# Coupled-mode theory of the polarization dynamics inside a microring resonator with a uniaxial core

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**Abstract:** We develop a coupled-mode theory for polarization coupling inside a LiNbO $_3$  microring resonator. We use analytical techniques from quantum mechanics to characterize the polarization dynamics in such a resonator. © 2021 The Author(s)

### 1. Introduction

The material birefringence of a LiNbO<sub>3</sub> microring resonator couples its transverse-electric (TE) and transverse-magnetic (TM) modes as they propagate along its circumference [1], resulting in unusual polarization dynamics that are not yet fully understood. We develop a theoretical framework to study the polarization evolution inside such resonators.

Modes of uniaxial planar waveguides have been discussed for over forty years [2–6]. The results show that the modes depend on the relative orientation of the optic axis with respect to the direction of propagation. As the light propagates along the ring, the angle that the optic axis makes with the direction of propagation changes continuously. This rotation complicates the polarization evolution inside such a curved waveguide. We have developed a novel approach to model the resulting polarization dynamics.

### 2. Zero-bending model

We model the ring waveguide by introducing a "zero-bending model" (ZBM). The ZBM simplifies the mathematical analysis by replacing the curved ring with a straight waveguide along which the optical axis rotates with  $\phi = z/r$  at a distance z. Fig. 1 (left) shows the microring resonator together with the bus waveguide used to couple light into it. The optical axis of the uniaxial material lies in the plane of the ring along a fixed direction  $\hat{\mathbf{u}}_e$ . As light travels along the ring, its state-of-polarization (SOP) changes with the angle  $\phi$  because of a continuous reorientation of the optical axis with respect to the direction of propagation. Fig. 1 (right) shows the microring after the ZBM is applied.

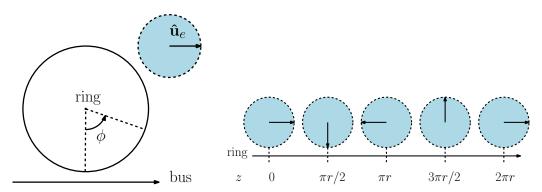


Fig. 1. Ring resonator under study (left) and ring resonator under zero-bending model (right)

# 3. Polarization dynamics

We show that the coupled-mode equations for the Jones vector become identical to the Schrödinger equation of a two-level atom driven by a classical optical field and undergoing frequency modulation [7]. We exploit this analogy and use mathematical techniques of quantum mechanics to analyze the SOP evolution. Employing the ZBM, we treat the reorientation of the optical axis as a perturbation to an otherwise longitudinally-invariant birefringent

waveguide and develop a coupled-mode theory for the evolution of the SOP of light injected into the microring. The theory describes coupling between the TE and TM modes of the microring and includes both the material and geometrical birefringences. We identify three regimes of the parameter space where the SOP dynamics are approximately integrable; namely a perturbative regime, a resonant regime, and an adiabatic regime.

## 4. Application to LiNbO<sub>3</sub> ring

We use our coupled-mode description to analyze polarization coupling in an air-cladded LiNbO<sub>3</sub> waveguide made on a silica substrate and operating at the 1550 nm wavelength. We obtain close agreement between our simplified model and numerical results obtained from commercial software. We verify that the approximate solutions in each dynamical regime is accurate. For instance, we solve the coupled-mode equations numerically and also analytically in the rotating-wave approximation (RWA) of quantum mechanics, valid in the resonant regime. We show in Fig. 2 the SOP evolution on the Poincare sphere (left) and the power coupled into the TM mode over one round trip inside the microring. We show that the SOP dynamics can be switched between the resonant and adiabatic regimes by changing the waveguide thickness.

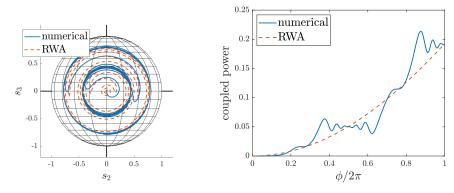


Fig. 2. Evolution of the SOP on the Poincaré sphere of initially TE-polarized light (left) for a waveguide thickness of 800 nm. Fractional power transfer as a function of angle  $\phi$  (right). The solid trace shows the exact evolution; the dashed trace shows the RWA solution.

# 5. Conclusion

By introducing a zero-bending model, we formulate a coupled-mode description of polarization coupling in an uniaxial microring resonator, owing to the continuous reorientation of the direction of propagation relative to the optical axis. We show that the resulting coupled-mode equations are identical to the Schrödinger equation of a two-level atom under optical excitation and external modulation. We leverage this isomorphism and use analytical tools of quantum mechanics to study the polarization dynamics inside the microring. Our formalism can be used to characterize the polarization properties of LiNbO<sub>3</sub> microrings used as the building block of electro-optical and nonlinear optical devices.

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