Finite-Horizon H_∞ Fault-Tolerant Constrained Consensus for Multiagent Systems With Communication Delays

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***Abstract*—This article focuses on the fault-tolerant constrained consensus problem for multiagent systems with communication delays. The communication graphs are first assumed to be directed and fixed. Then, a novel delay-dependent fault-tolerant controller is designed such that, in the presence of communication delays and randomly occurring actuator failures, the influence of the projections and the initial states on the closed-loop system can be attenuated with a prespecified level. Based on the provided performance requirement, the initial state of each agent does not need to be identical. The proposed control algorithms ensure that sufficient conditions are met for the fault-tolerant constrained consensus to be achieved according to the prespecified performance index. After this, the controller gains are computed by employing an iterative linear matrix inequality scheme. Finally, a numerical example is provided to show the effectiveness of the proposed method.**

***Index Terms*—Communication delays, constrained consensus, H_∞ control, failures, fault-tolerant control, multiagent systems.**

I. INTRODUCTION

OVER the past few decades, there has been a growing interest in solving the consensus problem of multiagent systems due to its potential applications in formation control, flocking, coordination, rendezvous, sensor networks, and swarming. (See [1], [4]–[7], [12], and their associated references.) Most of the above research results are derived without any constraints. However, in some situations, each agent needs to remain in a certain region, and seeking constrained consensus is important [3], [29]. For instance, in the rescue problem with unmanned aerial vehicles (UAVs), the position of each UAV has to be located in a constraint set due to hazardous areas [10], [30]. Recently, some effective research methods have been developed that can solve the constrained consensus problem. More specifically, a distributed control algorithm is provided in [9] to guarantee the constrained optimal consensus with prespecified cost functions. The proposed algorithm is based on local averaging, local projection, and local subgradients. In [10], a multiagent system with multicluster networks is considered that uses a hierarchical projection-based consensus algorithm to achieve a constrained consensus. In the context of multiagent systems with unbalanced networks and communication delays, [2] solves the constrained consensus problem by deriving an equivalent delay-free system. By utilizing the H_∞ theory, a distributed algorithm is proposed with a nonlinear cost function in [11] to solve the constrained consensus problem. A type of constrained consensus algorithm with a fixed step size is presented in [31], which concentrates on improving the convergence rate. Margellos *et al.* [32] dealt with the constrained consensus problem with time-varying networks and uncertainties by using a scenario-based methodology. Considering agents with single-integrator and double-integrator dynamics, the corresponding constrained consensus algorithm is provided in [33]. In order to solve more general constrained distributed optimization problems, a second-order multiagent network is given in [34]. It should be noted that most of the results on the constrained consensus problem obtained so far have been achieved without any failure. However, actuator failures are inevitable in practice.

The multiagent systems are likely to suffer from sudden failures resulting from unknown phenomena. This will lead to degradation in control performance, and the closed-loop systems may even become unstable [17], [23], [45], [46]. Current research on fault-tolerant control can be divided into two categories, namely, passive fault-tolerant control [13]–[16], [18] and active fault-tolerant control [19]–[22], [25]. A fault-tolerant control strategy for multiagent systems is provided in [23], where the controller consists of a healthy control protocol and an estimator to predict the fault severity. In [26], a high-order multiagent system with actuator failures is examined. To overcome the problem of network disconnections, an end-to-end communication rate-based adaptive fault-tolerant control method is given. For faulty multiagent systems with a bidirectional communication topology, Khalili *et al.* [8] developed a local fault-tolerant control algorithm for each agent. The controller consists of a baseline controller and two

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adaptive fault-tolerant controllers. Working on the assumption of a completely unknown system model, Shi *et al.* [35] introduced a novel system called weighted edge dynamics to ensure the fault-tolerant consensus under the designed controller. For multiagent systems with nonidentical unknown nonlinear dynamics and undetectable actuator failures, Wang *et al.* [24] provided a robust adaptive fault-tolerant consensus algorithm that can compensate for uncertain dynamics and unpredictable actuator failures simultaneously. To cover the case of general linear multiagent systems, a distributed adaptive event-triggered fault-tolerant consensus algorithm is developed in [36]. This algorithm uses an online updating strategy to avoid the computation of the minimum eigenvalue of a Laplacian matrix while a consensus controller is designed with an event-triggered mechanism. For a class of nonlinear second-order leader-following multiagent systems with multiple actuator failures, a distributed controller based on the circuit theory is proposed in [37]. Since the multiagent systems are usually connected via communication networks and communication delays are inevitable, the stability of multiagent systems would be affected by the delays [2], [12], [44]. At present, the corresponding research problem for the fault-tolerant constrained consensus with communication delays has not been fully investigated yet, which motivates this article.

In this article, we endeavor to investigate the finite-horizon H_∞ fault-tolerant constrained consensus problem for multiagent systems with communication delays. One application of the theoretical result could be the rescue problem using UAVs mentioned earlier. Over the course of our exposition, we will address the following issues.

1) In the classical H_∞ control theory, the zero initial condition has to be satisfied in advance. To meet this requirement, [11], [38], and [39] have assumed that all agents' initial conditions are the same. Unfortunately, this assumption is rather conservative in practice, especially when confronted with a constrained consensus problem.
2) It is well known that partial actuator failures can lead to performance deterioration of the investigated systems. The moving directions of the agents can be affected by partial failures, and hence it is hard to achieve a constrained consensus without proper controllers.
3) In practice, communication delays between agents will exist. To overcome this phenomenon, Lin and Ren [2] proposed a delay-free system that uses an equivalent method. However, the analysis is performed without considering the size of the time delay. In other words, it is a delay-independent approach. The delay-dependent methods, in contrast, focus on the size of the delay, making them less conservative than the delay-independent techniques [40].

By tackling the above issues, the main contributions of this article can be summarized as follows.

1) Motivated by the finite-horizon theory [27], a performance index is constructed that uses projection information and initial states. This bypasses the need for zero initial conditions during the controller design procedure. As a result, the influence of the initial states on a closed-loop system can be attenuated at a prespecified level γ.
2) A novel finite-horizon H_∞-constrained consensus scheme is provided for multiagent systems subject to abrupt partial actuator failures. By using the upper and lower bounds of the failure coefficients, an iterative linear matrix inequality algorithm is proposed that can update the controller gains.
3) A delay-dependent fault-tolerant controller is proposed that has the advantage that time delays would have no direct impact on the stability of the system. By using the delay-dependent iterative linear matrix inequality approach proposed in this article, the finite-horizon H_∞ performance is achieved.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. Given symmetric square matrices X and Y, $X \geq Y$ (respectively, $X > Y$) means that $X - Y$ is positive semidefinite (respectively, positive definite). Similarly, $X \leq Y$ (respectively, $X < Y$) means that $X - Y$ is negative semidefinite (respectively, negative definite). A^{T} represents the transpose of the matrix A. diag$\{\ldots\}$ stands for a block-diagonal matrix. "$*$" denotes the symmetric term in a symmetric matrix. The symbol $\otimes$ means the Kronecker product. For a square matrix X, we denote $\mathrm{He}(X) = X + X^{\mathrm{T}}$. $\mathbf{1}_n$ denotes $[1, 1, \ldots, 1]^{\mathrm{T}} \in \mathbb{R}^n$. $P_Y\{\chi\}$ denotes the projection of a vector χ on a closed convex set Y.

II. Problem Formulation and Preliminaries

A. Graph Theory

A directed graph of order n is denoted as $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is the set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{A} = [a_{ij}]_{n\times n}$ is an adjacency matrix. If agent i can receive information from agent j, $(j, i) \in \mathcal{E}$ and $a_{ij} > 0$; otherwise, $a_{ij} = 0$. Given any $i \neq j, i, j = 1, 2, \ldots, n$, the Laplacian matrix $L = [l_{ij}]_{n\times n}$ of $\mathcal{G}$ is denoted as

$$l_{ij} = -a_{ij}, \quad l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}. \tag{1}$$

B. Problem Formulation

Consider a group of n nonlinear agents described by the following discrete-time dynamics:

$$x_i(k+1) = P_{\Omega_i}\left\{u_i^F(k)\right\} \tag{2}$$

where $x_i(k) \in \mathbb{R}^{n_x}$ denotes the state vector of the ith agent, and $u_i^F(k) \in \mathbb{R}^{n_u}$ is the fault-tolerant controller of the agent. The state vector of each agent is constrained to lie in a nonempty closed convex set Ω_i. Define $\Omega = \cap_{i=1}^{n}\Omega_i$ as the intersection set of all Ω_i. The controller has the following form:

$$u_i^F(k) = \rho_i u_i(k)$$

where $u_i(k)$ is the nominal controller to be designed, and ρ_i denotes the coefficient matrix of partial actuator failures of agent i that satisfies

$$\rho_i = \mathrm{diag}\left\{\rho_{i1}, \rho_{i2}, \ldots, \rho_{in_u}\right\}$$

$$0 < \rho_i^{\min} \leq \rho_{im} \leq \rho_i^{\max} \leq 1$$
$$i = 1, 2, \ldots, n, \quad m = 1, 2, \ldots, n_u.$$

If $\rho_{im} = 1$, the mth actuator of agent i is running without failure. If $0 < \rho_{im} < 1$, the mth actuator of agent i has a partial failure with the effectiveness of the actuator being reduced. In practice, the moving direction of the agents can be changed due to the effect of the partial actuator failures.

