptic cameras
 46 (order f/2 – f/4). Notably, the reduction in effective
 47 aperture is perfectly balanced by the reduction in
 48 perspective-view's resolution, which effectively
 49 increases the "pixel" size. Hence, a perspective-view
 50 pixel has the same signal-to-noise ratio as a
 51 conventional camera's pixel, *but* at a much lower

52 total pixel count, thus only using a fraction of the
 53 collected light. If all perspective-views participate in
 54 volumetric reconstruction, the contribution from all
 55 collected light is naturally regained, without losing
 56 the benefit of extended DOF.

57 A key limitation of plenoptic-PIV (and any
 58 plenoptic 3D measurements) is the limited parallax
 59 angle straddled by its single main-lens. As shown in
 60 bottom of Fig. 8, an on-axis point source at the
 61 nominal focal-plane fills all pixels under the center
 62 microlens. If rays are projected backwards from
 63 these pixels, finite rays of light-cones are formed
 64 (purple, green, orange and red cones in Fig. 8). All
 65 rays within a cone will fall on the same pixel; hence,
 66 a cone marks a zone of ray ambiguity. The
 67 intersections of all cones form a diamond-shaped
 68 region around the real point source, within which we
 69 cannot determine the point source's true location
 70 with certainty. Hence, the width of the "diamond"
 71 represents the system's lateral resolution (Δx) and its
 72 length the depth resolution (Δz). Notably, Δx and Δz
 73 are both depth-dependent.

74 A tomographic algorithm such as MART would
 75 thus reconstruct a circular flow particle as an
 76 elongated "diamond" filling the zone of ambiguity.
 77 Elongation grows worse with particle size and
 78 distance between camera and object. Using
 79 commercial lenses where the F-number is generally
 80 limited to 1.2, plenoptic cameras are usually only
 81 suitable for volumes with lateral dimension smaller
 82 than the order of 100 mm before elongation begins to
 83 severely impact PIV accuracy. Details on the
 84 accuracy of plenoptic cameras in PIV application are

given by [58–60], while [61] directly compares a single-camera plenoptic-PIV against four-camera tomo-PIV. A direct solution to elongation involves adding a second plenoptic camera at 70–90° to the first, which drastically reduces the zone of ambiguity to the intersection region of both cameras’ “diamonds” [35,58,62,63].

Future developments of plenoptic-PIV will naturally benefit from continued accelerations in reconstruction, either through algorithmic improvements or optimizing the number of required perspectives. Reconstruction accuracy is also expected to increase with improvements in calibration scheme, higher-resolution image-decoding and incorporation of prior knowledge into reconstruction. Advanced cross-correlation algorithms with optimized kernels [64] are also in development to reduce the impact of elongations and other reconstruction artifacts.

4. Application to 3D Tracking

The development of plenoptic-PTV and 3D-tracking can be traced to three motivators: (i) it is an extension of plenoptic cameras’ earliest application in depth-sensing [14], (ii) under many scenarios 3D-PTV require less compute and storage costs than tomographic reconstruction, and (iii) the Lagrangian approach localizes particles to a specific value in lieu of an elongated group of voxels in plenoptic-PIV (though a higher Δz uncertainty could still manifest). Like plenoptic-PIV the earliest approach to 3D tracking for both plenoptic 1.0 and 2.0 involve

creating a focal-stack, whereby depth localization of objects or particles is performed through determination of the focal slice with the sharpest image edges (see Fig. 9A) [65,66]. However, this approach is often slow and has limited resolution in z .

Present approaches to 3D tracking differ substantially between plenoptic 1.0 and 2.0 (see Fig. 10). In the former, the raw image is first decoded into perspective-views. Image segmentation then tags particles/objects of interest with associated (s, t, u, v) coordinates. Next, coordinates belonging to the same object appearing across multiple perspective-views are found, similar to the “correspondence problem” in multi-camera 3D-PTV or stereo-photogrammetry, except in this case ~ 100 perspectives exist. Finally, the object’s (x, y, z) position is triangulated by projecting its rays to where they intersect and originated based on (s, t, u, v) , with the associated camera calibration in the loop.

As shown in Fig. 10A, an early K-means clustering approach developed by Hall et al. [54,67] for large fragment tracking lays out all identified object centroids on a 2D (s, t) plot. Each point still retains its (u, v) identity. A K-means clustering algorithm performed on the 2D space then identifies sets of centroids belonging to the same object. Finally, (x, y, z) is solved based on the clustered (s, t, u, v) sets. Though precise, K-means clustering in 2D was not robust for flow velocimetry with high particle densities. The approach is more applicable to large fragment tracking, such as shown in Fig. 9B.

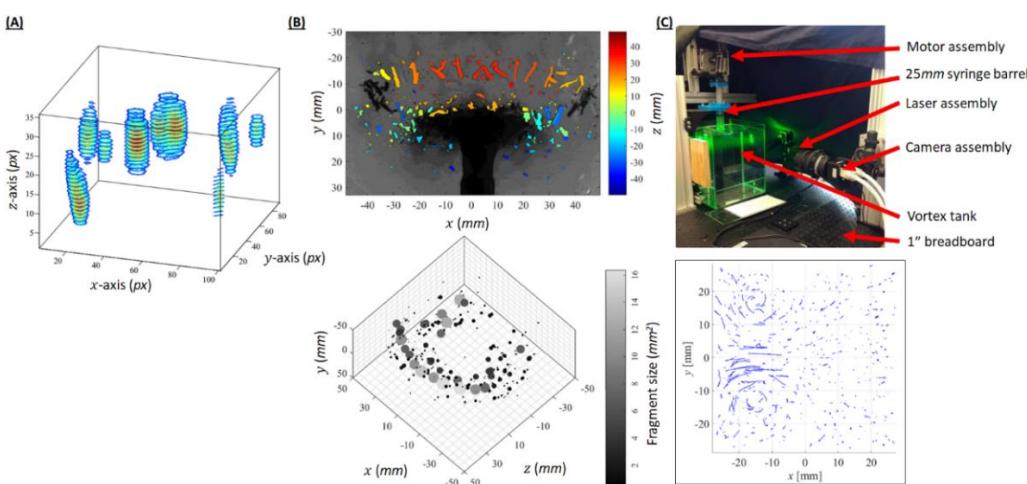


Fig. 9 (A) Example of 3D tracking via focal-stack [66]. (B) Example of explosive fragment tracking and sizing by K-means clustering of plenoptic 1.0 perspective-views [67]. (C) Vortex ring 3D PTV via plenoptic 2.0’s ETC method [68].

