Imitation of Success Leads to Cost of Living Mediated Fairness in the Ultimatum Game

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August 23, 2021

Abstract

In this paper, we analyze a social imitation model that incorporates internal energy caches (e.g., food/money savings), cost of living, death, and reproduction into the Ultimatum Game. We show that when imitation (and death) occurs, a natural correlation between selfishness and cost of living emerges. However, in all societies that do not collapse, non-Nash sharing strategies emerge as the de facto result of imitation. We explain these results by constructing a mean-field approximation of the internal energy cache informed by time-varying distributions extracted from experimental data.

1 Introduction

Cooperation is clearly critical for the emergence of societies (e.g., ants, cetaceans, humans, etc.). However cooperation, and by extension fairness, frequently appears to be an irrational response to an environment with survival pressures, such as the cost of living. Consequently, modeling mechanisms for the emergence of cooperation and fairness continues to be active area of research in social and biological theory [1–12]. The *Ultimatum Game* (UG) is an archetypal game illustrating the difficulties in modeling concepts of fairness. In the game, one player is given a sum of money which she must divide in some proportion between herself and a second player. The second player may then accept the offer, in which case the pot is divided accordingly, or reject the offer, in which case each player receives nothing [13]. This is like a continuous variation of the Stag-Hunt game, in which individual gain competes against mutual benefit. The notional *money* can act as a stand-in for a cooperative hunt, business venture etc. Here we introduce an additional UG variable, individual wealth, which drives the dynamic *imitate the successful*.

A considerable amount of theoretical and experimental research has been done on the ultimatum game (see e.g., [14–24]). Classical game theory asserts the most rational, sub-game perfect solution is for the dividing player to keep as much of the prize as possible, while the deciding player accepts any offer. However, almost all experiments with humans (but not chimpanzees [19]) show that individuals will offer far more than the minimum quantity and deciding players will frequently reject offers at the expense of their own well-being (presumably as an act of punishment for unfair or non-cooperative behavior). In particular, Oosterbeek *et al.* conducted a meta-analysis of 37 papers with 75 results from various countries [16], and concluded that there is not a significant difference in proposers' behavior, although there is a difference in responders' behaviors across (geographic) regions.

Mathematical models by Nowak *et al.* approach the Ultimatum Game from an evolutionary game theory perspective, by including the reputation of agents as part of the offer making process [25] and in a one-shot game context [26]. More recently, Gale *et al.* construct a discrete strategy evolutionary game representation [13], and show that evolutionary stable strategies exist in this game. In addition to this, a substantial amount of work has been done on spatial ultimatum games [27–35], which shows that fairness can arise in various contexts, driven by spatial population structures. In this paper, we study imitation rather than evolution to ascertain the effect on Ultimatum Game strategies. Our imitation dynamics are taken from [36, 37].

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However, unlike [37], which uses a metric to determine imitation, we use success imitation as in [36], with the net individual winnings (savings) of each player used as a proxy for past success.

2 Model

Our proposed model is an agent-based simulation in the spirit of [25, 26], and similar to the approach taken in [38, 39]. We introduce a dynamic wealth variable [40] for each player, as an integrated measure of success. In our model, agents interact randomly and each interaction is an instance of UG with a possible prize P. Agents are chosen at random to be the offerer or decider. The state of agent i is specified by internal variables $(\lambda^i, \theta^i, B^i)$ where $\lambda^i \in [0, 1]$ is the demanded offer level of Agent i, $\theta^i \in [0, 1]$ is the offer provided by Agent i, and $B^i > 0$ is the savings (energy cache) held by each agent, which keeps track of the winnings from each interaction. Energy loss in the system is set by the cost of living parameter κ, which is subtracted at each time step from the energy cache of each player.

If Agents i and j interact and i is the offerer, then Agent j rejects the offer whenever $\theta^i < \lambda^j$. In the case of acceptance, Agent i keeps $(1 - \theta^i)P$ and Agent j keeps $\theta^i P$. When $P = 1$, then all parameters can be expressed as ratios of P. Let $\chi_{ij}(t)$ be an indicator function that is 1 at time t exactly when i and j interact. The discrete time agent-based model dynamics are given by:

$$B^i(t + \epsilon) = B^i + \frac{\epsilon}{2} \sum_{j \neq i} \chi_{ij}(t) P \left(1 - \theta^i\right) U(\theta^i - \lambda^j) + \frac{\epsilon}{2} \sum_{j \neq i} \chi_{ij}(t) P \theta^j U\left(\theta^j - \lambda^i\right) - \kappa, \quad (1)$$

where

$$U(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

is the unit step function, and we take $\kappa \in [0, 1]$. For simulation purposes, we set $\epsilon = 1$. Taking the expected value of these equations, a mean-field approximation of the agent energy dynamics can be derived:

$$\Delta \hat{B}^i(t) = \frac{\epsilon}{2q} \sum_{j \neq i} \left[\left(1 - \theta^i\right) U\left(\theta^i - \lambda^j\right) + \theta^j U\left(\theta^j - \lambda^i\right)\right] - \kappa, \quad (2)$$