Our main purpose is to design a fault-tolerant control algorithm for each agent modeled by (2) to ensure that the finite-horizon H_∞ fault-tolerant constrained consensus can be achieved. We first recall the following definition and lemmas, which are necessary to present our main results.

Definition 1 [2]: For any initial condition, the constrained consensus problem of the multiagent systems (2) is said to be reached if

$$\lim_{k\to\infty} \|x_i(k) - \tilde{z}\| = 0 \quad \forall i = 1, 2, \ldots, n \tag{3}$$

where $\tilde{z}$ is a constant vector that belongs to the intersection set Ω.

Lemma 1 (Schur Complement [43]): Given a symmetric matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22} \end{bmatrix}$$

the following statements are equivalent:

1) $S < 0$
2) $S_{11} < 0, S_{22} - S_{12}^{\mathrm{T}} S_{11}^{-1} S_{12} < 0$
3) $S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^{\mathrm{T}} < 0$.

Lemma 2 [41]: Given matrices Υ, Γ, and Λ of appropriate dimensions with Υ being symmetrical, then $\Upsilon + \Gamma F(k)\Lambda + \Lambda^{\mathrm{T}} F^{\mathrm{T}}(k)\Gamma < 0$ for all $F(k)$ satisfying $F^{\mathrm{T}}(k)F(k) \leq I$, if and only if there exists some $\sigma > 0$ such that $\Upsilon + \sigma\Gamma\Gamma^{\mathrm{T}} + \sigma^{-1}\Lambda^{\mathrm{T}}\Lambda < 0$.

Proof: Setting $R = I$ in [41, Lemma 2.4], the proof is obtained. ∎

Lemma 3 [42]: For any constant matrix $R \in \mathbb{R}^{n\times n}$ with $R = R^{\mathrm{T}} > 0$, integers r_1 and r_2 with $r_2 > r_1 > 0$, and vector function $\omega : \{r_1, r_1{+}1, \ldots, r_2\} \to \mathbb{R}^n$, the following inequality holds:

$$\sum_{j=r_1}^{r_2-1} \omega^{\mathrm{T}}(j)R\omega(j) \geq \frac{1}{r_2 - r_1}\left(\sum_{j=r_1}^{r_2-1}\omega(j)\right)^{\mathrm{T}} R \left(\sum_{j=r_1}^{r_2-1}\omega(j)\right).$$

Lemma 4 [29]: Suppose that $\mathcal{W} \neq \varnothing$ is a closed convex set in $\mathbb{R}^m$. For any $y, z \in \mathbb{R}^m$, the following inequalitiy holds:

$$\|P_{\mathcal{W}}(y) - P_{\mathcal{W}}(z)\| \leq \|y - z\|.$$

III. Main Results

A. Delay-Free Fault-Tolerant Control

In this section, consider the following control law for (2) as:

$$u_i(k) = K_k \sum_{j=1, j\neq i}^{n} a_{ij}\big(x_j(k) - x_i(k)\big) + P_\Omega\{x_i(k)\} \tag{4}$$

where K_k is the controller gain to be designed. It is assumed that each agent knows the intersection set Ω. Substituting (4) into (2), we have

$$x_i(k+1) = P_{\Omega_i}\left\{\rho_i K_k \sum_{j=1, j\neq i}^{n} a_{ij}\big(x_j(k) - x_i(k)\big) + \rho_i P_\Omega\{x_i(k)\}\right\}.$$

The projection error is shown as

$$e_{1i}(k) = P_{\Omega_i}\{u_i^F(k)\} - u_i^F(k).$$

Define $e_i(k) = e_{1i}(k) + \rho_i P_\Omega\{x_i(k)\}$. The closed-loop system can be described as

$$x_i(k+1) = \rho_i K_k \sum_{j=1, j\neq i}^{n} a_{ij}\big(x_j(k) - x_i(k)\big) + e_i(k). \tag{5}$$

Denote

$$x(k) = \begin{bmatrix} x_1^{\mathrm{T}}(k) & x_2^{\mathrm{T}}(k) & \ldots & x_n^{\mathrm{T}}(k)\end{bmatrix}^{\mathrm{T}}$$
$$e(k) = \begin{bmatrix} e_1^{\mathrm{T}}(k) & e_2^{\mathrm{T}}(k) & \ldots & e_n^{\mathrm{T}}(k)\end{bmatrix}^{\mathrm{T}}$$
$$\Theta = \mathrm{diag}\{\rho_1, \quad \rho_2, \quad \ldots, \quad \rho_n\}$$
$$L = \begin{bmatrix} l_{11} & l_{12} & \cdots & l_{1n} \\ l_{21} & l_{22} & \cdots & l_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & \cdots & l_{nn} \end{bmatrix}.$$

The augmented closed-loop system can be rewritten as

$$x(k+1) = [-(\Theta L) \otimes K_k]x(k) + e(k). \tag{6}$$

Another variable $\hat{x}(k)$ is chosen as $\hat{x}(k) = (H \otimes I_{n_x})x(k)$, where $H = I_n - (1/n)\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}$. It follows that $\hat{x}(k) = 0$ if and only if $x_1(k) = x_2(k) = \cdots = x_n(k)$. We have

$$\hat{x}(k+1) = [-(H\Theta L) \otimes K_k]\hat{x}(k) + \big(H \otimes I_{n_x}\big)e(k). \tag{7}$$

Based on the H_∞ control approach, a new variable $z_i(k) = x_i(k) - (1/n)\sum_{j=1}^{n} x_j(k)$ is defined as the controlled output, where $0 \leq k \leq N - 1$. Here, N is a positive constant number and hence $N - 1$ denotes the end of time. We can conclude that $z(k) = [z_1^{\mathrm{T}}(k) \quad z_2^{\mathrm{T}}(k) \quad \ldots \quad z_n^{\mathrm{T}}(k)]^{\mathrm{T}} = \hat{x}(k)$. The control performance requirement is constructed with the error attenuation lever $\gamma > 0$, as

$$J = \left\{\sum_{k=0}^{N-1}\Big(z^{\mathrm{T}}(k)z(k) - \gamma^2 e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}} W_1 \tilde{H} e(k)\Big)\right\} - \gamma^2 \sum_{k=0}^{N-1}\big\{\hat{x}^{\mathrm{T}}(0)W\hat{x}(0)\big\} < 0 \ \forall \hat{x}(0) \neq 0. \tag{8}$$

where $\tilde{H} = H \otimes I_{n_x}$, and W and W_1 are given positive-definite matrices with appropriate dimensions.

Remark 1: In the classical H_∞ control theory, due to the requirement on the zero initial condition, $x_1(0) = x_2(0) = \cdots = x_n(0) \in \Omega$ in (6) and hence $\hat{x}(0) = 0$ in (7) must be satisfied in advance. In practice, such a requirement is conservative. The proposed requirement (8) means that the projection error $\tilde{H}e(k)$ and arbitrary initial state $\hat{x}(0) \neq 0$ are attenuated with the level γ, and hence the requirement on the zero initial condition is avoided.

In the next stage, we shall tackle the finite-horizon H_∞ fault-tolerant constrained consensus problem. We have the following theorems.

Theorem 1: Suppose that the directed graph is fixed. Let the attenuation level $\gamma > 0$. Consider the multiagent systems (2) with fault-tolerant controller (4). Given appropriate controller gain matrices $\{K_k\}_{k\in[0,N-1]}$, failure coefficient ρ_{im}, and positive-definite matrices $W > 0$ and $W_1 > 0$, the overall systems can achieve fault-tolerant constrained consensus with an H_∞ performance γ, if there exists a positive-definite matrix $\{Q_k\}_{k\in[0,N]} > 0$ satisfying

$$\Xi_k = \begin{bmatrix} \Gamma_k^{\mathrm{T}} Q_{k+1}\Gamma_k - Q_k + I & \Gamma_k^{\mathrm{T}} Q_{k+1} \\ * & -\gamma^2 W_1 + Q_{k+1} \end{bmatrix} < 0 \tag{9}$$

with the initial condition

$$0 < Q_0 < \gamma^2 W \tag{10}$$

where

$$\Gamma_k = -(H\Theta L)\otimes K_k.$$

Proof: Choose the Lyapunov function candidate as

$$V(k) = \hat{x}^{\mathrm{T}}(k) Q_k \hat{x}(k). \tag{11}$$

Calculate the difference of $V(k)$ along the solution of system (7). We have

$$\Delta V(k) = \hat{x}^{\mathrm{T}}(k+1)Q_{k+1}\hat{x}(k+1) - \hat{x}^{\mathrm{T}}(k)Q_k\hat{x}(k). \tag{12}$$