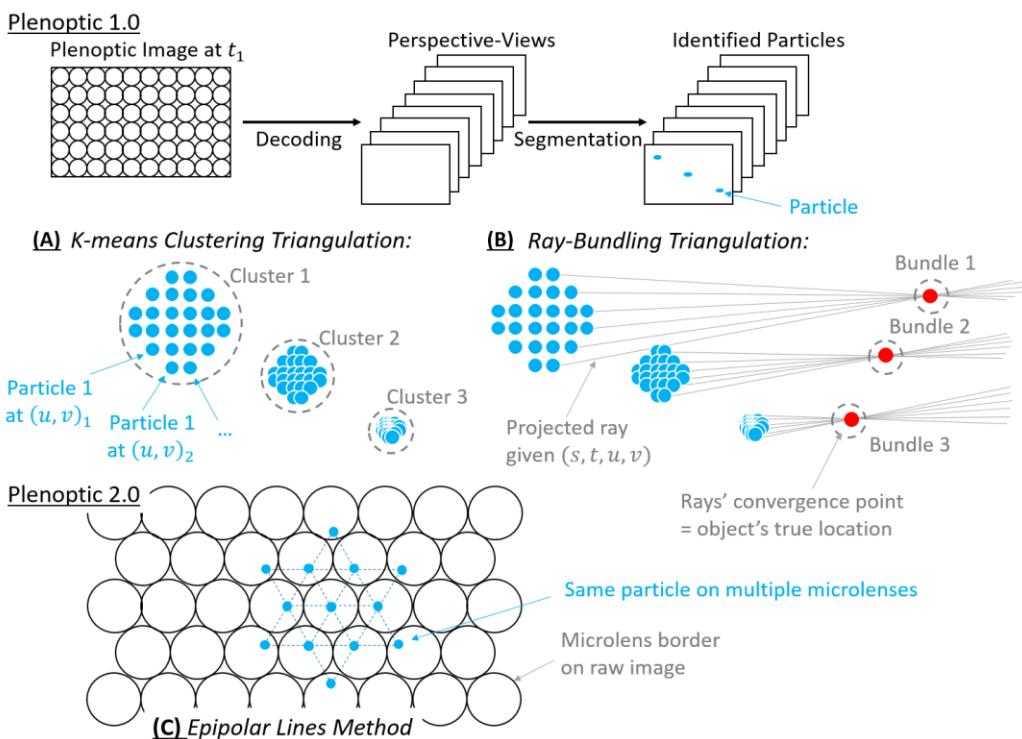


Fig. 10 Different approaches to plenoptic PTV and particle-tracking: (A) K-means 2D clustering and (B) “Ray-Bundling” 3D clustering for plenoptic 1.0, (C) Epipolar Triangular Connections for plenoptic 2.0.

In response, Clifford et al. [69] developed the “Ray-Bundling” method that extended K-means clustering into 3D space by treating each (s, t, u, v) as a projected ray. The underlying assumption being that the plenoptic camera samples hourglass-shaped bundles of rays from every particle (Fig. 10B). Expansion to 3D space makes the projected rays relatively sparse, and robust clustering can be performed to identify bundles based on their minimum crossing distances. Clifford et al. [69] successfully demonstrated the Ray-Bundling algorithm at 3D-PTV levels of particle densities. And, though computational cost scales with the number of particles, the aggregate computations for practical experiments remain lower than plenoptic-PIV. A direct comparison of accuracies for MART-reconstruction and tracking remains to be done. Preliminary efforts by [69] suggest Ray-Bundling result in lower errors than MART in all directions, though errors in the z-axis remain up to z5 times higher than xy -axes.

PTV via plenoptic 2.0 cameras uses a distinct method (Fig. 10C) that is more akin to multi-camera 3D-PTV’s epipolar line approach. The plenoptic 2.0 approach proposed by [68] called “Epipolar

Triangular Connections (ETC)” method leverages its in-focus raw image to bypass perspective-view decoding. Instead, particle segmentation occurs directly on the raw plenoptic image. The correspondence problem is solved beginning with an identified particle, followed by extension of epipolar lines outwards from this particle to adjacent microlenses, which effectively act as neighboring micro-cameras. Corresponding images of the particle are sought in adjacent microlenses, and if found the epipolar lines are extended outwards again, until a diameter corresponding to the maximum CoC in the measured volume is reached. The physical particle location is then found by triangulating from the found set of (s, t, u, v) . This approach has the advantage of bypassing image-decoding, which not only expedites computation but also avoids any decoding artifacts, especially those associated with the perspective-view’s low resolution (a major impediment to segmenting dense particle fields in plenoptic 1.0). Example of applying the ETC method to measure a vortex ring flow is shown in Fig. 9C. At the point of writing the method is still undergoing refinement.

