This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRS.2021.3100782, IEEE Transactions on Power Systems

An Incentive Compatible Iterative Mechanism for Coupling Electricity Markets

Alfredo Garcia, Roohallah Khatami, Ceyhun Eksin and Furkan Sezer

***Abstract*—The coordinated operation of interconnected but locally controlled electricity markets is generally referred to as a "coupling". In this paper we propose a new decentralized market mechanism for efficient coupling of independent electricity markets. The mechanism operates after each individual market has settled (e.g. hour-ahead). Based upon the reported supply and demand functions for internal market optimization (clearing), each market operator is asked to iteratively quote the terms of energy trade (on behalf of the agents participating in its market) across the transmission lines connecting to other markets. We show the mechanism's outcome converges to the optimal flows between markets given the reported supply and demand functions from each individual market clearing. In light of incentive compatibility issues that result from pricing power flows across interconnection lines with locational marginal prices, the mechanism features incentive transfers (updated at each iteration) that compensate each given market with its marginal contribution, i.e. the cost reduction to all other participating markets. We show that these transfers imply truthful participation in the mechanism is a Nash equilibrium. The proposed decentralized mechanism is implemented on the three-area IEEE Reliability Test System where the simulation results showcase the performance guarantees of the proposed mechanism.**

I. Introduction

The control and operation of many interconnected electricity markets (e.g. Western Europe, Northeast US) is conducted in a decentralized manner by independent agencies (e.g. system operators ISOs in the US, power exchanges PXs and transmission system operators TSOs in Europe). In a decentralized market architecture, each individual market operator has the necessary control authority over an area to ensure the associated power system operation achieves an acceptable trade-off between economic efficiency and reliability [1], [2]. However, the interconnection of decentralized electricity markets by means of transmission lines (or tielines) provides increased efficiency and/or reliability by enabling access to cheaper generation and/or flexibility in the form of reserves. The coordinated operation of interconnected but locally controlled electricity markets is generally referred to as market coupling. Such coupling can be seen as a consensus amongst independent market operators on the technical and economic terms associated to energy flows between markets.

In this paper we introduce a decentralized market coupling mechanism that operates after each individual market has settled (e.g. hour ahead). Based upon the reported supply and demand cost functions, each market operator is asked to iteratively quote the terms of energy trade (quantities and prices) across interties (on behalf of the agents participating in its market) by recomputing internal market clearing dispatch, a task that involves solving a DC optimal power flow (DC-OPF), which is a convex optimization problem. Each market operator exchanges voltage phase angles of boundary buses with adjacent market operators. This information is needed to ensure the flows across markets are feasible. A capacity price for each intertie is updated at every round (or iteration) to reflect excess demand and each market operator exchanges voltage phase angles of boundary buses with adjacent area operators (Section IV-C). The proposed market coupling design allows individual area operators to retain local control and can be easily implemented *after* existing short-run markets (e.g. hour ahead) have cleared. We show that the sequence of outcomes of the proposed market coupling mechanism converges to the optimal allocation and pricing of intertie capacity *given* the reported supply and demand functions from each individual market clearing (Theorem 1). In other words, the limit outcome corresponds to the solution of the joint (centralized) DC-OPF for all areas combined.

In the second result of the paper (Theorem 2) we show that any deviation from reported supply costs functions will result in (approximately) a lower net cost reduction. This result relies on incentive (monetary) transfers to each market operator which are set to compensate each market's *marginal* contribution to the coupling, i.e. the change in costs that a given area's participation induces in all other areas. Such estimate is a function of the reported information by market operators (i.e. desired intertie flows and locational marginal prices at boundary nodes). Setting incentive transfers approximately equal to each individual's market marginal contribution to the coupling, aligns the incentives of each individual market with that of minimizing total cost for all areas. Assuming supply costs function for internal market clearing are a Nash equilibrium, a corollary of Theorem 2 states that any allocation of cost reduction obtained from coupling in fixed proportions ensures the original internal equilibrium remains an approximate Nash equilibrium (after the coupling of areas is implemented). Thus, in this case, the proposed market coupling design has *negligible effect* on the structure of competition within each market.

II. Literature Review

The iterative mechanism considered in this paper is related to decentralized approaches for solving the joint interconnected economic dispatch problem in a distributed fashion via primal decomposition methods [3], [4], [5], [6] or dual decomposition methods [7], [8], [9]. Improvements to existing market coupling designs are considered in [10], [11]. However, this literature does not address incentive compatibility concerns arising from potential strategic behavior by participating

A. Garcia, C. Eksin and F. Sezer are with the Department of Industrial and Systems Engineering, Texas A&M University, College Station, TX 77843 USA (e-mails: alfredo.garcia@tamu.edu, eksinc@tamu.edu, furkan.sezer@tamu.edu). R. Khatami is with the Department of Electrical and Computer Engineering, University of British Columbia, Vancouver, BC V6T 1Z4 Canada (e-mail: roohallah.khatami@ece.ubc.ca).

