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Robust Event-triggered Distributed Average Tracking for Double-integrator Agents Without Velocity Measurements

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Abstract— **This paper investigates the distributed average tracking problem for a group of double-integrator agents. In some practical applications, velocity measurements may be unavailable due to technology and space limitations, and it is also usually less accurate and more expensive to implement. To this end, a distributed average tracking algorithm without using velocity measurements and correct initialization is established. It is worth noting that no global information is needed for parameter design. Then, in order to remove the requirement on continuous interaction, and reduce the communication cost and improve the energy efficiency, an event-triggered distributed average tracking algorithm is designed by incorporating an event-triggered communication strategy without using velocity measurements. To be exact, the idea of a dynamic event-triggered strategy is used to construct a triggering condition for each agent to guarantee the exclusion of Zeno behavior. Finally, simulations are provided to illustrate the obtained results.**

I. INTRODUCTION

During the recent decades, the distributed average tracking problem, which includes consensus and distributed tracking as special cases, is formulated and addressed in the literature. In the distributed average tracking problem, each agent in the group has a time-varying reference signal, and the goal is to design controllers such that the agents can track the average of the group's reference signals. It has useful and practical applications, such as region following formation control [1], [2], coordinated path planning [3], and distributed convex optimization [4]. It is noted that the distributed average tracking problem is theoretically more challenging compared with consensus and distributed tracking problems.

When each agent aims to only estimate the average of the group reference signals instead of designing controllers for physical agents, the problem is often termed as dynamic average consensus in the literature. There are several applications, such as feature-based map merging [5], and distributed Kalman filtering [6], reported in the literature. Some distributed algorithms are established to deal with the dynamic average consensus problem for certain types of reference signals. See [7]–[12] for instance.

In this paper, we focus on the distributed average tracking problem in the context of controller design for physical agents. Several distributed average tracking algorithms are proposed for agents with double-integrator dynamics [13], general linear dynamics [2], [14], nonlinear dynamics [15]–[17], and Euler-Lagrange dynamics [18]. The distributed average tracking algorithms mentioned above need full state information (e.g., both positions and velocities for double-integrator agents) to update the controllers. However, in some practical applications, only partial states are available due to technology and space limitations. Moreover, it is usually less accurate and more expensive to implement velocity measurements compared with position measurements. Hence, it is worth investigating the distributed average tracking problem for double-integrator agents without using velocity measurements. In [19], the authors investigate the same problem described above. However, the lower bounds of the design parameters depend on the bounds related to the reference signals and the graph information including the number of agents in the network, and the largest and smallest nonzero eigenvalues of the Laplacian matrix, which are global information and inaccessible to the agents.

All these aforementioned continuous-time distributed average tracking algorithms require continuous interaction among the agents. However, because of the constrained bandwidth of the communication network and limited power supply, continuous communication may not be practical in reality. On the other hand, discrete-time distributed average tracking algorithms require agents to interact with each other periodically, which may result in a waste of network resources. Moreover, there usually exist tracking errors by using the discrete-time algorithms for general bounded reference signals. Thus, it makes sense to employ event-triggered communication strategies in distributed average tracking algorithms to reduce the communication cost. In [20], building on the algorithm in [9], the authors establish an event-triggered dynamic average consensus algorithm, but specific initialization is needed for a certain variable and there exist tracking errors for general bounded reference signals. A dynamic average consensus algorithm under dynamic communication is proposed in [21] without correct initialization. These two works focus on the estimation aspect of the distributed average tracking problem.

The objective of this paper is to design controllers without using any velocity measurements for double-integrator agents such that their positions and velocities are able to, respectively, track the averaged reference signal and reference signals' averaged velocity. First, a robust distributed average tracking algorithm without velocity measurements is presented. This algorithm distinguishes itself from [19] by removing any requirement of global information for parameter design. Then an event-triggered distributed average

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tracking algorithm is proposed, which further distinguishes this paper from [19] by removing the continuous interaction requirement. To be exact, in the event-triggered algorithm, we borrow the idea of the dynamic event-triggered strategy in [22] and construct a dynamic event-triggering condition to exclude Zeno behavior. This event-triggered algorithm is able to drive double-integrator agents to track the averaged reference signal without velocity measurements and correct initialization, which is a more complicated problem compared with [20], [21]. Finally, we present simulations to illustrate the results obtained.

II. Preliminaries and Problem Statement

For a given vector $x \in \mathbb{R}^p$, $\|x\|_2$, $\|x\|_1$, and $\|x\|_\infty$ denote the two-norm, one-norm, and infinity norm of x, respectively. For a set $\mathcal{S}$, $|\mathcal{S}|$ denotes the cardinality of $\mathcal{S}$. The transpose of matrix A is denoted by A^T. For matrices A and B, the Kronecker product is denoted by $A \otimes B$. Let $\mathbf{0}_{m\times n} \in \mathbb{R}^{m\times n}$ denote the $m \times n$ dimensional zero matrix, and for simplicity, let $\mathbf{0}_m = \mathbf{0}_{m\times 1}$. $I_n \in \mathbb{R}^{n\times n}$ denotes the identity matrix. For $x = [x_1, \ldots, x_p]^T \in \mathbb{R}^p$, we denote $\mathrm{sgn}(x) = [\mathrm{sgn}(x_1), \ldots, \mathrm{sgn}(x_p)]^T$, where $\mathrm{sgn}(x_i) = 1$ if $x_i > 0$, $\mathrm{sgn}(x_i) = -1$ if $x_i < 0$, and $\mathrm{sgn}(x_i) = 0$ if $x_i = 0$. In the rest of the paper, we omit the argument t for notational simplicity.

