An Efficient Algorithm for Routing and Recharging of Electric Vehicles

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Abstract. In this paper, we address the routing and recharging problem for electric vehicles, where charging nodes have heterogeneous prices and waiting times, and the objective is to minimize the total recharging cost. We prove that the problem is NP-hard and propose two algorithms to solve it. The first, is an algorithm which obtains the optimal solution in pseudo-polynomial time. The second, is a polynomial time algorithm that obtains a solution with the total cost of recharging not greater than the optimal cost for a more constrained instance of the problem with the maximum waiting time of $(1-\epsilon)\cdot W$, where W is the maximum allowable waiting time.

Keywords: Routing · Electric Vehicles · Recharging · Rounding.

1 Introduction

In the last decade, the increased awareness of the global warming brought attention toward transportation, as this sector accounts for a large amount of air pollution. In 2018, the share of transportation in greenhouse gas emissions was 28.2% [11]. The policy of replacing conventional gasoline vehicles with All-Electric Vehicles (EVs) has been followed by many countries as an effective approach toward a greener transportation system. EVs, in addition to being environmental friendly, are more energy efficient. In these vehicles, over than 77% of the electrical energy from the grid is transmitted to the wheels, while in conventional gasoline vehicles only about 12% - 30% of the energy stored in gasoline is converted to power at the wheels [3]. Despite the developments in the EVs technologies over the last decade, these vehicles represent a very small fraction of the overall vehicle market, even in the countries with the largest emission of carbon dioxide. In 2018, the penetration rate of EV in the light-vehicle market of the US and China was only 2.1 and 3.9 percent, respectively [5]. The low public interest in EVs is partially attributed to the driving range anxiety and to the lack of extensive charging infrastructure. This challenge has motivated researchers to devote their efforts to developing efficient optimization methods for recharging and routing policies for EVs.

In fact, recharging policy optimization for EVs is analogous to refueling policy optimization for gasoline vehicles. There are factors such as overcharging cost and charging waiting time that do not apply to refueling policy optimization for gasoline vehicles, but need to be taken into account while optimizing the recharging policies for EVs. The first effort on investigating the problem of refueling policy optimization dates back to 1980s, where the aim was to find the shortest path between two nodes while the vehicle has to visit some intermediate nodes for refueling [6]. Since then, researchers have attempted to study the properties of the problem and take multiple factors into account when designing the refueling policies. Lin [8] studied the properties of the refueling policy optimization problem, and based on these properties showed that the problem of finding the optimal refueling policy can be reduced to the classical shortest path problem. Khuller et al. [7] studied refueling and routing optimization problems for conventional gasoline vehicles, assuming that each gas station has a certain price for gasoline. They considered the problem with the objective of minimizing the cost of a fixed route and showed that it can be solved in polynomial time. The authors showed that the problem of finding the cheapest tour while a given set of locations are visited is NP-complete and developed approximation algorithms for this problem. Arslan et al. [1] formulated the refueling/recharging policy optimization for plug-in hybrid EVs where the vehicle has to visit both refueling and recharging stations, and the objective is to minimize the total cost which includes fuel and energy costs, stopping costs, depreciation costs, and battery degradation costs. Nejad et al. [9] developed one approximation and two exact algorithms for the routing problem of plug-in hybrid EVs. In addition to the optimal route, their proposed algorithms identify the predominant operating mode for each segment of the path in order to minimize the fuel consumption. In a recent work, Sweda et al. [10] considered the availability of a charging station at any point in time as a probabilistic parameter and developed two heuristic methods to obtain an a priori routing and recharging policy. In real world, recharging stations might have heterogeneous prices and waiting times. To the best of our knowledge, no research has been done on the routing and recharging problem for EVs with heterogeneous prices and waiting times. In this paper, we address the routing and recharging problem for electric vehicles with the objective of minimizing the total recharging cost, where charging nodes have heterogeneous prices and waiting times. We prove that the problem is NP-hard. We propose a pseudo-polynomial algorithm to obtain the optimal solution. We also propose a polynomial time algorithm and prove that it obtains a solution with the total cost of recharging not greater than the optimal cost for a more constrained instance of the problem with the maximum waiting time of $(1 - \epsilon) \cdot W$, where W is the maximum allowable waiting time.

2 Problem Definition

We formulate the Electric Vehicle Routing and Recharging Problem (EVRRP). We consider an EV which is initially fully charged that is going to travel through a road network (i.e., a directed graph) having n charging nodes $v_1, \ldots, v_n$. We do not consider any restriction such as acyclicity and predetermined order of the nodes. The EV travels from the start node v_1 to the destination node v_n. During the trip from the source to the destination, the EV may need to be recharged at the charging nodes. The goal is to find a path from the start node to the destination node as well as a recharging policy for the EV such that the total cost of recharging is minimized, while the total waiting time for recharging does not exceed a given value, W.

As the driver selects a path from the start node to the destination node, she/he must decide whether to stop and recharge at each node, and how much to recharge at each stop. We assume that the maximum capacity of the battery is F units. We denote by h_{ij} the amount of charge consumed from node i to node j, if they are adjacent. We assume that the braking energy recuperation is negligible and thus, the amount of charge consumption of the EV to pass a road segment is non-negative (i.e., $h_{ij} \geq 0$). To have a feasible solution, we assume that for adjacent nodes i and j, h_{ij} is less than the maximum capacity of the battery, F. Also, we assume that $h_{ij} = \infty$ if there is no road segment from node i to node j. Note that in the paper, we use v_i and i alternatively when we refer to a charging node v_i.

