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Practical output synchronization for asynchronously switched multi-agent systems with adaption to fast-switching perturbations ☆

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ABSTRACT

The asynchronously switched multi-agent systems comprising switched agents of different dynamics and switching signals are considered under arbitrarily switching communication topologies. The practical output synchronization problem is studied for such a kind of systems due to the heterogeneity brought by both the dynamics and the switchings of agents. A switching-dependent controller with an embedded virtual reference system is proposed for each agent. The original problem is then converted into tracking problems between each agent and its reference system. The analysis of resultant tracking error systems involves the analysis of switched systems with bounded but non-attenuating state impulses. By satisfying sufficient conditions featuring the average dwell time (ADT) and the newly proposed piecewise ADT, the practical output synchronization can be achieved and the ultimate bound of the output errors can also be obtained for the considered systems. Furthermore, a realistic case where the agent switching signals undergo adverse fast-switching perturbations is studied. The perturbations may potentially invalidate the "slow-switching" based method. A regulation strategy is thus developed for each agent to render it adaption to such adversity. A payload transport task is taken as the practical example to illustrate the effectiveness of the proposed method and the adaption strategy.

1. Introduction

Multi-agent systems have received tremendous attention in recent years due to increasing demand in studying the group behaviors of autonomous objects, such as unmanned vehicles and robots. The consensus problems are one of the central topics for multi-agent systems, see, e.g., Knorn, Chen, and Middleton (2016), Li, Duan, Chen, and Huang (2010), Olfati-Saber and Murray (2004), Ren and Beard (2008), Ren, Beard, and Atkins (2007) and Yu, Chen, Cao, and Kurths (2010). The goal is to seek a control strategy for each agent such that all the agent states asymptotically converge to a certain value or a dynamic trajectory. However, in some real applications, the asymptotical consensus performance for a multi-agent system may not always be expected since there might exist environmental restrictions or disturbances that impede the states from converging to a single point or trajectory. In such cases, one usually expects the practical consensus performance instead, i.e., the agent states finally reach a bounded consensus region. Some recent results regarding such a topic are collected as Back and Kim (2017), Chen, Ho, Li, and Liu (2015), Ding and Zheng (2017) and Dong, Xi, Shi, and Zhong (2013). On the other hand, for state consensus problems, most multi-agent models considered are homogeneous, that is, all the agents have identical dynamics. Nevertheless, when there are demands of studying the more practical heterogeneous models which exhibit different agent dynamics, the state consensus would remain senseless. For such a case, the output synchronization (Almeida, Silvestre, & Pascoal, 2017; Chen & Chen, 2017; Wieland, Sepulchre, & Allgöwer, 2011) and the output regulation problems (Meng, Yang, Dimarogonas, & Johansson, 2015; Su & Huang, 2012) were extensively studied instead. Generally, the leader-following framework (Liu & Huang, 2018) was often applied in these problems (there may be no actual leader existing in the system, e.g., in some output synchronization

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problems Wieland et al., 2011), which usually led to the analysis of some tracking-like problems.

As another popular topic in the control community, the switched systems have seen fast development over the last few decades. Representative works, e.g., Hespanha (2004), Hespanha and Morse (1999), Kundu, Chatterjee, and Liberzon (2016), Liberzon (2012), Liberzon and Morse (1999), Lin and Antsaklis (2009) and Zhang, Zhuang, and Shi (2015) have provided some useful approaches and tools for analyzing various kinds of switched systems. Switched systems have also long been applied to the problems of the multi-agent systems. One of the well-known applications is in modeling the multi-agent systems with switching communication topologies. Fundamental results, such as Olfati-Saber and Murray (2004) and Ren and Beard (2005), have paved the way for numerous subsequent works on such a topic, e.g., Dong and Hu (2016), Liu and Huang (2018), Meng et al. (2015) and Saboori and Khorasani (2014).

On the other hand, the switching behavior can also occur in agent dynamics. This is motivated by some practical scenarios in which an agent is required to work in different modes, such as the payload transport tasks (Foehn, Falanga, Kuppuswamy, Tedrake, & Scaramuzza, 2017; Lee, Sreenath, & Kumar, 2013). For instance, in Lee et al. (2013) the cooperating load tracking control was studied for a group of quadrotors performing a series of pick-up and drop-off maneuvers on the cable-suspended payloads. This process features variations in the agent dynamics (mass, geometry, etc.) and is thus described by the hybrid (switched) systems. Recently, some attention has been paid to such a kind of multi-agent systems, see, e.g., Jia and Zhao (2015), Lin and Zheng (2017), Yoo (2018) and Zhang, Ho, Tang, and Liu (2019). A finite-time consensus problem has been addressed in Lin and Zheng (2017) as the authors considered a two-mode switched multi-agent system comprising both continuous-time and discrete-time single integrators. The authors in Zhang et al. (2019) addressed the quasi-consensus problem for a class of synchronously switched heterogeneous multi-agent systems. Note that all the agents considered in these works share a common switching signal. However, it is usually unpractical since it is hard to make all the agents switch simultaneously in existence of some factors like delays, faults and etc. This has been referred to as the asynchronous switching problems for the classic switched systems, e.g., Zhang and Gao (2010), though, to date only a few number of works have considered the multi-agent systems with such asynchronous switchings. In Jia and Zhao (2015), the output regulation problems were considered for a class of asynchronously switched linear heterogeneous multi-agent models and the authors used the agent-dependent average dwell time (ADT) approach to deal with the case where the stabilizable and unstabilizable modes co-exist. However, the method only applies when the solutions to the pivotal output regulation equations are independent of switchings. The obtained asymptotical convergence result implicitly restricts the output matrices to be non-switching and is thus rather ideal. A kind of asynchronously switched multi-agent systems with special nonlinear agent dynamics was considered in Yoo (2018) and the function approximation technique was utilized to address the practical tracking consensus problem. Despite that the work has presented a decent method for nonlinear switched multi-agent systems, the result for the arbitrarily switching signals still entails the search of a common Lyapunov function, which would be difficult to find for the general linear or nonlinear switched model.

Furthermore, as the application scenarios become much more complicated, there can be a considerable number of cooperation tasks executed in adverse or hostile environments. These elements, such as device malfunctions (Yang, Jiang, & Cocquempot, 2014) or deliberate attacks (Wu, Wang, Xiao, & Guan, 2010), would constantly affect the agent dynamics and force it to vary at unexpected time. For the agent system with switched dynamics, such unexpected changes would lead to an uncontrollable increase of the switching frequency as we call these changes a kind of "fast-switching" perturbations. In such a case, the coordination approaches that are based on the classic "slow-switching" principle (Liberzon, 2012), such as the ADT-based methods used in Jia and Zhao (2015) and Zhang et al. (2019) could not be effective anymore when the switching frequency increases to a certain value. It is notable that although the "fast-switching" perturbations can be deemed the arbitrary switchings such that one can handle them with some classic tools like the common Lyapunov function, it turns out to be rather conservative and costly for the controller design since it is always required to cover all the possible switching patterns once applied. In a practical sense, to allow for all the switching situations remains unnecessary or even not affordable. Therefore, how to handle the "fast-switching" perturbations in a way that the system can dynamically react to such changes is worth investigating. All these points have motivated this work.

In this work, we consider the practical output synchronization problem of asynchronously switched multi-agent systems with switching topologies and take into account the case where the switched agents suffer the "fast-switching" perturbations. We summarize the contributions of this work as follows:

(1) The asynchronously switched multi-agent system under switching topologies is considered. The system is of high heterogeneity as it consists of switched agents of different dynamics and switching signals. The practical output synchronization problem is thus addressed by employing a switching-dependent controller which converts the original problem into tracking problems between each agent and its virtual reference.
(2) The corresponding tracking error system is analyzed by respectively studying its zero-input response and zero-state response. The zero-input response system can be modeled as an impulsive switched linear system. To handle the non-attenuating impulsive effects, we propose the piecewise ADT which generalizes the classic ADT method to guarantee the desired performance of such a system.
(3) We take into account a practical scenario where there are fast-switching perturbations against the switched agents that would undermine the slow-switching patterns of agents. Since it is unable to ensure the desired performance by constraining the switching frequency of the agent in this case, we develop a real-time strategy that dynamically regulates the dwell-time lower bound for switched agents such that they can adapt to such fast-switching perturbations.

The rest of this work are organized as follows: the graph theory and the system formulation with some preliminaries are given in Section 2; the main results are collected as two parts in Section 3; practical examples and simulation results are presented in Section 4 to show the effectiveness of the proposed methods; the conclusion of this work is given in Section 5.

Notations: The notations are summarized as follows: $\mathbb{R}_{\geq 0}$ denotes the non-negative real number set, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ represent the nth dimensional Euclidean vector space and $m \times n$ matrix space, respectively; $\mathbb{C}_-$ denotes the open left-half complex plane and $\mathbb{C}_+$ denotes the open right-half complex plane; $\overline{\mathbb{C}}$ denotes the complement of set $\mathbb{C}$; the identity matrix without explicitly specifying its dimension is denoted by I and the nth dimensional identity matrix is denoted by I_n; for any square matrix $R \in \mathbb{R}^{n\times n}$, $\lambda(R)$ denotes the spectrum of R; $\lambda_j(R)$ denotes the jth eigenvalue of

R, Re(...) denotes the real part of a complex number; $P > 0$ means that P is real symmetric and positive definite; $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote the minimum and maximum eigenvalues of a symmetric matrix P, respectively; diag(...) stands for a block-diagonal matrix; $\|\ldots\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix; min(...) and max(...) are the minimization and the maximization operators, respectively.

2. Problem statement

In this section we will give some preliminaries for the considered problem and present the mathematical model for the asynchronously switched multi-agent systems with switching topologies.

2.1. Communication topology

Given a piecewise continuous switching signal $\sigma(t), \sigma : \mathbb{R}_{\geq 0} \to \mathcal{P} = \{1, 2, \ldots, s\}$ where s is the number of switching modes of $\sigma(t)$, the switching communication topology of an N-agent system can be represented by a switching directed graph $\mathcal{G}_{\sigma(t)} = \{\mathcal{V}, \mathcal{E}_{\sigma(t)}\}$ in which each switching mode indicates a possible switching topology. $\mathcal{V} = \{1, 2, \ldots, N\}$ denotes the vertex set of the graph $\mathcal{G}_{\sigma(t)}$ and $\mathcal{E}_{\sigma(t)} \subseteq \mathcal{V}\times\mathcal{V}$ is the edge set. It is assumed that the graph contains no self loops, i.e., $(i, i) \notin \mathcal{E}_{\sigma(t)}$. The joint graph of $\mathcal{G}_{\sigma(t)}$ over the certain time interval $[t_1^*, t_2^*)$, $0 \leq t_1^* < t_2^* \leq +\infty$ is defined by $\bigcup_{t\in[t_1^*,t_2^*)} \mathcal{G}_{\sigma(t)} = \{\mathcal{V}, \bigcup_{t\in[t_1^*,t_2^*)} \mathcal{E}_{\sigma(t)}\}$. Let $\mathcal{A}_{\sigma(t)} = [a_{ij}(\sigma(t))] \in \mathbb{R}^{N\times N}$ denote the switching adjacency matrix of $\mathcal{G}_{\sigma(t)}$ in which $a_{ij}(\sigma(t)) = 0$ if $(j, i) \notin \mathcal{E}_{\sigma(t)}$, i.e., there is not a directed edge from j to i, otherwise $a_{ij}(\sigma(t)) = 1$. A Laplacian matrix of $\mathcal{G}_{\sigma(t)}$ is denoted by $L_{\sigma(t)} = [l_{ij}(\sigma(t))] = \mathcal{D}_{\sigma(t)} - \mathcal{A}_{\sigma(t)}$, where $\mathcal{D}_{\sigma(t)} = [\tilde{d}_{ij}(\sigma(t))]$ is the corresponding switching in-degree matrix with $\tilde{d}_{ij}(\sigma(t)) = 0,\ \forall i \neq j$ and $\tilde{d}_{ii}(\sigma(t)) = \sum_{j=1}^{N} a_{ij}(\sigma(t))$.