Equation (12) can be rewritten as

$$\begin{aligned}\Delta V(k) = {} & \big[\Gamma_k\hat{x}(k) + \tilde{H}e(k)\big]^{\mathrm{T}} Q_{k+1}\big[\Gamma_k\hat{x}(k) + \tilde{H}e(k)\big] \\ & - \hat{x}^{\mathrm{T}}(k)Q_k\hat{x}(k).\end{aligned} \tag{13}$$

Define $\tilde{J} = \Delta V(k) + (z^{\mathrm{T}}(k)z(k) - \gamma^2 e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}}W_1\tilde{H}e(k))$ and denote $\zeta(k) = [\hat{x}^{\mathrm{T}}(k) \quad e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}}]^{\mathrm{T}}$. $\tilde{J}$ can be described as

$$\tilde{J} = \zeta^{\mathrm{T}}(k)\Xi_k\zeta(k). \tag{14}$$

It is easy to conclude that $\tilde{J} < 0$ when $\Xi_k < 0$. Summing up $\tilde{J}$ from 0 to $N-1$ with respect to k, we have

$$\begin{aligned}\sum_{k=0}^{N-1}\tilde{J} = {} & \sum_{k=0}^{N-1}\Big(z^{\mathrm{T}}(k)z(k) - \gamma^2 e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}}W_1\tilde{H}e(k)\Big) \\ & + \sum_{k=0}^{N-1}[V(k+1) - V(k)] < 0.\end{aligned}$$

We can conclude that

$$\begin{aligned}& V(N) - V(0) + \sum_{k=0}^{N-1}\Big(z^{\mathrm{T}}(k)z(k) - \gamma^2 e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}}W_1\tilde{H}e(k)\Big) < 0 \\ & \Rightarrow V(N) + \sum_{k=0}^{N-1}\Big(z^{\mathrm{T}}(k)z(k) - \gamma^2 e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}}W_1\tilde{H}e(k)\Big) \\ & \quad - \gamma^2\sum_{k=0}^{N-1}\big\{\hat{x}^{\mathrm{T}}(0)W\hat{x}(0)\big\} \\ & \quad + \left(\gamma^2\sum_{k=0}^{N-1}\big\{\hat{x}^{\mathrm{T}}(0)W\hat{x}(0)\big\} - \hat{x}^{\mathrm{T}}(0)Q_0\hat{x}(0)\right) < 0.\end{aligned}$$

Due to $V(N) > 0$, $\gamma^2 W - Q_0 > 0$, and $W > 0$, the following result is obtained:

$$\begin{aligned}& \sum_{k=0}^{N-1}\Big(z^{\mathrm{T}}(k)z(k) - \gamma^2 e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}}W_1\tilde{H}e(k)\Big) \\ & \quad - \gamma^2\sum_{k=0}^{N-1}\big\{\hat{x}^{\mathrm{T}}(0)W\hat{x}(0)\big\} < 0\end{aligned}$$

which is equivalent to (8). Hence, the performance is achieved.

Next, we will discuss the fault-tolerant constrained consensus problem. System (7) is stable with a prespecified performance index γ, which implies that $\lim_{k\to\infty}\hat{x}(k) = \lim_{k\to\infty}\hat{x}(k+1) = 0$. We can obtain that

$$\begin{aligned}& \lim_{k\to\infty} x_1(k) = \lim_{k\to\infty} x_2(k) = \cdots = \lim_{k\to\infty} x_n(k) \in \Omega \\ & \lim_{k\to\infty} x_1(k+1) = \lim_{k\to\infty} x_2(k+1) \\ & \qquad = \cdots = \lim_{k\to\infty} x_n(k+1) \in \Omega.\end{aligned}$$

Define $\phi_i(k) = \rho_i K_k \sum_{j=1, j\neq i}^{n} a_{ij}(x_j(k) - x_i(k))$. By using Lemma 4, we have the following results:

1) $\rho_{im} = 1$, $m = 1, 2, \ldots, n_u$

$$\begin{aligned}& \lim_{\chi\to\infty}\Big\|x_i\Big(\hat{k}+\chi\Big) - x_i\Big(\hat{k}+\chi-1\Big)\Big\| \\ & = \lim_{\chi\to\infty}\Big\|P_{\Omega_i}\Big\{\phi_i\Big(\hat{k}+\chi-1\Big) + P_{\Omega}\Big\{x_i\Big(\hat{k}+\chi-1\Big)\Big\}\Big\} \\ & \qquad - P_{\Omega_i}\Big\{x_i\Big(\hat{k}+\chi-1\Big)\Big\}\Big\| \\ & \le \lim_{\chi\to\infty}\Big\|\phi_i\Big(\hat{k}+\chi-1\Big) \\ & \qquad + P_{\Omega}\Big\{x_i\Big(\hat{k}+\chi-1\Big)\Big\} - x_i\Big(\hat{k}+\chi-1\Big)\Big\| \\ & = \lim_{\chi\to\infty}\Big\|\phi_i\Big(\hat{k}+\chi-1\Big)\Big\|.\end{aligned}$$

2) $0 < \rho_{im} < 1$.

For all $u_i(k) \neq 0$, we can conclude that $\|\rho_i u_i(k)\| < \|u_i(k)\|$. There exist scalars $0 < \varepsilon_k < 1$ such that $\|\rho_i u_i(k)\| \le \varepsilon_k\|u_i(k)\|$ holds. We have

$$\begin{aligned}& \lim_{\chi\to\infty}\Big\|x_i\Big(\hat{k}+\chi\Big) - x_i\Big(\hat{k}+\chi-1\Big)\Big\| \\ & = \lim_{\chi\to\infty}\Big\|P_{\Omega_i}\Big\{\rho_i u_i\Big(\hat{k}+\chi-1\Big)\Big\} - P_{\Omega_i}\Big\{\rho_i u_i\Big(\hat{k}+\chi-2\Big)\Big\}\Big\| \\ & \le \lim_{\chi\to\infty}\Big\|\rho_i u_i\Big(\hat{k}+\chi-1\Big) - \rho_i u_i\Big(\hat{k}+\chi-2\Big)\Big\| \\ & \le \lim_{\chi\to\infty}\Big(\Big\|\phi_i\Big(\hat{k}+\chi-1\Big) - \phi_i\Big(\hat{k}+\chi-2\Big)\Big\| \\ & \qquad + \varepsilon_{\hat{k}+\chi-2}\Big\|P_{\Omega}\Big\{x_i\Big(\hat{k}+\chi-1\Big)\Big\} \\ & \qquad\qquad - P_{\Omega}\Big\{x_i\Big(\hat{k}+\chi-2\Big)\Big\}\Big\|\Big) \\ & \le \lim_{\chi\to\infty}\Big(\Big\|\phi_i\Big(\hat{k}+\chi-1\Big) - \phi_i\Big(\hat{k}+\chi-2\Big)\Big\| \\ & \qquad + \varepsilon_{\hat{k}+\chi-2}\Big\|x_i\Big(\hat{k}+\chi-1\Big) - x_i\Big(\hat{k}+\chi-2\Big)\Big\|\Big) \\ & = \lim_{\chi\to\infty}\Big(\Big\|\phi_i\Big(\hat{k}+\chi-1\Big) - \phi_i\Big(\hat{k}+\chi-2\Big)\Big\| \\ & \qquad + \varepsilon_{\hat{k}+\chi-2}\Big\|P_{\Omega_i}\Big\{\rho_i u_i\Big(\hat{k}+\chi-2\Big)\Big\} \\ & \qquad\qquad - P_{\Omega_i}\Big\{\rho_i u_i\Big(\hat{k}+\chi-3\Big)\Big\}\Big\|\Big)\end{aligned}$$

$$
\begin{aligned}
&\le \lim_{\chi\to\infty}\Big(\Big\|\phi_i\Big(\hat{k}+\chi-1\Big)-\phi_i\Big(\hat{k}+\chi-2\Big)\Big\| \\
&\quad + \varepsilon_{\hat{k}+\chi-2}\Big\|\rho_i u_i\Big(\hat{k}+\chi-2\Big) \\
&\quad - \rho_i u_i\Big(\hat{k}+\chi-3\Big)\Big\|\Big) \\
&\cdots \\
&\le \lim_{\chi\to\infty}\left(\sum_{l=0}^{\chi-2}\Big\|\phi_i\Big(\hat{k}+\chi-1-l\Big)-\phi_i\Big(\hat{k}+\chi-2-l\Big)\Big\|\right) \\
&\quad + \lim_{\chi\to\infty}\varepsilon^{\chi-2}\Big\|x_i\Big(\hat{k}+1\Big)-x_i\Big(\hat{k}\Big)\Big\|
\end{aligned}
$$

where $\varepsilon = \max\{\varepsilon_{\hat{k}+\chi-2-l}\}, l = 0, 1, \ldots, \chi-3$. Notice that $x_i(\hat{k}) = x_j(\hat{k})\ \forall i, j = 1, 2\ldots, n, i \neq j$ when $\hat{k} \to \infty$, which implies that $\lim_{\hat{k}\to\infty}\phi_i(\hat{k}) = 0$ and

$$\lim_{\hat{k}\to\infty}\left\{\lim_{\chi\to\infty}x_i\Big(\hat{k}+\chi\Big)\right\} = \lim_{\hat{k}\to\infty}\left\{\lim_{\chi\to\infty}x_i\Big(\hat{k}+\chi-1\Big)\right\}.$$