Consider a path p that contains two consecutive nodes v_i and v_j. The charge level of the EV's battery at node j is denoted by q_j, and is recursively defined based upon: q_i, the charge level of the battery in the previously visited node i; h_{ij}, the amount of charge consumption to reach node j from node i; and r_j, the amount of recharging at node j, as follows,

$$q_j = r_j + q_i - h_{ij}. \tag{1}$$

The EV must have enough charge to travel from node i to node j,

$$h_{ij} \leq q_i. \tag{2}$$

The charge level of the EV at a node i cannot exceed the maximum capacity F,

$$q_i \leq F. \tag{3}$$

The waiting time to access node i is denoted by ω_i. The total waiting time for recharging cannot exceed a given value W,

$$\sum_{i \in p, r_i > 0} \omega_i \leq W. \tag{4}$$

The charging nodes are heterogeneous in terms of their charging price. At node i, the EV is charged at a fixed price per unit of charge, μ_i. The objective is to find a path p over all possible paths P from node 1 to node n, and a recharging policy r over all feasible recharging polices on path p, R_p, such that the total recharging cost is minimized,

$$\min_{p \in P, r \in R_p} \quad \sum_{i \in p} \mu_i \cdot r_i. \tag{5}$$

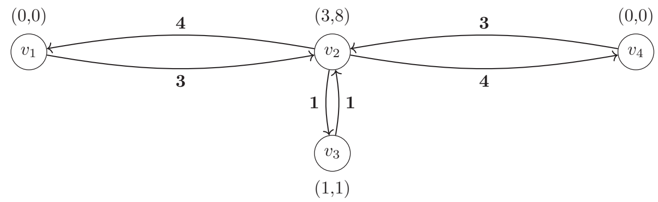


Fig. 1: An illustrative example: a road network with four charging nodes, where the start node is v_1 and the destination node is v_4. The weight on each edge indicates the amount of charge consumption, while the pair on the vertices indicates the waiting time and the price per unit of charge, respectively.

Note that in the objective function, we include the cost of recharging at the source node and the destination. In fact, since the EV is full at node v_1 at the beginning of the trip, without loss of optimality, we can assume that $\mu_1 = 0$. Therefore, the cost of recharging at this node is zero and the EV is fully charged at this node. Similarly, we do not need to recharge the EV at the destination node, we can assume that $\mu_n = 0$. Therefore, the cost of recharging at this node is zero and will not affect the value of the objective function. It is straightforward to extend the results in this paper to settings where the EV starts with a non-full charge.

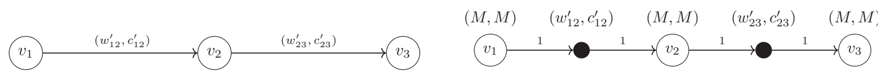
EVRRP can be represented as a directed graph $G(V, E)$, where V is the set of vertices representing charging nodes and E is the set of edges representing the road segments between the nodes. Figure 1 shows an example of such a road network. In this example, there are four charging nodes. The vehicle must travel from the start node v_1 to the destination node v_4. The weight on each edge shows the amount of charge consumption required to travel the corresponding road. The pair on the vertices shows the waiting time and the price per unit of charge, respectively.

2.1 Complexity of EVRRP

In this section, we prove that EVRRP is NP-hard by showing that: (i) the decision version (EVRRP-D) of EVRRP belongs to NP, and, (ii) a well known NP-complete problem is reduced to EVRRP-D in polynomial time.

For the first condition, we can easily show that EVRRP-D is in NP. We only need to guess a solution and a value C, compute the total value of the objective function (Equation (5)), and verify if the solution is feasible and the associated objective value is at most C. Obviously, this can be done in polynomial time. For the second condition, we show that the Shortest Weight-Constrained Path problem (SWCP), a well-known NP-complete problem (problem ND 30 in [4]), is reduced to EVRRP-D in polynomial time.

An instance of EVRRP-D is represented by a graph $G(V, E)$, where $V = \{v_1, \ldots, v_n\}$ is the set of vertices representing charging nodes, and E is the set of edges representing the road segments between the nodes. Each node is char-



Graph G' Graph G

Fig. 3: Transforming graph G', an arbitrary instance of SWCP, to graph G, an instance of EVRRP-D.

acterized by the charging price per unit of charge, μ_i, and the waiting time, ω_i. The weight of each edge (i, j) is the amount of charge consumed when traveling road segment (i, j) (i.e., h_{ij}). The decision question is whether there is a path in G from the source node v_1 to the destination node v_n with a feasible recharging policy over the path so that the total cost of recharging (Equation (5)) does not exceed C and the total waiting time for recharging does not exceed W.

An instance of SWCP consists of a graph $G'(V', E')$, where $V' = \{v'_1, \ldots, v'_m\}$ is the set of vertices and E' is the set of edges with cost c'_{ij} and weight w'_{ij} for each $(i, j) \in E'$. The decision question is whether there is a path in G' from v'_1 to v'_m such that the total cost does not exceed C', while the total weight is not greater than W'.

Theorem 1. *EVRRP-D is NP-complete.*

Proof. We show that an arbitrary instance of SWCP is transformed into an instance of EVRRP-D. Let $F = 2$, $C = 2C'$, $W = W'$, and $n = m$. First, we build graph G with the same set of vertices as graph G' (i.e., $V = V'$), where $v_1 = v'_1$ and $v_n = v'_m$. We call these nodes the primary nodes of G. Then, we add some other nodes to the graph as the secondary nodes.

For every edge (i, j) in G', we add a secondary node in G. We denote this node by v_{ij}. Then, we add one edge from node v_i to node v_{ij}, another edge from node v_{ij} to node v_j. We set the amount of charge consumption of these edges to one. Therefore, the amount of charge consumption on path $\{v_i, v_{ij}, v_j\}$ is two.