For the switching graphs $\mathcal{G}_{\sigma(t)}$, the following assumption on switching topologies is commonly adopted in the related literature (see, e.g., Back & Kim, 2017; Liu & Huang, 2018; Ren & Beard, 2005; Scardovi & Sepulchre, 2009):

Assumption 1. It is assumed that there exists $T^* \geq 0$, such that for any $t \geq 0$ the joint graph of the switching directed graphs $\mathcal{G}_{\sigma(t)}$ contains a directed spanning tree over the time interval $[t, t+T^*]$.

2.2. System formulation and preliminaries

The asynchronously switched multi-agent system with heterogeneous linear dynamics is presented as follows:

$$\begin{aligned}\dot{x}_i(t) &= A_{i,\sigma_i(t)}x_i(t) + B_{i,\sigma_i(t)}u_i(t),\\ y_i(t) &= C_{i,\sigma_i(t)}x_i(t),\end{aligned} \tag{1}$$

where for each agent $i \in \mathcal{V}$, $x_i(t) \in \mathbb{R}^{n_i}$ is the agent state; $u_i(t) \in \mathbb{R}^{l_i}$ is the control input and $y_i(t) \in \mathbb{R}^q$ is the output of interest; $A_{i,\sigma_i(t)} \in \mathbb{R}^{n_i\times n_i}$, $B_{i,\sigma_i(t)} \in \mathbb{R}^{n_i\times l_i}$ and $C_{i,\sigma_i(t)} \in \mathbb{R}^{q\times n_i}$; the matrix pairs $(A_{i,\phi_i}, B_{i,\phi_i})$ and $(A_{i,\phi_i}, C_{i,\phi_i})$ for any $i \in \mathcal{V}$, $\phi_i \in \mathcal{P}_i$ are assumed to be stabilizable and detectable, respectively; $\sigma_i : \mathbb{R}_{\geq 0} \to \mathcal{P}_i$; $\mathcal{P}_i = \{1, 2, \ldots, s_i\}$ is the switching signal which is assumed to be right-continuous at each switching instant. Note that by calling them "asynchronously switched", we mean that all the agents do not share a common switching signal. Then for (1) we employ the following distributed observer-based switching controller for each agent

$$u_i(t) = K_{i,\sigma_i(t)}(\hat{x}_i(t) - \Omega_{i,\sigma_i(t)}\xi_i(t)) + \Theta_{i,\sigma_i(t)}\xi_i(t), \tag{2}$$

where $K_{i,\phi_i} \in \mathbb{R}^{l_i\times n_i}$, $\Omega_{i,\sigma_i(t)} \in \mathbb{R}^{n_i\times p}$ and $\Theta_{i,\sigma_i(t)} \in \mathbb{R}^{l_i\times p}$ are constant matrices to be calculated. $\hat{x}_i(t) \in \mathbb{R}^{n_i}$ is the observer state satisfying

$$\begin{aligned}\dot{\hat{x}}_i(t) &= A_{i,\sigma_i(t)}\hat{x}_i(t) + B_{i,\sigma_i(t)}u_i(t) + G_{i,\sigma_i(t)}(\hat{y}_i(t) - y_i(t)),\\ \hat{y}_i(t) &= C_{i,\sigma_i(t)}\hat{x}_i(t),\end{aligned} \tag{3}$$

where $\hat{y}_i(t) \in \mathbb{R}^q$, $G_{i,\sigma_i(t)} \in \mathbb{R}^{n_i\times q}$ is the gain matrix to be determined. Note that to handle the heterogeneity in the agent dynamics (1), in the above we have introduced to each agent i a virtual exogenous reference: $\xi_i(t) \in \mathbb{R}^p$, whose dynamics is subject to:

$$\dot{\xi}_i(t) = S\xi_i(t) - \sum_{j=1}^{N} a_{ij}(\sigma(t))(\xi_i(t) - \xi_j(t)),\ i \in \mathcal{V}, \tag{4}$$

in which $S \in \mathbb{R}^{p\times p}$ is a matrix that can be determined according to certain performance objectives, $\lambda_j(S) \in \overline{\mathbb{C}}_+$ with at least one eigenvalue on the imaginary axis and all such eigenvalues having the identical algebraic and geometric multiplicity. Assume there exists a matrix $H \in \mathbb{R}^{q\times p}$ such that the matrix pair (S, H) is observable.

For the reference system (4), the next lemma ensures its consensus performance under the switching topologies satisfying Assumption 1.

Lemma 1 (Scardovi & Sepulchre, 2009)**.** *Consider (4) with switching topologies represented by the switching directed graphs $\mathcal{G}_{\sigma(t)}$ satisfying Assumption 1. Then the consensus can be reached for (4), i.e., $\lim_{t\to+\infty} \|\xi_i(t) - \xi_j(t)\| = 0$, $\forall i, j \in \mathcal{V}$ given that no eigenvalue of S has the positive real part.*

Moreover, we have the following assumption:

Assumption 2. The matrices A_{i,ϕ_i}, B_{i,ϕ_i} and $C_{i,\phi}$ for any $i \in \mathcal{V}$, $\phi_i \in \mathcal{P}_i$ satisfy the rank property:

$$\operatorname{rank}\begin{bmatrix} A_{i,\phi_i} - \lambda_S I & B_{i,\phi_i} \\ C_{i,\phi_i} & 0 \end{bmatrix} = n_i + q,\ \forall \lambda_S \in \lambda(S). \tag{5}$$

Note that (5) is often called the transmission zeros condition (Su & Huang, 2012). With the above, we have the following lemma:

Lemma 2 (Huang, 2004)**.** *Given systems (1) and (4) under Assumption 2, then there exist $\Omega_{i,\phi_i} \in \mathbb{R}^{n_i\times p}$ and $\Theta_{i,\phi_i} \in \mathbb{R}^{l_i\times p}$, $\forall i \in \mathcal{V}$, $\forall \phi_i \in \mathcal{P}_i$ such that*

$$\begin{aligned}\Omega_{i,\phi_i}S &= A_{i,\phi_i}\Omega_{i,\phi_i} + B_{i,\phi_i}\Theta_{i,\phi_i},\\ C_{i,\phi_i}\Omega_{i,\phi_i} &= H.\end{aligned} \tag{6}$$

Remark 1. The condition (5) ensures the existence of solutions to (6). The detailed proof of this conclusion for a general case can be found in Huang (2004). Note that since the virtual reference system matrix S is in essence a parameter that can be freely set under corresponding assumptions, then given the system dynamics (1) one can always find such a matrix that satisfies the requirement in Assumption 2.

Remark 2. As was stated in Wieland et al. (2011), the method of constructing the virtual reference system (4) reflects the spirit of the internal model principle (Francis & Wonham, 1976), since its state $\xi_i(t)$ is embedded in (2), which indicates (4) can be regarded as an internal model in (1). This method has been proved effective in handling multi-agent systems with heterogeneous dynamics as shown in Wieland et al. (2011). Moreover, for the asynchronously switched multi-agent system (1), its heterogeneity not only lies in the agent dynamics but also lies in the asynchronous switching signals. Thus we made the controller (2) depend on the switching of each agent as well such that one can well handle the heterogeneity in switching signals.

With the introduced virtual reference system, the closed-loop system can be given as follows:

$$\begin{aligned}\dot{x}_i(t) =& A_{i,\sigma_i(t)}x_i(t) + B_{i,\sigma_i(t)} \\ & \times (K_{i,\sigma_i(t)}(\hat{x}_i(t) - \Omega_{i,\sigma_i(t)}\xi_i(t)) + \Theta_{i,\sigma_i(t)}\xi_i(t)),\\ y_i(t) =& C_{i,\sigma_i(t)}x_i(t),\\ \dot{\hat{x}}_i(t) =& A_{i,\sigma_i(t)}\hat{x}_i(t) + B_{i,\sigma_i(t)}u_i(t) + G_{i,\sigma_i(t)}(C_{i,\sigma_i(t)}\hat{x}_i(t) \\ & - y_i(t)),\\ \dot{\xi}_i(t) =& S\xi_i(t) - \sum_{j=1}^{N} a_{ij}(\sigma(t))(\xi_i(t) - \xi_j(t)),\ i \in \mathcal{V}. \end{aligned} \tag{7}$$

Before proceeding, some definitions and useful conclusions necessary for further analysis are given as follows:

Definition 1 (*Hespanha, 2004*)**.** For $t > t_0 \geq 0$, let $N_\sigma(t_0, t)$ be the switching times in the time interval (t_0, t), if there holds

$$N_\sigma(t_0, t) \leq N_0 + \frac{t - t_0}{\tau_a}, \tag{8}$$

in which $\tau_a > 0$, $N_0 \geq 0$ is the chatter bound, then τ_a is called the ADT of the switching signal $\sigma(t)$.

A switching signal satisfying (8) is often referred to as the "slow switching" (Liberzon & Morse, 1999). Note that if one reverses the inequality symbol then the corresponding switching signal can be referred to as the "fast switching" (Zhao, Shi, Yin, & Nguang, 2017). A similar concept based on it will be introduced next.

For the convenience of further development, based on Definition 1 we are introducing a new framework for describing the switching characteristics of $\sigma(t)$:

Definition 2. For any $t > t_k > t_0 \geq 0$, let $N_\sigma(t_k, t)$ be the number of switchings in the time interval (t_k, t), where t_k is the kth switching instant of $\sigma(t)$. The piecewise ADT of $\sigma(t)$ on (t_k, t) is defined as

$$\tau_a(t_k, t) = \frac{t - t_k}{N_\sigma(t_k, t) + 1},\ k = 1, 2, \ldots, N_\sigma(t_0, t). \tag{9}$$

Moreover, we call $\tau_a(t) = \tau_a(t_0, t)$ the current ADT at t.

Remark 3. The piecewise ADT depicts the average activating period of the switching modes on a specific time interval (t_k, t). It can be seen that a piecewise ADT satisfies (8) if choosing $t_k = t_0$. Thus the ADT can be deemed a special case of the piecewise ADT. Note that one can also regard the piecewise ADT as the ADT that has the initial instant at switching instant t_k.

In what follows, $N_i(\ldots)$ indicates the number of switchings during a certain time span of the switched agent i.

If on any certain time interval $[t_0, t_f]$, denoting a switching sequence as $t_0 < t_1 < \cdots < t_k < \cdots < t_{N_\sigma(t_0,t_f)} < t_f$, then with the above definitions, we immediately have the following definition for the "fast-switching" perturbation:

Definition 3. Given a positive constant τ_a^*, then a switching signal $\sigma(t)$ is said to suffer a "fast-switching" perturbation on $[t^*, t_f]$, $t^* \in (t_k, t_{N_\sigma(t_0,t_f)})$ that violates τ_a^*, if its current ADT and all the piecewise ADTs satisfy $\tau_a(t^*) \geq \tau_a^*$ and $\tau_a(t_{k-1}, t^*) \geq \tau_a^*$; $\tau_a(t_f) < \tau_a^*$ or $\tau_a(t_{k-1}, t_f) < \tau_a^*$, $\forall k \in \{1, \ldots, N_\sigma(t_0, t_f)\}$.