Overall, plenoptic-PTV via both 1.0 and 2.0 approaches are still in their infancy relative to multi-camera 3D-PTV. In addition to gradual improvements in accuracy and computation costs, we expect the next step in development of plenoptic-PTV to include integration of proven advanced 3D-PTV algorithms such as iterative particle reconstruction (IPR) and Shake-the-Box (STB), as well as customization of these algorithms to exploit plenoptic cameras' perspective redundancy. We also note that plenoptic-PTV and 3D tracking is conceptually very similar to plenoptic depth estimation, which contains a vast literature partially covered in [22]. The depth estimation literature includes Adelson & Wang's landmark paper [14] on plenoptic 1.0 camera, as well as many plenoptic 2.0 algorithms [17,20] where depth calculation from disparity map is integrated into the image-decoding workflow. The implications of these algorithms

have not been fully explored for fluid diagnostics and further work is required.

5. Application to Microscopy

The application of plenoptic cameras in microfluidic measurements is primarily motivated by the lack of optical access. Many biological processes are dynamic and 3D, but placement of multiple microscope objective lenses and illuminators around a microscopic subject is inherently difficult [70]. Additionally, microscope objective lenses have very shallow DOF, while many of them are also object-space telecentric, thus offering no perspective parallax when translated relative to the subject. Thus, a plenoptic camera's ability to refocus and shift perspective within a single image is highly sought after in microscopy.

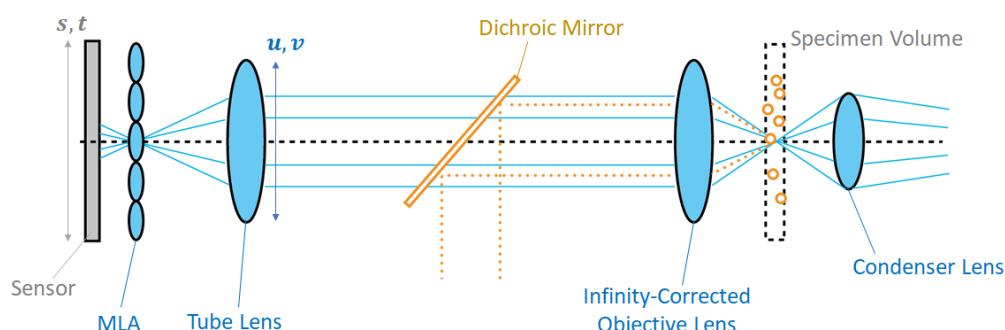


Fig. 11 Typical architecture of plenoptic microscope.

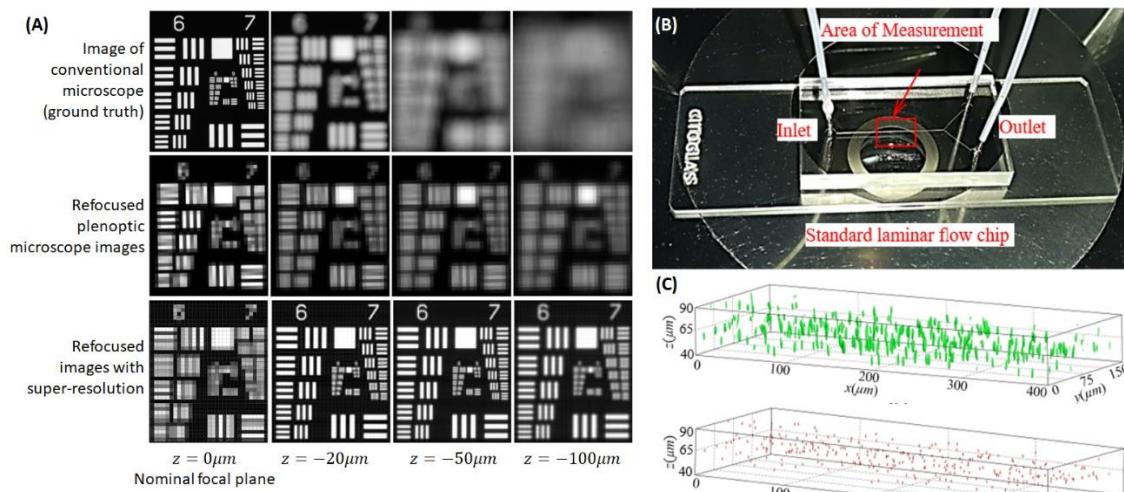


Fig. 12 (A) Plenoptic microscope images refocused to a translating target's corresponding planes, without and with super-resolution, as compared to a non-refocusing conventional microscope image [71]. (B) Example of plenoptic micro-PIV on a flow chip. (C) 3D particle reconstruction (green) from the flow-chip, with the additional step of particle centroid-finding (red) prior to cross-correlation [30].

5 A typical plenoptic microscope's layout is
6 schematically illustrated in Fig. 11 [30,66,72–74],
7 comprising of a condenser lens for focusing
8 illumination onto the subject, an infinity-corrected
9 objective lens and a tube lens. The MLA is placed at
10 the tube lens' image-plane, followed by the sensor
11 at its usual placement behind the MLA. (s, t) and
12 (u, v) are mapped in the typical manner. For micro-
13 PIV applications where particles are seeded into the
14 specimen volume, illumination can also be
15 introduced along the optical path via an angled
16 dichroic mirror to elicit back-scattering signals from
17 particles.

18 Though the standard plenoptic refocusing and
19 perspective-view procedures apply to microscopy,
20 wave optics must be considered for accuracy due to
21 diffraction at this scale. Compared to macro-scale
22 volumetric reconstruction with tomography
23 equations, the microscopy community has a longer
24 history with the deconvolution approach tracing
25 back to the use of non-plenoptic cameras [71]. The
26 original approach consists of focusing a
27 conventional camera on a microscopic subject, and
28 then physically translating the subject in depth to
29 generate a focal-stack. Deconvolving the focal-stack
30 with the system's point spread function (PSF),
31 which is conveniently shift-invariant for a
32 telecentric microscope objective, then reconstructs
33 the subject volume. Notably, Levoy et al. [72]
34 showed that deconvolution is fundamentally
35 equivalent to limited-angle tomographic
36 reconstruction.