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRS.2021.3100782, IEEE Transactions on Power Systems

agents, an important consideration for coupling electricity markets with locational marginal prices.

The paper is also related to the literature on distributed solutions to the economic dispatch problem in power systems to increase scalability, robustness to failure and cyber-security [2], [8], [12]–[19]. In contrast, in this paper we focus on coupling the operation of multiple independently operated markets via interties wherein each market optimizes its own internal operation but consensus must be achieved on the flows across markets.

Finally, the paper is also related to the literature on mechanism design on networks [20], [21] and [22] which studies incentives for efficient allocation of resources to individual agents interacting in a network. In contrast, in this paper we focus on incentives for the efficient interconnection of multiple networks (i.e. markets) via interties. Our overall goal is different as we seek to identify pricing and allocation rules for interconnection capacity that are guaranteed to not distort internal equilibrium conditions in each market and enable the identification of optimal allocation and pricing of intertie capacity *given* equilibrium supply and demand functions from each individual market clearing.

III. Incentives for Market Coupling: A Toy Illustration

To motivate the intricacies of pricing market coupling transactions we examine a toy illustration with independent markets A, B, and C each consisting of a single node with generation costs, and demand and interconnection lines illustrated in Figure 1.

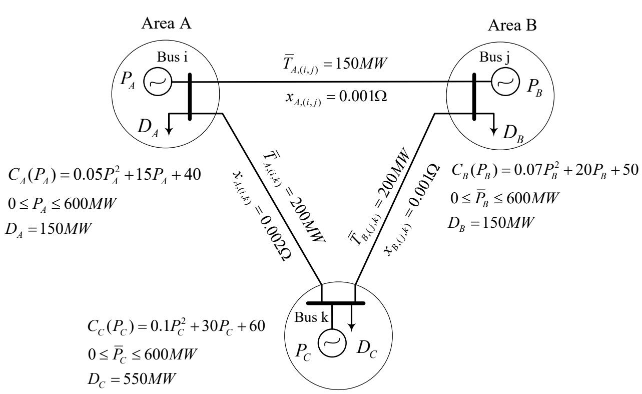

Fig. 1. Illustration on a Three-bus system

Assuming complete information, two coupling scenarios are characterized in Table I. First, a *full* coupling scenario (with *non-zero* flows across interconnection lines) labeled as $(A+B+C)$. Secondly, a *no coupling* scenario labeled as (A,B,C) in which there are no flows across interconnection lines (i.e. tielines between markets are switched off).

A *full coupling* of all markets brings about a cost reduction $46,810 - 9,436 = \$37,374$ for market C. This is achieved at the expense of cost increases for *individual* markets, namely $15,568 - 3,415 = \$12,153$ for A and $8,454 - 4,625 = \$3,920$ for B. Let π_i denote the monetary transfer to market $i \in \{A,B,C\}$ due to coupling. To implement a full market

TABLE I
Coupling Scenarios

	$(A+B+C)$	(A,B,C)
Operation Cost of A (\$)	15,568	3,415
Operation Cost of B (\$)	8,454	4,625
Operation Cost of C (\$)	9,436	46,810
Total Operation Cost (\$)	33,457	54,850

coupling, each individual market must be (weakly) better off. These participation constraints can be expressed as follows:

$$\begin{aligned} 15,568 - \pi_A &\leq 3,415 \\ 8,454 - \pi_B &\leq 4,625 \\ 9,436 - \pi_C &\leq 46,810 \end{aligned} \quad (1)$$

The details for the optimal DC power flow for full coupling scenario are given in Table II.

In an approach to coupling based upon locational marginal pricing, an independent third party (with complete information) can reproduce the information in Table II and compute payments of the form $\pi_A = \alpha_A(T_{A,B} + T_{A,C}) = 15,992$, $\pi_B = \alpha_B(T_{B,C} - T_{A,B}) = 4,298$ and $\pi_C = -\alpha_C(T_{B,C} + T_{A,C}) = -24,483$. It can be readily verified that these payments satisfy condition (1). However, in practice, internal cost and demand information for each market is *private*. To compute an optimal coupling, an independent third party would need to elicit reports from each market operator which in turn gather cost and demand information from market participants. However, using locational marginal prices to settle market coupling incentivizes *strategic manipulation* of reported information. For example, let us consider the case in which participants in market A report information corresponding to a cost function $C_A^D(P_A)$ that is a scaled version of the *true* cost function by a factor of 15%, i.e., $C_A^D(P_A) = 1.15C_A(P_A)$. The corresponding solution in Table II (under coupling scenario (A^D+B+C)) implies a reduction of power generation in market A with *true* cost equal to $C_A(383.7) = 13,157$. The payment to market A is $\pi_A^D = \alpha_A(T_{A,B} + T_{A,C}) = 14,319$. Since $C_A(383.7) - \pi_A^D < 15,568 - \pi_A$, the manipulation of cost reports pays off. This example illustrates how locational marginal pricing of interconnection power flows *will not induce efficient coupling of markets*.