Remark 4. The "fast-switching" perturbations indicate the disruptions against a "slow-switching" signal that result in an increase of its switching frequency during a period of time. The "fast-switching" perturbations can be confirmed only if the ADT or the piecewise ADT of the switching signal has violated the "slow-switching" lower bound τ_a^*. It is thus also clear from Definition 3 that the "fast-switching" perturbed switching signal will recover into the "slow-switching" one if the corresponding ADT and any piecewise ADT do not violate τ_a^* any more.

The definition of the practical output synchronization of the system (1) is stated as follows:

Definition 4. For the system (1), the practical output synchronization is said to be achieved if for any given $\epsilon > 0$, it holds that

$$\lim_{t\to+\infty} \|y_i(t) - y_j(t)\| \leq \epsilon,\quad \forall i, j \in \mathcal{V}. \tag{10}$$

Some similar definitions on the practical convergence can be found in Back and Kim (2017) and Ding and Zheng (2017).

The following lemma is also useful for the analysis.

Lemma 3 (*Squashing Lemma Bernstein, 2009*)**.** *For the asymptotically stable system $\dot{x}(t) = Ax(t)$ with the initial state $x_0 = x(t_0)$, there exist positive constants $\bar{\lambda}$, $\bar{c}$ such that*

$$\|e^{At}\| \leq \bar{c}\|P^{-1}\|\|P\|e^{-\bar{\lambda}t} \tag{11}$$

in which $x(t) \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $\max_j \mathrm{Re}(\lambda_j(A)) < 0$, $\bar{c}$ is a scalar related to A and $-\bar{\lambda} \in (\max_j \mathrm{Re}(\lambda_j(A)), 0)$. $P \in \mathbb{R}^{n\times n}$ is a nonsingular matrix satisfying $P^{-1}AP = J_A$ with J_A the Jordan canonical form of A.

With the system formulation provided, we are ready to present the main results of this work.

3. Main results

In this section, the main results are presented twofold. In Section 3.1, the standard practical output synchronization problem is addressed for (1). Then based on the obtained results, the strategy for dealing with the fast-switching perturbations is presented in Section 3.2.

3.1. Practical output synchronization for asynchronously switched multi-agent systems

In this subsection, we will present the result on the practical output synchronization for (1), which is captured by the following theorem.

Theorem 1. *Considering the system (1) with switching topologies $\mathcal{G}_{\sigma(t)}$, $\sigma : \mathbb{R}_{\geq 0} \to \mathcal{P}$ under Assumptions 1 and 2, then the practical output synchronization can be achieved via (2) if there exist positive definite matrices P_{i,ϕ_i}, $\hat{P}_{i,\phi_i}$, $\in \mathbb{R}^{n_i\times n_i}$, $\forall \phi_i \in \mathcal{P}_i$ such that $\forall t \in [t^i_{k_i}, t^i_{k_i+1})$, $k_i \in \mathbb{N}_{>0}$, the ADT and the piecewise ADTs of $\sigma_i(t)$ satisfy:*

$$\tau_a^i \geq \tau_a^{i*} = \max\left(\frac{\ln\hat{\mu}_i}{\underline{\hat{\gamma}}_i}, \frac{\ln\mu_i + \ln\tilde{d}_i}{\underline{\gamma}_i}\right) \tag{12}$$

$$\tau_a^i(t_j^i, t) \geq \frac{\ln\mu_i + \ln\tilde{d}_i}{\underline{\gamma}_i},\ j = 1, 2, \ldots, k_i - 1, \tag{13}$$

where μ_i and $\hat{\mu}_i$ can be derived from the following constrained optimization problems, respectively:

$$\min \mu_i,\quad s.t.\ P_{i,\breve{\phi}_i} < \mu_i P_{i,\hat{\phi}_i},\ \forall \breve{\phi}_i \neq \hat{\phi}_i \in \mathcal{P}_i, \tag{14}$$

$$\min \hat{\mu}_i,\quad s.t.\ \hat{P}_{i,\breve{\phi}_i} < \hat{\mu}_i \hat{P}_{i,\hat{\phi}_i},\ \forall \breve{\phi}_i \neq \hat{\phi}_i \in \mathcal{P}_i. \tag{15}$$

In the above, t_j^i denotes the jth switching instant of $\sigma_i(t)$, $\underline{\hat{\gamma}}_i = \min_{\phi_i\in\mathcal{P}_i}(\hat{\gamma}_{i,\phi_i})$, $\hat{\gamma}_{i,\phi_i} = \frac{1}{\lambda_{\max}(\hat{P}_{i,\phi_i})} + 2\hat{\lambda}_{i,\phi_i}$, $\tilde{d}_i = 2\max_{\forall\phi_i\in\mathcal{P}_i}\left(\frac{\lambda_{\max}(P_{i,\phi_i})}{\lambda_{\min}(P_{i,\phi_i})}\right)$, $\underline{\gamma}_i = \min_{\phi_i\in\mathcal{P}_i}(\gamma_{i,\phi_i})$, $\gamma_{i,\phi_i} = \frac{1}{\lambda_{\max}(P_{i,\phi_i})} + 2\lambda_{i,\phi_i}$. Moreover, the ultimate error bound defined in (10) can be calculated *by* $\epsilon = 2\max_{\forall h\in\mathcal{V},\phi_h\in\mathcal{P}_h}\|C_{h,\phi_h}\|\epsilon_h$, $\epsilon_i = \frac{\max_{\forall\hat{\phi}_i,\check{\phi}_i\in\mathcal{V}}\|\Omega_{i,\hat{\phi}_i}-\Omega_{i,\check{\phi}_i}\|\|\xi_i(t_0)\|}{\sqrt{e^{\frac{\gamma_i}{}\varsigma_i}-1}}$, $\Omega_{i,\hat{\phi}_i}$, $\Omega_{i,\check{\phi}_i}$ *are solutions to* (6) *for any* $\hat{\phi}_i$ *and* $\check{\phi}_i$, $\underline{\varsigma}_i = \min_{\forall j}\tilde{\varsigma}_j^i$, $\tilde{\varsigma}_j^i = \lim_{t\to+\infty}\tau_a^i(t_j^i,t) - \frac{\ln\mu_i+\ln\tilde{d}_i}{\underline{\gamma}_i}$, $j = 1,2,\ldots,k_i-1$.

Proof of Theorem 1. See the Appendix. ■

In addition, the above conditions only require the local information ($\sigma_i(t)$, A_{i,ϕ_i}, etc.) of the agent i, $i\in\mathcal{V}$, in which sense, they are all distributed.

Remark 5. The eigenvalue settings for S are slightly different from some existing works, e.g., Wieland et al. (2011), as we further require $\xi_i(t)$ in (4) to be bounded. Note that this is an essential requirement given the control scheme (2) since it can be seen from (39) in the proof of Theorem 1 that one must guarantee the boundedness of the impulsive jumps $\Delta\varepsilon_i(t_{k_i}^i)$ of $\varepsilon_i(t)$ at any switching instant $t_{k_i}^i$, which entails the boundedness of $\xi_i(t)$.

Remark 6. The impulsive jumps of the tracking error state $\varepsilon_i(t)$ are brought by the switching solution $\Omega_{i,\sigma_i(t)}$ to (6), which is the result of the switching $C_{i,\sigma_i(t)}$. Different from the impulsive effects that would attenuate to zero as the state converges to zero (e.g., Xu & Teo, 2010), the impulses in (39) would never reach zero since they are always invoked by the switchings of each agent. This impedes us from seeking for the asymptotical synchronization performance of the outputs. Thus the ultimate boundedness synchronization performance is studied instead. Nevertheless, the asymptotical convergence result can be expected if one considers a special case where all the output matrices remain non-switching. This has also been reflected in the proof of Theorem 1. It is also notable that the obtained multiple Lyapunov function condition (44) at the switching (impulsive) instant is more general than that in Hespanha, Liberzon, and Teel (2008) and Yang et al. (2014) since an extra constant term is added, which implies the non-attenuating impulsive effects. This entails the development of the piecewise ADT condition (13).

Based on the obtained result, next we will consider a practical case where the agent switching $\sigma_i(t)$, $\forall i\in\mathcal{V}$ suffers the "fast-switching" perturbations.

3.2. Regulation strategy for the existence of fast-switching perturbations

As has been defined in Definition 3, the "fast-switching" perturbations may cause an increase of the switching frequency in a short time. This would potentially make the conditions (12) and (13) be violated. Since it has been pointed out in Section 1 that it is practically unnecessary to handle such perturbations as the arbitrary switching, then the challenge is to develop a control strategy that contains such "fast-switching" perturbations while does not merely statically allow for all the switching patterns. To attack these problems, in this subsection we will develop such a strategy that enables each switched agent to dynamically react to the "fast-switching" perturbations.

The following assumption is prerequisite for the upcoming development.

Assumption 3. The matrix pairs $(A_{i,\phi_i}, B_{i,\phi_i})$ and $(A_{i,\phi_i}, C_{i,\phi_i})$ for $i\in\mathcal{V}$, $\phi_i\in\mathcal{P}_i$ are controllable and observable, respectively.

Since the "fast-switching" perturbations feature the violations of the conditions (12) and (13), then the key in neutralizing the perturbations is to "recalibrate" these conditions such that they can still be satisfied when the "fast-switching" hits. Observing conditions (12) and (13), it can be seen that the lower bound of the ADT is in fact related to the convergence rate of the subsystem state (Liberzon, 2012). It enables one to "regulate" (hence the name of the "regulation strategy") the lower bound of the ADT of a switched system by properly affecting the subsystem dynamics such that the violated conditions can be "recalibrated".

Note that for (41) if perturbing $A_{i,\phi_i}+B_{i,\phi_i}K_{i,\phi_i}$ into $A_{i,\phi_i}+B_{i,\phi_i}K_{i,\phi_i}-\tilde{\lambda}_iI$, $\tilde{\lambda}_i>0$, since the Hurwitzness still holds, one can derive that

$$(A_{i,\phi_i}+B_{i,\phi_i}K_{i,\phi_i}-\tilde{\lambda}_iI)^TP_{i,\phi_i}+P_{i,\phi_i}(A_{i,\phi_i}+B_{i,\phi_i}K_{i,\phi_i} - \tilde{\lambda}_iI)+2(\lambda_{i,\phi_i}+\tilde{\lambda}_i)P_{i,\phi_i} = -I, \tag{16}$$

which yields an identical solution to (41), i.e., the solution P_{i,ϕ_i} remains unchanged under such a structure perturbation. Apparently, this conclusion also applies to (26). Such a feature makes the lower bounds (12) and (13) only vary with the change of $\tilde{\lambda}_i$ which as a result plays a central role in the regulation strategy.

In addition, we introduce to the switching signal of each agent i a slow-switching threshold ζ_i which satisfies $\zeta_i = \frac{\tau_a^{i*}}{\Delta_i}$, where $0<\Delta_i<1$. In the regulation strategy the threshold ζ_i mainly serves as an early alert for the switching's potential violation of the ADT lower bound. Note that this threshold is dependent on τ_a^{i*}.

Remark 7. It can be seen clearly from (9) that the current ADT is monotonously increasing during the activating period of each switching mode except at the switching instants. In terms of the switching frequency, a slow-switching features a small number of switchings during a certain time span while a fast switching features an increasing number of switchings during the same time span. When the occurrence of the fast switchings makes the current ADT decrease towards the corresponding lower bound, there should be a mechanism that always keeps the current ADT over the corresponding alerting threshold.