37 Acquisition of focal-stack via physical
38 translation is not possible for highly dynamic
39 subjects; thus, plenoptic microscopy offers a
40 valuable alternative where the light-field is
41 instantaneously captured, and a focal-stack can be
42 synthetically generated. In early implementation by
43 Levoy et al. [72], the plenoptic system's PSF was
44 empirically determined by imaging a sub-pixel
45 fluorescent bead to approximate a point-source; later,
46 Broxton et al. [71] proposed a more comprehensive
47 model that accounts for plenoptic systems' non-
48 uniform sampling of a scene at different depths and,
49 consequently, its shift-variant PSF. Additionally, as
50 Fig. 12A shows, using super-resolution procedures
51 that exploit the system's non-uniform sampling
52 pattern, Broxton et al. [71] was also able to gain
53 resolutions that were 8 times higher than a naïve 1:1
54 (s, t) - (u, v) tradeoff would otherwise offer.

55 More recent developments in plenoptic
56 microscopy include the concept of selective volume
57 illumination (SVM), which found that 3D
58 reconstruction and refocusing have lower artifacts
59 when the illumination is confined to the depths of
60 interest in lieu of a back-light that permeates the
61 volume [70,75]. On the other hand, Levoy et al. [73]
62 proposed that in addition to imaging, a second MLA
63 can be installed and operated in "reverse" as an
64 illuminator to achieve depth-modulated illumination.
65 Finally, successful demonstrations of velocimetry
66 on the micro scale include [30,74] for PIV (see Fig.
67 12B-C) and [66] for PTV. Given its vast potentials,
68 plenoptic microscopy is developing into a field of its
69 own. The future of plenoptic flow diagnostics will
70 likely benefit from adopting unique techniques
71 developed from the general plenoptic microscopy
72 community.

6. Application to Scalar-Field Measurements

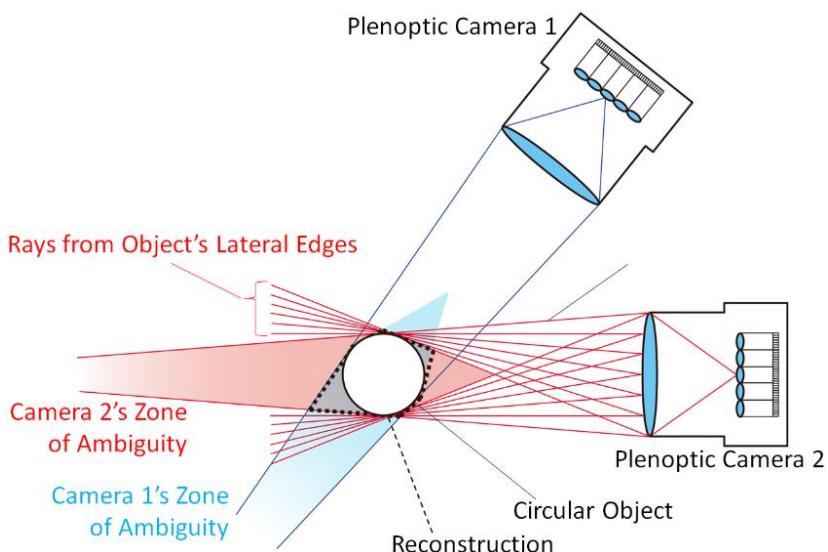
73 The use of plenoptic cameras for 3D scalar-
74 field measurements involves reconstruction of a 3D
75 luminescent field such as flame or fluorescing flow
76 [76–81], and in some instances, further
77 specialization into simultaneous multi-spectral
78 measurements by installing color filters within the
79 plenoptic camera (see next section) [78,79], or
80 derivation of physical quantities such as 3D flame
81 temperature-fields by assuming proportionality
82 between luminescence and temperature [80,81].

83 Similar to plenoptic-PIV, early works in
84 plenoptic scalar-field measurements only achieved
85 qualitative 3D reconstruction by refocusing the
86 scene to create a focal-stack, and subsequently
87 applying image segmentation to localize a subject in
88 z . Subsequently, quantitative reconstructions were
89 achieved by adopting a similar tomographic
90 workflow as plenoptic-PIV, with the critical
91 difference that the subject is no longer sparse
92 particles. This has the immediate ramification that
93 large scalar-field objects create proportionately
94 giant zones of ambiguity. Consider a uniform-
95 intensity spherical object imaged by just one camera
96 ("Camera 2") in Fig. 13. The two red fans of rays
97 define the side-most edges of the objects that
98 Camera 2 sees, and their associated captured rays.
99 The shaded region between the fans contain the
100 object's zone of ambiguity, within which the
101 plenoptic camera cannot distinguish between the
102 presence or absence of object. Hence, a naïve

5 tomographic reconstruction will fill the shaded
6 region.

7 Thus, complex scalar-fields are not easily
8 resolved in spite of the plenoptic camera's
9 perspective-view redundancy. This problem relates
10 to the principle of limited-angle tomography, which
11 stipulates that not all perspective-views are created
12 equal [82]. For a given number of perspectives,
13 views that are spaced far apart contribute
14 significantly more quality to reconstruction than
15 views of limited angles. Present research work

16 around this issue by implementing multiple
17 plenoptic cameras (or a camera with split view) to
18 increase the effective measurement angle. I.e. as
19 illustrated in Fig. 13, the zone of ambiguity is
20 significantly reduced by adding just a second
21 camera- though the resulting shape would still be far
22 from smooth. The exact equivalence between the
23 number of regular cameras versus plenoptic cameras
24 required for scalar-field measurement remains to be
25 determined.