Let us now consider a different approach to pricing based upon an estimate of the marginal contribution of each market $i \in \{A,B,C\}$, say M_i to the coupling. Formally, let M_i be defined as the aggregate cost reduction to markets $j \in \{A,B,C\} \backslash i$ that is made possible by *full coupling*. In this sense, the marginal contribution of each market can be computed as:

$$\begin{aligned} M_A &= 4,625 + 46,810 - 8,454 - 9,436 = 33,545 \\ M_B &= 3,415 + 46,810 - 15,568 - 9,436 = 25,221 \\ M_C &= 3,415 + 4,625 - 15,568 - 8,454 = -15,982 \end{aligned}$$

With transfers of the form $\pi_i := M_i - R$, $i \in \{A,B,C\}$ where $R > 0$ is participation fee, there is no incentive to manipulate reported information *because* π_i *does not depend on the information reported by market* i. With a participation fee

TABLE II
DC POWER FLOW

Coupling Scenario	Locational Marginal Prices ($/MWh)			Tieline Power Flows (MW)			Power Generation (MW)		
	α_A	α_B	α_C	$T_{A,B}$	$T_{A,C}$	$T_{B,C}$	P_A	P_B	P_C
$(A+B+C)$	57.7	52.5	68.2	118.1	159.0	200.0	427.1	231.9	190.9
(A^D+B+C)	61.3	56.5	71.0	89.1	144.5	200.0	383.7	260.8	205.4

$R = 14,261.33$, the incentive transfers exceed the increased costs in the lower cost markets A and B as per (1), i.e:

$$\pi_A = M_A - R = 19,283.67 > 15,568 - 3,415$$
$$\pi_B = M_B - R = 10,959.67 > 8,454 - 4,625,$$

so that consumers can be fully compensated by means for example of a rebate. Similarly, consumers in the higher cost market C are better off since the cost for *full coupling* scenario (including incentive payment $\pi_C = M_C - R = -30,243.33$) is less than the cost associated with *no coupling* scenario:

$$9,436 - \pi_C < 46,810$$

Finally, the mechanism is balanced since $\pi_A + \pi_B + \pi_C = 0$.

IV. MARKET COUPLING MECHANISM

Consider a power system composed of $A > 1$ operational areas, as schematically shown in Fig. 2, where the areas correspond to the associated electricity markets governed by separate ISOs[1]. Each area $a \in \mathcal{A} = \{1, 2, \ldots, A\}$ is represented by a directed graph $(\mathcal{N}_a, \mathcal{F}_a)$ where $\mathcal{N}_a = \{1, 2, \ldots, N_a\}$ and $\mathcal{F}_a = \{(i,j)|i,j \in \mathcal{N}_a, j \equiv j(i)\}$ respectively represent the set of nodes (buses) and edges (transmission lines). For area $a \in \mathcal{A}$, the bus voltage angles and nodal loads form respectively the vectors $\boldsymbol{\theta}_a = (\theta_{a,1}, \theta_{a,2}, \ldots, \theta_{a,N_a})^T$ and $\mathbf{D}_a = (D_{a,1}, D_{a,2}, \ldots, D_{a,N_a})^T$. The $N_a \times N_a$ admittance matrix of each area is denoted by $\mathbf{B}_a$. The set of G_a generating units at each area is represented by $\mathcal{G}_a = \{1, \ldots, G_a\}$, the power generation of units form the vector $\mathbf{P}_a = (P_{a,1}, P_{a,2}, \ldots, P_{a,G_a})^T$, and the $N_a \times G_a$ incidence matrix $\mathbf{M}_a$ maps the generating units to buses. The tielines entering/leaving area a are defined by set $\mathcal{T}_a = \{(i,j)|i \in \mathcal{N}_a, j \in \mathcal{N}_{a'}, a' \in \mathcal{A} \backslash a\}$, the associated tieline power flows form the vector $\mathbf{T}_a = \left(T_{a,(i,j)}\right), \ (i,j) \in \mathcal{T}_a$, and the incidence matrix mapping tieline power flows to buses is shown by $\mathbf{R}_a$.

A. Centralized DC-OPF

In describing the operation of the proposed market coupling mechanism, it will be useful to refer to the centralized DC-OPF problem where a single system operator optimally operates the entire power system with complete information regarding all the areas' generators supply costs. [2] This is a hypothetical exercise as in practice there is no single market operator and information on generators costs is not available to any single party. In the formulation below, the areas are separated and the transmission line power flows are distinguished from tieline power flows. This arrangement would facilitate formulating the decentralized DC-OPF, which is the backbone of the proposed market coupling mechanism. To sum up, in the centralized DC-OPF formulation the total operational cost of power system in (2) is minimized subject to the operation constraints:

[1] In the rest of this paper we use the terms "areas", "markets", and "ISOs" interchangeably.

[2] For ease of exposition, we leave out demand offers and assume demand is inelastic and known. This implies the internal market clearing corresponds to solving the DC power flow that minimizes total cost. The results can be extended with active demand participation and market clearing modeled as DC power flow maximizing aggregate surplus.