Defining $k = \hat{k}+\chi-1$, we have

$$\lim_{k\to\infty}x_i(k) = \lim_{k\to\infty}x_i(k+1).$$

By using the same method, one has

$$
\begin{aligned}
\lim_{k\to\infty}x_i(k) &= \lim_{k\to\infty}x_i(k+1) \\
&= \lim_{k\to\infty}x_i(k+2) = \cdots = \tilde{z}_i \in \Omega
\end{aligned}
$$

and

$$
\begin{aligned}
\lim_{k\to\infty}x_m(k) &= \lim_{k\to\infty}x_m(k+1) = \lim_{k\to\infty}x_m(k+2) = \cdots \\
&= \tilde{z}_m \in \Omega \quad \forall m \neq i.
\end{aligned}
$$

When $k \to \infty$, one has $x_i(k) = x_m(k)\ \forall m \neq i$. It is not difficult to conclude that $\tilde{z}_i = \tilde{z}_m = \tilde{z} \in \Omega$. The corresponding result $\lim_{k\to\infty}\|x_i(k)-\tilde{z}\| = 0$ can be obtained, which means that the constrained consensus is achieved. The proof is completed. ■

Next, we will design the fault-tolerant controller gain K_k for system (7). Assume that the failure matrix Θ is already known. The theorem is provided as follows.

Theorem 2: Suppose that the directed graph is fixed. Let the attenuation level $\gamma > 0$. Consider the multiagent system (2) with fault-tolerant controller (4). Given appropriate failure coefficient ρ_{im} and positive-definite matrices $W > 0$ and $W_1 > 0$, the overall systems can achieve fault-tolerant consensus with an H_∞ performance γ, if there exist positive-definite matrices $0 < \{X_k\}_{k\in[0,N-1]} < I$ and $\{Q_k\}_{k\in[0,N]} > 0$ and real-valued matrices $\{K_k\}_{k\in[0,N-1]}$ satisfying

$$\begin{bmatrix} -Q_k+I & 0 & \Lambda_k \\ * & -\gamma^2 W_1 & I \\ * & * & -X_k \end{bmatrix} < 0 \tag{15}$$

with the initial condition

$$0 < Q_0 < \gamma^2 W \tag{16}$$

where

$$\Lambda_k = -[I_n \otimes K_k^{\mathrm{T}}]L^{\mathrm{T}}\Theta H.$$

Proof: By using Lemma 1, it follows from (9) that:

$$\begin{bmatrix} -Q_k+I & 0 & \Gamma_k^{\mathrm{T}}Q_{k+1} \\ * & -\gamma^2 W_1 & Q_{k+1} \\ * & * & -Q_{k+1} \end{bmatrix} < 0. \tag{17}$$

Premultiplying and postmultiplying (17) by $\mathrm{diag}\{I, I, X_k^{\mathrm{T}}\}$ and $\mathrm{diag}\{I, I, X_k\}$, respectively, we have

$$\begin{bmatrix} -Q_k+1 & 0 & \Gamma_k^{\mathrm{T}}Q_{k+1}X_k \\ * & -\gamma^2 W_1 & Q_{k+1}X_k \\ * & * & -X_k^{\mathrm{T}}Q_{k+1}X_k \end{bmatrix} < 0.$$

Defining $Q_{k+1}X_k = I$, the corresponding result in (15) can be obtained directly. The proof is completed. ■

In Theorems 1 and 2, we have assumed that the failure matrix Θ is already known. However, in practice, the failure matrix might not be known in advance. According to the characteristic of ρ_{im} as

$$0 < \rho_i^{\min} \le \rho_{im} \le \rho_i^{\max} \le 1$$

denote the following variables:

$$\tilde{\rho}_{i0} = \frac{\rho_i^{\max}+\rho_i^{\min}}{2}, \quad \tilde{g}_{im} = \frac{\rho_{im}-\tilde{\rho}_{i0}}{\tilde{\rho}_{i0}}, \quad \tilde{r}_i = \frac{\rho_i^{\max}-\rho_i^{\min}}{\rho_i^{\max}+\rho_i^{\min}}.$$

We have

$$\rho_{im} = \tilde{\rho}_{i0}(1+\tilde{g}_{im}), |\tilde{g}_i| \le \tilde{r}_i < 1.$$

Denote

$$\rho_{i0} = I_{n_u}\otimes\tilde{\rho}_{i0}, g_i = \mathrm{diag}\{\tilde{g}_{i1}, \ldots, \tilde{g}_{in_u}\}, r_i = I_{n_u}\otimes\tilde{r}_i.$$

We then obtain

$$\rho_i = \rho_{i0}(I+g_i), |g_i| \le r_i < I.$$

Similarly, define

$$
\begin{aligned}
\Theta_0 &= \mathrm{diag}\{\rho_{01}, \rho_{02}, \ldots, \rho_{0n}\}, \quad R = \mathrm{diag}\{r_1, r_2, \ldots, r_n\} \\
G &= \mathrm{diag}\{g_1, g_2, \ldots, g_n\}, \quad |G| = \mathrm{diag}\{|g_1|, |g_2|, \ldots, |g_n|\}.
\end{aligned}
$$

The failure matrix Θ can be rewritten as

$$\Theta = \Theta_0(I+G), \quad |G| \le R < I. \tag{18}$$

The following theorem is provided for system (7) with an unknown failure matrix Θ.

Theorem 3: Suppose that the directed graph is fixed. Let the attenuation level $\gamma > 0$. Consider the multiagent system (2) with fault-tolerant controller (4). Given positive-definite matrices $W > 0$ and $W_1 > 0$, the overall systems can achieve fault-tolerant consensus with an H_∞ performance γ, if there exist positive-definite matrices $0 < \{X_k\}_{k\in[0,N-1]} < I$ and $\{Q_k\}_{k\in[0,N]} > 0$, real-valued matrices $\{K_k\}_{k\in[0,N-1]}$, and positive scalars $\{\sigma_k\}_{k\in[0,N-1]} > 0$ satisfying

$$\begin{bmatrix} -Q_k+I & 0 & \Phi_k H & \Phi_k \\ * & -\gamma^2 W_1 & I & 0 \\ * & * & -X_k+\sigma_k HRR^{\mathrm{T}}H^{\mathrm{T}} & 0 \\ * & * & * & -\sigma_k \end{bmatrix} < 0 \tag{19}$$

with the initial condition

$$0 < Q_0 < \gamma^2 W \tag{20}$$

Algorithm 1 Delay-Free Fault-Tolerant Control Scheme

Step 1: Specify the attenuation level γ and positive definite matrices $W > 0$ and $W_1 > 0$.
Step 2: Setting $k = 0$, solve the inequalities (19)-(20) to get the values of Q_0, X_0 and K_0.
Step 3: Given $Q_1 = X_0^{-1}$, substitute Q_1 into (19) to compute K_1.
Step 4: Setting $k - 1 = k$, substitute $Q_k = X_{k-1}^{-1}$ into (19) to get the controller gain K_k.

where

$$\Phi_k = -[I_n \otimes K_k^{\mathrm{T}}]L^{\mathrm{T}}\Theta_0.$$

Proof: Substitute (18) into (15). Equation (15) can be divided into two subparts as Ξ_k and $\Delta\Xi_k$

$$\begin{aligned}\Xi_k &= \begin{bmatrix} -Q_k + I & 0 & \tilde{\Lambda}_k \\ * & -\gamma^2 W_1 & I \\ * & * & -X_k \end{bmatrix}\\ \tilde{\Lambda}_k &= -\big[I_n \otimes K_k^{\mathrm{T}}\big]L^{\mathrm{T}}\Theta_0 H\\ \Delta\Xi_k &= \begin{bmatrix} 0 & 0 & \Delta\tilde{\Lambda}_k \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix}\\ \Delta\tilde{\Lambda}_k &= -\big[I_n \otimes K_k^{\mathrm{T}}\big]L^{\mathrm{T}}\Theta_0 GH. \end{aligned} \tag{21}$$

It follows from (21) that:

$$\Delta\Xi_k = \mathrm{He}\left\{\begin{bmatrix}\Phi_k \\ 0 \\ 0\end{bmatrix} G[0 \quad 0 \quad H]\right\}.$$

Based on Lemma 2, $\Xi_k + \Delta\Xi_k < 0$ holds if and only if there exists a positive scalar σ_k such that

$$\Xi_k + \sigma_k \begin{bmatrix}0 \\ 0 \\ H\end{bmatrix} RR^{\mathrm{T}}[0 \quad 0 \quad H] + \sigma_k^{-1}\begin{bmatrix}\Phi_k \\ 0 \\ 0\end{bmatrix}\big[\Phi_k^{\mathrm{T}} \quad 0 \quad 0\big] < 0.$$

By using Lemma 1, the corresponding result in Theorem 3 can be obtained directly. The proof is completed. ■

The delay-free fault-tolerant control scheme is described in Algorithm 1.