A. Graph Theory

For a multi-agent system consisting of N agents, the interaction topology can be modeled by an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denote the node set and edge set, respectively. An edge denoted by $(i, j) \in \mathcal{E}$, means that agents i and j can obtain information from each other. In an undirected graph, the edges (i, j) and (j, i) are equivalent. It is assumed that $(i, i) \notin \mathcal{E}$. The neighbor set of node i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\}$. The adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ of the graph G is defined such that $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. For an undirected graph, $a_{ij} = a_{ji}$. By arbitrary assigning an orientation for every edge in $\mathcal{G}$, let $B = [B_{ij}] \in \mathbb{R}^{N\times|\mathcal{E}|}$ denote the incidence matrix associated with graph $\mathcal{G}$, where $B_{ij} = -1$ if edge e_j leaves node i, $B_{ij} = 1$ if it enters node i, and $B_{ij} = 0$ otherwise.

An undirected path between node i_1 and i_k is a sequence of edges of the form (i_1, i_2), (i_2, i_3), $\ldots$, (i_{k-1}, i_k), where $i_k \in \mathcal{V}$. A connected graph means that there exists an undirected path between any pair of nodes in $\mathcal{V}$.

B. Problem Formulation

In this paper, we consider N physical agents, and the interaction topology among these agents is characterized as the undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Unless otherwise stated, throughout this paper, we assume a time-invariant graph. Each agent i is modeled by double-integrator dynamics

$$\dot{x}_i = v_i, \quad \dot{v}_i = u_i, \quad i \in \mathcal{V}, \tag{1}$$

where $x_i \in \mathbb{R}^p$ and $v_i \in \mathbb{R}^p$ are the ith agent's position and velocity, respectively, and u_i is its control input.

Each agent has a time-varying reference signal $x_i^r(t) \in \mathbb{R}^p$, $i \in \mathcal{V}$ satisfying

$$\dot{x}_i^r = v_i^r, \quad \dot{v}_i^r = u_i^r, \quad i \in \mathcal{V}, \tag{2}$$

where $v_i^r, u_i^r \in \mathbb{R}^p$ are the velocity and acceleration of the ith agent's reference signal, respectively. Clearly, the reference signal x_i^r, $i \in \mathcal{V}$, is twice differentiable. We assume that the reference signals are generated internally by the agents, and that each agent has access to its own reference signal, and the velocity and acceleration of the reference signal. In this paper, we make the following assumption on the reference signals.

Assumption 1: For any two connected agents, the local difference in reference signals x_i^r, their velocities v_i^r and their accelerations a_i^r are bounded, i.e., $\sup_{\substack{t\in[0,\infty) \\ \forall(i,j)\in\mathcal{E}}} \left\|x_i^r - x_j^r\right\|_\infty \le \bar{x}^r$, $\sup_{\substack{t\in[0,\infty) \\ \forall(i,j)\in\mathcal{E}}} \left\|v_i^r - v_j^r\right\|_\infty \le \bar{v}^r$, and $\sup_{\substack{t\in[0,\infty) \\ \forall(i,j)\in\mathcal{E}}} \left\|u_i^r - u_j^r\right\|_\infty \le \bar{a}^r$.

In the distributed average tracking for a group of double-integrator agents, the objective is to design controller u_i for agent $i \in \mathcal{V}$ such that each agent's position (velocity) is capable of tracking the group average of their reference signals (their reference signals' velocities). That is, for any $i \in \mathcal{V}$, it is achieved that $\lim_{t\to\infty} \left\|x_i - \frac{1}{N}\sum_{j=1}^N x_j^r\right\|_2 = 0$ and $\lim_{t\to\infty} \left\|v_i - \frac{1}{N}\sum_{j=1}^N v_j^r\right\|_2 = 0$. In this paper, we are particularly interested in developing a controller for each agent without velocity measurements and any correct initialization. The motivation behind is that employing velocity measuring device is usually costly in the aspect of finance and energy. Also, the velocity measurements are less accurate compared with position measurements. On the other hand, perfect initialization is hard to achieve in reality.

III. Robust Distributed Average Tracking without Velocity Measurements

In this section, we introduce a distributed average tracking algorithm for double-integrator agents without using the velocity measurements and in the absence of any correct initialization.

For each agent i, we design a filter as

$$\begin{aligned} \dot{\phi}_i &= -\kappa(x_i - x_i^r) - 2\kappa(w_i - v_i^r) + u_i^r(t) \\ &\quad - \sum_{j=1}^N a_{ij}\pi_{ij}(t)\mathrm{sgn}(x_i - x_j + w_i - w_j) \\ w_i &= \phi_i + \kappa(x_i - x_i^r), \quad i \in \mathcal{V}, \end{aligned} \tag{3}$$

where $\kappa \in \mathbb{R}$ is a positive constant to be determined, $\phi_i(t) \in \mathbb{R}^p$ is the internal state of the filter, $w_i \in \mathbb{R}^p$ is the output of the filter, and $\pi_{ij}(t)$ is a time-varying gain for the edge (i, j) satisfying the following adaptation law

$$\dot{\pi}_{ij} = a_{ij}\left\|x_i - x_j + w_i - w_j\right\|_1, \tag{4}$$

with $\pi_{ij}(0) > 0$ if $(i, j) \in \mathcal{E}$. In addition, each agent i needs to coordinate with its neighbor $j \in \mathcal{N}_i$ to ensure $\pi_{ij}(0) =$