To perform the aforementioned regulation operation, each agent will be equipped with the so-called internal regulators $\nu_i(t)$, $\hat{\nu}_i(t)$, i.e., the system (1) becomes

$$\begin{aligned}
\dot{x}_i(t) =& A_{i,\sigma_i(t)}x_i(t)+B_{i,\sigma_i(t)}u_i(t)+\nu_i(t),\\
y_i(t) =& C_{i,\sigma_i(t)}x_i(t),\\
\dot{\hat{x}}_i(t) =& A_{i,\sigma_i(t)}\hat{x}_i(t)+B_{i,\sigma_i(t)}u_i(t)+G_{i,\sigma_i(t)}(C_{i,\sigma_i(t)}\hat{x}_i(t)\\
&-y_i(t))+\hat{\nu}_i(t),\\
u_i(t) =& K_{i,\sigma_i(t)}(\hat{x}_i(t)-\Omega_{i,\sigma_i(t)}\xi_i(t))+\Theta_{i,\sigma_i(t)}\xi_i(t),\\
\dot{\xi}_i(t) =& S\xi_i(t)-\sum_{j=1}^{N}a_{ij}(\sigma(t))(\xi_i(t)-\xi_j(t)),\ i\in\mathcal{V}.
\end{aligned} \tag{17}$$

In the above,

$$\begin{cases}\nu_i(t) = -\tilde{\lambda}_i(\tau_a^i((t_{k_i}^i)^+))x_i(t)\\ \hat{\nu}_i(t) = -\tilde{\lambda}_i(\tau_a^i((t_{k_i}^i)^+))\hat{x}_i(t),\end{cases}\quad \forall t\in[t_{k_i}^i, t_{k_i+1}^i), \tag{18}$$

where $\tilde{\lambda}_i(\tau_a^i((t_{k_i}^i)^+))$, $\tilde{\lambda}_i:\mathcal{S}_i\to\mathbb{R}_{\geq0}$ is the regulation coefficient subject to a certain procedure which will be presented later. Note that by calling them "internal regulators", we mean that the regulators are able to directly acquire the information of the agent states instead of obtaining via the observers. Clearly, the Hurwitz requirements in Theorem 1 are still satisfied since the introduction of the regulators only drives the value of the real part of the relevant eigenvalues towards the negative direction of the real axis. Additionally, define the gap between the dwell-time lower bounds of the zero-input tracking error system (38) and the observation error system (25) as $\tilde{\Delta}_i = \frac{\ln\mu_i+\ln\tilde{d}_i}{2\lambda_i} - \frac{\ln\hat{\mu}_i}{2\hat{\lambda}_i}$. For a concise implementation of the regulation process, here suppose that the lower bound w.r.t. (38) is always no smaller than the lower bound w.r.t. (25), i.e., there exists a desired bound $\tilde{\Delta}_i^* \geq 0$ for each agent such that $\tilde{\Delta}_i^* \leq \tilde{\Delta}_i$. This can be readily achieved by properly placing the poles of (25) and (38) in advance. With this setup, we have $\tau_a^{i*} = \frac{\ln \mu_i + \ln \tilde{d}_i}{2(\lambda_i + \tilde{\lambda}_i(\tau_a^*(t_{k_i}^i)))}$ and the regulation process can thus be implemented by tuning only $\tilde{\lambda}_i(\tau_a^*(t_{k_i}^i))$. Moreover, according to (16), the sign of $\tilde{\Delta}_i$ turns out unchanged under the variation of $\tilde{\lambda}_i(\tau_a^*(t_{k_i}^i))$. Next we are ready to present the regulation procedure for $\tilde{\lambda}_i(\tau_a^i(t_{k_i}^i))$ as the following algorithm.

Algorithm 1. Regulation of $\tilde{\lambda}_i(\tau_a^i(t_{k_i}^i))$ for agent i at switching instant $t_{k_i}^i$.

Input: $\tilde{\zeta}_i \leftarrow \zeta_i$, $\tilde{\tau}_a^{i*} \leftarrow \tau_a^{i*}$.
for $k_i \leftarrow 1$ **to** $N_i(t_0, t_f) - 1$ **do**
  if $t \in [t_{k_i}^i, t_{k_i+1}^i)$ **then**
    if $\tau_a^i((t_{k_i}^i)^+) < \tilde{\zeta}_i$ **then**
      $\tilde{\zeta}_i \leftarrow \tau_a^i((t_{k_i}^i)^+)$
      $\bar{\tilde{\tau}}_a^{i*} \leftarrow \tilde{\zeta}_i \times \Delta_i$
    else
      $\bar{\tilde{\tau}}_a^{i*} \leftarrow \tau_a^{i*}$
    end if
    for $j = 1$ **to** $k_i - 1$ **do**
      if $\tau_a^i((t_j^i, t)) < \tilde{\zeta}_i$ **then**
        $\tilde{\zeta}_i(j) \leftarrow \tau_a^i((t_j^i, t))$
        $\underline{\tilde{\tau}}_a^{i*}(j) \leftarrow \tilde{\zeta}_i(j) \times \Delta_i$
      else
        $\underline{\tilde{\tau}}_a^{i*}(j) \leftarrow \tau_a^{i*}$
      end if
    end for
    $\underline{\tilde{\tau}}_a^{i*} \leftarrow \min_{\forall j} \underline{\tilde{\tau}}_a^{i*}(j)$
    $\tilde{\tau}_a^{i*} \leftarrow \min(\bar{\tilde{\tau}}_a^{i*}, \underline{\tilde{\tau}}_a^{i*})$
    $\tilde{\lambda}_i(\tau_a^i(t_{k_i}^i)) \leftarrow \frac{\ln \mu_i + \ln \tilde{d}_i}{2\tilde{\tau}_a^{i*}} - \frac{\ln \mu_i + \ln \tilde{d}_i}{2\tau_a^{i*}}$
  end if
end for
Output: $\tilde{\lambda}_i(\tau_a^i(t_{k_i}^i))$.

Remark 8. The execution of Algorithm 1 is on-demand since it only regulates $\tilde{\lambda}_i(\tau_a^i(t_{k_i}^i))$ when there is a violation of τ_a^{i*} occurring. Similar to the event-triggered control (Almeida et al., 2017), such a on-demand scheme is favored by some scenarios where the resource saving is required. The algorithm assigns new values to the current lower bounds of the ADT and the piecewise ADT according to the current ADT it calculates. The Algorithm 1 is only executed at the switching instant since the current ADT never decreases during any non-switching period and thus rules out any potential violation of the lower bound. The piecewise ADT requires the test at every switching instant which indeed brings about some increase on the computation complexity. Nevertheless, such complexity is acceptable since it is no higher than that of the fixed dwell time (Liberzon, 2012) which remains a special case of the piecewise ADT. Moreover, since the lower bound of the (piecewise) ADT of an agent's switching signal is regulated only according to its own current ADT which is the local information, the algorithm is in a distributed manner.

The next theorem illustrates that the practical output synchronization of the asynchronously switched multi-agent systems can still be maintained when the fast-switching perturbations hit under the above regulation strategy.

Theorem 2. *Under Assumptions 1 and 3, considering (1) with (2) suffering the fast-switching perturbations against $\sigma_i(t)$ on $[t_0, t_f]$, then the practical output synchronization can be achieved for (1) and (2) with (18) applied on $[t_0, t_f]$, provided that $\forall \lambda_S \in \lambda(S)$ the matrices A_{i,ϕ_i}, B_{i,ϕ_i} and $C_{i,\phi}$ for any $i \in \mathcal{V}$, $\phi_i \in \mathcal{P}_i$ satisfy the following rank property:*

$$\operatorname{rank} \begin{bmatrix} A_{i,\phi_i} - (\tilde{\lambda}_i(\tau_a^*(t_{k_i}^i)) + \lambda_S)I & B_{i,\phi_i} \\ C_{i,\phi_i} & 0 \end{bmatrix} = n_i + q, \tag{19}$$

where the regulation of $\tilde{\lambda}_i(\tau_a^*(t_{k_i}^i)) > 0$ in (18) is performed by Algorithm 1.

Proof of Theorem 2. Following a similar procedure to the proof of Theorem 1, since (19) holds then with Lemma 2 one concludes there exist $\Omega_{i,\phi_i} \in \mathbb{R}^{n_i \times p}$, $\Theta_{i,\phi_i} \in \mathbb{R}^{l_i \times p}$ and $\tilde{\lambda}_i(\tau_a^*(t_{k_i}^i)) > 0$ such that $\forall t_{k_i}^i \in \mathcal{S}_i$ the following matrix equalities hold:

$$\begin{aligned} &\Omega_{i,\phi_i} S = (A_{i,\phi_i} - \tilde{\lambda}_i(\tau_a^*(t_{k_i}^i)) I_{n_i}) \Omega_{i,\phi_i} + B_{i,\phi_i} \Theta_{i,\phi_i}, \\ &C_{i,\phi_i} \Omega_{i,\phi_i} = H, \ i = 1, \ldots, N. \end{aligned} \tag{20}$$

Then with (20) and (17) the tracking error system of each switching mode ϕ_i of agent i activated during $[t_{k_i}^i, t_{k_i+1}^i]$ is given by:

$$\begin{aligned} \dot{\varepsilon}_i(t) =& \dot{x}_i(t) - \Omega_{i,\phi_i} \dot{\xi}_i(t) \\ =& (A_{i,\phi_i} + B_{i,\phi_i} K_{i,\phi_i} - \tilde{\lambda}_i(\tau_a^*(t_{k_i}^i)) I_{n_i}) \varepsilon_i(t) \\ &+ \Omega_{i,\phi_i} \sum_{j=1}^{N} a_{ij}(\sigma(t))(\xi_i(t) - \xi_j(t)) \\ &- B_{i,\phi_i} K_{i,\phi_i} \hat{\varepsilon}_i(t). \end{aligned} \tag{21}$$

Correspondingly, the observation error system is formulated as the follows:

$$\begin{aligned} \dot{\hat{\varepsilon}}_i(t) =& \dot{x}_i(t) - \dot{\hat{x}}_i(t) \\ =& (A_{i,\phi_i} + G_{i,\phi_i} C_{i,\phi_i} - \tilde{\lambda}_i(\tau_a^*(t_{k_i}^i)) I_{n_i}) \hat{\varepsilon}_i(t). \end{aligned} \tag{22}$$

Clearly, since $\tilde{\lambda}_i(\tau_a^*(t_{k_i}^i)) > 0$, the Hurwitzness of $A_{i,\phi_i} + B_{i,\phi_i} K_{i,\phi_i} - \tilde{\lambda}_i(\tau_a^*(t_{k_i}^i)) I_{n_i}$ and $A_{i,\phi_i} + G_{i,\phi_i} C_{i,\phi_i} - \tilde{\lambda}_i(\tau_a^*(t_{k_i}^i)) I_{n_i}$ retains only if $A_{i,\phi_i} + B_{i,\phi_i} K_{i,\phi_i}$ and $A_{i,\phi_i} + G_{i,\phi_i} C_{i,\phi_i}$ are both Hurwitz, which is ensured by Assumption 3. Furthermore, at any switching instant $t_{k_i}^i$ Algorithm 1 always outputs the regulation parameter $\tilde{\lambda}_i(\tau_a^*(t_{k_i}^i))$ that yields a lower bound τ_a^{i*} smaller than the current ADT $\tau_a^i(t_0, t_{k_i}^i)$ and the piecewise ADTs $\tau_a^i(t_j, t_{k_i}^i)$, $j = 1, \ldots, k_i - 2$. This implies the satisfaction of the conditions (12) and (13) over the given time span $[t_0, t_f]$ with Algorithm 1 applied. The rest of the proof are identical to that of Theorem 1. ■

Remark 9. The proof of Theorem 2 has followed a similar fashion to that of Theorem 1, though, the considered system (17) and condition (19) are slightly different from those of Theorem 1. Moreover, the proof of Theorem 2 is presented given that Algorithm 1 has been applied, which implies the proof performs under the case that there have been regulators introduced to neutralize the "fast-switching" perturbations. Thus in the proof of Theorem 2, one only needs to deal with the system structure changes brought by the introduction of (18) and ensures they would not change the way of proving the stability of $\varepsilon_i(t)$, $\hat{\varepsilon}_i(t)$ as that in the proof of Theorem 1.