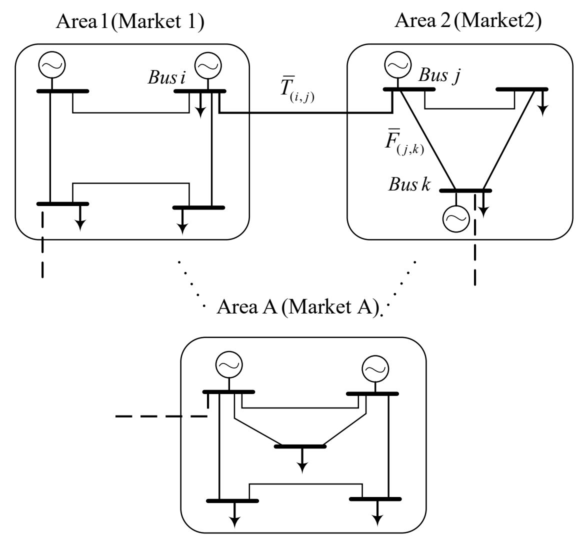

Fig. 2. Generic multi-area power transmission network.

$$\min_{P_{a,g}} \quad \sum_{a\in\mathcal{A}} \sum_{g\in\mathcal{G}_a} C_{a,g}(P_{a,g}), \tag{2}$$

$$\mathbf{B}_a\boldsymbol{\theta}_a + \mathbf{R}_a\mathbf{T}_a = \mathbf{M}_a\mathbf{P}_a - \mathbf{D}_a, \quad (\boldsymbol{\alpha}_a), \quad \forall a \in \mathcal{A}, \tag{3}$$

$$\underline{\mathbf{P}}_a \leq \mathbf{P}_a \leq \overline{\mathbf{P}}_a, \quad (\nu_{a,g}, \lambda_{a,g}), \quad \forall a \in \mathcal{A}, \tag{4}$$

$$-\overline{F}_{a,(i,j)} \leq \frac{\theta_{a,i} - \theta_{a,j}}{x_{a,(i,j)}} \leq \overline{F}_{a,(i,j)}, \quad (\kappa_{a,(i,j)}, \eta_{a,(i,j)}), \quad \forall a \in \mathcal{A}, \forall (i,j) \in \mathcal{F}_a, \tag{5}$$

$$T_{a,(i,j)} = \frac{\theta_{a,i} - \theta_{a',j}}{\bar{x}_{a,(i,j)}}, \quad (\xi_{a,(i,j)}), \quad \forall a \in \mathcal{A}, \ \forall (i,j) \in \mathcal{T}_a, \tag{6}$$

$$-\overline{T}_{a,(i,j)} \leq T_{a,(i,j)} \leq \overline{T}_{a,(i,j)}, \quad (\bar{\kappa}_{a,(i,j)}, \bar{\eta}_{a,(i,j)}), \ \forall a \in \mathcal{A}, \ \forall (i,j) \in \mathcal{T}_a, \tag{7}$$

$$\theta_{1,1} = 0, \tag{8}$$

where $C_{a,g}(P_{a,g})$ denotes the strictly convex cost functions of generating units, the vectors $\underline{\mathbf{P}}_a$ and $\overline{\mathbf{P}}_a$ respectively represent the minimum and maximum generation limits of units, and $\overline{F}_{a,(i,j)}$ and $\overline{T}_{a,(i,j)}$ respectively represent the power flow limits of transmission lines and tielines. The nodal power balance is secured through (3), the generation of units are confined to their limits in (4), transmission line power flows are maintained within thermal limits in (5) where $x_{a,(i,j)}$ represents the reactance of each line, and the tieline power flows are calculated and constrained to their thermal limits in (6) and (7) where $\bar{x}_{a,(i,j)}$ represents the tieline reactances. Further, the voltage phase angle of the slack bus, which is assumed to be the bus 1 of area 1, is set to zero in (8). Note that $\overline{T}_{a,(i,j)} = \overline{T}_{a',(j,i)}$ and $\bar{x}_{a,(i,j)} = \bar{x}_{a',(j,i)}$.

B. Decentralized DC-OPF

Given locational marginal prices ($\alpha_{a',j}$) and angles ($\theta_{a',j}$) of adjacent areas and the capacity price $\mu_{(i,j)}$, the decentralized DC-OPF problem for area $a \in \mathcal{A}$ is given by:

$$\min \sum_{g\in\mathcal{G}_a} C_{a,g}(P_{a,g}) - \sum_{(i,j)\in\mathcal{T}_a} \alpha_{a',j} T_{a,(i,j)} + \sum_{(i,j)\in\mathcal{T}_a} \frac{\mu_{(i,j)}}{2}(|T_{a,(i,j)}| - \bar{T}_{a,(i,j)}) \quad (9)$$

$$\mathbf{B}_a\boldsymbol{\theta}_a + \mathbf{R}_a\mathbf{T}_a = \mathbf{M}_a\mathbf{P}_a - \mathbf{D}_a, \qquad (\hat{\boldsymbol{\alpha}}_a), \quad (10)$$