$\pi_{ji}(0)$. In this way, the gains $\pi_{ij}(t)$ and $\pi_{ji}(t)$ remain equal to each other. For each agent i, we design the controller as

$$\begin{aligned} u_i = & -\kappa(x_i - x_i^r) - \kappa(w_i - v_i^r) + u_i^r \\ & - \sum_{j=1}^N a_{ij}\pi_{ij}(t)\mathrm{sgn}(x_i - x_j + w_i - w_j), \quad i \in \mathcal{V}. \end{aligned} \tag{5}$$

Essentially, the filter is designed such that its output is capable of tracking the average of the reference signals' velocities, and the controller is applied to drive each agent's position to the average of the reference signals and velocity to the output of the filter. Note that the designs of the filter (3) and the controller (5) for each agent i depend on only local information and the positions and filter's outputs from its neighbors. Therefore, it is implementable in reality.

Remark 1: Note that there is no requirement on the initialization of each agents' position and velocity, as well as the internal state of the filter. Thus, the proposed algorithm (3)-(5) is called robust distributed average tracking algorithm.

Let $x = \left[x_1^T, \ldots, x_N^T\right]^T$, $v = \left[v_1^T, \ldots, v_N^T\right]^T$, $w = \left[w_1^T, \ldots, w_N^T\right]^T$, $x^r = \left[(x_1^r)^T, \ldots, (x_N^r)^T\right]^T$, $v^r = \left[(v_1^r)^T, \ldots, (v_N^r)^T\right]^T$, and $u^r = \left[(u_1^r)^T, \ldots, (u_N^r)^T\right]^T$. Define $\tilde{x} = (M \otimes I_p)x$, $\tilde{v} = (M \otimes I_p)v$, and $\tilde{w} = (M \otimes I_p)w$, where $M = I_N - \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^T$. Then we have

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{v} \\ \dot{\tilde{v}} &= -\kappa\tilde{x} + (M \otimes I_p)\kappa x^r - \kappa\tilde{w} + (M \otimes I_p)\kappa v^r \\ &\quad + (M \otimes I_p)u^r - (B\Pi \otimes I_p)\mathrm{sgn}\left[\left(B^T \otimes I_p\right)(\tilde{x} + \tilde{w})\right] \\ \dot{\tilde{w}} &= -\kappa\tilde{x} + (M \otimes I_p)\kappa x^r - 2\kappa\tilde{w} + (M \otimes I_p)\kappa v^r + \kappa\tilde{v} \\ &\quad + (M \otimes I_p)u^r - (B\Pi \otimes I_p)\mathrm{sgn}\left[\left(B^T \otimes I_p\right)(\tilde{x} + \tilde{w})\right], \end{aligned} \tag{6}$$

where $\Pi(t) \in \mathbb{R}^{|\mathcal{E}|\times|\mathcal{E}|}$ is a time-varying diagonal matrix, and the sth diagonal entry, denoted by $\Pi_{ss}(t)$, represents the weight on the sth edge. That is, if the sth edge is between agent i and agent j, then $\Pi_{ss}(t) = \pi_{ij}(t)$.

Theorem 1: Suppose that the undirected graph $\mathcal{G}$ is connected, and Assumption 1 holds. Using the algorithm (3)-(5) for (1), distributed average tracking is achieved asymptotically if $\kappa > \frac{3+2\sqrt{3}}{3}$.

Proof: We prove this statement in two steps, which are denoted by consensus and sum-tracking steps. In consensus step, we prove that for any $i \in \mathcal{V}$, $x_i \to \frac{1}{N}\sum_{j=1}^N x_j$ and $v_i \to \frac{1}{N}\sum_{j=1}^N v_j$ as $t \to \infty$. In sum-tracking step, we prove that for any $i \in \mathcal{V}$, $\sum_{j=1}^N x_j \to \sum_{j=1}^N x_j^r$ and $\sum_{j=1}^N v_j \to \sum_{j=1}^N v_j^r$ as $t \to \infty$. Combining these two steps, it can be concluded that distributed average tracking is achieved.

Consider a Lyapunov function candidate as

$$V = \frac{1}{2}X^T P X + \sum_{i=1}^N\sum_{j=1}^N \frac{(\pi_{ij} - \pi_m)^2}{4}, \tag{7}$$

where $X = \left[\tilde{x}^T, \tilde{v}^T, \tilde{w}^T\right]^T$, π_m is a positive constant to be determined, and $P = \begin{bmatrix} \mu I_{Np} & \mathbf{0}_{Np\times Np} & I_{Np} \\ \mathbf{0}_{Np\times Np} & I_{Np} & -I_{Np} \\ I_{Np} & -I_{Np} & 2I_{Np} \end{bmatrix}$. It holds that P is positive definite if and only if $\mu > 1$. Taking the derivative of V along (6) yields

$$\begin{aligned} \dot{V} = & -X^T Q X + (\tilde{x} + \tilde{w})^T (M \otimes I_p)\alpha \\ & - \frac{1}{2}\sum_{i=1}^N\sum_{j=1}^N a_{ij}\pi_{ij}\left\|x_i - x_j + w_i - w_j\right\|_1 \\ & + \frac{1}{2}\sum_{i=1}^N\sum_{j=1}^N \pi_{ij}\dot{\pi}_{ij} - \frac{\pi_m}{2}\sum_{i=1}^N\sum_{j=1}^N \dot{\pi}_{ij}. \end{aligned}$$

where $\alpha = \kappa x^r + \kappa v^r + u^r$, and

$$Q = \begin{bmatrix} \kappa I_{Np} & -\frac{\mu+\kappa}{2}I_{Np} & \frac{3\kappa}{2}I_{Np} \\ -\frac{\mu+\kappa}{2}I_{Np} & \kappa I_{Np} & -\frac{1+3\kappa}{2}I_{Np} \\ \frac{3\kappa}{2}I_{Np} & -\frac{1+3\kappa}{2}I_{Np} & 3\kappa I_{Np} \end{bmatrix}. \tag{8}$$