The proposed regulation strategy will be proved effective in neutralizing the fast-switching perturbations by the practical example to be presented.

4. Illustrative examples

For the illustration of the proposed methods, the example presented in this section is based on the payload transport problem studied in Lee et al. (2013). Consider four different drones equipped with electromagnetic grippers (a similar setting can be found in Foehn et al., 2017) as each gripper performs a series of pick-up and drop-off maneuvers to the magnetic payloads. Similar to Lee et al. (2013), we also regard such maneuvers as switchings between two operation modes. Particularly, each drone i, $i = 1, 2, 3, 4$ is set to work in mode $\phi_i = 1$ if it is unloaded and in mode $\phi_i = 2$ if it carries the payload. It is assumed that all the drones work in a vertical plane as each of them tracks and catches a moving payload on the ground using the electromagnetic attractions.

Specifically, drones 1 and 2 are modeled as typical quadrotors with both translational and rotational states; drones 3 and 4 are modeled as tiny-sized quadrotors that only translational states are considered. Hence, the considered states can be given as $x_i(t) = [z_i(t), v_i(t), \theta_i(t), \omega_i(t)]^T$, $i = 1, 2$, $x_i(t) = [z_i(t), v_i(t)]^T$, $i = 3, 4$. For any drone i, z_i, $v_i \in \mathbb{R}$ denote the position and the velocity on a specific axis in the horizontal plane, respectively; $\theta_i, \omega_i \in \mathbb{R}$ denote the angular position and velocity w.r.t. the perpendicular axis in the same plane, respectively. The interested outputs of each drone are two certain combinations of its translational position and velocity. These combinations correspond to the two different payload operation modes described above. The objective is to seek the practical synchronization of the outputs of all the drones during given payload transport processes.

With the above settings and a modified configuration of that in Lee et al. (2013), we have the following asynchronously switched model for the payload transport task: for drone $i = 1, 2$,

$$A_{i,\phi_i} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha^z_{i,\phi_i} & \frac{\beta^z_{i,\phi_i}}{m_{i,\phi_i}} & \alpha^\theta_{i,\phi_i} & \frac{\beta^\theta_{i,\phi_i}}{m_{i,\phi_i}} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \alpha^\theta_{i,\phi_i} & \frac{\beta^\theta_{i,\phi_i}}{m_{i,\phi_i}} \end{bmatrix}, B_{i,\phi_i} = \begin{bmatrix} 0 \\ \frac{1}{m_{i,\phi_i}} \\ 0 \\ \frac{1}{m_{i,\phi_i}} \end{bmatrix},$$
$$C_{i,\phi_i} = \begin{bmatrix} 1.6 - 0.6\phi_i & -0.2 + 0.6\phi_i & 0 & 0 \end{bmatrix}, \tag{23}$$

for drone $i = 3, 4$,

$$A_{i,\phi_i} = \begin{bmatrix} 0 & 1 \\ \alpha^z_{i,\phi_i} & \frac{\beta^z_{i,\phi_i}}{m_{i,\phi_i}} \end{bmatrix}, B_{i,\phi_i} = \begin{bmatrix} 0 \\ \frac{1}{m_{i,\phi_i}} \end{bmatrix},$$
$$C_{i,\phi_i} = \begin{bmatrix} 1.6 - 0.6\phi_i & -0.2 + 0.6\phi_i \end{bmatrix}. \tag{24}$$

In the above, $\alpha^z_{i,1} = -0.1$, $\forall i$; $\beta^z_{i,\phi_i} = -0.5$, $i = 1, 2$, $\forall \phi_i$; $\beta^z_{i,\phi_i} = -0.2$, $i = 3, 4$, $\forall \phi_i$; $\alpha^\theta_{i,2}$=0.15, $\forall i$; $\beta^\theta_{i,\phi_i} = -0.5$, $i = 1, 2$, $\forall \phi_i$; $\beta^\theta_{i,\phi_i} = -0.2$, $i = 3, 4$, $\forall \phi_i$. These parameters denote the friction coefficients regarding the translational and the rotational variables in different operation modes of drone i. $m_{i,\phi_i} = m^i_D + (\phi_i - 1)m^i_L$, where m^i_D and m^i_L denote the mass of the drone and the payload, respectively. Specifically, $m^1_D = 1$ kg, $m^2_D = 0.92$ kg, $m^3_D = 0.5$ kg, $m^4_D = 0.49$ kg; $m^1_L = 0.2$ kg, $m^2_L = 0.192$ kg, $m^3_L = 0.1$ kg, $m^4_L = 0.097$ kg. Moreover, the parameters of (4) are given as the harmonic oscillator: $S = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $H = [1\ 0]$.

The switching topologies are depicted in Fig. 1 with the fourth subplot denoting the joint graph of the switching graphs $\mathcal{G}_{\sigma(t)}$ on $[0, 60]$.

Applying the controller (2) and deriving the corresponding feedback gain matrices K_{i,ϕ_i} and G_{i,ϕ_i} by specifying the stable poles, one can readily obtain the lower bounds for the ADT and the piecewise ADTs for each agent by solving corresponding Lyapunov equation (41). Fig. 2 shows the switching signal for each agent with the corresponding lower bounds of the ADT and the ADT over $[t_0, t_f]$. The piecewise ADTs are also shown in the captions. Clearly the ADT and the piecewise ADT series of each agent satisfy conditions (12) and (13). Fig. 3 depicts the outputs $y_i(t)$, $i = 1, 2, 3, 4$ and the output errors $y_j(t) - y_1(t)$, $j = 2, 3, 4$ under switching signals in Fig. 2. It can be seen from Fig. 3 that the outputs track the reference systems well but only with some fluctuations at each switching instant, this implies the ultimate boundedness of the output errors which thus validates the practical synchronization of all the outputs.

The equipped electromagnetic grippers are considered to be prone to the electromagnetic inference (EMI). The EMI can be exploited as a deliberate attack against the electromagnetic equipment (Ängskog, Näsman, & Mattsson, 2019). When the payload transport undergoes a constant series of external EMI, the grippers would be forced to lose and regain the attractions frequently. Such malfunction operations would consequently generate fast-switching patterns that undermine the original "slow-switching" ones.

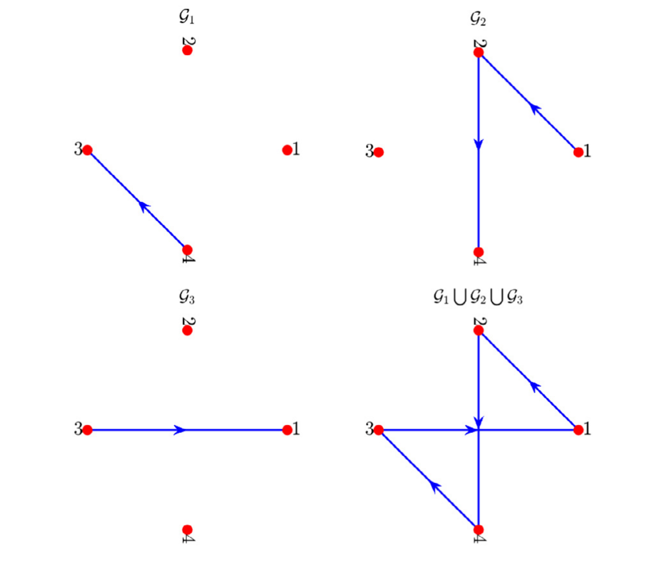

Fig. 1. Directed graphs $\mathcal{G}_\phi$, $\phi = 1, 2, 3$ of switching topologies and their joint graph on $[0, t_f]$, $t_f = 60$ s. The numbers shown on red nodes denote the agent labels.

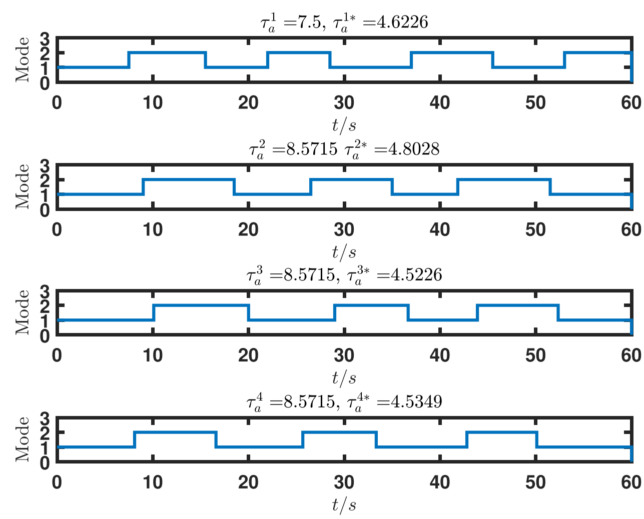

Fig. 2. Switching signals $\sigma_i(t)$, $i = 1, 2, 3, 4$ without fast switching perturbations. The piecewise ADTs of $\sigma_i(t)$ on $[0, t_f]$ are $\tau^1_a(t^1_1, t_f) = 7.50$, $\tau^1_a(t^1_2, t_f) = 7.42$, $\tau^1_a(t^1_3, t_f) = 7.60$, $\tau^1_a(t^1_4, t_f) = 7.88$, $\tau^1_a(t^1_5, t_f) = 7.67$, $\tau^1_a(t^1_6, t_f) = 7.25$; $\tau^2_a(t^2_1, t_f) = 8.50$, $\tau^2_a(t^2_2, t_f) = 8.30$, $\tau^2_a(t^2_3, t_f) = 8.38$, $\tau^2_a(t^2_4, t_f) = 8.33$, $\tau^2_a(t^2_5, t_f) = 9.08$; $\tau^3_a(t^3_1, t_f) = 8.32$, $\tau^3_a(t^3_2, t_f) = 8$, $\tau^3_a(t^3_3, t_f) = 7.75$, $\tau^3_a(t^3_4, t_f) = 7.78$, $\tau^3_a(t^3_5, t_f) = 8.05$; $\tau^4_a(t^4_1, t_f) = 8.65$, $\tau^4_a(t^4_2, t_f) = 8.68$, $\tau^4_a(t^4_3, t_f) = 8.58$, $\tau^4_a(t^4_4, t_f) = 8.89$, $\tau^4_a(t^4_5, t_f) = 8.60$.