The normalizing value q is given by:

$$q = \begin{cases} n & \text{n is odd} \\ n - 1 & \text{otherwise} \end{cases},$$

which models the random choice of two agents from a completely connected population of n agents. We next propose dynamics that drive the population towards a statistical equilibrium $(\theta^i, \lambda^i) \to (\lambda^*, \theta^*)$. However, independent of any game dynamics for the population, we can already derive certain relations that characterize the dynamics of the energy cache B using Eq. (2). If $\lambda^* > \theta^*$, then $U(\theta^* - \lambda^*) = 0$ and $\Delta\hat{B} < 0$. Populations of this type will collapse. On the other hand, if $\lambda^* < \theta^*$, then as $n \to \infty$:

$$\Delta \hat{B}(t) = \frac{\epsilon}{2} \left(1 - 2\kappa\right), \quad (3)$$

which also holds in general for even n. Thus, if $\kappa > \frac{1}{2}$, the population will collapse in the mean. For $\kappa = \frac{1}{2}$, the population energy caches will stabilize in the mean and for $\kappa < \frac{1}{2}$, the population energy caches will increase without bound.

In discrete time, the dynamics of (θ, λ) are given by:

$$\lambda^i(t + \epsilon) = \lambda^i(t) + \epsilon \sum_{j \neq i} (\lambda^j - \lambda^i) p_{ij} \quad (4)$$

$$\theta^i(t + \epsilon) = \theta^i(t) + \epsilon \sum_{j \neq i} (\theta^j - \theta^i) p_{ij}, \quad (5)$$

where p_{ij} are imitation probabilities, to be defined by the model dynamics. Let:

$$Q^i = \sum_j U(B^j - B^i), \tag{6}$$

which is the cumulative difference in energy values for all agents j with $B^j > B^i$. For the discrete time simulation, we set:

$$p_{ij} = \begin{cases} \frac{(B^j - B^i)U(B^j - B^i)}{\sum_h (B^h - B^i)U(B^h - B^i)} & \text{if } Q^i > 0 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Eqs. (4) and (5) are imitation dynamics in which agents imitate those who outperform them. Thus Agent j does not rationally choose (λ^j, θ^j), but adjusts these values based on observations weighted towards other, more successful agents.

For imitation systems like Eqs. (4) and (5), Griffin *et al.* proved that a sufficient condition for convergence is the emergence of a fixed leader i^* imitated (directly or indirectly) by all agents [36], which readily occurs in this system as a result of the total ordering of B^i. As $\epsilon \to 0$, Eqs. (4) and (5) become the continuous time consensus equations as surveyed in [37], but with state-varying coefficients. The proof of convergence in [36] for discrete time updates suggests that exact values of p_{ij} are irrelevant, as long as Agent i is imitating those agents who outperform it.

Whether in continuous or discrete dynamics, these systems have an infinite set of fixed points $\theta^k = \theta^*$, $\lambda^k = \lambda^*$ for $\theta^*, \lambda^* \in [0,1] \times [0,1]$. It is clear that not all such fixed points are equally likely or even realistic, since all systems with $\lambda^* > \theta^*$ would lead to population collapse for any cost of living $\kappa > 0$. Therefore, the distributions of long-run behavior in these systems should provide insights into the emergence of cooperative or fair behaviors.

We assume agents are initialized with θ^k and λ^k uniformly distributed in $[0,1]$. Again consider the case when $n \to \infty$. From Eq. (2) the expected per-round energy increase near $t = 0$ for an agent with parameters (θ, λ) is:

$$\Delta B(\theta, \lambda, \kappa) = \frac{1}{2}\int_0^\theta (1-\theta)\, d\lambda + \frac{1}{2}\int_\lambda^1 \theta \, d\theta - \kappa =$$

$$\frac{1}{2}(1-\theta)\theta + \frac{1}{2}\left(\frac{1}{2} - \frac{\lambda^2}{2}\right) - \kappa. \tag{8}$$

Maximizing this expression subject to the constraints $0 \leq \theta, \lambda \leq 1$, suggests the optimal fairness demand is $\lambda^+ = 0$, while the best offer is $\theta^+ = \frac{1}{2}$. This is consistent with the classical Nash equilibrium ($\lambda^+ = 0$) but also consistent with fairness considerations ($\theta^+ = \frac{1}{2}$), since an agent can never be certain whether she will interact with an agent with high or low λ. If the players were perfectly rational, then a true Nash equilibrium would be $\theta^{\mathrm{NE}} = \lambda^{\mathrm{NE}} = 0$, since rational players realizing $\lambda^+ = \lambda^{\mathrm{NE}} = 0$ would make $\theta^{\mathrm{NE}} = 0$. Our empirical results show that this equilibrium does not result from imitation.