$$\underline{\mathbf{P}}_a \le \mathbf{P}_a \le \overline{\mathbf{P}}_a, \quad (11)$$

$$-\overline{F}_{a,(i,j)} \le \frac{\theta_{a,i} - \theta_{a,j}}{x_{a,(i,j)}} \le \overline{F}_{a,(i,j)}, \quad \forall (i,j) \in \mathcal{F}_a, \quad (\hat{\kappa}_{a,(i,j)}, \hat{\eta}_{a,(i,j)}) \quad (12)$$

$$T_{a,(i,j)} = \frac{\theta_{a,i} - \theta_{a',j}}{\bar{x}_{a,(i,j)}}, \quad \forall (i,j) \in \mathcal{T}_a, \left(\hat{\xi}_{a,(i,j)}\right), \quad (13)$$

$$\theta_{a,1} = 0, \text{ if } a = 1. \quad (14)$$

Note the objective function in (9) not only includes area a's total generation cost but also accounts for the capacity costs of incident tielines as well as the cost/revenue of the energy import/export from/to neighboring areas. It is assumed that the interconnected areas split the capacity cost over tielines, meaning that the two incident areas of a tieline are both charged for the excess capacity usage at the rate of $\frac{\mu_{(i,j)}}{2}$. This is done with no loss of generality: other conventions (e.g. exporter is assigned capacity cost, importer is assigned capacity cost) can also be used without altering the results in this paper.

The following lemma makes precise in which way decentralization is made possible in this formulation.

Lemma 1 *If the locational marginal prices, angles of adjacent areas and the capacity price of interties correspond to the optimal solution of centralized OPF (i.e. $\alpha^*_{a',j}$, $\theta^*_{a',j}$ and $\frac{\mu^*_{(i,j)}}{2} sign(T^*_{a,(i,j)}) = \bar{\eta}^*_{a,(i,j)} - \bar{\kappa}^*_{a,(i,j)}$), then the solution to the decentralized OPF problem for area $a \in \mathcal{A}$ corresponds to the solution of the centralized OPF.*

Proof: See Appendix. ■

C. A Decentralized, Iterative Market Mechanism for Coupling

In this section we describe the design for coupling the operation of independent locally controlled markets—see Algorithm 1. At each iteration $k > 0$ of Algorithm 1,

1) (*Information exchange*) Each area reports the terms of trade for each interconnection in the form of intertie flows ($T^{k-1}_{a',(i,j)}$), locational marginal prices ($\alpha^{k-1}_{a',j}$) and angles of adjacent areas ($\theta^{k-1}_{a',j}$), where $\alpha^0_{a',j} = 0$.
2) (*Updating Intertie Flows*) Given a capacity price $\mu^{k-1}_{i,j}$, each area $a \in \mathcal{A}$ solves (9) - (14) and reports the desired flows $\hat{\mathbf{T}}^k_a$, corresponding angles at interties $\hat{\boldsymbol{\theta}}^k_a$ and locational marginal prices $\hat{\boldsymbol{\alpha}}^k_a$ to the agent in charge of executing the mechanism which updates the flows, angles and locational marginal prices along the interties according to:
$$\mathbf{T}^k_a = (1-\rho_k)\mathbf{T}^{k-1}_a + \rho_k \hat{\mathbf{T}}^k_a \quad (15)$$
$$\boldsymbol{\theta}^k_a = (1-\rho_k)\boldsymbol{\theta}^{k-1}_a + \rho_k \hat{\boldsymbol{\theta}}^k_a \quad (16)$$
$$\boldsymbol{\alpha}^k_a = (1-\rho_k)\boldsymbol{\alpha}^{k-1}_a + \rho_k \hat{\boldsymbol{\alpha}}^k_a \quad (17)$$
with $\rho_k \to 0^+$, $\sum_k \rho_k = +\infty$ and $\sum_k \rho_k^2 < \infty$.
3) (*Updating Intertie Capacity Prices*)
$$\mu^k_{(i,j)} = \max\{\mu^{k-1}_{(i,j)} + \beta(\frac{\left|T^k_{a,(i,j)}\right| + \left|T^k_{a',(i,j)}\right|}{2} - \bar{T}_{a,(i,j)}), 0\} \quad (18)$$
where $\beta \in (0,1)$ and $\mu^0_{(i,j)} = 0$.
4) An incentive transfer $\Delta\pi_{a,k}$ is allocated to each area $a \in \mathcal{A}$. Upon stopping after $T > 0$ iterations a coupling participation fee $R > 0$ is assessed to each area $a \in \mathcal{A}$.