Note that

$$\begin{aligned} \left\|\tilde{x} + \tilde{w}\right\|_1 &\leq \frac{1}{N}\sum_{i=1}^N\sum_{j=1,j\neq i}^N \left\|x_i - x_j + w_i - w_j\right\|_1 \\ &\leq \frac{N-1}{2}\sum_{i=1}^N\sum_{j=1}^N a_{ij}\left\|x_i - x_j + w_i - w_j\right\|_1, \end{aligned}$$

and by Assumption 1, it holds that

$$\begin{aligned} \left\|(M \otimes I_p)\alpha\right\|_\infty &\leq \frac{1}{N}\max_{i\in\mathcal{V}}\left\{\sum_{j=1,j\neq i}^N \left\|\alpha_i - \alpha_j\right\|_\infty\right\} \\ &\leq \frac{N-1}{2N}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\left\|\alpha_i - \alpha_j\right\|_\infty \leq \frac{\beta}{2}, \end{aligned}$$

where

$$\beta = N_{\max}(N-1)(\kappa\bar{x}^r + \kappa\bar{v}^r + \bar{a}^r), \tag{9}$$

and $N_{\max} = \max_{i\in\mathcal{V}}|\mathcal{N}_i|$. It follows that $\dot{V} \leq -X^T Q X - \left[\frac{\pi_m}{2} - \frac{(N-1)\beta}{4}\right]\sum_{i=1}^N\sum_{j=1}^N a_{ij}\left\|x_i - x_j + w_i - w_j\right\|_1$. Selecting π_m such that $\pi_m > \frac{(N-1)\beta}{2}$, one has $\dot{V} \leq -X^T Q X := -W[X(t)]$. The matrix Q is positive definite if and only if $\kappa > \mu + \frac{1}{3(\mu-1)} = f(\mu)$, which implies that Q is positive definite if $\kappa > \min_{\mu>1} f(\mu) = \frac{3+2\sqrt{3}}{3}$. Thus $\dot{V} \leq 0$, which implies that V is nonincreasing. Then it follows that $\tilde{x}(t)$, $\tilde{v}(t)$, $\tilde{w}(t)$, and $\pi_{ij}(t)$ are bounded. Note that $V(t)$ is bounded from below by zero. Thus, $\lim_{t\to\infty} V(t)$ exists and is finite. It also holds that $\lim_{t\to\infty}\int_0^t W[X(\tau)]d\tau$ exists and is finite. Note that $\dot{\tilde{x}}(t)$, $\dot{\tilde{v}}(t)$ and $\dot{\tilde{w}}(t)$ are also bounded. Hence, $\tilde{x}(t)$, $\tilde{v}(t)$, and $\tilde{w}(t)$ are uniformly continuous. Consequently, $W[X(t)]$ is uniformly continuous. By Barbalat's Lemma, it can be concluded that $W[X(t)] \to 0$ as $t \to \infty$, which implies that $x_i \to \frac{1}{N}\sum_{j=1}^N x_j$, $v_i \to \frac{1}{N}\sum_{j=1}^N v_j$, and $w_i \to \frac{1}{N}\sum_{j=1}^N w_j$, as $t \to \infty$. This completes consensus step.

Second, define $S_x = \sum_{j=1}^N x_j - \sum_{j=1}^N x_j^r$, $S_v = \sum_{j=1}^N v_j - \sum_{j=1}^N v_j^r$, and $S_w = \sum_{j=1}^N w_j - \sum_{j=1}^N v_j^r$. Then we have that $\dot{S} = \left(\begin{bmatrix} 0 & 1 & 0 \\ -\kappa & 0 & -\kappa \\ -\kappa & \kappa & -2\kappa \end{bmatrix} \otimes I_p\right) S = (A \otimes I_p)S$, where $S = \left[S_x^T, S_v^T, S_w^T\right]^T$. Note that $\kappa > \frac{3+2\sqrt{3}}{3} > 0$.

According to the Routh-Hurwitz stability criterion, it is easy to verify that A is Hurwitz, which indicates $\lim_{t\to\infty} S(t) = \mathbf{0}_{3p}$. This completes sum-tracking step. ■

Remark 2: Theorem 1 shows that the agents are capable of achieving distributed average tracking under any fixed connected undirected communication network. It is actually able to extend to the case of arbitrarily switching connected communication networks with positive dwelling time. The function defined in (7) can be used as a common Lyapunov function during the proof process.

Remark 3: Note that the dynamics (6) is discontinuous due to the introduction of the signum function in the controller and filter design (3)-(5). Then, the solutions should be understood in terms of differential inclusion by using non-smooth analysis [23], [24]. However, since the signum function is measurable and locally essentially bounded, the Filippov solutions for the closed-loop dynamics always exist. The Lyapunov function used in the proof is continuously differentiable. Then its set-valued Lie derivative is a singleton at the discontinuous points. Therefore, the proof is valid as in the case without discontinuities.