We assume that the waiting time at the secondary node v_{ij} is w'_{ij}, while at each primary node it is $M >> W$, a very large value. Furthermore, the charging price per unit of charge of the secondary node v_{ij} is c'_{ij}. The charging price rate at each primary node is $M >> C$, a very large value. Therefore, it is more preferred to recharge the EV at the secondary nodes.

Figure 3 shows how the graph G is built based on graph G'. Figure 2a shows graph G' that has three nodes v_1, v_2, and v_3. The label on each edge represents the weight and the cost of that edge, respectively. Figure 2b shows graph G with three primary nodes v_1, v_2, and v_3. We add one secondary node between nodes v_1 and v_2, and one secondary node between nodes v_2 and v_3. The secondary nodes are represented by black filled circles.

Now, we show that the solution for EVRRP-D can be constructed based on the solution for SWCP. Let us assume that U' is the routing path obtained for SWCP in G'. To obtain the corresponding path U in G, we choose the same path for the primary nodes. The path from a primary node v_i to the next primary node v_j is $\{v_i, v_{ij}, v_j\}$.

The EV starts the route from node v_1 with an initial charge level F. Since the recharging price rate at primary nodes is relatively high, the EV recharges only at the secondary nodes. Furthermore, the amount of charge consumption between every two adjacent nodes on the path (primary/secondary) is $\frac{F}{2}$. Thus, an optimal recharging policy of the EV is to stop at the first secondary node after v_1 and recharge the EV to level F. The amount of recharging at this node is $\frac{F}{2}$. For the remaining path, the policy is to pass the next primary (without recharging) and stop at the next secondary node and recharge the battery fully. For the last secondary node, the node immediately before the destination node, we recharge the EV to level $\frac{F}{2}$, which is enough to reach the destination node. Thus, the amount of recharging at this node is $\frac{F}{2}$. The total waiting time of the path is equivalent to the total waiting time of the secondary nodes on the path, $\sum_{v_{ij} \in U} w'_{ij} = W'$. Since $W = W'$, Constraint (4) is satisfied.

The price per unit of charge at each secondary node v_{ij} is c'_{ij}. Since we recharge the EV for at most two units, the total cost of recharging to reach the destination is at most $\sum_{v_{ij} \in U} 2 \cdot c'_{ij} = 2C'$. Thus, we obtain a solution for EVRRP with objective value less than $2C'$. Since $C = 2C'$, the total recharging cost for EVRRP does not exceed C.

Conversely, suppose that U is the routing path in G obtained for EVRRP. To obtain path U' in G', we choose the sequence of primary nodes of path U. Since the total waiting time of path U does not exceed W, the total weight of the corresponding edges in G' does not exceed W, too. Since $W' = W$, the total weight on path U' does not exceed W'.

Furthermore, in path U, the amount of recharging at the first secondary node and the last secondary node is $\frac{F}{2}$. The amount of recharging at other secondary nodes is $F = 2$. Thus, the cost of recharging at the first secondary node and the last secondary node is equivalent to the cost of the corresponding edge in G'. The cost of recharging at any other secondary node, is two times greater than the cost of the corresponding edge in G. Since the total cost of recharging in G does not exceed C, the total cost of the corresponding edges in graph G' does not exceed $\frac{C}{2}$. Since $C = 2C'$, the total cost of edges of path U' does not exceed C'.

3 Optimal Solution for EVRRP

Here, we present an algorithm that obtains the optimal solution for EVRRP in pseudo-polynomial time. We transform the original directed graph $G(V, E)$ into a directed graph $\tilde{G}(\tilde{V}, \tilde{E})$. In the transformed graph, we consider all possible sequences of stops for recharging. We denote by $H(i, j)$, the minimum amount of charge consumed from stop i to stop j. The value of $H(i, j)$ is obtained based on the shortest path (in terms of the amount of charge consumption) from node i to node j in G. We show that finding the optimal routing and recharging in G is equivalent to finding the shortest weighted constrained path in $\tilde{G}$. Then, we provide an algorithm to solve the problem.

In the following, we describe the Transform-Graph procedure which obtains the transformed graph $\tilde{G}(\tilde{V}, \tilde{E})$ from the original graph G. This procedure is based on the recharging rules described in the following lemmas which are extensions of the gas filling policy for the gas station problem [7].

Lemma 1. *Let node i and node j be two consecutive stops (for recharging) in the optimal solution. The path from node i to node j is the shortest path with the minimum amount of charge consumed from node i to node j in G. The following rules provide the optimal recharging policy at node i,*
(i) if $\mu_i < \mu_j$, then recharge the battery fully.
(ii) if $\mu_i \geq \mu_j$, then recharge the battery just enough to reach node j.

Proof. We can prove this by contradiction. If in the optimal solution, the path from node i to node j is not the shortest path, we can replace this path with the shortest path. Since the shortest path has the minimum amount of charge consumed from node i to node j, the level of the battery upon arriving to node j is higher than that in the optimal solution. Thus, the amount of recharging needed at node j is less than that in the optimal solution. This means that we improve the cost of recharging which is a contradiction with the optimality assumption.

Furthermore, if $\mu_i < \mu_j$ and the optimal solution does not recharge the battery fully at node i, then we can improve the cost of recharging by increasing the amount of recharging at node i and decreasing the amount of recharging at node j, which is a contradiction with the optimality assumption. Similarly, in the second case, if the optimal solution recharges the battery more than the charge amount needed to reach node j, then, we can improve the cost of recharging by decreasing the amount of recharging at node i and increasing the amount of recharging at node j.