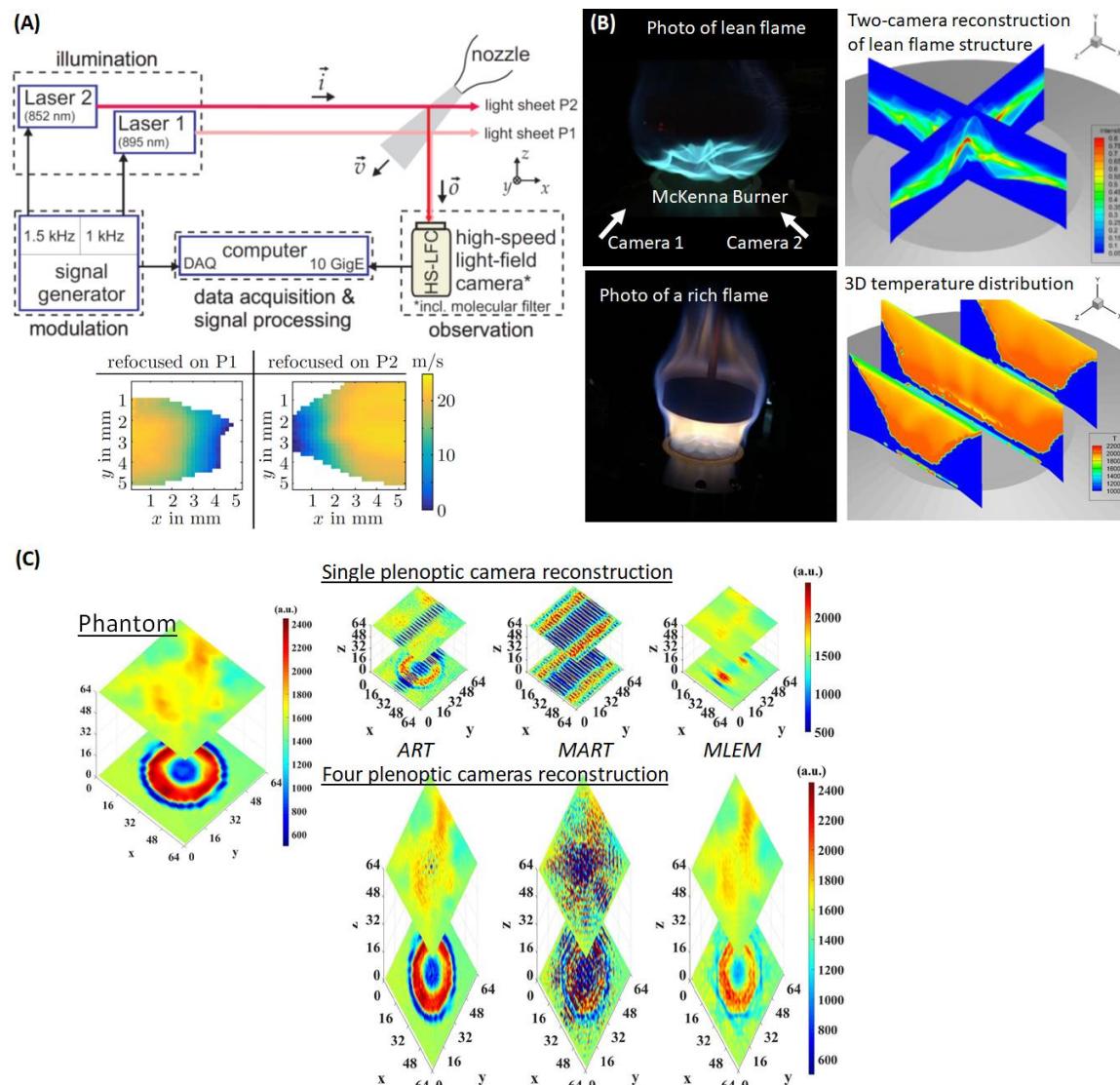
33 Fig. 13 The configuration and challenges of plenoptic 3D scalar-field measurement. Colored zones
34 represent the zones of ambiguity for each camera. Whereas, the dotted diamond represents the combined
35 zone of ambiguity for two cameras.

36 Some existing examples of plenoptic scalar-
37 field measurements are shown in Fig. 14. Fig.
38 14A by Fischer et al. [29] represents an early
39 unique application, where laser-scattering signals
40 were elicited from two *planes* within a spray. The
41 images resembled a smooth scalar-field due to
42 sub-pixel droplet sizes. Fischer et al. then used
43 refocusing to localize signals from both planes,
44 after which the refocused images were interpreted
45 through frequency modulation Doppler global
46 velocimetry (FM-DGV) to determine each
47 plane's droplets velocities. Fig. 14B by George et
48 al. [79] describes a combustion experiment with
49 two plenoptic cameras. A filtered variant of the
50 focal-stack method was used for reconstruction.
51 In addition to flame luminescence, George et al.
52 was able to derive 3D distributions of soot loading,
53 temperature and other quantities by filtering the
54 cameras' wavelengths. Filtered focal-stack was
55

56 ultimately concluded as insufficient and
57 tomography recommended for future
58 reconstructions. Fig. 14C also shows a scalar-
59 field/flame study. This work by Liu et al. [76]
60 compared the effects of camera number and three
algorithms (namely, Algebraic Reconstruction
Technique, ART; MART; and Maximum
Likelihood Expectation Maximization, MLEM)
on reconstruction quality. Both simulation and
experimental data were employed. They
confirmed that a single plenoptic camera was
insufficient for 3D scalar-field measurement, and
additionally concluded that MART was
unsuitable for non-sparse scalar-fields. ART and
MLEM performed similarly. In conclusion, Liu et
al. also demonstrated improved results using a
single Lytro plenoptic 1.0 camera modified with a
three-view splitter for added parallax angle.