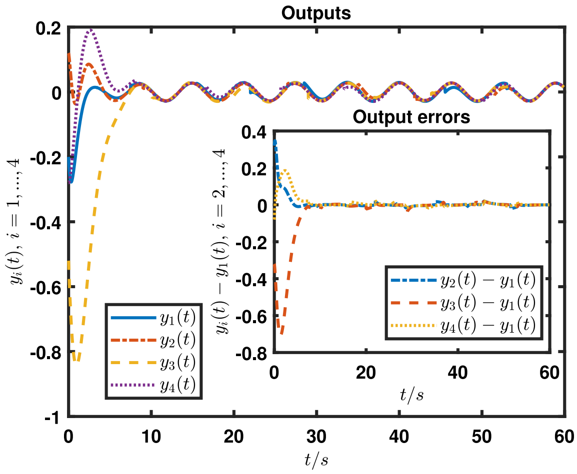

Fig. 3. Outputs $y_i(t)$, $i = 1, 2, 3, 4$ under slow-switching signals. The subplot depicts the corresponding output errors $y_i(t) - y_1(t)$, $i = 2, 3, 4$.

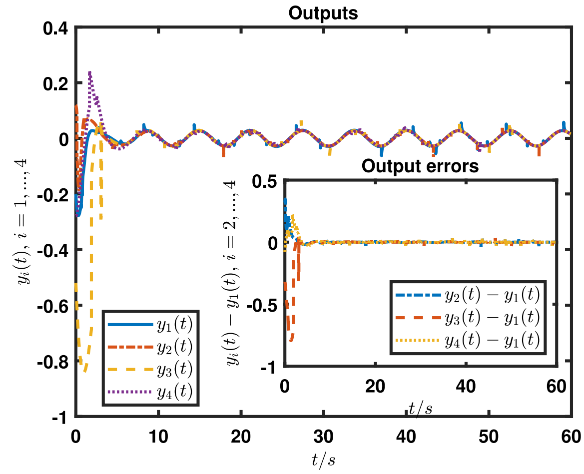

Fig. 6. Outputs $y_i(t)$, $i = 1, 2, 3, 4$ under fast-switching perturbations with regulation strategy. The subplot depicts the corresponding output errors $y_i(t) - y_1(t)$, $i = 2, 3, 4$.

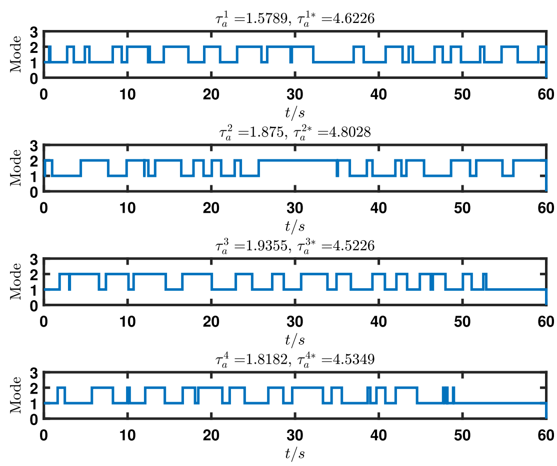

Fig. 4. Switching signals $\sigma_i(t)$, $i = 1, 2, 3, 4$ under fast-switching perturbations starting from $t = 0$.

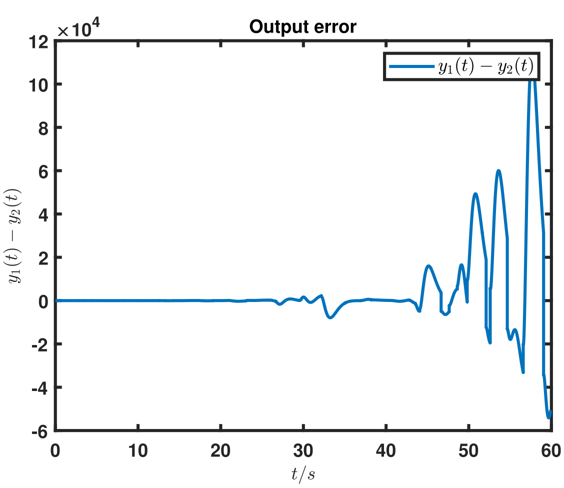

Fig. 5. Output error $y_1(t) - y_2(t)$ under fast-switching perturbations without regulation strategy.

Assume such EMI-induced malfunctions have happened to all the drones at $t = 0$. The resultant fast-switching perturbations against $\sigma_i(t)$ are depicted in Fig. 4. It can be seen that in such a case the conditions (12) and (13) are apparently violated. If there are no regulators applied, then it is shown by Fig. 5 that the output error is divergent which means the practical output synchronization cannot be achieved. Nevertheless, when one applies the regulators (18), then it is demonstrated in Fig. 6 that the all the outputs of drones have successfully reached the practical synchronization. This indicates that the EMI-induced "fast-switching" perturbations are well contained.

5. Conclusion

The practical output synchronization problem of the asynchronously switched multi-agent system under switching topologies has been addressed. The proposed distributed switching-dependent controllers with embedded virtual reference states convert the original problem into tracking problems between each agent and its reference. To handle the non-attenuating impulses in the resultant tracking error system, the new concept of piecewise ADT has been introduced to ensure the ultimate boundedness of the tracking error state. Furthermore, for a realistic case where each agent is subject to fast-switching perturbations, the same performance can be maintained by the proposed dynamic regulation strategy. For future works, related experiments of the payload transport can be performed for a more reliable validation of the considered problem; new control structures like the dynamic output feedback controller can be considered to improve the performance; more general agent dynamics such as those are arbitrarily switching or uncertain can be considered; the piecewise ADT can also be further improved to reduce the computation complexity and to eliminate the need of testing every switching instant.

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Appendix

Proof of Theorem 1. Denote a certain switching instant set of the agent i by $\mathcal{S}_i = \{t_1^i, t_2^i, \ldots, t_{k_i+1}^i\}$. On the time interval $[t_0, t_f]$,

$t_{k_i}^i$ denotes the instant of the k_ith switching of agent i. Assume on the period of $[t_{k_i}^i, t_{k_i+1}^i)$ the switching mode $\phi_i \in \mathcal{P}_i$ is activated.

(I) Observation error analysis

For any switching mode $\phi_i \in \mathcal{P}_i$ of each agent $i \in \mathcal{V}$, denote the observation error $\hat{\varepsilon}_i(t) = x_i(t) - \hat{x}_i(t)$. A differentiating operation on both sides yield

$$\dot{\hat{\varepsilon}}_i(t) = \dot{x}_i(t) - \dot{\hat{x}}_i(t) = (A_{i,\phi_i} + G_{i,\phi_i}C_{i,\phi_i})\hat{\varepsilon}_i(t). \tag{25}$$

Since $(A_{i,\phi_i}, C_{i,\phi_i})$ is detectable, then there always exists a matrix G_{i,ϕ_i} such that $A_{i,\phi_i}+G_{i,\phi_i}C_{i,\phi_i}$ is Hurwitz for any switching mode ϕ_i of agent i. Then it is concluded that there exists a positive definite matrix $\hat{P}_{i,\phi_i}$ such that the following Lyapunov equation holds

$$(A_{i,\phi_i} + G_{i,\phi_i}C_{i,\phi_i})^T\hat{P}_{i,\phi_i} + \hat{P}_{i,\phi_i}(A_{i,\phi_i} + G_{i,\phi_i}C_{i,\phi_i}) + 2\hat{\lambda}_{i,\phi_i}\hat{P}_{i,\phi_i} = -I, \tag{26}$$

where $0 > -\hat{\lambda}_{i,\phi_i} > \max_j \mathrm{Re}(\lambda_j(A_{i,\phi_i} + G_{i,\phi_i}C_{i,\phi_i}))$. If constructing a multiple Lyapunov function for each agent i as $\hat{V}_{i,\phi_i}(\hat{\varepsilon}_i(t)) = \hat{\varepsilon}_i^T(t)\hat{P}_{i,\phi_i}\hat{\varepsilon}_i(t)$, then one gets

$$\dot{\hat{V}}_{i,\phi_i}(\hat{\varepsilon}_i(t)) \le -\underline{\hat{\gamma}}_i\hat{V}_{i,\phi_i}(\hat{\varepsilon}_i(t)),\ \forall t \in [t_{k_i}^i, t_{k_i+1}^i), \tag{27}$$

with $\underline{\hat{\gamma}}_i = \min_{\phi_i\in\mathcal{P}_i}(\hat{\gamma}_{i,\phi_i})$, $\hat{\gamma}_{i,\phi_i} = \frac{1}{\lambda_{\max}(\hat{P}_{i,\phi_i})} + 2\hat{\lambda}_{i,\phi_i}$. Moreover, if $\forall k_i > 0$, $\sigma((t_{k_i}^i)^+) = \hat{\phi}_i$, $\sigma((t_{k_i}^i)^-) = \breve{\phi}_i$, then with the corresponding solutions to (26) obtained, there exists a positive scalar $\hat{\mu}_i > 1$ which is given by (15) such that:

$$\hat{V}_{i,\hat{\phi}_i}(\hat{\varepsilon}_i((t_{k_i}^i)^+)) \le \hat{\mu}_i\hat{V}_{i,\breve{\phi}_i}(\hat{\varepsilon}_i((t_{k_i}^i)^-)). \tag{28}$$

By Definition 2 as well as the well-known result fulfilled by Liberzon and Morse (1999), one can readily specify the lower bound of the ADT for agent i, i.e., $\tau_a^{i*} = \frac{\ln\hat{\mu}_i}{\underline{\hat{\gamma}}_i}$ and with (12) satisfied, $\hat{V}_{i,\sigma_i(t_{k_i}^i)}(\hat{\varepsilon}_i(t)) \le \hat{V}_{i,\sigma_i(t_0)}((t_0)^+)\mathrm{e}^{(\frac{\ln\hat{\mu}_i}{\tau_a^i}-\underline{\hat{\gamma}}_i)(t-t_0)} \to 0$ as $t \to +\infty$, which guarantees that there exists a positive constant $\hat{c}_i$ such that $\forall t \in [t_0, +\infty)$,

$$\|\hat{\varepsilon}_i(t)\| \le \hat{c}_i\|\hat{\varepsilon}_i(t_0)\|\mathrm{e}^{-\underline{\hat{\gamma}}_i(t-t_0)}. \tag{29}$$

(II) Tracking error analysis

Similarly, let an activating period of ϕ_i be $[t_{k_i}^i, t_{k_i+1}^i)$. Following the same terming manner as in Wieland et al. (2011), $\forall t \in [t_{k_i}^i, t_{k_i+1}^i)$, defining the tracking error as $\varepsilon_i(t) = x_i(t) - \Omega_{i,\phi_i}\xi_i(t)$, then for any ϕ_i, a differentiating operation on both sides with applying (6) of Lemma 2 yields:

$$\begin{aligned}\dot{\varepsilon}_i(t) =& \dot{x}_i(t) - \Omega_{i,\phi_i}\dot{\xi}_i(t)\\ =& (A_{i,\phi_i} + B_{i,\phi_i}K_{i,\phi_i})\varepsilon_i(t) + \Omega_{i,\phi_i}\sum_{j=1}^N a_{ij}(\sigma(t))\\ &\times (\xi_i(t) - \xi_j(t)) - B_{i,\phi_i}K_{i,\phi_i}\hat{\varepsilon}_i(t).\end{aligned} \tag{30}$$

Then the corresponding solution or the complete response to (30) on $[t_{k_i}^i, t]$ is given by

$$\begin{aligned}\varepsilon_i(t) =& \mathrm{e}^{(A_{i,\phi_i}+B_{i,\phi_i}K_{i,\phi_i})(t-t_{k_i}^i)}\varepsilon_i((t_{k_i}^i)^+)\\ &+ \int_{(t_{k_i}^i)^+}^t \mathrm{e}^{(A_{i,\phi_i}+B_{i,\phi_i}K_{i,\phi_i})(t-\tau)}\Bigg(\Omega_{i,\phi_i}\sum_{j=1}^N a_{ij}(\sigma(t))\\ &\times (\xi_i(\tau) - \xi_j(\tau)) - B_{i,\phi_i}K_{i,\phi_i}\hat{\varepsilon}_i(\tau)\Bigg)d\tau.\end{aligned} \tag{31}$$