From Eq. (8), when $\kappa > \frac{3}{8}$, the expected increase for even an optimal player is negative. This will lead to a mean decrease in energy caches until imitation leads to higher success rates in UG. Let $\chi_{\Delta B}(\theta, \lambda, \kappa)$ be an indicator function that is 1 just in case, Eq. (8) is positive. Numerical evaluation shows that when $\kappa^* \approx 0.26246$:

$$\int_0^1 \int_0^1 \chi_{\Delta B}(\theta, \lambda, \kappa^*)\, d\lambda \, d\theta = \tfrac{1}{2}.$$

For $\kappa > \kappa^*$, the median energy cache value will decrease in early interactions before imitation can contract the strategy space. Individuals whose energy cache reaches zero are assumed dead and can no longer interact in the system. Reproduction or replacement of players is used to maintain a constant population, and the specific rule we use is described in the simulation details below.

3 Simulation Results

We simulate a population with N agents. Agents are initialized with an energy cache value B^i, and uniformly randomly assigned values θ^i and λ^i. Agents enter a *game loop*, where each agent plays UG with another randomly selected agent. Once all agents have played, energy caches are updated accordingly. In the agent-based simulation, we introduce a reproductive step into the mimicking process to account for agents with non-positive energy cache and to identify population collapse prior to convergence. If all agents have $B^i < 0$ after subtracting the cost of living, the simulation ends immediately. Otherwise, all agents with $B^i > 0$, mimic others using Eqs. (4) and (5) in a *mimic/reproduce* loop. If all agents have survived, agents return to the game loop. Otherwise, agents are randomly chosen to reproduce with probability proportional to their energy cache; i.e., the fittest reproduce with higher probability. Reproduction continues until the population reaches N. If the population never collapses, the process is terminated after T rounds. The size of T is chosen to ensure convergence. To ensure numerical validity, the model was implemented both in Python and Mathematica, and results were compared to ensure statistical consistency. We experimented with varying numbers of agents ranging from 50 to 300. We restricted the size to this level to ensure simulations could run on a personal computer within a reasonable amount of time.

Fig. 1 shows simulation results for $N = 150$ players and running time $T = 300$. All agent energy caches are initially set to 1. We used 100 realizations (replications). Distribution plots for B, θ and λ are shown, with cost of living κ ranging from 0.05 to 0.5. Density plots showing the joint converged (θ, λ) distributions are shown in Fig. 3. The convergence of $\theta^i(t)$ and $\lambda^i(t)$ is illustrated in Fig. 2 for 300 agents, $T = 500$ and $\kappa = 0.1$. To create this figure, 100 replications were constructed and $\theta^i(t)$ and $\lambda^i(t)$ were sorted at each round. These sorted lists where then averaged (over replication) to obtain $\bar{\theta}^{[i]}(t)$ and $\bar{\lambda}^{[i]}(t)$, where $\bar{\theta}^{[i]}(t)$ is the mean offer value of the agent with the i^{th} smallest offer value. The quantity $\bar{\lambda}^{[i]}(t)$ is defined analogously.

The simulation shows downward pressure on the offer value correlated with the energy cost of living κ with consistent values of λ^* between (approximately) 0.1 and 0.4. As is expected, the value of $\hat{B}^i(t)$, the mean energy store value decreases as a function of cost of living. We derive an empirical linear approximation for the mean, which we discuss in the sequel. Understanding the origin of this relationship is complicated by the fact that there is no convenience closed form expression for $\lambda^i(t)$ or $\theta^i(t)$. To remedy this, we use a combination of empirical distribution modeling and closed form analysis of $\Delta\hat{B}^i$ to explain the observed behavior.