5 Ongoing works in this field continue to
 6 explore variations of algorithms to improve
 7 reconstruction, including adaptive simultaneous
 8 algebraic reconstruction technique (ASART) with
 9 total variation (TV) regularization [78], dynamic
 10 masking [78], and potential incorporation of prior
 11 knowledge among others. In our opinion,
 12 plenoptic cameras may not maximally
 13 demonstrate its potential in single-physics 3D
 14 scalar-field measurement, where large
 15 perspective-view redundancy does not equate to
 16 significant quality improvement. I.e. while a

17 plenoptic approach may still lead to a reduction in
 18 the number of cameras, the reduction will not be
 19 proportional to the number of perspective-views
 20 per camera. Instead, a plenoptic camera may show
 21 its true advantage when employed in future multi-
 22 physics measurements such as combined 3D and
 23 multispectral measurement, where portions of the
 24 redundant perspective-views are devoted to
 25 sampling different physics. The groundwork for
 26 incorporating more physics into plenoptic flow
 27 diagnostics is touched upon in the section on
 28 plenoptic spectroscopic imaging.



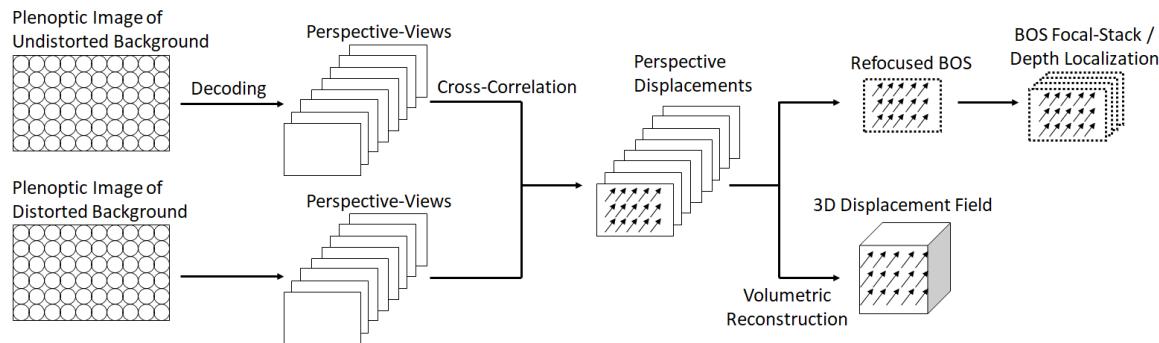
53 Fig. 14 Examples of plenoptic scalar-field measurements: (A) High-speed multi-plane frequency
 54 modulation Doppler global velocimetry (FM-DGV) by [29]. (B) 3D reconstruction of flame structure,
 55 temperature-field and other quantities through filtered-refocusing and two plenoptic cameras [79]. (C)
 56 [76]'s assessment of the effects of algorithm and camera number on reconstruction quality.

5 **7. Application to Background-Oriented**
6 **Schlieren (BOS)**

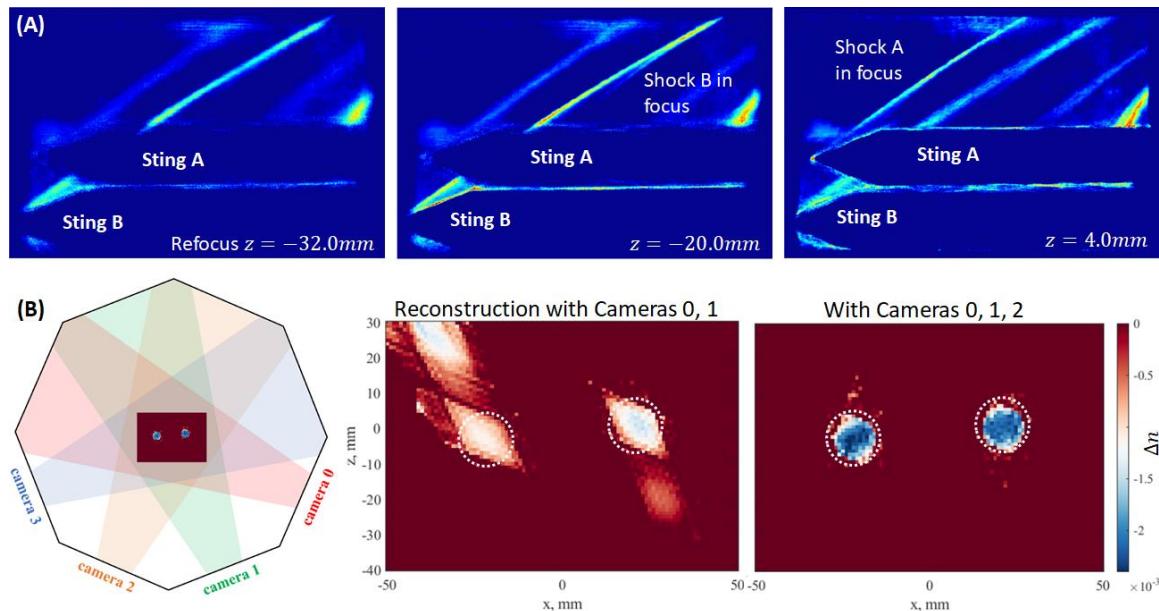
7 BOS is a prevalent technique in flow diagnostics, especially for studying supersonic aerodynamics and mixture interfaces, due to its simplicity and non-intrusiveness, where only a patterned background is required to determine density gradients around a subject. However, similar to scalar-field tomography, the camera apparatus to acquire 3D BOS measurements is cumbersome [83] and plenoptic-BOS [84–88] represents a potential for cost and complexity reduction. Like plenoptic-PIV, plenoptic-PTV and scalar-field measurement, early plenoptic-BOS efforts employed focal-stack for qualitative 3D localization [85,86]. The associated workflow is illustrated on the top-half of Fig. 15: two separate plenoptic images are first taken, with and without distortion of the patterned background. Subsequently, these images are decoded into perspective-views. Cross-correlation between the two sets of perspective-views provide 2D background displacement field in each perspective. From these “displacement perspectives,” a refocusing procedure that treats displacements as equivalent intensities is implemented to create a BOS focal-stack. An example of such a stack is shown in Fig. 16A for two stings at different depths in a supersonic flow, Sting B at around $z \approx -20\text{mm}$ and Sting A at $z \approx 4\text{mm}$. Color represents

displacement magnitude. It is evident that displacement features appear sharpest when the BOS image is refocused to their corresponding depth. Thus, a low resolution 3D localization can be achieved based on image sharpness in the focal-stack.