Lemma 2. *Let nodes i, j, and k be three consecutive stops (for recharging) in the optimal solution. If $\mu_i < \mu_j$ and $\mu_j \geq \mu_k$, then $H(i,j) + H(j,k) > F$.*

Proof. According to Lemma 1, the level of the battery when the EV leaves node i is F. By contradiction, if we assume that $H(i,j)+H(j,k) \leq F$, then, the EV can reach node k without stopping at node j. In other words, the EV can improve the cost of recharging by decreasing the amount of recharging at node j (to level zero) and increasing the amount of recharging at node k. This is a contradiction with the optimality assumption.

Transform-Graph procedure. In this procedure, we transform the original graph $G(V,E)$ into a new graph $\tilde{G}(\tilde{V},\tilde{E})$. In the transformed graph $\tilde{G}$, each vertex represents two possible consecutive stops of the EV, and each edge represents three consecutive recharging stops. For every node i and node j in G, we add a node $< i,j >$ in $\tilde{G}$, if node j is reachable from node i (i.e., $H(i,j) \leq F$).

We also add a dummy source node $< 0,1 >$ and a dummy destination node $< n, n+1 >$ to $\tilde{G}$. Since the EV is full at node v_1 at the beginning of the trip, it will not go back to this node during the trip. Therefore, we do not need to add any node $< i,1 >$ (where $i > 1$) to $\tilde{G}$. Similarly, since the goal is to reach node v_n, the EV will not go back from this node to any other node. Therefore, we do not add any node $< n,i >$ to $\tilde{G}$.

For every pair of nodes $< i, j >$ and $< j, k >$, we add an edge from node $< i, j >$ to node $< j, k >$ based on a set of conditions. Each edge has a label (w_{ijk}, c_{ijk}), where w_{ijk} is the waiting time for recharging at node j, and c_{ijk} is the cost of recharging at node j.

We add an edge with label $(0, 0)$ from node $< 0, 1 >$ to every adjacent node $< 1, j >$. In fact, since the cost of recharging at node v_1 is zero (i.e., $\mu_1 = 0$), the battery will be fully charged at node v_1 with cost and waiting time equal to zero. We also add an edge from every node $< i, n >$ to the destination node $< n, n+1 >$ with label $(0, 0)$.

For every node $< i, j >$ and node $< j, k >$, where $i > 0$ and $k \leq n$, we consider all possible cases for the values of μ_i, μ_j, and μ_k. Based on these values, we add an edge from node $< i, j >$ to node $< j, k >$, as follows:

Case I ($\mu_i < \mu_j < \mu_k$): By Lemma 1, we should fully fill the battery at node i when node j is the next stop. Therefore, the level of the battery when arriving at node j is $F - H(i, j)$. Given that, $\mu_j < \mu_k$, we should again fill up the battery fully at node j. Thus, the cost of edge $(< i, j >, < j, k >)$ is $c_{ijk} = \mu_j \cdot H(i, j)$, and the waiting time of this edge is $w_{ijk} = \omega_j$.

Case II ($\mu_i < \mu_j$ and $\mu_j \geq \mu_k$): According to Lemma 2, node k is the next stop after node j only if $H(i, j) + H(j, k) > F$. Therefore, we add an edge from node $< i, j >$ to node $< j, k >$ if $H(i, j) + H(j, k) > F$. By Lemma 1, the level of the battery upon arriving at node j from node i should be $F - H(i, j)$. Given that $\mu_j \geq \mu_k$, the battery should only be filled up just enough to reach node k from node j. Thus, the cost of the edge is $c_{ijk} = \mu_j \cdot (H(j, k) + H(i, j) - F)$, and the waiting time is $w_{ijk} = \omega_j$.

Case III ($\mu_i \geq \mu_j$ and $\mu_j < \mu_k$): In this case, we have an empty battery when reaching node j from node i. Also, we want to recharge the battery fully at node j since $\mu_j < \mu_k$. Thus, we add an edge with cost $c_{ijk} = \mu_j \cdot F$, and waiting time $w_{ijk} = \omega_j$.

Case IV ($\mu_i \geq \mu_j$ and $\mu_j \geq \mu_k$): In this case, we have an empty battery when reaching node j; however, we only want to recharge enough to reach node k. Thus, the cost of the edge is $c_{ijk} = \mu_j \cdot H(j, k)$, and the waiting time is $w_{ijk} = \omega_j$.

Theorem 2. *The optimal solution for* EVRRP *in graph* G *is equivalent to the optimal solution for* EVRRP *in graph* $\tilde{G}$.

Proof. We need to show that: (i) for any feasible sequence of recharging stops in G, the corresponding sequence in $\tilde{G}$ is a feasible sequence and the amount of recharging at each node is the same as in G; (ii) for any feasible sequence of recharging stops in $\tilde{G}$, the corresponding sequence in G is a feasible sequence and the EV has the same recharging policy as in $\tilde{G}$.

Let $p = \{p_1, \ldots, p_s\}$ be the sequence of stops of a feasible path in G, where s is the number of nodes in the sequence. We need to show that (1) p is a feasible sequence of stops in $\tilde{G}$; and (2) the level of the battery when the EV arrives at node p_i in both graphs is the same. We prove this by induction.

According to our assumption, the EV reaches node v_1 with an empty battery. Then, it will be recharged to level F with zero cost/waiting time ($\mu_1 = \omega_1 = 0$).

Thus, node $p_1 = v_1$ is the first stop for recharging. In both G and $\tilde{G}$, node v_1 is reachable and the level of the battery when the EV reaches this node is zero.