Remark 4: In the proposed algorithm (3)-(5), the adaptive gain $\pi_{ij}(t)$ is introduced to remove the requirement of knowledge on global information, such as the interaction graph information and the bounds related to the reference signals. Although the related works in [19] also studied the distributed average tracking problem without velocity measurements, the lower bounds of the design parameters depend on the reference signals' bounds, the total number of agents, and the graph information including the largest and smallest nonzero eigenvalues of the Laplacian matrix. In the present paper, as long as $\kappa > \frac{3+2\sqrt{3}}{3}$, the proposed algorithm (3)-(5) achieves distributed average tracking with zero error. That is, no global information is needed for parameter design.

Remark 5: If each agent chooses its own $\kappa_i(0) > \frac{3+2\sqrt{3}}{3}$, then the agents can run the finite-time max consensus algorithm in [25] as $\dot{\kappa}_i(t) = \text{sgn}_+\left\{\sum_{j\in\mathcal{N}_i} a_{ij}[\kappa_i(t) - \kappa_j(t)]\right\}$, where $\text{sgn}_+ : \mathbb{R} \to \mathbb{R}$ is defined as $\text{sgn}_+(z) = 1$ if $z > 0$ and $\text{sgn}_+(z) = 0$ if $z \leq 0$. Then, $\kappa_i(t)\ \forall i \in \mathcal{V}$ converges to $\bar{\kappa} = \max_{j\in\mathcal{V}} \kappa_j(0) > \frac{3+2\sqrt{3}}{3}$ in finite time.

IV. Event-Triggered Distributed Average Tracking without Velocity Measurements

The algorithm (3)-(5) in Section III requires each agent i to continuously exchange the position and the filter output with its neighbors, which may not be practical in reality. To this end, we design an event-triggered distributed average tracking algorithm without velocity measurements and initialization requirements to remove the requirement of continuous communication.

Inspired by the dynamic event-triggered mechanism in [22], we propose a dynamic event-triggered distributed average tracking algorithm in this section. We consider the following event-triggered version of the filter, controller, and adaptation law presented in Section III as follows. For agent i, the filter is given by

$$\begin{aligned} \dot{\phi}_i &= -\kappa(x_i - x_i^r) - 2\kappa(w_i - v_i^r) + u_i^r \\ &\quad - \sum_{j=1}^N a_{ij}\pi_{ij}\text{sgn}(\hat{x}_i - \hat{x}_j + \hat{w}_i - \hat{w}_j) \\ w_i &= \phi_i + \kappa(x_i - x_i^r), \quad i \in \mathcal{V}, \end{aligned} \tag{10}$$

and the controller is given by

$$\begin{aligned} u_i &= -\kappa(x_i - x_i^r) - \kappa(w_i - v_i^r) + u_i^r \\ &\quad - \sum_{j=1}^N a_{ij}\pi_{ij}\text{sgn}(\hat{x}_i - \hat{x}_j + \hat{w}_i - \hat{w}_j), \quad i \in \mathcal{V}, \end{aligned} \tag{11}$$

and π_{ij} satisfies the adaptation law

$$\dot{\pi}_{ij} = a_{ij}\left\|\hat{x}_i - \hat{x}_j + \hat{w}_i - \hat{w}_j\right\|_1, \quad i \in \mathcal{V}, \tag{12}$$

where $\hat{x}_j(t) = x_j(t_{k_j}^j)$ and $\hat{w}_j(t) = w_j(t_{k_j}^j)$ $t \in [t_{k_j}^j, t_{k_j+1}^j)$, denote the last broadcast position and filter's output of agent j, respectively, and $t_{k_j}^j = \max\left\{t_k^j \mid t_k^j \leq t\right\}$ is the latest triggering time instant of agent j. The agent i's sequence of triggering time instants t_1^i, t_2^i, $\ldots$, is to be determined later. For agent $i \in \mathcal{V}$, define $e_{x_i} = \hat{x}_i - x_i$, $e_{w_i} = \hat{w}_i - w_i$, and let $\hat{q}_i = \sum_{j=1}^N a_{ij}\pi_{ij}\text{sgn}(\hat{x}_i - \hat{x}_j + \hat{w}_i - \hat{w}_j)$. Each agent i maintains the following additional internal dynamics

$$\dot{y}_i = -\gamma_i y_i - \sigma_{1i}\left[\theta_i\left\|e_{x_i} + e_{w_i}\right\|_1 + (e_{x_i} + e_{w_i})^T\hat{q}_i\right] \tag{13}$$

with $y_i(0) > 0$, where $\gamma_i > 0$, $\sigma_{1i} \geq 1$ and $\theta_i > 0$ are design parameters to be determined.

Theorem 2: Suppose that the undirected graph $\mathcal{G}$ is connected, and Assumption 1 holds. Selecte the positive scalar θ_i such that $\theta_i \geq \frac{\beta}{2}$ for any $i \in \mathcal{V}$, where β is defined in (9), then using the algorithm (10)-(12) for (1), the distributed average tracking is achieved with zero tracking error if $\kappa > \frac{3+2\sqrt{3}}{3}$, and the triggering time instant is determined by

$$\begin{aligned} t_{k+1}^i = \min\Big\{t > t_k^i \mid \sigma_{2i}\big[\theta_i\left\|e_{x_i} + e_{w_i}\right\|_1 \\ + (e_{x_i} + e_{w_i})^T\hat{q}_i\big] \geq y_i\Big\} \end{aligned} \tag{14}$$

with $t_1^i = 0\ \forall i \in \mathcal{V}$, where $\sigma_{2i} \in (0,1)$. In addition, the dynamic triggering law (14) excludes Zeno behavior while running the algorithm (10)-(12).