If taking $\Omega_{i,\phi_i}\sum_{j=1}^N a_{ij}(\sigma(t))(\xi_i(t)-\xi_j(t))-B_{i,\phi_i}K_{i,\phi_i}\hat{\varepsilon}_i(t)$ as the input and setting $\varepsilon_i((t_{k_i}^i)^+) = 0$, then (31) becomes

$$\begin{aligned}\underline{\varepsilon}_i(t) =& \int_{(t_{k_i}^i)^+}^t \mathrm{e}^{(A_{i,\phi_i}+B_{i,\phi_i}K_{i,\phi_i})(t-\tau)}\Bigg(\Omega_{i,\phi_i}\sum_{j=1}^N a_{ij}(\sigma(t))\\ &\times (\xi_i(\tau) - \xi_j(\tau)) - B_{i,\phi_i}K_{i,\phi_i}\hat{\varepsilon}_i(\tau)\Bigg)d\tau,\end{aligned} \tag{32}$$

which is termed the zero-state response of (30). Correspondingly, denoting the zero-input response as

$$\bar{\varepsilon}_i(t) = \mathrm{e}^{(A_{i,\phi_i}+B_{i,\phi_i}K_{i,\phi_i})(t-t_{k_i}^i)}\varepsilon_i\left((t_{k_i}^i)^+\right),\ \forall t \in [t_{k_i}^i, t_{k_i+1}^i) \tag{33}$$

by setting the input to 0, then it follows that there exist positive constant scalars c_{i,ϕ_i} and κ_{i,ϕ_i} such that $\forall t \in [t_{k_i}^i, t_{k_i+1}^i)$,

$$\|\bar{\varepsilon}_i(t)\| \le c_{i,\phi_i}\|\varepsilon_i((t_{k_i}^i)^+)\|\mathrm{e}^{-\kappa_{i,\phi_i}(t-t_{k_i}^i)}, \tag{34}$$

provided $A_{i,\phi_i}+B_{i,\phi_i}K_{i,\phi_i}$ is Hurwitz for any ϕ_i. Clearly, there holds $\varepsilon_i(t) = \underline{\varepsilon}_i(t) + \bar{\varepsilon}_i(t),\ \forall t \ge t_0$.

(II-1) Zero-state response analysis

For (32), according to Lemma 1 one readily concludes that there exist positive constant scalars $\breve{c}_i$ and $\breve{\kappa}$ such that $\forall t \ge t_0$,

$$\begin{aligned}&\|\sum_{j=1}^N a_{ij}(\sigma(t))(\xi_i(t) - \xi_j(t))\|\\ &\le \breve{c}_i\sum_{j=1}^N \|\xi_i(t_0) - \xi_j(t_0)\|\mathrm{e}^{-\breve{\kappa}(t-t_0)}.\end{aligned} \tag{35}$$

Note here we use the fact that $a_{ij}(\sigma(t)) \le 1,\ \forall i, j \in \mathcal{V}$. Furthermore, since for any switching mode of agent i, Ω_{i,ϕ_i} is a unique solution to (6), then one can always select a solution of the maximum norm from the solution pool generated by different switching modes, i.e., $\Omega_i = \max_{\phi_i}\|\Omega_{i,\phi_i}\|$, $\forall\phi_i \in \mathcal{P}_i$. Additionally, one can derive the following via the mean value theorem of definite integrals for (32):

$$\begin{aligned}\underline{\varepsilon}_i(t) =& \mathrm{e}^{(A_{i,\phi_i}+B_{i,\phi_i}K_{i,\phi_i})(t-t_{k_i}^{i*})}\Bigg(\Omega_{i,\phi_i}\sum_{j=1}^N a_{ij}(\sigma(t))\\ &\times \left(\xi_i(t_{k_i}^{i*}) - \xi_j(t_{k_i}^{i*})\right) - B_{i,\phi_i}K_{i,\phi_i}\hat{\varepsilon}_i(t_{k_i}^{i*})\Bigg)(t - t_{k_i}^i),\end{aligned} \tag{36}$$

where $t_{k_i}^{i*} \in (t_{k_i}^i, t_{k_i+1}^i]$. Then given (29), one concludes from (36) via Lemma 3 and the property of the compatible matrix norm that $\forall\phi_i \in \mathcal{P}_i$,

$$\begin{aligned}\|\underline{\varepsilon}_i(t)\| \le& \bar{c}_i\mathrm{e}^{-\breve{\lambda}_{i,\phi_i}(t-t_{k_i}^{i*})}\Bigg(\|\Omega_i\|\sum_{j=1}^N \|\xi_i(t_0) - \xi_j(t_0)\|\\ &\times \breve{c}_i\mathrm{e}^{-\breve{\kappa}(t_{k_i}^{i*}-t_0)} + \hat{c}_i\|B_iK_i\|\|\hat{\varepsilon}_i(t_0)\|\mathrm{e}^{-\underline{\hat{\gamma}}_i(t_{k_i}^{i*}-t_0)}\Bigg)\\ &\times (t - t_{k_i}^i),\ \forall t \in [t_{k_i}^i, t_{k_i+1}^i)\\ \le& \Psi_i\mathrm{e}^{-\underline{\lambda}_i(t-t_0)},\ \forall t \in [t_0, \infty)\end{aligned} \tag{37}$$

where $\|B_iK_i\| = \max_{\phi_i\in\mathcal{P}_i}\|B_{i,\phi_i}K_{i,\phi_i}\|$ since K_{i,ϕ_i} can always be calculated as a matrix with finite norm for any switching mode ϕ_i. Moreover, $\underline{\lambda}_i = \min_{\phi_i}(\breve{\lambda}_{i,\phi_i}, \underline{\hat{\gamma}}_i, \breve{\kappa})$, in which, $\breve{\lambda}_{i,\phi_i} \in (\max_j \mathrm{Re}(\lambda_j(A_{i,\phi_i} + B_{i,\phi_i}K_{i,\phi_i})), 0)$, $\Psi_i = \breve{c}_i\bar{c}_i\max_{k_i}(t_{k_i+1}^i - t_{k_i}^i)\|\Omega_i\|\sum_{j=1}^N\|\xi_i(t_0) - \xi_j(t_0)\| + \hat{c}_i\bar{c}_i\|B_iK_i\|\|\hat{\varepsilon}_i(t_0)\|$, $\bar{c}_i = \max_{\phi_i}(h_{i,\phi_i}\|\tilde{Q}_{i,\phi_i}\|\|\tilde{Q}_{i,\phi_i}^{-1}\|)$ in which h_{i,ϕ_i} is a positive constant and $\tilde{Q}_{i,\phi_i}$ is a non-singular matrix satisfying $\tilde{Q}_{i,\phi_i}^{-1}(A_{i,\phi_i} + B_{i,\phi_i}K_{i,\phi_i})\tilde{Q}_{i,\phi_i} = \tilde{J}_{i,\phi_i}$, where $\tilde{J}_{i,\phi_i}$ denotes the Jordan canonical form of $A_{i,\phi_i} + B_{i,\phi_i}K_{i,\phi_i}$.

(II-2) Zero-input response analysis

Given the zero-input version of (30):

$$\dot{\varepsilon}_i(t) = (A_{i,\phi_i} + B_{i,\phi_i}K_{i,\phi_i})\varepsilon_i(t),\ \ i \in \mathcal{V}, \tag{38}$$

clearly (33) is the solution to (38), thus we will slightly abuse the denotation $\varepsilon_i(t)$ instead of $\overline{\varepsilon}_i(t)$ as the zero-input state in part of the following proof.

Note that the error state $\varepsilon_i(t)$ suffers the jumping dynamics brought by the switching-dependent parameter Ω_{i,ϕ_i} at each switching instant $t^i_{k_i}$, $k_i = 1, \ldots, N_i(t_0, t_f)$. This means (38) is essentially a switched system with state jumps or an impulsive switched system (Xu & Teo, 2010). The instant variation or the impulse of the state at $t^i_{k_i}$ is given by:

$$\begin{aligned}\Delta\varepsilon_i(t^i_{k_i}) =&\varepsilon_i((t^i_{k_i})^+) - \varepsilon_i((t^i_{k_i})^-)\\ =&(\Omega_{i,\sigma_i((t^i_{k_i})^-)} - \Omega_{i,\sigma_i((t^i_{k_i})^+)})\xi_i(t^i_{k_i}),\end{aligned} \tag{39}$$

in which one can also note that $\Omega_{i,\sigma_i(t)}$ always switches at any $t^i_{k_i}$ and $\xi_i(t)$ does not converge to 0. These imply that the variation $\Delta\varepsilon_i(t^i_{k_i})$ at the switching instant would never diminish to 0 as the impulsive effects that have been commonly considered in, e.g., Xu and Teo (2010). Consequently, the methods in addressing classic impulsive switched systems turn out less effective. It thus brings the challenge of using a different method to study such an impulsive switched system with non-attenuating impulses.

Moreover, given the eigenvalue setting in Section 2.2 for S, one can conclude the boundedness characteristics of $\xi_i(t)$, i.e., $\|\xi_i(t^i_{k_i})\| \le c_S\|\xi_i(t_0)\|$, where $c_S > 0$ is a constant scalar related to the selection of S. This implies $\|\Delta\varepsilon_i(t^i_{k_i})\| \le c_S \max_{\forall k_i} \|\Omega_{i,\sigma_i((t^i_{k_i})^-)} - \Omega_{i,\sigma_i((t^i_{k_i})^+)}\|\|\xi_i(t_0)\|$ given the boundedness of $\|\Omega_{i,\sigma_i((t^i_{k_i})^-)} - \Omega_{i,\sigma_i((t^i_{k_i})^+)}\|$ over a certain time span $[t_0, t_f]$. Furthermore, one has

$$\begin{aligned}\|\varepsilon_i((t^i_{k_i})^+)\|^2 \le&2(c_S \max_{\forall k_i}\|\Omega_{i,\sigma_i((t^i_{k_i})^-)} - \Omega_{i,\sigma_i((t^i_{k_i})^+)}\|\\ &\times \|\xi_i(t_0)\|)^2 + 2\|\varepsilon_i((t^i_{k_i})^-)\|^2.\end{aligned} \tag{40}$$