Let us assume that $\{p_1, \ldots, p_i\}$ is a feasible sequence of stops in both graphs; and for every node $j \leq i$, the level of the battery is the same in both graphs. Now, we need to show that node $< p_i, p_{i+1} >$ in $\tilde{G}$ is reachable from $< p_{i-1}, p_i >$ via edge $(< p_{i-1}, p_i, < p_i, p_{i+1} >)$ and the level of the battery when the EV arrives at node p_{i+1} is the same in both graphs.

Since p_{i+1} is the next stop in the sequence p in G, $H(p_i, p_{i+1}) \leq F$. Thus, according to transformation rules, there is an edge from node $< p_{i-1}, p_i >$ to node $< p_i, p_{i+1} >$ in $\tilde{G}$. This implies that node $< p_i, p_{i+1} >$ is reachable from node $< p_{i-1}, p_i >$. For the second condition, we consider the possible values of μ_i and μ_{i+1},

Case 1 ($\mu_i < \mu_{i+1}$): According to Lemma 1, the level of the battery when the EV leaves node p_i in G is F. Thus, upon arriving at node p_{i+1}, the level of the battery is $F - H(p_i, p_{i+1})$. On the other hand, according to transformation rules (I) and (III), the level of the battery corresponding to edge $(< p_{i-1}, p_i >, < p_i, p_{i+1} >)$ is F. Thus, the level of the battery upon arriving at node p_{i+1} in both graphs is the same and equal to $F - H(p_i, p_{i+1})$.

Case 2 ($\mu_i \geq \mu_{i+1}$): According to Lemma 1, the level of the battery when the EV leaves node p_i in G is $H(p_i, p_{i+1})$. Thus, upon arriving at node p_{i+1}, the level of the battery is zero. On the other hand, according to transformation rules (II) and (IV), the level of the battery corresponding to edge $(< p_{i-1}, p_i >, < p_i, p_{i+1} >)$ is $H(p_i, p_{i+1})$. Thus, the level of battery upon arriving at node p_{i+1} in both graphs is the same and equal to zero.

Similarly, we can show that a feasible sequence of recharging in $\tilde{G}$ is a feasible sequence of stops in G and the level of the battery upon arriving at each node of the sequence in both graphs is the same.

Now, the problem is to find a path from the source node $< 0, 1 >$ to the destination node $< n, n+1 >$ in the transformed graph $\tilde{G}$ such that the total cost of the path is minimized, while the total waiting time does not exceed W. Therefore, EVRRP can be viewed as an SWCP problem. In order to obtain the optimal solution, we use a dynamic programming algorithm [2], called DP-SWCP, introduced for the SWCP problem.

The general idea of DP-SWCP is to use a set of labels for each node of the graph. Each label (W_{ijl}, C_{ijl}) corresponds to a path l from the source node $< 0, 1 >$ to node $< i, j >$ and is composed of two elements: W_{ijl}, the total waiting time of the path when the EV leaves node j, and C_{ijl}, the total cost of that path. DP-SWCP finds all non-dominated labels on every node. The dominance relation is defined based on the total waiting time and the total cost on each label. For a given node $< i, j >$, let us assume we find two labels (W_{ijl}, C_{ijl}) and $(W_{ijl'}, C_{ijl'})$ such that $W_{ijl} \leq W_{ijl'}$, and $C_{ijl} < C_{ijl'}$. Then, path l' cannot be a part of the optimal solution, because we could replace it with path l which has a lower cost and a lower weight. Therefore, we can disregard this path. In this case, we say that label (W_{ijl}, C_{ijl}) *dominates* label $(W_{ijl'}, C_{ijl'})$ and denote it by $(W_{ijl}, C_{ijl}) \triangleright (W_{ijl'}, C_{ijl'})$.

Algorithm 1 OPT-EVRRP Algorithm

Input: $G(V, E)$: Graph representing the road network
W: Maximum allowable waiting time
Output: $p = \{p_i\}$: Routing vector
$r = \{r_i\}$: Recharging vector
cost: Total cost
1: $\tilde{G} \leftarrow$ Transform-Graph(G)
2: $p \leftarrow$ DP-SWCP$(\tilde{G}, W)$
3: $q_1 \leftarrow F$
4: $i \leftarrow 1$
5: **for each** $u = 2, \ldots, |p| - 1$ **do**
6: $\quad j \leftarrow p_u$
7: $\quad k \leftarrow p_{u+1}$
8: $\quad$ **if** $\mu_j \leq \mu_k$ **then**
9: $\quad\quad q_j \leftarrow F$
10: $\quad$ **else**
11: $\quad\quad q_j \leftarrow H(j, k)$
12: $\quad r_j \leftarrow q_j - q_i + H(i, j)$
13: $cost \leftarrow \sum_{i \in p} r_i \cdot \mu_i$

DP-SWCP starts with the source node $< 0, 1 >$ and assigns a label $(0, 0)$. The algorithm extends the set of labels by treating a label with the minimum cost. In the treatment of a label l, the algorithm extends the path corresponding to the label l along all outgoing edges. In fact, the treatment of a label (W_{ijl}, C_{ijl}) on node $< i, j >$ considers each adjacent node $< j, k >$ such that $W_{ijl} + w_{ijk} \leq W$: if $(W_{ijl} + w_{ijk}, C_{ijl} + c_{ijk})$ is not dominated by any label on node $< j, k >$, adds it to the set of labels on node $< j, k >$. DP-SWCP continues the procedure until all non-dominated labels are treated. Finally, it picks the path that corresponds to the label with minimum cost at the destination node $< n, n + 1 >$ as the optimal solution.