B. Delay-Dependent Fault-Tolerant Control

In the networked multiagent systems, the time delays may exist between agents. In this section, we will give a type of delay-dependent fault-tolerant control algorithm for multiagent systems. The controller is designed as

$$u_i(k) = K_k \sum_{j=1, j\neq i}^{n} a_{ij}\big(x_j(k-\tau) - x_i(k-\tau)\big) + P_\Omega\{x_i(k)\} \tag{22}$$

where τ denotes the time delay. By using the same augmented method as delay-free system in (5) and (6), we have

$$x(k+1) = -[(\Theta L) \otimes K_k]x(k-\tau) + e(k). \tag{23}$$

The closed-loop system of state $\hat{x}(k)$ can be described as

$$\hat{x}(k+1) = -[(H\Theta L) \otimes K_k]\hat{x}(k-\tau) + \tilde{H}e(k). \tag{24}$$

We have the following theorems.

Theorem 4: Suppose that the directed graph is fixed. Let the attenuation level $\gamma > 0$. Consider the multiagent system (2) with delay-dependent fault-tolerant controller (22). Given appropriate controller gain $\{K_k\}_{k\in[0,N-1]}$, failure coefficient ρ_{im}, and positive-definite matrices $U > 0$ and $U_1 > 0$, the overall systems can achieve fault-tolerant consensus with an H_∞ performance γ, if there exist positive-definite matrices $\{Q_k\}_{k\in[0,N]} > 0$, $\{W_k\}_{k\in[0,N-1]} > 0$, $\{Z_k\}_{k\in[0,N-1]} > 0$, $\{S_{1k}\}_{k\in[0,N-1]} > 0$, $\{S_{2k}\}_{k\in[0,N-1]} > 0$, and $\{S_{3k}\}_{k\in[0,N-1]} > 0$ and matrices $\{\mathcal{N}_{1k}\}_{k\in[0,N-1]}$, $\{\mathcal{N}_{2k}\}_{k\in[0,N-1]}$, and $\{\mathcal{N}_{3k}\}_{k\in[0,N-1]}$ with appropriate dimensions such that the following inequalities hold:

$$\begin{aligned}\hat{\Xi}_{1k} &= \begin{bmatrix} \hat{\Xi}_{11k} & \hat{\Xi}_{12k} & \mathcal{N}_{3k}^{\mathrm{T}} & 0 & -\tau Z_k \\ * & \hat{\Xi}_{22k} & -\mathcal{N}_{3k}^{\mathrm{T}} & \Gamma_k^{\mathrm{T}}Q_{k+1} & \tau\Gamma_k^{\mathrm{T}}Z_k \\ * & * & \hat{\Xi}_{33k} & Q_{k+1} & \tau Z_k \\ * & * & * & -Q_{k+1} & 0 \\ * & * & * & * & -Z_k \end{bmatrix} < 0\\ \hat{\Xi}_{2k} &= \begin{bmatrix} S_k & \mathcal{N}_k \\ * & \min_{l\in[k-\tau,k-1]} Z_l \end{bmatrix} > 0 \end{aligned} \tag{25}$$

with the initial conditions

$$\begin{aligned} &Q_0 < \hat{\gamma}_1^2 U, \quad W_0 < \hat{\gamma}_2^2 U \quad \forall Z_0 > 0\\ &\hat{\gamma}_1^2 = \gamma_1^2, \quad \hat{\gamma}_2^2 = \frac{1}{\tau}\gamma_2^2, \quad \gamma_1^2 + \gamma_2^2 = \gamma^2 \end{aligned} \tag{26}$$

where

$$\begin{aligned} \hat{\Xi}_{11k} &= -Q_k + W_k + \mathcal{N}_{1k} + \mathcal{N}_{1k}^{\mathrm{T}} + S_{1k} + I\\ \hat{\Xi}_{12k} &= \mathcal{N}_{2k}^{\mathrm{T}} - \mathcal{N}_{1k}\\ \hat{\Xi}_{22k} &= -W_{k-\tau} - \mathcal{N}_{2k}^{\mathrm{T}} - \mathcal{N}_{2k} + S_{2k}\\ \hat{\Xi}_{33k} &= S_{3k} - \gamma^2 U_1\\ \mathcal{N}_k &= \big[\mathcal{N}_{1k}^{\mathrm{T}} \quad \mathcal{N}_{2k}^{\mathrm{T}} \quad \mathcal{N}_{3k}^{\mathrm{T}}\big]^{\mathrm{T}}, \quad S_k = \mathrm{diag}\{S_{1k}, S_{2k}, S_{3k}\}. \end{aligned}$$

Proof: We consider the Lyapunov function candidates as

$$\mathcal{V}_1(k) = \hat{x}^{\mathrm{T}}(k)Q_k\hat{x}(k) + \sum_{l=k-\tau}^{k-1}\hat{x}^{\mathrm{T}}(l)W_l\hat{x}(l) + \tau\sum_{\theta=-\tau+1}^{0}\sum_{l=k-1+\theta}^{k-1}\eta^{\mathrm{T}}(l)Z_l\eta(l) \tag{27}$$

where $\eta(l) = \hat{x}(l+1) - \hat{x}(l)$. Calculate the difference of $\mathcal{V}_1(k)$ along the solution of system (24). We have

$$\begin{aligned}\Delta\mathcal{V}_1(k) \le \big\{&\hat{x}^{\mathrm{T}}(k+1)Q_{k+1}\hat{x}(k+1) - \hat{x}^{\mathrm{T}}(k)Q_k\hat{x}(k)\\ &+ \tau^2\eta^{\mathrm{T}}(k)Z_k\eta(k) - \tau\sum_{l=k-\tau}^{k-1}\eta(l)Z_l\eta(l)\\ &+ \hat{x}^{\mathrm{T}}(k)W_k\hat{x}(k) - \hat{x}^{\mathrm{T}}(k-\tau)W_{k-\tau}\hat{x}(k-\tau)\big\}. \end{aligned} \tag{28}$$

By using the free-weighting matrix approach [28], the following equation holds for any matrices $\mathcal{N}_k$ with appropriate dimensions:

$$0 = 2\zeta_1^{\mathrm{T}}(k)\mathcal{N}_k\left[\hat{x}(k) - \hat{x}(k-\tau) - \sum_{l=k-\tau}^{k-1}\eta(l)\right] \tag{29}$$

where

$$\zeta_1(k) = \begin{bmatrix}\hat{x}^{\mathrm{T}}(k) & \hat{x}^{\mathrm{T}}(k-\tau) & e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}.$$

The control performance requirement J_1 is constructed as

$$J_1 = \left\{\sum_{k=0}^{N-1}\Big(z^{\mathrm{T}}(k)z(k) - \gamma^2 e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}}U_1\tilde{H}e(k)\Big)\right\} - \gamma^2\sum_{k=0}^{N-1}\left\{\hat{x}^{\mathrm{T}}(0)U\hat{x}(0)\right\} < 0 \ \forall \hat{x}(0) \neq 0. \tag{30}$$

Define $\tilde{J}_1 = \Delta\mathcal{V}_1(k) + (z^{\mathrm{T}}(k)z(k) - \gamma^2 e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}}U_1\tilde{H}e(k))$ and denote

$$\zeta_2(k) = \left[\zeta_1^{\mathrm{T}}(k) \quad \sum_{l=k-\tau}^{k-1}\eta^{\mathrm{T}}(l)\right]^{\mathrm{T}}.$$

By using Lemma 3, we have

$$\begin{aligned}
&-\tau\sum_{l=k-\tau}^{k-1}\eta^{\mathrm{T}}(l)Z_l\eta(l) - \zeta_1^{\mathrm{T}}(k)S_k\zeta_1(k)\\
&\quad - 2\zeta_1^{\mathrm{T}}(k)\mathcal{N}_k\sum_{l=k-\tau}^{k-1}\eta(l)\\
&\le -\left[\sum_{k-\tau}^{k-1}\eta^{\mathrm{T}}(l)\right]\min_{l\in[k-\tau,k-1]}Z_l\left[\sum_{k-\tau}^{k-1}\eta(l)\right]\\
&\quad - 2\zeta_1^{\mathrm{T}}(k)\mathcal{N}_k\sum_{l=k-\tau}^{k-1}\eta(l) - \zeta_1^{\mathrm{T}}(k)S_k\zeta_1(k).
\end{aligned}$$