More recent plenoptic-BOS studies attempt to acquire higher resolution 3D reconstruction of density field by adopting the tomographic approach [88]. As shown in the bottom-half of Fig. 15, the displacement perspectives are processed through a tomographic reconstruction routine much alike scalar-field reconstruction in this approach. Consequently, a 3D displacement-field is generated, from which 3D density-field can be acquired. This tomographic approach suffers from the same challenges as scalar-field tomography, where the field is too information-rich for a single plenoptic camera’s limited parallax angle. The effect is exhibited in Fig. 16B of an experiment with two translucent cylinders immersed in *nearly* index-matched liquid, and four plenoptic cameras. Reconstruction with only two cameras resulted in rough diamond-shapes objects. The incorporation of a third camera substantially rounded the reconstructed objects, making them closer to the ground truth (dotted circles). Summarily, as they face similar challenges, we expect development in plenoptic scalar-field measurement and BOS to be parallel and mutually beneficial in the near future.



48 Fig. 15 Two plenoptic-BOS work-flows: refocusing (top) and tomography (bottom).
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 50
 51
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 60



24 Fig. 16 Examples of plenoptic-BOS in (A) qualitative 3D localization two stings' shockwaves as part of
25 Klemkowsky et al.'s developmental work [85,86] and (B) quantitative tomographic reconstruction [88].
26

27 8. Application to Spectroscopic Imaging

28 While previous applications of plenoptic
29 camera have focused on multiplexing numerous
30 perspectives onto a single sensor, a recent line of
31 research proposed wavelength-multiplexing. As
32 shown in Fig. 17, the fundamental architecture
33 involves a plenoptic 1.0 system, which conveniently
34 decouples (s, t) and (u, v) . A color filter is ideally
35 placed at the main-lens' aperture plane where (u, v)
36 is mapped, such that rays from an object are filtered
37 by wavelengths λ depending on their incident angles.
38 I.e. λ is mapped to (u, v) . And, since (u, v) is two
39 dimensional, a color filter with 2D pattern can be
40 used. The result of this mapping is shown on the
41 (s, t) vs. (u, v) sampling diagram on the right of Fig.
42 17, where images of specific λ can be rendered
43 identically as perspective-views. Notably, this
44 approach only works on objects within the DOF of
45 the nominal focal-plane, since objects away from the
46 focal-plane will defocus and spread across multiple
47 microlens, losing its direct (u, v) multiplexing
48 pattern.

49 Key works in this area include [89–91], which
50 separately used discrete and continuous color filters
51 to gain spectral information of a scene. In both cases,
52 the spectral information is further mapped against a

53 blackbody distribution to gain insights into a scene's
54 temperature, in a manner more precise than single-
55 wavelength infrared thermometry. Used for
56 spectroscopy, plenoptic multi-band imaging would
57 also compare favorably against traditional point-
58 measurement spectrometers, against imagers that
59 separate wavelengths by prisms/dichroic-mirrors
60 and requires one camera per wavelength, or the use
61 of filter wheels that compromise the time-resolved
62 capability of the sensor, or against wavelength-
63 filtering on the sensor's Bayer filter, which is not
64 currently amenable to customization from a cost
65 standpoint.

66 Multi-band plenoptic imaging is only in its
67 infancy, and while valuable on its own, we see the
68 next step in development as extending the multi-
69 band imaging capability to 3D. Breaking the
70 limitation of confining objects to the DOF will offer
71 greater potential in adopting it as a doorway to more
72 complex measurements such as combined multi-
73 spectral scalar-field tomography or FSI
74 measurements where surface and flow tracers are
75 filtered by wavelength, as well as a doorway to
76 rendering colors on otherwise monochromatic
77 devices such as night-vision image intensifier.

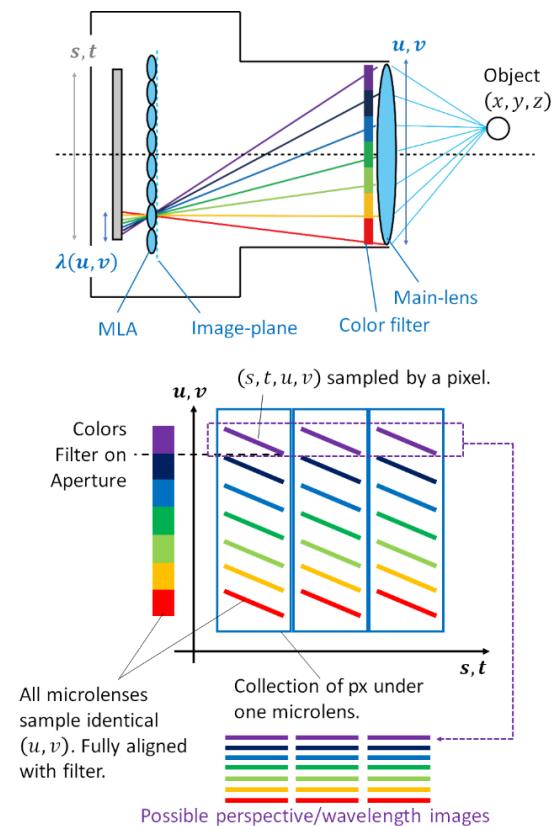


Fig. 17 Principles of plenoptic spectroscopic imaging.