Proof: The proof process for convergence are similar to the proof of Theorem 1. For the consensus step, consider the Lyapunov candidate function $V_1 = V + \sum_{i=1}^N y_i$, where V is given in (7). From (14), it holds that $\sigma_{2i}\big[\theta_i\left\|e_{x_i} + e_{w_i}\right\|_1 + (e_{x_i} + e_{w_i})^T\hat{q}_i\big] \leq y_i$. Thus, $\dot{y}_i \geq -\gamma_i y_i - \frac{\sigma_{1i}}{\sigma_{2i}} y_i$, which implies that $y_i > 0\ \forall t \geq 0$. Use the same definitions of $\tilde{x}$, $\tilde{v}$ and $\tilde{w}$ as in Section III, and let $\hat{x} = \left[\hat{x}_1^T, \ldots, \hat{x}_N^T\right]^T$, $\hat{w} = \left[\hat{w}_1^T, \ldots, \hat{w}_N^T\right]^T$, $e_x = \left[e_{x_1}^T, \ldots, e_{x_N}^T\right]^T$ and $e_w =$

$\left[e_{w_1}^T, \ldots, e_{w_N}^T\right]^T$. It follows that

$$\begin{aligned}
\dot{\tilde{x}} &= \tilde{v} \\
\dot{\tilde{v}} &= -\kappa\tilde{x} - \kappa\tilde{w} + (M \otimes I_p)\alpha \\
&\quad - (B\Pi \otimes I_p)\text{sgn}\left[\left(B^T \otimes I_p\right)(\hat{x} + \hat{w})\right] \\
\dot{\tilde{w}} &= -\kappa\tilde{x} - 2\kappa\tilde{w} + \kappa\tilde{v} + (M \otimes I_p)\alpha \\
&\quad - (B\Pi \otimes I_p)\text{sgn}\left[\left(B^T \otimes I_p\right)(\hat{x} + \hat{w})\right].
\end{aligned} \tag{15}$$

Taking the derivative of V_1 along (15) yields

$$\begin{aligned}
\dot{V}_1 &= -X^TQX + (\hat{x} + \hat{w})^T(M \otimes I_p)\alpha \\
&\quad - (e_x + e_w)^T(M \otimes I_p)\alpha \\
&\quad - (\hat{x} + \hat{w})^T(B\Pi \otimes I_p)\text{sgn}[(B^T \otimes I_p)(\hat{x} + \hat{w})] \\
&\quad + (e_x + e_w)^T(B\Pi \otimes I_p)\text{sgn}[(B^T \otimes I_p)(\hat{x} + \hat{w})] \\
&\quad + \frac{1}{2}\sum_{i=1}^N\sum_{j=1}^N \pi_{ij}\dot{\pi}_{ij} - \frac{\pi_m}{2}\sum_{i=1}^N\sum_{j=1}^N \dot{\pi}_{ij} + \sum_{i=1}^N \dot{y}_i \\
&\leq -X^TQX + \sum_{i=1}^N (e_{x_i} + e_{w_i})^T\hat{q}_i + \sum_{i=1}^N \dot{y}_i \\
&\quad - (e_x + e_w)^T(M \otimes I_p)\alpha.
\end{aligned}$$

where the equality follows by noting that $M^2 = M$ and $MB = B$, and the inequality follows by selecting π_m such that $\pi_m \geq \frac{(N-1)\beta}{2}$. Applying the triggering law (14) with the internal dynamics (13) yields

$$\begin{aligned}
\dot{V}_1 &\leq -X^TQX - \sum_{i=1}^N(\sigma_{1i} - 1)(e_{x_i} + e_{w_i})^T\hat{q}_i \\
&\quad - \sum_{i=1}^N \gamma_i y_i - \sum_{i=1}^N(\sigma_{1i} - 1)\theta_i \left\|e_{x_i} + e_{w_i}\right\|_1 \\
&\leq -X^TQX - \sum_{i=1}^N \gamma_i y_i := W_1,
\end{aligned}$$

Apparently, $W_1 \leq 0$. It then follows that V_1 is nonincreasing, and hence $\tilde{x}$, $\tilde{v}$, $\tilde{w}$, Π, and y_i $\forall i \in \mathcal{V}$ are all bounded. Note that $\dot{V}_1 \leq -X^TQX$. Following the similar analysis in the proof of Theorem 1 and Barbalat's Lemma, it can be concluded that $x_i \to \frac{1}{N}\sum_{j=1}^N x_j$, $v_i \to \frac{1}{N}\sum_{j=1}^N v_j$, and $w_i \to \frac{1}{N}\sum_{j=1}^N w_j$, as $t \to \infty$, which means consensus is achieved.

The sum-tracking step follows directly from that of Theorem 1. The proof of Zeno behavior exclusion part follows a similar line of analysis to that in [21],which is omitted here. ■

Remark 6: In [20], [21], event-triggered distributed average tracking schemes are proposed for the estimation purpose. However, the proposed algorithm in this section aims to solve the distributed average tracking problem for double-integrator agents without using velocity measurements, which is more complicated compared with [20], [21]. Moreover, compared with the work in [20], the proposed algorithm (10)-(12) requires no correct initialization and achieves distributed average tracking with zero errors.