The iterative mechanism described above is a synchronous algorithm. That is, each market solves its local DC-OPF problem at each step after receiving intertie flows and locational marginal prices from all neighboring areas. In updating the intertie flows and angles, and locational marginal prices, each area uses an inertial update in (15)-(17) by weighting the current optimal solution, i.e., $\hat{\mathbf{T}}^k_a$, $\hat{\boldsymbol{\theta}}^k_a$, $\hat{\boldsymbol{\alpha}}^k_a$, with the previous step, rather than using the current optimal solution to the decentralized DC-OPF problem. From the updated capacity price in (18), both sending and receiving areas use the same capacity price in their DC-OPF problem at each iteration k. Moreover, the capacity price is updated with a constant step size (β), while the inertial updates assume a diminishing step size. The convergence analysis is based on the exploitation of this fast update of the capacity prices versus the slow update of intertie flows and angles, and locational marginal prices. The update mechanism is complemented by an *incentive (monetary) transfer* $\Delta\pi_{a,k}$ and a participation fee $R > 0$ which will be analyzed in Section VI.

Remark 1 *(Computational Feasibility) To examine real-time implementation issues, consider interconnected energy markets that schedule intertie transactions in 15 minute blocks. Recall that each iteration of the market coupling algorithm*

Algorithm 1: Market Coupling Mechanism

Require: Initialize $\boldsymbol{x}_a^0 = (\mathbf{T}_a^0, \boldsymbol{\theta}_a^0, \boldsymbol{\alpha}_a^0)$, and $\mu_{(i,j)}^0$ for all areas and tielines.
Require: Maximum number of iterations $T+1$. Set $k = 1$
while $k \leq T+1$ **do**
- Run DC-OPF problems of all areas and obtain $\hat{\boldsymbol{x}}_a^k = (\hat{\mathbf{T}}_a^k, \hat{\boldsymbol{\theta}}_a^k, \hat{\boldsymbol{\alpha}}_a^k)$.
- Update $\boldsymbol{x}_a^k = (1-\rho_k)\boldsymbol{x}_a^{k-1} + \rho_k \hat{\boldsymbol{x}}_a^k$ for all areas.
- Update $\mu_{(i,j)}^k$ as in (18) for all tielines.
- Exchange $\boldsymbol{x}_a^k$ between neighboring areas and compute incentive transfers $\Delta\pi_{a,k}$.
- Set $k = k+1$.

end

requires each market operator to solve anew the internal DC-OPF problem (a convex optimization problem with a "hot start" solution). Assuming each iteration takes 0.25 *seconds (inclusive of communication overhead), a budget of* 1 *minute would allow a total of* $T = \frac{60}{0.25} = 240$ *iterations of the market coupling algorithm.*

Remark 2 *(Non-Participation) We assume that when a market operator does not participate in the coupling mechanism all existing interconnection lines with that market are physically disconnected or switched-off. The ability to maintain (fixed) power flows over tielines with a given market may also enable other forms of non-participation. However, this ability may require additional control measures (e.g., phase shifter, automatic generation control (AGC)).*

V. Convergence Analysis

We make the following standing assumption on the feasibility of each area in the absence of tielines.

Assumption (Area feasibility): There exists a feasible solution to the decentralized DC-OPF problem (9)-(14) for area $a \in \mathcal{A}$ when $T_{a,(i,j)} = 0$ for all $(i,j) \in \mathcal{T}_a$.

The above assumption states that each market can meet the demand in its own area without relying on the intertie power flows. A manifestation of this assumption on coupled markets is the existence of a maximum intertie capacity price $\bar{\mu}_{(i,j)} > 0$ for $(i,j) \in \mathcal{T}_a$ such that for $\mu_{(i,j)} \geq \bar{\mu}_{(i,j)}$ the optimal solution to (9) - (14) will have no flow along the intertie, i.e. $T_{a,(i,j)} = 0$—which we summarize as Proposition 1 below.

Proposition 1 *If there exists a feasible solution to the decentralized DC-OPF problem (9)-(14) for area* $a \in \mathcal{A}$ *when* $T_{a,(i,j)} = 0$ *for all* $(i,j) \in \mathcal{T}_a$*, then for each intertie* (i,j)*, there exists a critical price* $\bar{\mu}_{(i,j)} > 0$ *such that for* $\mu_{(i,j)} \geq \bar{\mu}_{(i,j)}$ *the optimal solution to (9) - (14) will have no flow along the intertie, i.e.* $T_{a,(i,j)} = 0$.

Proof: See appendix. ■

In the first part of the convergence analysis, we show that the capacity multipliers converge.

Lemma 2 *For every intertie* (i,j) *it holds that* $\mu_{(i,j)}^k \to \mu_{(i,j)}^\infty \geq 0$ *and*

$$\mu_{(i,j)}^k\left(\frac{\left|T_{a,(i,j)}^k\right| + \left|T_{a',(i,j)}^k\right|}{2} - \bar{T}_{a,(i,j)}\right) \to 0 \tag{19}$$

Proof: See appendix. ■

The above result implies that the asymptotic capacity price $\mu_{(i,j)}^\infty$ is dual feasible and satisfies a complementary slackness condition (19).

Our next result shows that the sequence of outcomes of the proposed market coupling mechanism converges to the optimal allocation and pricing of intertie capacity *given* the reported supply and demand functions from each individual market clearing.