3.1 Mixed Empirical and Closed form System Modeling

At arbitrary time t when the distribution of fairness demands and offers is given by probability density functions $f^t_\lambda(s)$ and $f^t_\theta(s)$ respectively, then Eq. (8) can be generalized as:

$$\Delta\hat{B}(t;\theta,\lambda,\kappa) = \frac{1}{2}\left(\int_0^\theta (1-\theta) f^t_\lambda(s)\,ds + \int_\lambda^1 s f^t_\theta(s)\,ds\right) - \kappa \tag{9}$$

This expression cannot be computed without the time-varying distributions in question, which cannot be computed without an appropriate Fokker-Plank equation, which is difficult to construct. To compensate, we can fit distributions to the data $\bar{\lambda}^{[i]}(t)$ and $\bar{\theta}^{[i]}(t)$ to obtain estimators $\hat{f}^t_\lambda(s)$ and $\hat{f}^t_\theta(s)$, which can be used in Eq. (9). These empirically estimated distributions stand in for the mean-field distributions[1]. We can then compute:

$$\hat{B}(t;\theta,\lambda,\kappa) = \begin{cases} B_0 & \text{if t} = 0 \\ \sum_{s\leq t} \Delta B(t;\theta,\lambda,\kappa) U[B(t-1;\theta,\lambda,\kappa)] & \text{otherwise} \end{cases}, \tag{10}$$

where the factor $U[B(t-1;\theta,\lambda,\kappa)]$ sets $\hat{B}(t;\theta,\lambda,\kappa) = 0$ if $\hat{B}(t-1;\theta,\lambda,\kappa) = 0$. That is, it models the death of a test agent with parameters (θ,λ). The imitation dynamics defined by Eqs. (4), (5) and (7) imply that the larger $\hat{B}(t;\theta,\lambda,\kappa)$ the more likely an agent with parameters (θ,λ) will be imitated. Thus, we can use $\hat{B}(t;\theta,\lambda,\kappa)$ at an appropriately large time (e.g., $t = 100$) to estimate which agents are most likely to be imitated for a given κ. We show this estimation for $\kappa = 0.1$ and $\kappa = 0.4$ in Fig. 4. To test this model,

[1] All distributions were estimated using Mathematica's `FindDistribution` function.

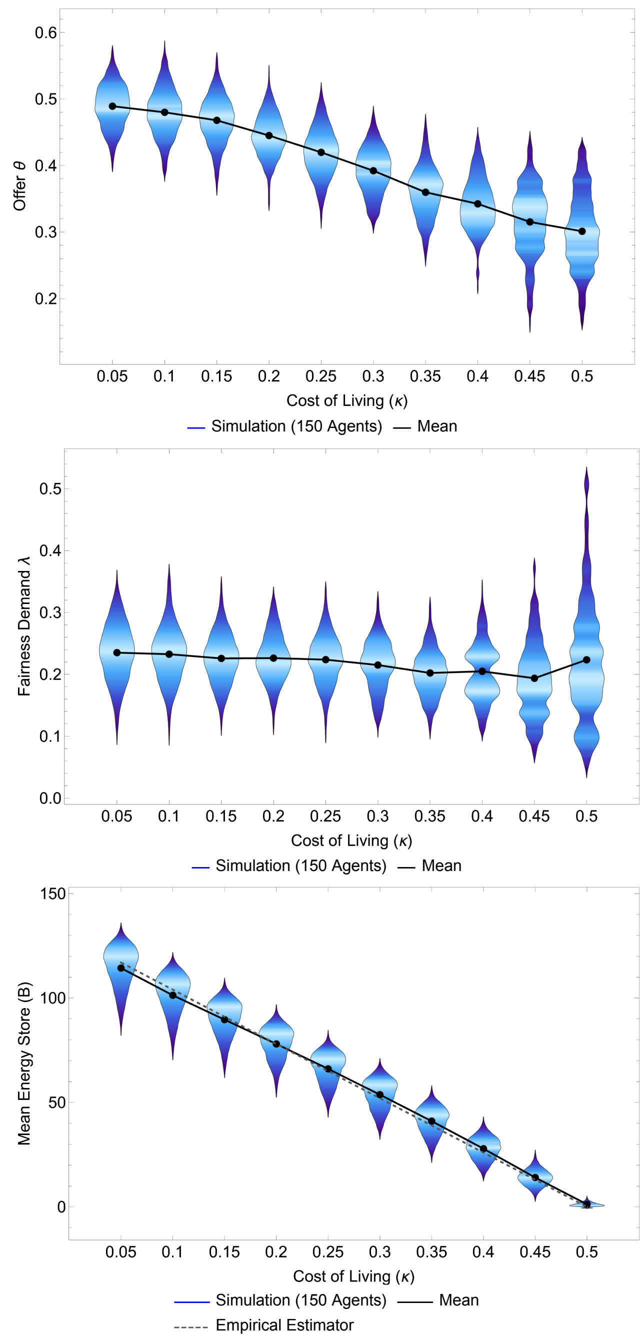


Figure 1: Simulation results using 150 agents, 300 rounds of play, and 100 replications, as a function of cost of living κ: (left) there is a well defined negative correlation between offers made and κ; (middle) demand levels are stable across κ values; (right) energy cache values (savings) follow an empirical trend (dashed line) derivable from the model.