Since $(A_{i,\phi_i}, B_{i,\phi_i})$ is stabilizable then there always exists a matrix K_{i,ϕ_i} such that $A_{i,\phi_i} + B_{i,\phi_i}K_{i,\phi_i}$ is Hurwitz for any $\phi_i \in \mathcal{P}_i$. Then one concludes that for each switching mode ϕ_i there exists a positive definite matrix P_{i,ϕ_i} such that the following Lyapunov equation holds:

$$\begin{aligned}(A_{i,\phi_i} + B_{i,\phi_i}K_{i,\phi_i})^T P_{i,\phi_i} + &P_{i,\phi_i}(A_{i,\phi_i} + B_{i,\phi_i}K_{i,\phi_i})\\ &+ 2\lambda_{i,\phi_i} = -I,\end{aligned} \tag{41}$$

where $0 > -\lambda_{i,\phi_i} > \max_j \mathrm{Re}(\lambda_j(A_{i,\phi_i} + B_{i,\phi_i}K_{i,\phi_i}))$. Constructing a multiple Lyapunov function for each agent i as $V_{i,\sigma_i(t)}(\varepsilon_i(t)) = \varepsilon_i^T(t)P_{i,\sigma_i(t)}\varepsilon_i(t)$, then one gets for $\sigma_i(t) = \phi_i \in \mathcal{P}_i$, $\forall t \in [t^i_{k_i}, t^i_{k_i+1})$, $k_i = 1, 2, \ldots, N_i(t_0, t_f)$,

$$\dot{V}_{i,\phi_i}(\varepsilon_i(t)) \le -\underline{\gamma}_i V_{i,\phi_i}(\varepsilon_i(t)), \tag{42}$$

with $\underline{\gamma}_i = \min_{\phi_i \in \mathcal{P}_i}(\gamma_{i,\phi_i})$, $\gamma_{i,\phi_i} = 2\lambda_{i,\phi_i} + \frac{1}{\lambda_{\max}(P_{i,\phi_i})}$. Moreover, if $\forall k_i > 0, \sigma((t^i_{k_i})^+) = \hat{\phi}_i, \sigma((t^i_{k_i})^-) = \breve{\phi}_i$, and with the corresponding solutions to (41) obtained, then there exists $\mu_i > 1$ which is derived from (14) such that:

$$V_{i,\hat{\phi}_i}(\varepsilon_i((t^i_{k_i})^+)) \le \mu_i V_{i,\breve{\phi}_i}(\varepsilon_i((t^i_{k_i})^-)). \tag{43}$$

In addition, with the multiple Lyapunov functions selected, one can derive from (40) that

$$\begin{aligned}V_{i,\sigma_i((t^i_{k_i})^-)}(\varepsilon_i((t^i_{k_i})^+)) \le&\tilde{d}_i V_{i,\sigma_i((t^i_{k_i})^-)}(\varepsilon_i((t^i_{k_i})^-))\\ &+ \tilde{C}_i \max_{\forall k_i}\left(\lambda_{\max}(P_{i,\sigma_i((t^i_{k_i})^-)})\right),\end{aligned} \tag{44}$$

in which $\tilde{d}_i = 2\max_{\forall k_i}\left(\frac{\lambda_{\max}(P_{i,\sigma_i((t^i_{k_i})^-)})}{\lambda_{\min}(P_{i,\sigma_i((t^i_{k_i})^-)})}\right)$, $\tilde{C}_i = 2(\max_{\forall k_i}\|\Omega_{i,\sigma_i((t^i_{k_i})^+)} - \Omega_{i,\sigma_i((t^i_{k_i})^-)}\|\|\xi_i(t_0)\|)^2$.

From (43) and (44), one can obtain the following by integrating (42) on both sides with respect to t:

$$\begin{aligned}V_{i,\sigma_i(t^i_{k_i})}(\varepsilon_i(t)) \le&V_{i,\sigma_i(t^i_{k_i})}(\varepsilon_i((t^i_{k_i})^+))\mathrm{e}^{-\underline{\gamma}_i(t-t^i_k)}\\ &\cdots\\ \le&\mu_i^{k_i}\tilde{d}_i^{k_i}V_{i,\sigma_i(t_0)}(\varepsilon_i((t_0)^+))\mathrm{e}^{-\underline{\gamma}_i(t-t_0)} + \mu_i\\ &\times\left(\sum_{j=0}^{k_i-1}\mu_i^j\tilde{d}_i^j\tilde{C}_i\max_{\forall k_i}\left(\lambda_{\max}(P_{i,\sigma_i((t^i_{k_i})^-)})\right)\right.\\ &\left.\times\ \mathrm{e}^{-\underline{\gamma}_i(t-t^i_{k_i-j})}\right).\end{aligned} \tag{45}$$

With (45) and (8), the following holds

$$\begin{aligned}V_{i,\sigma_i(t^i_{k_i})}(\varepsilon_i(t)) \le&V_{i,\sigma_i(t_0)}(\varepsilon_i((t_0)^+))\mathrm{e}^{k_i(\ln\mu_i+\ln\tilde{d}_i)-\underline{\gamma}_i(t-t_0)}\\ &+\ \mu_i\left(\sum_{j=0}^{k_i-1}\tilde{C}_i\max_{\forall k_i}\left(\lambda_{\max}(P_{i,\sigma_i((t^i_{k_i})^-)})\right)\right.\\ &\left.\times\ \mathrm{e}^{j(\ln\mu_i+\ln\tilde{d}_i)-\underline{\gamma}_i(t-t^i_{k_i-j})}\right).\end{aligned} \tag{46}$$

Next, to obtain the practical output synchronization for (1), one needs to prove the ultimate boundedness of $V_{i,\sigma_i(t^i_{k_i})}(\varepsilon_i(t))$ as $t \to +\infty$, which is presented as follows.

(III) Ultimate bound derivation

Note in (46), as $t \to +\infty$ the second additive term becomes an infinite series with non-negative terms because we rule out a trivial case where the number of switchings is finite on an infinite time span, i.e., we have $t^i_{k_i}, k_i \to +\infty$ as $t \to +\infty$. Then with (9) we can readily obtain the following limit:

$$\begin{aligned}&\lim_{\tilde{j}\to+\infty}\left(\lim_{t\to+\infty}\mathrm{e}^{\frac{(\tilde{j}-1)(\ln\mu_i+\ln\tilde{d}_i)-\underline{\gamma}_i(t-t^i_{k_i-(\tilde{j}-1)})}{\tilde{j}}}\right)\\ =&\lim_{\tilde{j}\to+\infty}\left(\lim_{t\to+\infty}\mathrm{e}^{\ln\mu_i\tilde{d}_i-\underline{\gamma}_i\tau^i_a(t^i_{k_i-(\tilde{j}-1)},t)-\frac{\ln\mu_i\tilde{d}_i}{\tilde{j}}}\right) = \varsigma_i,\end{aligned} \tag{47}$$

then one derives via (13) that $\varsigma_i < 1$. Furthermore, with the aid of the Cauchy's criterion for the convergence of the infinite series, one can conclude from (47) that the second additive term converges to a certain positive constant as $t \to +\infty$. For calculating such a constant, we specify the difference between the piecewise ADTs of (1) and their common lower bound as $t \to +\infty$ to be $\tilde{\varsigma}^i_j = \lim_{t\to+\infty}\tau^i_a(t^i_j, t) - \frac{\ln\mu_i+\ln\tilde{d}_i}{\underline{\gamma}_i}$, $j = 1, 2, \ldots, k_i - 1$. With (9), it is evident that $\forall j$, $\tilde{\varsigma}^i_j > 0$. Denoting $\underline{\varsigma}_i = \min_{\forall j}\tilde{\varsigma}^i_j$, then the constant can be calculated via (46) as follows

$$\begin{aligned}&\lim_{k_i\to+\infty}\left(\sum_{j=0}^{k_i-1}\mathrm{e}^{j(\ln\mu_i+\ln\tilde{d}_i)-\underline{\gamma}_i(t-t^i_{k_i-j})}\right)\\ =&\lim_{k_i\to+\infty}\left(\sum_{j=0}^{k_i-1}\mathrm{e}^{(j+1)\left((\ln\mu_i+\ln\tilde{d}_i)-\underline{\gamma}_i\tau^i_a(t^i_{k_i-j},t)\right)-(\ln\mu_i+\ln\tilde{d}_i)}\right)\\ \le&\lim_{k_i\to+\infty}\left(\sum_{j=0}^{k_i-1}\mathrm{e}^{-(j+1)\underline{\gamma}_i\underline{\varsigma}_i-(\ln\mu_i+\ln\tilde{d}_i)}\right)\\ =&\frac{1}{\mu_i\tilde{d}_i\left(\mathrm{e}^{\underline{\gamma}_i\underline{\varsigma}_i}-1\right)}.\end{aligned} \tag{48}$$

Therefore it follows: $V_{i,\sigma_i(t^i_{k_i})}(\varepsilon_i(t)) \rightarrow \overline{\epsilon}_i$ as $t \rightarrow +\infty$ where $\overline{\epsilon}_i = \frac{\tilde{C}_i \max_{\forall k_i}(\lambda_{\max}(P_{i,\sigma_i((t^i_{k_i})^-)}))}{\tilde{d}_i(e^{\underline{\gamma}_i \underline{\varsigma}_i}-1)}$ with conditions (12) and (13) holding. Recalling that in the above we have slightly abused $\varepsilon_i(t)$ to denote $\overline{\varepsilon}_i(t)$, then we have

$$\lim_{t\rightarrow+\infty} \|\overline{\varepsilon}_i(t)\| \leq \epsilon_i, \tag{49}$$

in which $\epsilon_i = \sqrt{\frac{\tilde{C}_i}{2(e^{\underline{\gamma}_i \underline{\varsigma}_i}-1)}}$. Recalling (31), with (37) and (49) then we have that the following holds:

$$\lim_{t\rightarrow+\infty} \|\varepsilon_i(t)\| \leq \lim_{t\rightarrow+\infty} \left(\|\overline{\varepsilon}_i(t)\| + \|\underline{\varepsilon}_i(t)\|\right) \leq \epsilon_i, \tag{50}$$

which implies $\lim_{t\rightarrow+\infty} \|x_i(t) - \Omega_{i,\phi_i}\xi_i(t)\| \leq \epsilon_i$. With (6) we have: $\lim_{t\rightarrow+\infty} \|y_i(t) - H\xi_i(t)\| \leq \max_{\forall\phi_i} \|C_{i,\phi_i}\|\epsilon_i$, which implies $\forall i, j \in \mathcal{V}$, $\lim_{t\rightarrow+\infty} \|y_i(t) - y_j(t)\| \leq \epsilon = 2\max_{\forall h\in\mathcal{V},\phi_h} \|C_{h,\phi_h}\|\epsilon_h$, i.e., the practical output synchronization is achieved. Note that $\epsilon_i = \max_{\forall k_i} \|\Omega_{i,\sigma_i((t^i_{k_i})^+)} - \Omega_{i,\sigma_i((t^i_{k_i})^-)}\| c_S \|\xi_i(t_0)\| / \sqrt{e^{\underline{\gamma}_i \underline{\varsigma}_i} - 1}$. This indicates that given certain dynamics settings of (1) and (4), the ultimate bound ϵ actually depends on the minimum margin between the piecewise ADT and its prescribed lower bound $\underline{\varsigma}_i$ among agents. It enables one to specify an arbitrarily small bound for the output synchronization error by specifying a switching signal with sufficiently large piecewise ADTs $\tau^i_a(t^i_j, t)$, $j = 1, 2, \ldots, k_i - 1$ for each agent i. Particularly, one can further conclude a special case where the asymptotical synchronization can be achieved for the outputs, i.e., $\epsilon = 0$. This can be implemented by requiring that for each agent i the output matrix C_{i,ϕ_i} does not switch. Since it will always yield a non-switching Ω_{i,ϕ_i} (see Huang, 2004 for detailed derivations), then it is obvious that $\|\Omega_{i,\sigma_i((t^i_{k_i})^+)} - \Omega_{i,\sigma_i((t^i_{k_i})^-)}\| = 0$, $\forall t^i_{k_i}$, which implies $\epsilon = 0$. Another trivial case is that there is not any switching occurring in a given time span, this would make $\varsigma_i \rightarrow +\infty$ which implies $\epsilon = 0$. The proof is complete. ■

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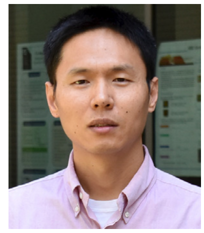

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