The algorithm for solving EVRRP, called OPT-EVRRP, is given in Algorithm 1. The input of the algorithm is the graph $G(V, E)$, while the output is the sequence of stops $p = \{p_i\}$, the recharging vector $r = \{r_i\}$, and the total cost of recharging. The algorithm calls the Transform-Graph procedure to obtain the transformed graph $\tilde{G}$ (Line 1). Then, it calls DP-SWCP to obtain the optimal sequence of recharging p in $\tilde{G}$ (Line 2). Based on Lemma 1, the OPT-EVRRP obtains the optimal amount of recharging at each stop (Lines 3-13). In Section 4, we provide an example on how OPT-EVRRP works on the EVRRP instance given in Figure 1.

Theorem 3. *OPT-EVRRP obtains the optimal solution for EVRRP and its time complexity is* $O(n^3 + n^2 \cdot W)$.

Proof. According to Theorem 2, the optimal solution for EVRRP in graph G is equivalent to the optimal solution in graph $\tilde{G}$. Since DP-SWCP obtains the optimal solution in $\tilde{G}$, this solution is also optimal for EVRRP in G.

To determine the time complexity of OPT-EVRRP, we need to determine the time complexity of Transform-Graph and DP-SWCP procedures. The time complexity of Transform-Graph is proportional to the number of edges in $\tilde{G}$ and is $O(n^3)$. The time complexity of DP-SWCP depends on the number of treated labels in $\tilde{G}$. DP-SWCP does not treat two labels with the same total waiting time (because one of them has a cost not less than the other one, and therefore, it dominates it). The maximum waiting time of a label is bounded by W (W is integer). Thus, there are at most $W + 1$ labels on node $< i, j >$. On the other hand, there are at most n^2 nodes in $\tilde{G}$. Thus, the total number of treated labels is $O(n^2 \cdot W)$. Thus, the time complexity of OPT-EVRRP is $O(n^3 + n^2 \cdot W)$.

4 An Illustrative Example

We provide a numerical example to illustrate how OPT-EVRRP works. In this example, we consider the road network given in Figure 1 as the original graph G. The maximum capacity of the battery is $F = 4$, and the maximum allowable waiting time is $W = 8$. We can easily see that the optimal solution for this example is to start the trip from node v_1, visit node v_2 without recharging, stop at node v_3 and recharge for 4 units, visit node v_2 again and recharge for one unit, and finally, visit node v_4. The total cost of this recharging policy is 12, while the total waiting time is 4.

Now, we show how OPT-EVRRP obtains the optimal solution for this example. To transform graph G into $\tilde{G}$, we determine the value of $H(i, j)$, the minimum amount of charge consumed from node i to node j in G. Table 1 shows the value of $H(i, j)$ for every node i and node j in G.

Figure 4 shows the transformed graph $\tilde{G}$. The source node is node $< 0, 1 >$ and the destination node is $< 4, 5 >$. For every nodes i and j in G, we add a node $< i, j >$ to $\tilde{G}$, if node j is reachable from node i (i.e., $H(i, j) \leq F$). For example, we add node $< 1, 2 >$ to $\tilde{G}$ because $H(1, 2) = 3$; but we do not add node $< 3, 4 >$ to $\tilde{G}$ because $H(3, 4) > F$.

For every pair of nodes $< i, j >$, and $< j, k >$, we add an edge from node $< i, j >$ to node $< j, k >$ based on the transformation rules in Transform-Graph procedure. In Figure 4, the pair on the edge $(< i, j >, < j, k >)$ shows the waiting time and the recharging cost of the edge.

We add an edge from node $< 0, 1 >$ to adjacent node $< 1, 2 >$ with label $(0, 0)$. Similarly, we add an edge from node $< 0, 1 >$ to node $< 1, 3 >$ with label $(0, 0)$. For nodes $< 1, 2 >$ and $< 2, 4 >$, since $\mu_1 < \mu_2$, and $\mu_2 > \mu_4$, we

i/j	1	2	3	4
1	0	3	4	7
2	4	0	1	4
3	5	1	0	5
4	7	3	4	0

Table 1: Example: The values of $H(i, j)$

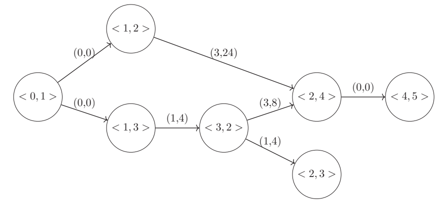


Fig. 4: Example: The transformed graph $\tilde{G}$.

follow the transformation rule (II). In this case, since $H(1,2) + H(2,4) > F$, we add an edge from node $< 1,2 >$ to node $< 2,4 >$. The cost of this edge is $\mu_2 \cdot (H(1,2) + H(2,4) - F) = 24$ and the waiting time is $\omega_2 = 3$. Similarly, for nodes $< 1,2 >$ and $< 2,3 >$, we follow the transformation rule (II). However, since $H(1,2) + H(2,3) = 4$ which is not greater than F, we do not add any edge from node $< 1,2 >$ to node $< 2,3 >$.

For nodes $< 1,3 >$ and $< 3,2 >$, we follow the transformation rule (I). We add an edge from node $< 1,3 >$ to node $< 3,2 >$ with cost $\mu_3 \cdot H(1,3) = 4$ and the waiting time $\omega_3 = 1$. We follow the transformation rule (II) to add an edge from node $< 3,2 >$ to node $< 2,4 >$. The cost of this edge is $\mu_2 \cdot (H(3,2) + H(2,4) - F) = 8$ and the waiting time is $\omega_2 = 3$. We also follow the transformation rule (III) and add an edge from node $< 2,3 >$ to node $< 3,2 >$. The cost of this edge is $\mu_3 \cdot F = 4$ and the waiting time is $\omega_3 = 1$. For nodes $< 3,2 >$ and $< 2,3 >$, we follow the transformation rule (II). Since $H(3,2) + H(2,3) = 2$ and is not greater than F, we do not add an edge from node $< 3,2 >$ to node $< 2,3 >$. Finally, we add an edge from node $< 2,4 >$ to destination node $< 4,5 >$ with label $(0,0)$.