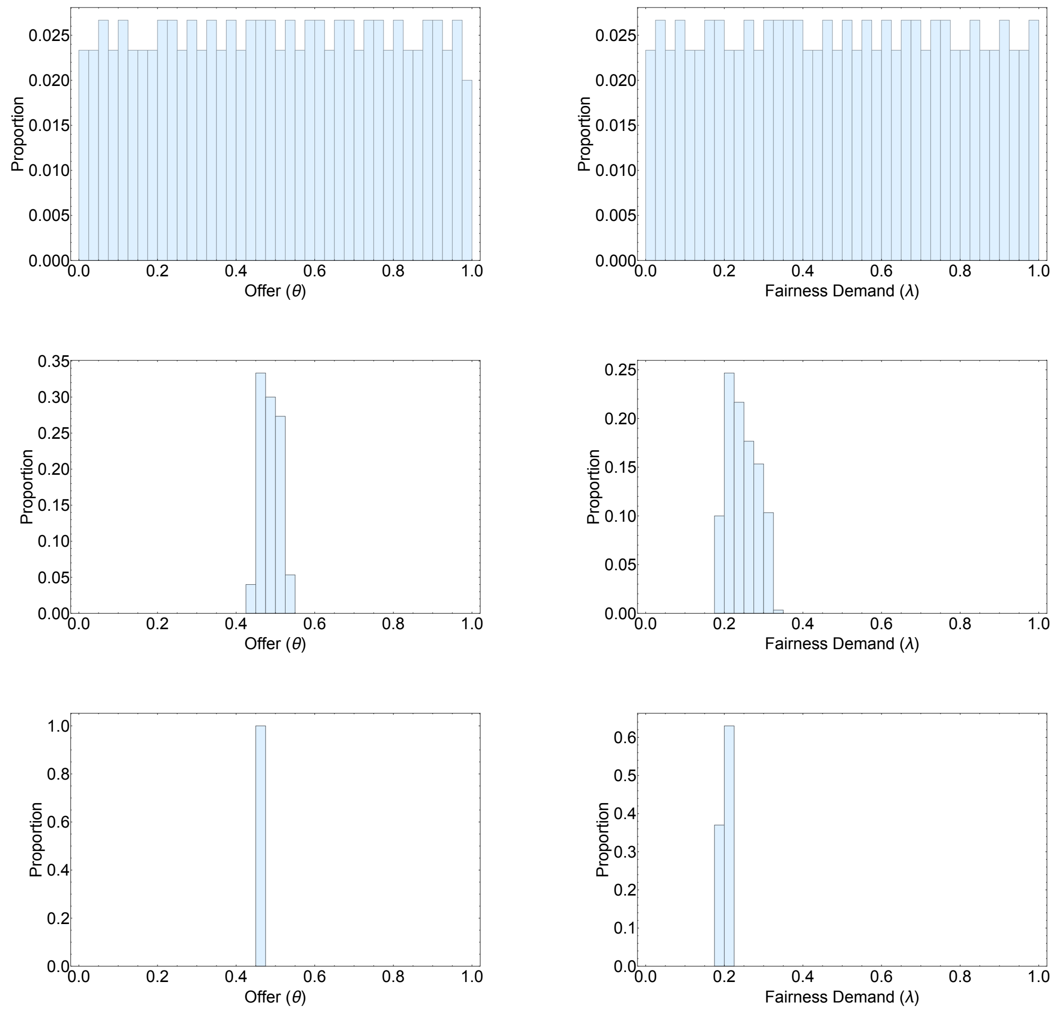


Figure 2: (top) Convergence of the distribution of θ^i from a uniform distribution to a delta distribution. (bottom) Convergence of the distribution of λ^i from a uniform distribution to a delta distribution. (both) $\kappa = 0.1$, 300 agents are simulated. Times go from $t = 0$ to $t = 500$ in increments of $\Delta t = 250$.

we use the top 5% of computed values of $\hat{B}(t, \theta, \lambda, \kappa)$ to compute estimated intervals on the values of θ^* and λ^* for $\kappa = 0.1$ and $\kappa = 0.4$. We compare these intervals with the $5\% - 95\%$ intervals computed from the experimental results shown in Fig. 1. This is shown in Table 1. These results are both consistent with and predictive of the distributions seen in Fig. 1; i.e., they explain both the downward slope of θ as a function of κ in Fig. 1 (left) and the relatively constant behavior of λ as a function of κ. We stress that estimations in Fig. 4 and Table 1 are generated by a model (Eq. (9) and Eq. (10)) with distribution constants determined empirically. Thus an area of future work is to replace these empirically determined distributions with modeled distributions.