Then, $\tilde{J}_1$ can rewritten as

$$\tilde{J}_1 = \zeta_1^{\mathrm{T}}(k)\tilde{\Xi}_{1k}\zeta_1(k) - \zeta_2^{\mathrm{T}}(k)\hat{\Xi}_{2k}\zeta_2(k) \tag{31}$$

where

$$\tilde{\Xi}_{1k} = \begin{bmatrix}\tilde{\Xi}_{11k} & \tilde{\Xi}_{12k} & \tilde{\Xi}_{13k}\\ * & \tilde{\Xi}_{22k} & \tilde{\Xi}_{23k}\\ * & * & \tilde{\Xi}_{33k}\end{bmatrix} < 0$$

$$\begin{aligned}
\tilde{\Xi}_{11k} &= -Q_k + W_k + \tau^2 Z_k + \mathcal{N}_{1k} + \mathcal{N}_{1k}^{\mathrm{T}} + S_{1k} + I\\
\tilde{\Xi}_{12k} &= -\tau^2 Z_k\Gamma_k + \mathcal{N}_{2k}^{\mathrm{T}} - \mathcal{N}_{1k}, \quad \hat{\Xi}_{13k} = -\tau^2 Z_k + \mathcal{N}_{3k}^{\mathrm{T}}\\
\tilde{\Xi}_{22k} &= \Gamma_k^{\mathrm{T}}Q_{k+1}\Gamma_k - W_{k-\tau} + \tau^2\Gamma_k^{\mathrm{T}}Z_k\Gamma_k - \mathcal{N}_{2k}^{\mathrm{T}} - \mathcal{N}_{2k} + S_{2k}\\
\tilde{\Xi}_{23k} &= \Gamma_k^{\mathrm{T}}Q_{k+1} + \tau^2\Gamma_k^{\mathrm{T}}Z_k - \mathcal{N}_{3k}^{\mathrm{T}}\\
\tilde{\Xi}_{33k} &= Q_{k+1} + \tau^2 Z_k + S_{3k} - \gamma^2 U_1.
\end{aligned}$$

By utilizing Lemma 1, $\tilde{\Xi}_{1k} < 0 \Leftrightarrow \hat{\Xi}_{1k} < 0$. Summing up $\tilde{J}_1$ from 0 to $N-1$, one has

$$\sum_{k=0}^{N-1}\tilde{J} = \sum_{k=0}^{N-1}\Big(z^{\mathrm{T}}(k)z(k) - \gamma^2 e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}}W_1\tilde{H}e(k)\Big) + \sum_{k=0}^{N-1}[\mathcal{V}(k+1) - \mathcal{V}(k)] < 0.$$

We can conclude that

$$\begin{aligned}
&\sum_{k=0}^{N-1}\Big(z^{\mathrm{T}}(k)z(k) - \gamma^2 e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}}W_1\tilde{H}e(k)\Big) + \mathcal{V}(N) - \mathcal{V}(0) < 0\\
&\Rightarrow \mathcal{V}(N) + \sum_{k=0}^{N-1}\Big(z^{\mathrm{T}}(k)z(k) - \gamma^2 e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}}W_1\tilde{H}e(k)\Big)\\
&\quad - \gamma^2\sum_{k=0}^{N-1}\left\{\hat{x}^{\mathrm{T}}(0)U\hat{x}(0)\right\}\\
&\quad + \left(\gamma^2\sum_{k=0}^{N-1}\left\{\hat{x}^{\mathrm{T}}(0)U\hat{x}(0)\right\} - \mathcal{V}(0)\right) < 0\\
&\Rightarrow \mathcal{V}(N) + \sum_{k=0}^{N-1}\Big(z^{\mathrm{T}}(k)z(k) - \gamma^2 e^{\mathrm{T}}(k)\tilde{H}^{\mathrm{T}}W_1\tilde{H}e(k)\Big)\\
&\quad - \gamma^2\sum_{k=0}^{N-1}\left\{\hat{x}^{\mathrm{T}}(0)U\hat{x}(0)\right\}\\
&\quad + \left(\gamma_1^2\sum_{k=0}^{N-1}\left\{\hat{x}^{\mathrm{T}}(0)U\hat{x}(0)\right\} - \hat{x}^{\mathrm{T}}(0)Q_0\hat{x}(0)\right)\\
&\quad + \left(\gamma_2^2\sum_{k=0}^{N-1}\left\{\hat{x}^{\mathrm{T}}(0)U\hat{x}(0)\right\} - \tau\hat{x}^{\mathrm{T}}(0)W_0\hat{x}(0)\right) < 0.
\end{aligned}$$

Based on the initial conditions (26) in Theorem 4, the required performance (30) is achieved.

Next, we will investigate the fault-tolerant constrained consensus problem. The stability of system (24) implies that

$$\begin{aligned}
&\lim_{k\to\infty}x_1(k) = \lim_{k\to\infty}x_2(k) = \cdots = \lim_{k\to\infty}x_n(k) \in \Omega\\
&\lim_{k\to\infty}x_1(k+1) = \lim_{k\to\infty}x_2(k+1)\\
&\qquad = \cdots = \lim_{k\to\infty}x_n(k+1) \in \Omega.
\end{aligned}$$

By using Lemma 4, one has

1) $\rho_{im} = 1$, $m = 1, 2, \ldots, n_u$

$$\begin{aligned}
&\lim_{\chi\to\infty}\left\|x_i\Big(\hat{k}+\chi+\tau+1\Big) - x_i\Big(\hat{k}+\chi+\tau\Big)\right\|\\
&= \lim_{\chi\to\infty}\left\|P_{\Omega_i}\left\{\phi_i\Big(\hat{k}+\chi+1\Big) + P_{\Omega}\left\{x_i\Big(\hat{k}+\chi+\tau\Big)\right\}\right\}\right.\\
&\qquad \left. - P_{\Omega_i}\left\{x_i\Big(\hat{k}+\chi+\tau\Big)\right\}\right\|\\
&\le \lim_{\chi\to\infty}\left\|\phi_i\Big(\hat{k}+\chi+1\Big) + P_{\Omega}\left\{x_i\Big(\hat{k}+\chi+\tau\Big)\right\}\right.\\
&\qquad \left. - x_i\Big(\hat{k}+\chi+\tau\Big)\right\|\\
&= \lim_{\chi\to\infty}\left\|\phi_i\Big(\hat{k}+\chi+1\Big)\right\|.
\end{aligned}$$

2) $0 < \rho_{im} < 1$.

Similar to Theorem 1, there exist scalars $0 < \varepsilon_k < 1$ to ensure that $\|\rho_i u_i(k)\| \le \varepsilon_k\|u_i(k)\| \ \forall u_i(k) \neq 0$. We have

$$\begin{aligned}
&\lim_{\chi\to\infty}\left\|x_i\Big(\hat{k}+\chi+\tau+1\Big) - x_i\Big(\hat{k}+\chi+\tau\Big)\right\|\\
&\le \lim_{\chi\to\infty}\left\|P_{\Omega_i}\left\{\phi_i\Big(\hat{k}+\chi+1\Big) + \rho_i P_{\Omega}\left\{x_i\Big(\hat{k}+\chi+\tau\Big)\right\}\right\}\right.\\
&\qquad \left. - P_{\Omega_i}\left\{\phi_i\Big(\hat{k}+\chi\Big) + \rho_i P_{\Omega}\left\{x_i\Big(\hat{k}+\chi+\tau-1\Big)\right\}\right\}\right\|
\end{aligned}$$

$$\begin{aligned}
&\le \lim_{\chi\to\infty}\Big\|\phi_i\Big(\hat{k}+\chi+1\Big)+\rho_i P_\Omega\Big\{x_i\Big(\hat{k}+\chi+\tau\Big)\Big\}\\
&\qquad -\phi_i\Big(\hat{k}+\chi\Big)-\rho_i P_\Omega\Big\{x_i\Big(\hat{k}+\chi+\tau-1\Big)\Big\}\Big\|\\
&\le \lim_{\chi\to\infty}\Big\|\phi_i\Big(\hat{k}+\chi+1\Big)-\phi_i\Big(\hat{k}+\chi\Big)\Big\|\\
&\quad+\lim_{\chi\to\infty}\varepsilon_{\hat{k}+\chi+\tau-1}\Big\|x_i\Big(\hat{k}+\chi+\tau\Big)-x_i\Big(\hat{k}+\chi+\tau-1\Big)\Big\|\\
&\cdots\\
&\le \lim_{\chi\to\infty}\sum_{l=0}^{\chi-1}\Big(\Big\|\phi_i\Big(\hat{k}+\chi+1-l\Big)-\phi_i\Big(\hat{k}+\chi-l\Big)\Big\|\Big)\\
&\quad+\lim_{\chi\to\infty}\varepsilon^{\chi}\Big\|x_i\Big(\hat{k}+\tau+1\Big)-x_i\Big(\hat{k}+\tau\Big)\Big\|
\end{aligned}$$

where $\varepsilon = \max\{\varepsilon_{\hat{k}+\chi+\tau-1-l}\}, l = 0, 1, \ldots, \chi - 1$. We can conclude that

$$\begin{aligned}
&\lim_{\hat{k}\to\infty}\Big\{\lim_{\chi\to\infty}x_i\Big(\hat{k}+\chi+\tau+1\Big)\Big\}\\
&\quad=\lim_{\hat{k}\to\infty}\Big\{\lim_{\chi\to\infty}x_i\Big(\hat{k}+\chi+\tau\Big)\Big\}.
\end{aligned}$$