Remark 7: As indicated in Theorem 2, the lower bound of θ_i depends on some global information, i.e., the bounds of the reference signals and some graph information. In reality, one can always select θ_i to be large enough. Moreover, when designing the distributed average tracking algorithm under event-triggered communication, in order to achieve distributed average tracking with zero error, the dependence on certain global information for the design parameter's bound might be unavoidable, which has usually been done in the literature [21]. In addition, one might use some algorithms in the literature [25], [26] to estimate N and the bounds on the reference signals $\bar{x}^r$, $\bar{v}^r$ and $\bar{a}^r$, so as to get a better estimate on the lower bound for the design parameter.

V. Illustrative Examples

We consider a group of seven physical agents ($N = 7$) given in (1), which are labeled as $1, \ldots, 7$. These seven agents form a ring topology In the simulation, we set $u_i^r = A_i \sin(\vartheta_i t + \varphi_i)$ in (2) with $A_i = -0.09(0.7i + 0.5)^2[6(i - 3.5) - 2(-1)^i]$, $\vartheta_i = 0.3(0.7i + 0.5)$, and $\varphi_i = \frac{2i\pi}{N} - \pi$.

Select $\kappa = 5$ and $\pi_{ij}(0) = 250$ for any i and j that are connected. Implement the algorithm (3)-(5) for (1). The position and velocity trajectories are shown in Fig. 1, which indicates that the agents' positions and velocities track, respectively, $\frac{1}{7}\sum_{j=1}^7 x_j^r(t)$ and $\frac{1}{7}\sum_{j=1}^7 v_j^r(t)$.

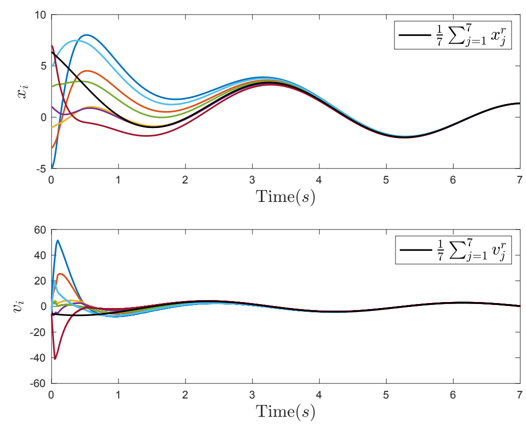

Fig. 1. Seven agents' position and velocity trajectories while using algorithm (3)-(5). The black lines denote the average of the reference signals and their velocities. The rest are the position and velocity trajectories of these seven agents.

In the following, we use the algorithm (10)-(12) for (1) with the same set of reference signals. The triggering time instants is determined as in (14). In this simulation, we use a fixed step solver to solve the Simulink model, and fixed-step size is 10^{-4}. For simplicity, we set $y_i(0) = 650$, $\theta_i = 700$, $\sigma_{1i} = 1$, $\sigma_{2i} = 0.99$ and $\gamma_i = 0.01$ for any $i = 1, \ldots, 7$. The position and velocity trajectories of the agents are shown in Fig. 2, which indicates that all the agents' positions and velocities track, respectively, $\frac{1}{7}\sum_{j=1}^7 x_j^r(t)$ and $\frac{1}{7}\sum_{j=1}^7 v_j^r(t)$, respectively. The number of triggering time instants for each agent is presented in Fig. 3. In the 7 seconds simulation time, agents $1-7$ are triggered 8.76%, 25.97%, 26.26%, 26.05%, 37.85%, 26.02%, and 25.60% of times. Therefore, the proposed algorithm (10)-(12) avoids continuous communication.

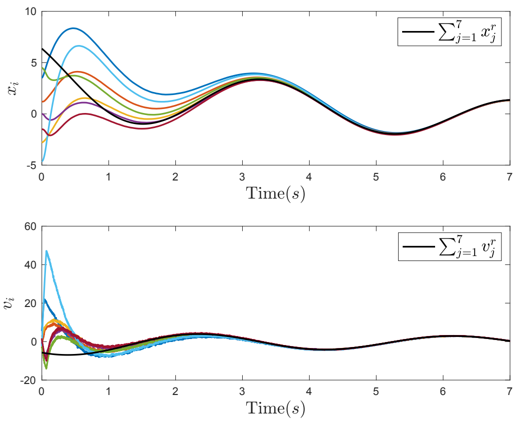

Fig. 2. Seven agents' position and velocity trajectories while using the algorithm (10)-(12).

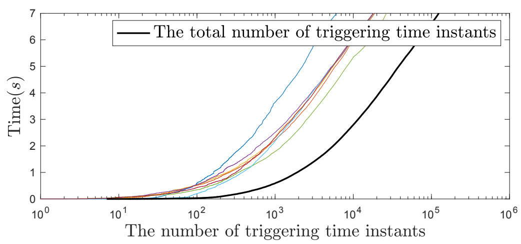

Fig. 3. The number of triggering time instants of the agents while using the algorithm (10)-(12).

VI. Conlusion

This paper investigated the distributed average tracking problem for double-integrator agents without velocity measurements and correction initialization. First, a distributed average tracking algorithm has been proposed without using velocity measurements, which removes the requirement of global information for parameter design. Then, building on this algorithm, an event-triggered distributed average tracking algorithm has been derived by using the idea of a dynamic event-triggered strategy to remove the continuous communication requirement. Zeno behavior is excluded in the event-triggered algorithm. Finally, we provided two examples to illustrate the results obtained in this paper.