Theorem 1 *Assume each market operator* $a \in \mathcal{A}$ *reports the desired terms of trade at each iteration k,* $\boldsymbol{x}_a^k = (\mathbf{T}_a^k, \boldsymbol{\theta}_a^k, \boldsymbol{\alpha}_a^k)$ *consistent with the solution to decentralized OPF in (9)-(14) with costs* $C_{a,g}(\cdot), g \in \mathcal{G}_a$*. Then for all* $a \in \mathcal{A}$*,* $\boldsymbol{x}_a^k \to \boldsymbol{x}_a^*$ *where* $\boldsymbol{x}^*$ *in the solution to the associated centralized DC-OPF problem.*

Proof: See appendix. ■

The main steps in the proof of Theorem 1 involves showing convergence of the capacity prices (Lemma 2) and the intertie flows, angles, and locational marginal prices ($\boldsymbol{x}_a^k = (\mathbf{T}_a^k, \boldsymbol{\theta}_a^k, \boldsymbol{\alpha}_a^k)$) to some finite value. In showing their convergence, we rely on the updates of $\boldsymbol{x}_a^k$ with diminishing step size in (15)-(17) and Lipschitz continuity of solutions to the decentralized DC-OPF problem. In the second part of the proof, we show that these convergence points have to correspond to the optimal solution of the centralized DC-OPF problem via the equivalence of KKT conditions between the decentralized and centralized DC-OPF problems.

VI. Incentive Compatibility

In this section we define the incentive transfers $\Delta\pi_{a,k}$ based upon market a's marginal contribution to the coupling defined as the aggregate cost reduction of all other markets $a' \in \mathcal{A} \backslash a$ from full coupling with a.

A. Incentive Transfers $\Delta\pi_a$

The cost of area a' associated with the updates $\boldsymbol{x}^k$ is denoted using $C_{a',k} := \sum_{g \in \mathcal{G}_{a'}} C_{a',g}(P_{a',g}^k)$. The incremental change in cost of area a' from iteration k to iteration $k+1$ is denoted by $\Delta C_{a',k} := C_{a',k+1} - C_{a',k}$. The marginal contribution of area a at step k is defined as the change in cost to all areas $a' \in \mathcal{A} \setminus \{a\}$ as follows:

$$\Delta M_{a,k} := \sum_{a' \in \mathcal{A} \setminus \{a\}} \Delta C_{a',k} \tag{20}$$

Using the definitions above, we define the incentive transfer for area a as follows:

$$\Delta\pi_{a,k} = \Delta M_{a,k} + r_{a,k+1} - r_{a,k} \tag{21}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRS.2021.3100782, IEEE Transactions on Power Systems

where

$$r_{a,k} := \sum_{(i,j)\in\mathcal{T}_a} \alpha^{k-1}_{a',j} \hat{T}^k_{a,(i,j)} - \sum_{(i,j)\in\mathcal{T}_a} \frac{\mu^{k-1}_{(i,j)}}{2} \left(\left| \hat{T}^k_{a,(i,j)} \right| - \bar{T}_{a,(i,j)} \right). \quad (22)$$

The incentive transfers give area a its marginal contribution to the coupling and offsets the net change value of intertie trades for area a, i.e. $r_{a,k+1} - r_{a,k}$. Hence the net change in cost for area a during iteration k of the market mechanism is:

$$C_{a,k+1} - r_{a,k+1} - (C_{a,k} - r_{a,k}) + \Delta\pi_{a,k} = C_{a,k+1} - C_{a,k} + \sum_{a'\in\mathcal{A}\setminus a} \Delta C_{a',k} \quad (23)$$

Hence, the net change in cost for area a corresponds to the net change in total cost for all coupled markets. Thus, individual incentives are aligned with the minimization of the total cost of coupled markets.

B. Estimating Cost Changes

Note that the incremental change in cost $\Delta C_{a,k}$ is *not reported* by market $a \in \mathcal{A}$. However, as we shall show in the next result it can be estimated using the reported terms of trade across interconnection lines.

Let $V_{a,k}$ denote the optimal value of area a's DC-OPF problem (9)-(14) at iteration k, i.e.:

$$V_{a,k} = C_{a,k} - r_{a,k}. \quad (24)$$

Note that $\Delta C_{a,k} = \Delta V_{a,k} + r_{a,k+1} - r_{a,k}$. The following result gives an estimate of $\Delta V_{a,k}$ based upon reported information so that an estimate of $\Delta C_{a,k}$ can be obtained relying only on reported information by market operators.