Define $k = \hat{k} + \chi + \tau$, one has

$$\lim_{k\to\infty}x_i(k)=\lim_{k\to\infty}x_i(k+1).$$

By using the same method as above, the following result can be obtained:

$$\begin{aligned}
\lim_{k\to\infty}x_i(k)&=\lim_{k\to\infty}x_i(k+1)\\
&=\lim_{k\to\infty}x_i(k+2)=\cdots=\tilde{z}_i\in\Omega.
\end{aligned}$$

Similarly, we have

$$\begin{aligned}
\lim_{k\to\infty}x_m(k)&=\lim_{k\to\infty}x_m(k+1)\\
&=\lim_{k\to\infty}x_m(k+2)=\cdots=\tilde{z}_m\in\Omega\quad\forall m\neq i.
\end{aligned}$$

We can conclude that $\tilde{z}_i = \tilde{z}_m = \tilde{z} \in \Omega$. The result of $\lim_{k\to\infty}\|x_i(k) - \tilde{z}\| = 0$ can be guaranteed. The proof is completed. ∎

Next, we will design the delay-dependent fault-tolerant controller gain K_k for system (24). We assume that the partial failure matrix Θ is already known. We have the following theorem.

Theorem 5: Suppose that the directed graph is fixed. Let the attenuation level $\gamma > 0$. Consider the multiagent system (2) with delay-dependent fault-tolerant controller (22). Given appropriate failure coefficient ρ_{im} and positive-definite matrices $U > 0$ and $U_1 > 0$, the overall systems can achieve fault-tolerant consensus with an H_∞ performance γ, if there exist positive-definite matrices $\{Q_k\}_{k\in[0,N]} > 0$, $\{W_k\}_{k\in[0,N-1]} > 0$, $\{Z_k\}_{k\in[0,N-1]} > 0$, $\{\mathcal{R}_k\}_{k\in[0,N-1]>0}$, $\{\mathcal{T}_{1k}\}_{k\in[0,N-1]} > 0$, $\{\mathcal{T}_{2k}\}_{k\in[N-1]} > 0$, $0 < \{X_k\}_{k\in[0,N-1]} < I$, $\{S_{1k}\}_{k\in[0,N-1]} > 0$, $\{S_{2k}\}_{k\in[0,N-1]} > 0$, and $\{S_{3k}\}_{k\in[0,N-1]} > 0$, matrices $\{\mathcal{N}_{1k}\}_{k\in[0,N-1]}$, $\{\mathcal{N}_{2k}\}_{k\in[0,N-1]}$, and $\{\mathcal{N}_{3k}\}_{k\in[0,N-1]}$ with appropriate dimensions, and real-valued matrices $\{K_k\}_{k\in[0,N-1]}$ such that the following inequalities hold:

$$\begin{bmatrix}
\hat{\Xi}_{11k} & \hat{\Xi}_{12k} & \mathcal{N}_{3k}^{\mathrm{T}} & 0 & -\tau\\
* & \Phi_{22k} & -\mathcal{N}_{3k}^{\mathrm{T}} & \Phi_{23k} & \Phi_{24k}\\
* & * & \hat{\Xi}_{33k} & I & \tau\\
* & * & * & -X_k & 0\\
* & * & * & * & -\mathcal{R}_k
\end{bmatrix}<0 \quad (32)$$

$$\begin{bmatrix} S_k & \mathcal{N}_k\\ * & \mathcal{T}_{2k}\end{bmatrix}>0 \quad (33)$$

with the initial conditions

$$\begin{aligned}
&Q_0<\hat{\gamma}_1^2U,\quad W_0<\hat{\gamma}_2^2U\quad \forall\mathcal{R}_0>0\\
&\hat{\gamma}_1^2=\gamma_1^2,\quad \hat{\gamma}_2^2=\frac{1}{\tau}\gamma_2^2,\quad \gamma_1^2+\gamma_2^2=\gamma^2
\end{aligned}\quad (34)$$

where

$$\begin{aligned}
&\Phi_{22k}=-\mathcal{T}_{1k}-\mathcal{N}_{2k}^{\mathrm{T}}-\mathcal{N}_{2k}+S_{2k}\\
&\Phi_{23k}=-\big[I_n\otimes K_k^{\mathrm{T}}\big]L^{\mathrm{T}}\Theta H\\
&\Phi_{24k}=-\tau\big[I_n\otimes K_k^{\mathrm{T}}\big]L^{\mathrm{T}}\Theta H\\
&\mathcal{N}_k=\big[\mathcal{N}_{1k}^{\mathrm{T}}\quad \mathcal{N}_{2k}^{\mathrm{T}}\quad \mathcal{N}_{3k}^{\mathrm{T}}\big]^{\mathrm{T}},\quad S_k=\mathrm{diag}\{S_{1k},S_{2k},S_{3k}\}.
\end{aligned}$$

Proof: Premultiplying and postmultiplying $\hat{\Xi}_{1k}$ in (25) by $\mathrm{diag}\{I, I, I, X_k^{\mathrm{T}}, Z_k^{-T}\}$ and $\mathrm{diag}\{I, I, I, X_k, Z_k^{-1}\}$, respectively, we then obtain

$$\begin{bmatrix}
\hat{\Xi}_{11k} & \hat{\Xi}_{12k} & \mathcal{N}_{3k}^{\mathrm{T}} & 0 & -\tau\\
* & \hat{\Xi}_{22k} & -\mathcal{N}_{3k}^{\mathrm{T}} & \Gamma_k^{\mathrm{T}}Q_{k+1}X_k & \tau\Gamma_k^{\mathrm{T}}\\
* & * & \hat{\Xi}_{33k} & Q_{k+1}X_k & \tau\\
* & * & * & -X_k & 0\\
* & * & * & * & -Z_k^{-1}
\end{bmatrix}<0.$$

Denoting $\mathcal{R}_k = Z_k^{-1}$, $Q_{k+1}X_k = I$, $\mathcal{T}_{1k} = W_{k-\tau}$, and $\mathcal{T}_{2k} = \min_{l\in[k-\tau,k-1]} Z_l$, (32) and (33) can be obtained directly. The proof is completed. ∎

Next, consider the unknown partial failure matrix Θ, which is defined in (18). We have the following theorem.

Theorem 6: Suppose that the directed graph is fixed. Let the attenuation level $\gamma > 0$. Consider the multiagent system (2) with delay-dependent fault-tolerant controller (22). Given positive-definite matrices $U > 0$ and $U_1 > 0$, the overall system can achieve the fault-tolerant consensus with an H_∞ performance γ, if there exist positive scalars $\{\sigma_k\}_{k\in[0,N-1]} > 0$, positive-definite matrices $U > 0$ and $U_1 > 0$, the overall systems can achieve the fault-tolerant consensus with an H_∞ performance γ, if there exist positive-definite matrices $\{Q_k\}_{k\in[0,N]} > 0$, $\{W_k\}_{k\in[0,N-1]} > 0$, $\{Z_k\}_{k\in[0,N-1]} > 0$, $\{\mathcal{R}_k\}_{k\in[0,N-1]>0}$, $\{\mathcal{T}_{1k}\}_{k\in[0,N-1]} > 0$, $\{\mathcal{T}_{2k}\}_{k\in[N-1]} > 0$, $0 < \{X_k\}_{k\in[0,N-1]} < I$, $\{S_{1k}\}_{k\in[0,N-1]} > 0$, $\{S_{2k}\}_{k\in[0,N-1]} > 0$, and $\{S_{3k}\}_{k\in[0,N-1]} > 0$, matrices $\{\mathcal{N}_{1k}\}_{k\in[0,N-1]}$, $\{\mathcal{N}_{2k}\}_{k\in[0,N-1]}$, and $\{\mathcal{N}_{3k}\}_{k\in[0,N-1]}$ with appropriate dimensions, and real-valued matrices $\{K_k\}_{k\in[0,N-1]}$ such that the following inequalities hold:

$$\begin{bmatrix}
\hat{\Xi}_{11k} & \hat{\Xi}_{12k} & \mathcal{N}_{3k}^{\mathrm{T}} & 0 & -\tau & 0\\
* & \Phi_{22k} & -\mathcal{N}_{3k}^{\mathrm{T}} & \Psi_{1k} & \Psi_{2k} & \Psi_{3k}\\
* & * & \hat{\Xi}_{33k} & I & \tau & 0\\
* & * & * & \Psi_{4k} & \Psi_{5k} & 0\\
* & * & * & * & \Psi_{6k} & 0\\
* & * & * & * & * & -\sigma_k
\end{bmatrix}<0 \quad (35)$$