Now, we use DP-SWCP to obtain the optimal solution in $\tilde{G}$. Due to the limited space, we do not illustrate the procedure of DP-SWCP. There are two possible paths from source node $< 0,1 >$ to the destination node $< 4,5 >$. DP-SWCP determines path $\{< 0,1 >, < 1,3 >, < 3,2 >, < 2,4 >, < 4,5 >\}$ as the optimal path in $\tilde{G}$. Thus, the optimal sequence of stops is v_1, v_3, v_2, v_4, the total waiting time is 4, and the total cost of recharging is 12, which corresponds to the optimal solution in the original graph G.

5 An Efficient Algorithm for EVRRP

In the previous section, we showed that OPT-EVRRP provides the optimal solution for EVRRP and is pseudo-polynomial (in terms of the maximum total waiting time). Here, we provide an efficient algorithm for EVRRP, called APX-EVRRP, by scaling down the waiting time of each edge in the transformed graph as well as the maximum waiting time. For this purpose, we scale down the values of w_{ijk} to $\bar{\omega}_j = \lceil \frac{n \cdot \omega_j}{\epsilon \cdot W} \rceil$, where $0 < \epsilon < 1$. The maximum waiting time is also scaled down to $\bar{W} = \frac{n}{\epsilon}$.

APX-EVRRP is given in Algorithm 2. The algorithm calls the Transform-Graph procedure to obtain the transformed graph $\tilde{G}$ (Line 1). Then, it scales down the waiting time of each edge of the transformed graph (Lines 2-3), and calls DP-SWCP to solve the rounded problem with maximum allowable waiting time $\frac{n}{\epsilon}$ (Line 4). Finally, given the the optimal solution p for the rounded problem, where p is the sequence of stops, it determines the recharging amount at each stop (Lines 5-15). We choose the sequence p with recharging policy r as the solution for EVRRP. In the next section, we show that this solution is feasible and the total cost obtained by this solution is bounded by the optimal cost for EVRRP with maximum waiting time $(1 - \epsilon) \cdot W$.

Algorithm 2 APX-EVRRP Algorithm

Input: $G(V, E)$: Graph representing the road network
W: Maximum allowable waiting time
Output: $p = \{p_i\}$: Routing vector
$r = \{r_i\}$: Recharging vector
cost: Total cost

$\tilde{G} \leftarrow$ Transform-Graph(G)
for each $(< i, j >< j, k >) \in \tilde{E}$ **do**
    $w_{ijk} \leftarrow \lceil \frac{n \cdot w_{ijk}}{\epsilon \cdot W} \rceil$
$p \leftarrow$ DP-SWCP$(\tilde{G}, \frac{n}{\epsilon})$
$q_1 \leftarrow F$
$i \leftarrow 1$
for each $u = 2, \ldots, |p| - 1$ **do**
    $j \leftarrow p_u$
    $k \leftarrow p_{u+1}$
    if $\mu_j \le \mu_k$ **then**
        $q_j \leftarrow F$
    else
        $q_j \leftarrow H(j, k)$
    $r_j \leftarrow q_j - q_i + H(i, j)$
$cost \leftarrow \sum_{i \in p} r_i \cdot \mu_i$

5.1 Properties of APX-EVRRP

In this section, we analyze the properties of the proposed algorithm. First, we prove the correctness of APX-EVRRP by showing that the algorithm obtains a feasible solution for EVRRP in polynomial time. Then, we show that the total recharging cost of the solution is not greater than the optimal cost for EVRRP with the maximum allowable waiting time $(1 - \epsilon) \cdot W$.

Let us denote the solution obtained by APX-EVRRP by (p, r), where p is the sequence of stops and r gives the recharging amount at each stop. We also denote by EVRRP$_{(1-\epsilon)}$, the EVRRP problem with the maximum allowable waiting time $(1 - \epsilon) \cdot W$, and by (p^*, r^*), the optimal solution for EVRRP$_{(1-\epsilon)}$.

Theorem 4. *APX-EVRRP obtains a feasible solution for EVRRP and its time complexity is* $O(\frac{n^3}{\epsilon})$.

Proof. Since the amount of charge consumption between every pair of nodes in the rounded problem is the same as in the original problem, the recharging policy r is feasible for the original problem (to reach the destination). Thus, we only need to show that the total waiting time of the solution does not exceed W. In the rounded solution, the total rounded waiting time is not greater than $\frac{n}{\epsilon}$ (i.e., $\sum_{i \in p} \bar{\omega}_i \le \frac{n}{\epsilon}$). On the other hand $\bar{\omega_i} = \lceil \frac{n \cdot \omega_i}{\epsilon \cdot W} \rceil$. Thus,

$$\sum_{i \in p} \frac{n \cdot \omega_i}{\epsilon \cdot W} \le \sum_{i \in p} \bar{\omega}_i \le \frac{n}{\epsilon}.$$

Thus,

$$\sum_{i \in p} \omega_i \leq W.$$

Therefore, (p, r) is a feasible solution for EVRRP with the total waiting time less than or equal W.