3.2 Asymptotic Behavior of $\hat{B}$

The dynamics of the energy cache values can be modeled asymptotically. As $t \to \infty$, $f_\lambda^t(s) \approx \delta(s - \lambda^*)$ and $f_\theta^t(s) \approx \delta(s - \theta^*)$, where (θ^*, λ^*) is the fixed point of the $(\theta(t), \lambda(t))$. This is illustrated in Fig. 2. As $t \to \infty$, the energy caches of each agent asymptotically approaches:

$$B^i(t) = (\tfrac{1}{2} - \kappa)t.$$

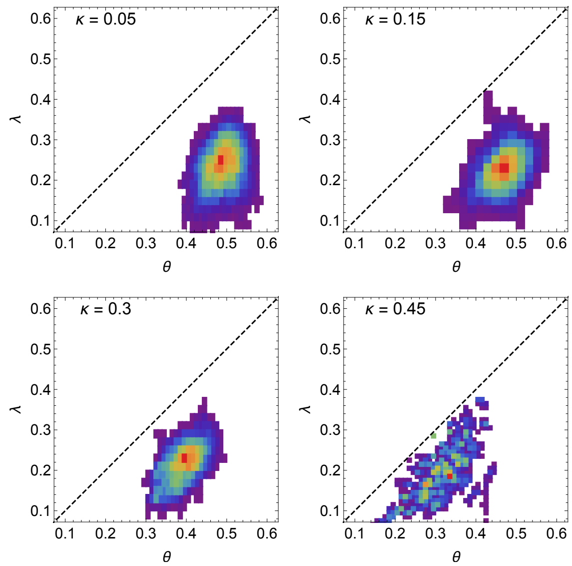


Figure 3: Density plots show the distributions of (θ^*, λ^*) over multiple replications with varying costs of living.

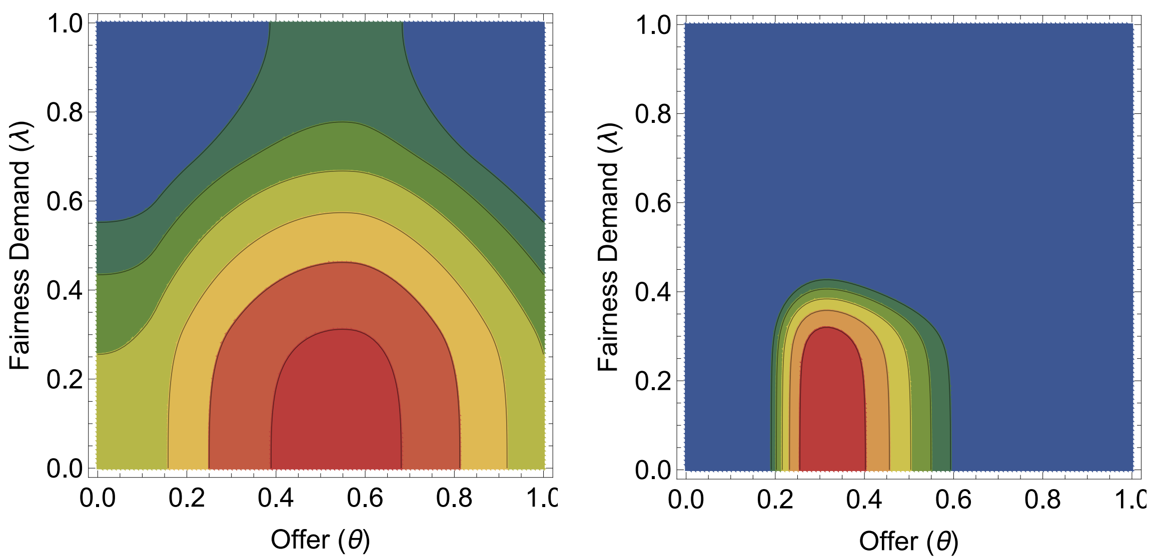


Figure 4: (left) Estimation of $\hat{B}(\infty; \theta, \lambda, \kappa)$ for $\kappa = 0.1$. (right) Estimation of $\hat{B}(\infty; \theta, \lambda, \kappa)$ for $\kappa = 0.4$.