Algorithm 2 Delay-Dependent Fault-Tolerant Control Scheme

Step 1: Specify the attenuation level γ and the positive definite matrices $U > 0$, $U_1 > 0$ and $\mathcal{R}_0 = Z_0^{-1} > 0$. Choose positive scalars γ_1 and γ_2 satisfying $\gamma_1^2 + \gamma_2^2 = \gamma^2$.
Step 2: Setting $k = 0$, solve (35-37) to obtain the values of Q_0, W_0, X_0, $\mathcal{N}_0$, S_0 and K_0.
Step 3: Given $Q_{k+1} = X_k^{-1}$, $\mathcal{T}_{1k} = W_0$ and $\mathcal{T}_{2k} = Z_0$, substitute these values into (35)-(36) to obtain K_k, W_k, and $\mathcal{R}_k$. Use $Z_k = \mathcal{R}_k^{-1}$ to obtain the value of Z_k. If $k \le 2$, back to Step 3; otherwise, go to Step 4.
Step 4: Given $Q_{k+1} = X_k^{-1}$, $\mathcal{T}_{1k} = W_0$ and $\mathcal{T}_{2k} = \min_{l\in[k-\tau,k-1]} Z_l$, solve (35)-(36) to obtain K_k. If $k \le \tau$, back to Step 4; otherwise, go to Step 5.
Step 5: Given $Q_{k+1} = X_k^{-1}$, $\mathcal{T}_{1k} = W_{k-\tau}$, and $\mathcal{T}_{2k} = \min_{l\in[k-\tau,k-1]} Z_l$, solve (35)-(36) to get the controller gain K_k.

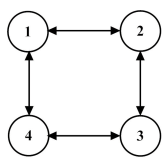

Fig. 1. Communication graph of the multiagent system.

$$\begin{bmatrix} S_k & \mathcal{N}_k \\ * & \mathcal{T}_{2k} \end{bmatrix} > 0 \tag{36}$$

with the initial conditions

$$Q_0 < \hat{\gamma}_1^2 U, \quad W_0 < \hat{\gamma}_2^2 U \quad \forall \mathcal{R}_0 > 0$$
$$\hat{\gamma}_1^2 = \gamma_1^2, \quad \hat{\gamma}_2^2 = \frac{1}{\tau}\gamma_2^2, \quad \gamma_1^2 + \gamma_2^2 = \gamma^2 \tag{37}$$

where

$$\Psi_{1k} = -[I_n \otimes K_k^{\mathrm{T}}]L^{\mathrm{T}}\Theta_0 H$$
$$\Psi_{2k} = -\tau[I_n \otimes K_k^{\mathrm{T}}]L^{\mathrm{T}}\Theta_0 H$$
$$\Psi_{3k} = -[I_n \otimes K_k^{\mathrm{T}}]L^{\mathrm{T}}\Theta_0$$
$$\Psi_{4k} = -X_k + \sigma_k HRR^{\mathrm{T}}H$$
$$\Psi_{5k} = \sigma_k \tau HRR^{\mathrm{T}}H$$
$$\Psi_{6k} = -\mathcal{R}_k + \sigma_k \tau^2 HRR^{\mathrm{T}}H$$
$$\mathcal{N}_k = [\mathcal{N}_{1k}^{\mathrm{T}} \quad \mathcal{N}_{2k}^{\mathrm{T}} \quad \mathcal{N}_{3k}^{\mathrm{T}}]^{\mathrm{T}}, \qquad S_k = \mathrm{diag}\{S_{1k}, S_{2k}, S_{3k}\}.$$

Proof: Similar to the prove procedure of Theorem 3, substitute (18) into (32). Then, (32) can be divided into the constant part and the uncertain part. By using Lemma 1, the corresponding results in Theorem 6 can be obtained directly. The proof is completed. ■

Next, the delay-dependent fault-tolerant control algorithm is described in Algorithm 2.

IV. Numerical Example

In this section, a numerical example is provided to show the effectiveness of the proposed methods. Consider a multiagent system with four agents, and the communication graph is shown in Fig. 1.

TABLE I
CONTROLLER GAINS K_k

k	K_k
0	$\begin{bmatrix} 0.0756 & 0.0361 \\ 0.0349 & 0.0897 \end{bmatrix}$
1	$\begin{bmatrix} 0.0527 & 0.0360 \\ 0.0352 & 0.0399 \end{bmatrix}$
2	$\begin{bmatrix} 0.0551 & 0.0399 \\ 0.0399 & 0.0707 \end{bmatrix}$
3	$\begin{bmatrix} 0.0280 & 0.0197 \\ 0.0192 & 0.0349 \end{bmatrix}$
4	$\begin{bmatrix} 0.0498 & 0.0345 \\ 0.0337 & 0.0622 \end{bmatrix}$
⋮	⋮

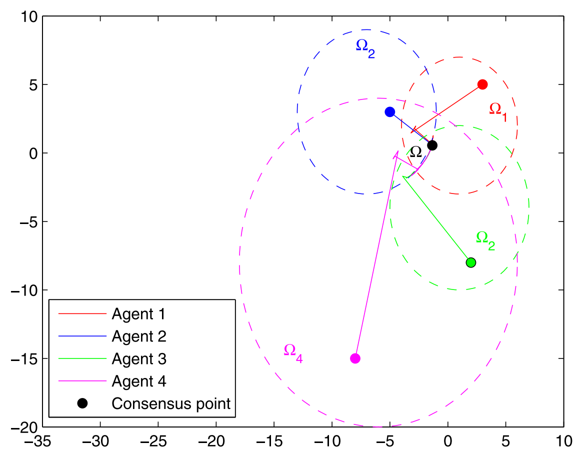

Fig. 2. State trajectories of all agents.

Each edge weight is chosen as 1, and the Laplacian matrix L in (1) is described as

$$L = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}.$$

The constraint set of each agent is denoted as

$$\Omega_1 = \left\{(x_1, y_1) \in \mathbb{R}^2 \middle| \|(x_1, y_1)^{\mathrm{T}} - (1, 2)^{\mathrm{T}}\| \le 5\right\}$$
$$\Omega_2 = \left\{(x_2, y_2) \in \mathbb{R}^2 \middle| \|(x_2, y_2)^{\mathrm{T}} - (-7, 3)^{\mathrm{T}}\| \le 6\right\}$$
$$\Omega_3 = \left\{(x_3, y_3) \in \mathbb{R}^2 \middle| \|(x_3, y_3)^{\mathrm{T}} - (1, -4)^{\mathrm{T}}\| \le 6\right\}$$
$$\Omega_4 = \left\{(x_4, y_4) \in \mathbb{R}^2 \middle| \|(x_4, y_4)^{\mathrm{T}} - (-6, -8)^{\mathrm{T}}\| \le 12\right\}.$$

Choose the partial failures as $0.8 \le \rho_{1m} \le 1$, $0.6 \le \rho_{2m} \le 1$, $0.9 \le \rho_{3m} \le 1$, and $0.9 \le \rho_{4m} \le 1$, $m = 1, 2$, and the time delay as $\tau = 2$. Set $\gamma = 2$, and U and U_1 as unit matrices. The controller gains K_k are computed with Algorithm 2. The corresponding results are listed in Table 1. At time 0, all the actuators are running without failure. Hence, we set $\rho_{im} = 1, i = 1, 2, 3, 4, m = 1, 2$. Then, at time $k \ge 7$, actuator failures exist in agent 2 and agent 4. We then set $\rho_{1m} = 1, \rho_{3m} = 1, m = 1, 2$. $\rho_{21} = 1$, $0.6 < \rho_{22} < 1$, $0.9 < \rho_{41} < 1$, and $\rho_{42} = 1$, where the values of ρ_{22} and ρ_{41} are unknown but belonging to the known ranges. Fig. 2 shows the trajectories of each agent. Under the designed controller, each agent i stays in its own constraint set Ω_i and all the agents finally reach the consensus point in $\Omega = \cap_{i=1}^{4} \Omega_i$. The simulation result shows that the control algorithm is suitable to solve the fault-tolerant constrained consensus problem.

V. Conclusion

In this article, a fault-tolerant constrained consensus problem is investigated for multiagent systems with communication delays. A novel finite-horizon H_∞-based delay-dependent fault-tolerant controller is designed. By utilizing the augmented and free-weighting matrix technologies, sufficient conditions have been provided to ensure that the closed-loop system under consideration satisfies a prespecified performance requirement and the time-varying controller gains are computed via iterative linear matrix inequalities. Finally, a numerical example is used to illustrate the effectiveness of the scheme presented in this article. The topic on the fault-tolerant constrained consensus for multiagent system with time-varying communication delays is interesting. This will be investigated in our future work.

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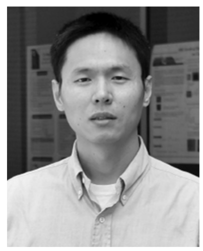

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