The time complexity of APX-EVRRP comes mainly from DP-SWCP. In the rounded problem, the possible value of the maximum waiting time is reduced to $\frac{n}{\epsilon}$. Therefore, the time complexity of DP-SWCP for the rounded problem is $O(\frac{n^3}{\epsilon})$. Thus, the time complexity of APX-EVRRP is $O(\frac{n^3}{\epsilon})$.

Now, we show that the total cost obtained by APX-EVRRP is not greater than the optimal cost for $\mathsf{EVRRP}_{(1-\epsilon)}$. For this purpose, we show that (p^*, r^*) is a feasible solution for the rounded problem and the recharging cost of this solution is not less than the cost obtained from solution (p, r).

Lemma 3. *The optimal solution (p^*, r^*) for $\mathsf{EVRRP}_{(1-\epsilon)}$ is a feasible solution for the rounded problem.*

Proof. Since the amount of charge consumption between every pair of nodes is the same in both the rounded problem and the original problem, the recharging policy r^* is feasible for the rounded problem. Thus, we only need to show that the total rounded waiting time obtained from solution (p^*, r^*) is not greater than $\frac{n}{\epsilon}$. The rounded waiting time at stop i is $\bar{\omega}_i = \lceil \frac{n \cdot \omega_i}{\epsilon \cdot W} \rceil$. Thus,

$$\sum_{i \in p^*} \bar{\omega}_i \leq \sum_{i \in p^*} (\frac{n \cdot \omega_i}{\epsilon \cdot W} + 1).$$

On the other hand, the total waiting time of $< p^*, r^* >$ is not greater that $(1-\epsilon) \cdot W$ (i.e., $\sum_{i \in p^*} \omega_i \leq (1-\epsilon) \cdot W$). Therefore,

$$\sum_{i \in p^*} \bar{\omega}_i \leq \sum_{i \in p^*} (\frac{n \cdot \omega_i}{\epsilon \cdot W} + 1) \leq \frac{n \cdot (1-\epsilon) \cdot W}{\epsilon \cdot W} + n \leq n \cdot (1 + \frac{1-\epsilon}{\epsilon}) \leq \frac{n}{\epsilon}.$$

Therefore, (p^*, r^*) is a feasible solution for the rounded problem.

Theorem 5. *The total cost of the solution obtained by APX-EVRRP is not greater than the optimal cost for $\mathsf{EVRRP}_{(1-\epsilon)}$.*

Proof. According to Lemma 3, the optimal solution for $\mathsf{EVRRP}_{(1-\epsilon)}$ is a feasible solution for the rounded problem. On the other hand, OPT-EVRRP obtains the optimal solution $< p, r >$ for the rounded problem. Therefore,

$$\sum_{i \in p} r_i \cdot \mu_i \leq \sum_{i \in p^*} r_i^* \cdot \mu_i.$$

Therefore, the total cost of the solution obtained by APX-EVRRP is not greater than the total cost of the optimal solution for $\mathsf{EVRRP}_{(1-\epsilon)}$.

6 Conclusion

We studied the routing and recharging optimization problem for electric vehicles, where the aim is to find the routing path from a starting point to destination such that the total recharging cost is minimized. We considered that charging nodes have heterogeneous prices and waiting times. We studied the properties of the problem and showed that the problem is NP-hard. We proposed a pseudo-polynomial algorithm for the optimal solution and a polynomial time algorithm that obtains a solution with the total recharging cost not greater than the optimal cost for the same problem but with the maximum waiting time of $(1-\epsilon)\cdot W$, where W is the maximum allowable waiting time. As a future research, we plan on considering the heterogeneity of charging stations in terms of charging speed. Another direction for future study is to take the uncertainty of waiting times at charging stations into account.

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References

1. Arslan, O., Yıldız, B., Karaşan, O.E.: Minimum cost path problem for plug-in hybrid electric vehicles. Transportation Research Part E: Logistics and Transportation Review **80**, 123–141 (2015)
2. Desrochers, M., Soumis, F.: A generalized permanent labelling algorithm for the shortest path problem with time windows. INFOR: Information Systems and Operational Research **26**(3), 191–212 (1988)
3. DoE: All Electric Vehicles (Accessed Jun 27, 2020), https://www.fueleconomy.gov/feg/evtech.shtml
4. Garey, M.R., Johnson, D.S.: Computers and intractability, vol. 174. Freeman San Francisco (1979)
5. Hertzke, P., Mller, N., Schaufuss, P., Schenk, S., Wu, T.: Expanding electric-vehicle adoption despite early growing pains. McKinsey & Company Insights (2019)
6. Ichimori, T., Ishii, H., Nishida, T.: Routing a vehicle with the limitation of fuel. Journal of the Operations Research Society of Japan **24**(3), 277–281 (1981)
7. Khuller, S., Malekian, A., Mestre, J.: To fill or not to fill: The gas station problem. ACM Transactions on Algorithms (TALG) **7**(3), 1–16 (2011)
8. Lin, S.H.: Finding optimal refueling policies in transportation networks. In: Int. Conf. on Algorithmic Applications in Management. pp. 280–291. Springer (2008)
9. Nejad, M.M., Mashayekhy, L., Grosu, D., Chinnam, R.B.: Optimal routing for plug-in hybrid electric vehicles. Transportation Science **51**(4), 1304–1325 (2017)
10. Sweda, T.M., Dolinskaya, I.S., Klabjan, D.: Adaptive routing and recharging policies for electric vehicles. Transportation Science **51**(4), 1326–1348 (2017)
11. US-EPA: Global Greenhouse Gas Emissions Data (Accessed Jun 27, 2020), https://www.epa.gov/ghgemissions/global-greenhouse-gas-emissions-data