κ	Model Est. Interval	Computed Interval
0.1	$[0.43, 0.64]$	[0.42,0.54]
0.4	$[0.25, 0.41]$	[0.280.41]

Offer Estimate

κ	Model Est. Interval	Computed Interval
0.1	$[0, 0.33]$	[0.15,0.31]
0.4	$[0, 0.26]$	[0.13,0.28]

Fairness Demand Estimate

Table 1: Comparison of estimated and computed intervals on θ^* and λ^* using information from Eq. (10).

This model is shown in Fig. 1 (right). This over-estimates the long-run energy cache value because of the initial time taken to converge. We can approximate the trend seen in Fig. 1 (right), by noting that the time for $\lambda^i(t)$ and $\theta^i(t)$ to converge so that most UG interactions are successful in approximately 80 rounds (out of the 300 rounds simulated). Assuming that prior to convergence, only half of all interactions result in a successful UG, we obtain a thermodynamic-type relationship between the mean wealth of the population and the cost of living:

$$\tilde{B}(\kappa) = 260\left(\frac{1}{2} - \kappa\right), \tag{11}$$

which explains the linear decrease with κ shown in Fig. 1 (right), where we show the fit of Eq. (11).

The global dynamics displayed in Fig. 1 are robust to changes in the speed of the underlying dynamics. In particular, we tested models in which (i) we replaced the discrete dynamics with continuous time differential equations (by letting $\epsilon \to 0$), (ii) an Euler step approximation of the resulting differential equations and (iii) a heterogenous starting energy cache value with the previously described dynamics. The ODE variants model fast imitation (on the time scale of the game play). In all cases except one, we included reproduction as a hybrid step by solving the ODEs for short time horizons, checking for death and then restarting the ODEs from the previous condition after removing agents with $B^k < 0$. All models used 100 replications except when reproduction was eliminated in which case 200 replications were used to ensure statistically signifiant sample sizes (samples with population collapse were discarded).

Results from robustness experiments are shown in Fig. 5 (top), where we show the mean values $\bar{\theta}^*$. The envelopes are 1σ. Similar tests were run for $\bar{\lambda}^*$ – see Fig. 5 (middle). For all cases, $\bar{\theta}$ is decreasing in κ. The mean fit line has negative slope as a function of κ ($p = 5.1 \times 10^{-6}$) and adjusted r^2 of 0.95, consistent with prior results and theoretical analysis. There is a difference in the behavior of $\bar{\lambda}^*$ for the the discrete time simulations and the continuous time (hybrid) variations. In the case of the hybrid ODE models (with or without Euler step approximations) $\bar{\lambda}^*$ increases as a function of κ ($p < 0.002$) while for discrete step simulations $\bar{\lambda}^*$ decreases as a function of κ ($p < 0.002$). When the data are combined, $\bar{\lambda}^*$ increases as a function of κ but with $p < 0.004$, suggesting this effect may disappear with larger samples. This would be the expected behavior as indicated by Fig. 4.

4 Conclusion

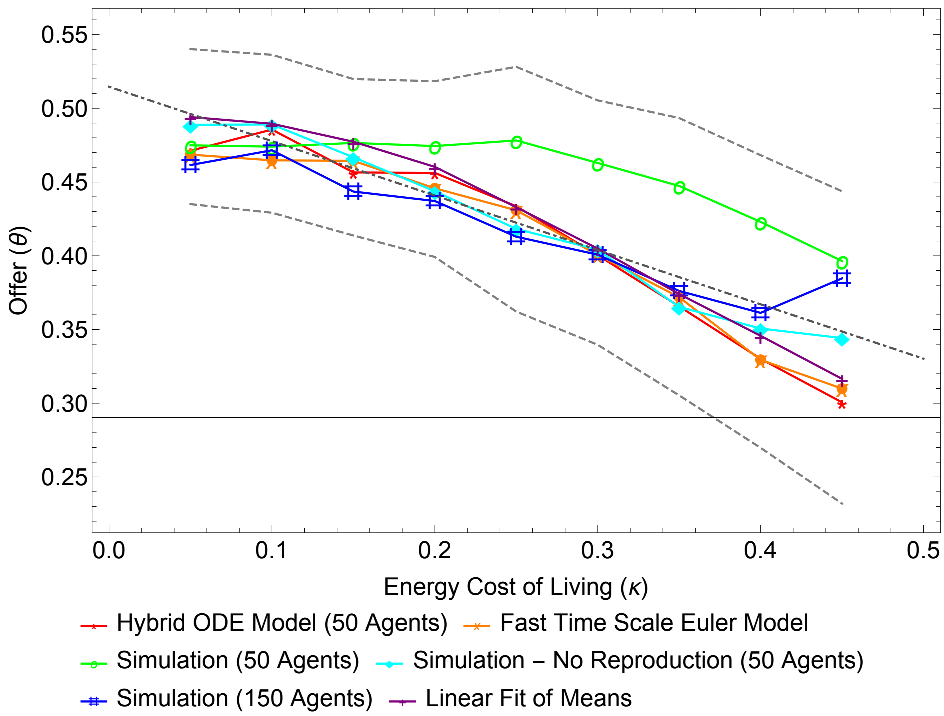
Game Theory finds application in biological and social sciences, yet well-known occurrences like cooperation and altruism remain challenging within its rational self-interest assumptions. Our paper presents a novel approach to the canonical Ultimatum Game (UG), introducing an individual savings or energy cache variable for each player, along with a universal cost of living.