References

[1] C. C. Cheah, S. P. Hou, and J. J. E. Slotine, "Region-based shape control for a swarm of robots," *Automatica*, vol. 45, no. 10, pp. 2406–2411, 2009.

[2] F. Chen and W. Ren, "A connection between dynamic region-following formation control and distributed average tracking," *IEEE Transactions on Cybernetics*, vol. 48, no. 6, pp. 1760–1772, 2018.

[3] P. Švestka and M. H. Overmars, "Coordinated path planning for multiple robots," *Robotics and autonomous systems*, vol. 23, no. 3, pp. 125–152, 1998.

[4] S. Rahili and W. Ren, "Distributed continuous-time convex optimization with time-varying cost functions," *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 1590–1605, 2017.

[5] R. Aragues, J. Cortes, and C. Sagues, "Distributed consensus on robot networks for dynamically merging feature-based maps," *IEEE Transactions on Robotics*, vol. 28, no. 4, pp. 840–854, 2012.

[6] H. Bai, R. A. Freeman, and K. M. Lynch, "Distributed kalman filtering using the internal model average consensus estimator," in *Proceedings of the 2011 American Control Conference*. IEEE, 2011, pp. 1500–1505.

[7] D. P. Spanos, R. Olfati-Saber, and R. M. Murray, "Dynamic consensus on mobile networks," in *IFAC World Congress*. Citeseer, 2005, pp. 1–6.

[8] R. A. Freeman, P. Yang, and K. M. Lynch, "Stability and convergence properties of dynamic average consensus estimators," in *Proceedings of the IEEE Conference on Decision and Control*. IEEE, 2006, pp. 338–343.

[9] S. S. Kia, J. Cortés, and S. Martinez, "Dynamic average consensus under limited control authority and privacy requirements," *International Journal of Robust and Nonlinear Control*, vol. 25, no. 13, pp. 1941–1966, 2015.

[10] S. Nosrati, M. Shafiee, and M. B. Menhaj, "Dynamic average consensus via nonlinear protocols," *Automatica*, vol. 48, no. 9, pp. 2262–2270, 2012.

[11] F. Chen, Y. Cao, and W. Ren, "Distributed average tracking of multiple time-varying reference signals with bounded derivatives," *IEEE Transactions on Automatic Control*, vol. 57, no. 12, pp. 3169–3174, 2012.

[12] J. George and R. Freeman, "Robust dynamic average consensus algorithms," *IEEE Transactions on Automatic Control*, vol. 64, no. 11, pp. 4615–4622, 2019.

[13] F. Chen, W. Ren, W. Lan, and G. Chen, "Distributed average tracking for reference signals with bounded accelerations," *IEEE Transactions on Automatic Control*, vol. 60, no. 3, pp. 863–869, 2015.

[14] Y. Zhao, Y. Liu, Z. Li, and Z. Duan, "Distributed average tracking for multiple signals generated by linear dynamical systems: An edge-based framework," *Automatica*, vol. 75, pp. 158–166, 2017.

[15] S. Ghapani, S. Rahili, and W. Ren, "Distributed average tracking of physical second-order agents with heterogeneous unknown nonlinear dynamics without constraint on input signals," *IEEE Transactions on Automatic Control*, vol. 64, no. 3, pp. 1178–1184, 2018.

[16] S. Ghapani, S. Rahili, and W. Ren, "Distributed average tracking for second-order agents with nonlinear dynamics," in *Proceedings of the American Control Conference*. IEEE, 2016, pp. 4636–4641.

[17] Y. Zhao, Y. Liu, G. Wen, and T. Huang, "Finite-time distributed average tracking for second-order nonlinear systems," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 30, no. 6, pp. 1780–1789, 2019.

[18] F. Chen, G. Feng, L. Liu, and W. Ren, "Distributed average tracking of networked Euler-Lagrange systems," *IEEE Transactions on Automatic Control*, vol. 60, no. 2, pp. 547–552, 2015.

[19] S. Ghapani, W. Ren, F. Chen, and Y. Song, "Distributed average tracking for double-integrator multi-agent systems with reduced requirement on velocity measurements," *Automatica*, vol. 81, pp. 1–7, 2017.

[20] S. S. Kia, J. Cortés, and S. Martínez, "Distributed event-triggered communication for dynamic average consensus in networked systems," *Automatica*, vol. 59, pp. 112–119, 2015.

[21] J. George, X. Yi, and T. Yang, "Distributed robust dynamic average consensus with dynamic event-triggered communication," in *Proceedings of the IEEE Conference on Decision and Control*. IEEE, 2018, pp. 434–439.

[22] A. Girard, "Dynamic triggering mechanisms for event-triggered control," *IEEE Transactions on Automatic Control*, vol. 60, no. 7, pp. 1992–1997, 2015.

[23] A. F. Filippov, *Differential equations with discontinuous righthand sides: control systems*. Springer Science & Business Media, 2013, vol. 18.

[24] J. Cortes, "Discontinuous dynamical systems," *IEEE Control systems magazine*, vol. 28, no. 3, pp. 36–73, 2008.

[25] J. Cortés, "Distributed algorithms for reaching consensus on general functions," *Automatica*, vol. 44, no. 3, pp. 726–737, 2008.

[26] D. Varagnolo, G. Pillonetto, and L. Schenato, "Distributed cardinality estimation in anonymous networks," *IEEE Transactions on Automatic Control*, vol. 59, no. 3, pp. 645–659, 2013.