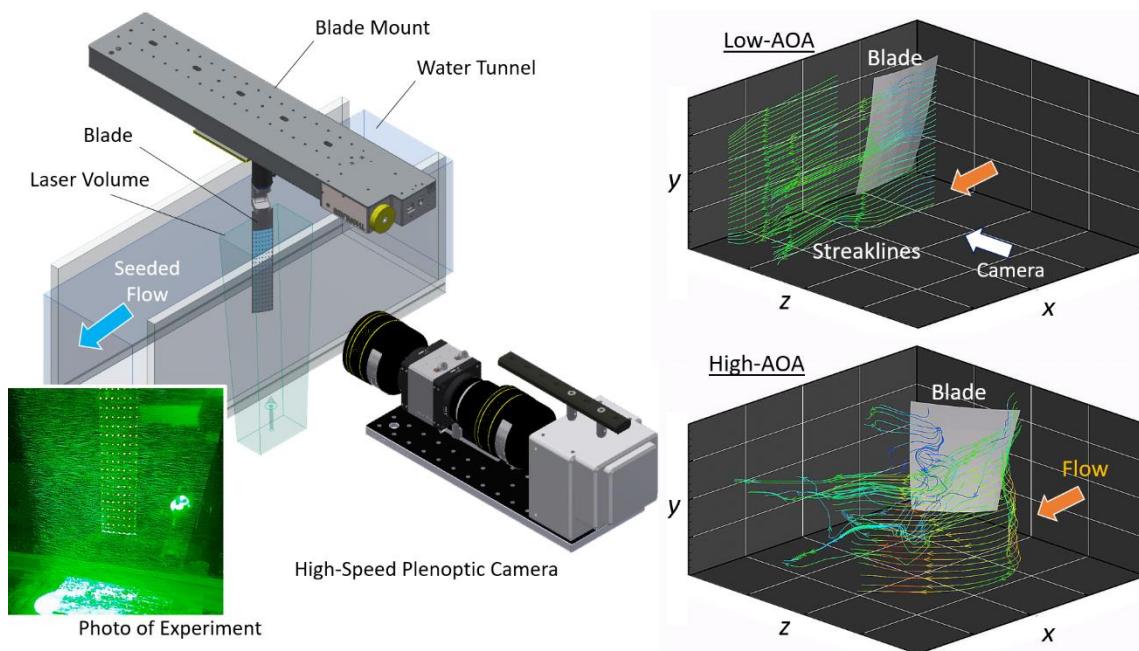
9. Future

The future of plenoptic flow diagnostics will benefit from both evolutionary hardware improvements as well as revolutionary architectural shifts. Evolution in image sensors will progressively allow for experiments with faster temporal dynamics, where currently $\sim 1000\text{fps}$ is near the upper limit due to plenoptic cameras' resolution requirement. Evolution in MLA fabrication will continue to enable cheaper and more precise devices - as well as more creative implementations such as the heterogeneous focal-length MLA [92]. At present, a large percentage of plenoptic flow diagnostics rely on the 1.0 architecture and its conveniently decoupled (s, t) and (u, v) data. A migration to plenoptic 2.0, especially with the ability to control $(s, t) : (u, v)$ trade-off on-the-fly will alleviate the resolution demand of plenoptic systems and possibly redefine the cost and capability of many plenoptic techniques. Finally, projecting further into the future, the advent of more powerful smartphone sensors will eventually see migration of

some users from large-sensor plenoptic camera to camera-array LF imaging based on small, cheap sensors- possibly in highly decentralized configurations optimized to particular experiments.

On the software side, the decoding of perspective-views from a raw image is the first step in most plenoptic techniques. Presently, decoded images suffer from low resolution, low SNR and interpolation artifacts that limit the range of data processing one can perform on perspective-views. Research in super-resolution, AI-driven data enhancement and refinement in decoding techniques will continue to improve image quality for a given hardware for some time to come. Outside of decoding, new calibration algorithms including higher-order corrections and auto-calibration will continue to improve 3D results and workflow ergonomics. Meanwhile, more advanced regularizations and tomography algorithms will improve and accelerate 3D reconstruction. Outside of predictable developments, we expect novel cross-disciplinary techniques to arise from rapid progress in the machine vision community, which has also taken interests in plenoptic and LF imaging. Finally, design and operation of these cameras still presently require significant experience and expertise. The simplification of plenoptic techniques into plug-and-play systems remain an industrial design challenge to be tackled both on hardware and software fronts.

Though exciting advancements are on the horizon, the challenge of limited parallax baseline and spatial-angular resolution tradeoff remain fundamental to plenoptic systems. Thus, for 3D applications where conventional approaches will suffice, plenoptic cameras provide some simplification and cost-saving, but not necessarily a transformative new capability. Instead, the biggest future value of plenoptic flow diagnostics may lie in its ability to fuse different diagnostics within one sensor, allowing it to achieve many conventionally impossible or impractical experiments. One early example is the fusion of plenoptic-PIV and 3D tracking under one camera to achieve 3D FSI measurement [93,94], as shown in Fig. 18. In this case, the equivalent conventional approach involves simultaneously exercising tomo-PIV and stereo-digital image correlation, which would have required six cameras, rendering the experiment prohibitive in cost and complexity [2,3]. Numerous other configurations of plenoptic diagnostics fusion remain to be explored in the near future.



25 Fig. 18 Example of plenoptic diagnostics-fusion: single-camera FSI measurement performed by the
 26 author's group. Figure shows simultaneous 3D tracking of a blade with 2.5cm chord and 3D velocimetry of
 27 its associated flow-field at different angles of attack (AOA). Tracking is based on surface markers while
 28 velocimetry is based on flow seeding. [93,94]

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