While this approach leads to a convergence towards fairness in the population, it is by no means the only model to do so. In particular, the inclusion of spatial structure in a population, whether on a grid or network, has been shown to lead to fairness in some models for the Ultimatum game [32–35]. Note that the inclusion of an empathy assumption alone, which would appear in our model as $\theta_i = \lambda_i$, has been shown to lead to fairness in the game, even without spatial structure [30]. An intriguing area of future work is to consider this dynamic in a spatial structure (i.e., without complete mixing) to determine what effect space has on the results presented in this paper.

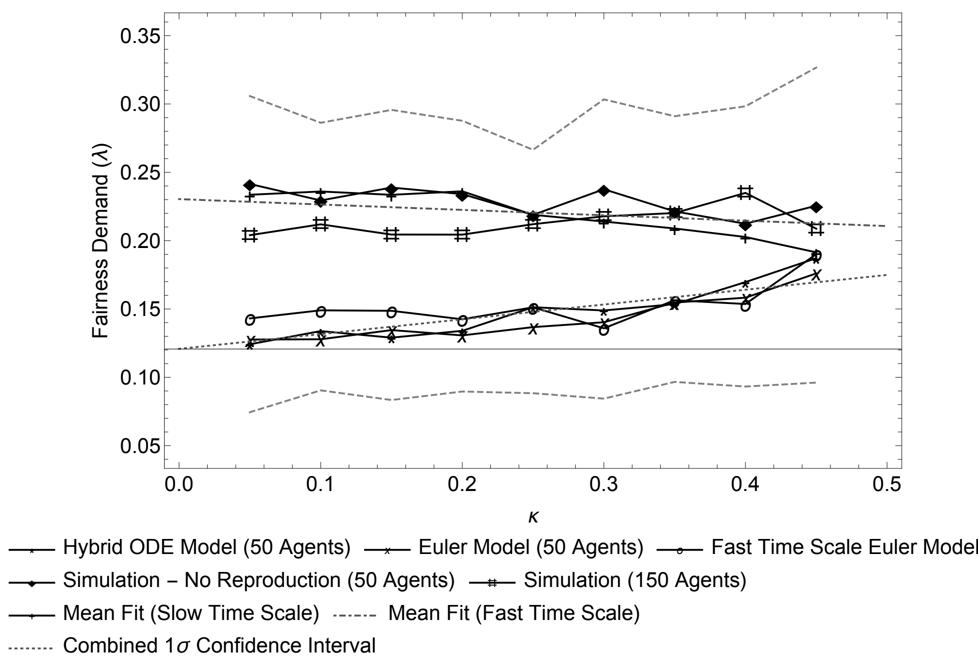
In our nonlinear agent-based model, this energy cache represents success, and drives the imitation dynamic. Agents are observed to evolve toward fair sharing, but are more selfish with higher costs of living, with consistently lower fairness demands of others. This behavior is explained and predicted using a model with empirical determined distribution parameters.

Our imitation dynamic is motivated by observations that children will imitate higher status individuals more selectively than lower status individuals [41]. Additionally, children will infer status based on observing imitation in adults [42]. More recently, humans in strategic settings have been shown to imitate behavior based on pay-off inequality [43]. This suggests an innate link between status and imitation, which we have incorporated here using the relative ordering of B^i as a proxy for status.

Future exploration of the dynamics of this model should indicate whether the exact structure of the distributions of θ and λ can be determined. This would remove the need to fit the distributions as a part of the modeling process, and provide a complete mean-field dynamics for this system.



(a) Offer



(b) Fairness Demand

Figure 5: (left) Model variations show the same trend in offer as a function of cost of living. (right) Model variations show differing trend in fairness demand depending on imitation speed (see text).

Acknowledgement

CG and AB were supported in part by the National Science Foundation under grant DMS-1814876. The authors would like to thank S. Rajtmajer for her feedback on earlier drafts. CG thanks R. Bailey (USN)

who ran the very first simulation of this phenomena in MATLAB while at the United States Naval Academy.

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