Lemma 3 *A first order approximation to the change in optimal value for market $a \in \mathcal{A}$ at iteration k is given by:*

$$\Delta V_{a,k} = -\sum_{(i,j)\in\mathcal{T}_a} \Big[\hat{T}^k_{a,(i,j)} (\alpha^k_{a',j} - \alpha^{k-1}_{a',j}) - \frac{1}{2} \left| \hat{T}^k_{a,(i,j)} \right| (\mu^k_{(i,j)} - \mu^{k-1}_{(i,j)}) - \frac{\hat{\xi}^k_{a,(i,j)}}{\bar{x}_{a,(i,j)}} (\theta^k_{a',j} - \theta^{k-1}_{a',j}) \Big] \quad (25)$$

with:

$$\hat{\xi}^k_{a,(i,j)} = \alpha^{k-1}_{a',i} - \frac{\mu^{k-1}_{(i,j)}}{2} sign(\hat{T}^k_{a,(i,j)}) - \hat{\alpha}^k_{a,i}. \quad (26)$$

Proof: See appendix. ■

C. Truthful Reporting is Approximate Nash Equilibrium

Our decentralized iterative mechanism is designed to operate after intra-day markets (e.g. hour-ahead) have cleared, that is, after active agents in each individual market have submitted offers to sell electricity in the next hour. In the next result, we examine the incentives that individual market participants may have to manipulate the offers submitted to the respective market operator in light of market coupling. Specifically, we consider the possibility of strategic behavior by generators in market $a \in \mathcal{A}$ in the form of reporting strictly convex functions $C^D_{a,g}(\cdot)$ that may not be *truthful*, i.e. $C^D_{a,g}(\cdot) \neq C_{a,g}(\cdot)$. Below we formalize the assumption that every market operator reports the results of their respective DC-OPF market clearing problem with respect to the strictly convex but possibly *untruthful* cost functions reported by active agents in that market. This is a natural assumption since for short-term markets (e.g. hour ahead) most electricity markets use bidding formats that ensure the reported cost functions are convex (e.g. increasing marginal costs for increasing levels of generation).

Definition 1 *(Convex reporting strategy) Area $a \in \mathcal{A}$ follows a convex reporting strategy if and only for every iteration $k > 0$, such area reports the values $\hat{\boldsymbol{x}}^{D,k}_a = (\hat{\mathbf{T}}^{D,k}_a, \hat{\boldsymbol{\theta}}^{D,k}_a, \hat{\boldsymbol{\alpha}}^{D,k}_a)$ corresponding to the solution to decentralized DC-OPF in (9)-(14) with strictly convex objective function $C^D_{a,g}(\cdot), g \in \mathcal{G}_a$.*

The following is the main result of this section:

Theorem 2 *Truthful reporting of information in the market coupling mechanism is an approximate Nash equilibrium, i.e. any unilateral deviation in the form of a convex reporting strategy with $C^D_{a,g}(\cdot) \neq C_{a,g}(\cdot)$ results in a total net cost reduction for market $a \in \mathcal{A}R$ of at most ϵ_T with $\epsilon_T \to 0$ as $T \to \infty$.*

Proof: See appendix. ■

Corollary *Assume the reported supply costs functions by participants for internal market clearing are a Nash equilibrium for each market $a \in \mathcal{A}$. The reported supply costs functions remain an approximate Nash equilibrium after coupling with neighboring markets.*

Proof : By Theorem 2, the incentive to manipulate the reported supply or demand function is bounded by ϵ_T which is negligible for large $T > 0$. ■

VII. Numerical Testbed

Here we provide the numerical results for the three-zone IEEE Reliability Test System (IEEE-RTS) with 96 generating units, 73 buses, 115 transmission lines, and 5 tielines. The topology of the test system under study, the operation limitations of generating units and lines/tielines, as well as the nodal load data are available in [23]. We make the following changes to the original network in order to simulate the occasion where transmission lines/tielines are congested:

- The capacity of transmission line located in area 1 which connects the buses $(16, 17)$ is reduced from 500MW to 200MW, e.g., $\overline{F}_{1,(16,17)}$=200MW.
- The capacity of transmission line located in area 2 which connects the buses $(3, 24)$ is reduced from 400MW to 150MW, e.g., $\overline{F}_{2,(3,24)}$=150MW.
- The capacity of tieline 4 is reduced from 500MW to 100MW.

For quick accessibility, we also provide the tieline data in Table VIII, where the set of areas is defined as $\mathcal{A} = \{\text{A}, \text{B}, \text{C}\}$, in accordance with [23].

We carry out the simulation for a single hour corresponding to the peak load of the three-zone IEEE-RTS, and assume all generating units are online.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRS.2021.3100782, IEEE Transactions on Power Systems

TABLE III
TIELINE DATA FOR THE THREE-AREA IEEE-RTS

Tieline Number	Sending End (Area-Bus)	Receiving End (Area-Bus)	Reactance (pu)	Capacity (MW)
TL1	A-7	B-3	0.16	175
TL2	A-13	B-15	0.08	500
TL3	A-23	B-17	0.07	500
TL4	C-25	A-21	0.10	100
TL5	C-18	B-23	0.10	500

A. Numerical Results of Market Coupling

Here we present the numerical results of implementing the proposed iterative market mechanism on the three-area IEEE-RTS. The constant step-size associated with the capacity price updates is $\beta = 0.3$, the diminishing step-size associated with intertie power flow updates as well as the voltage phase angles and locational marginal price updates are deemed as $\rho_k = \frac{1}{1+\log(k)}$. The simulation results indicate that the values obtained from the iterative market mechanism converge to that of the centralized DC-OPF model, confirming the efficiency of the proposed method.

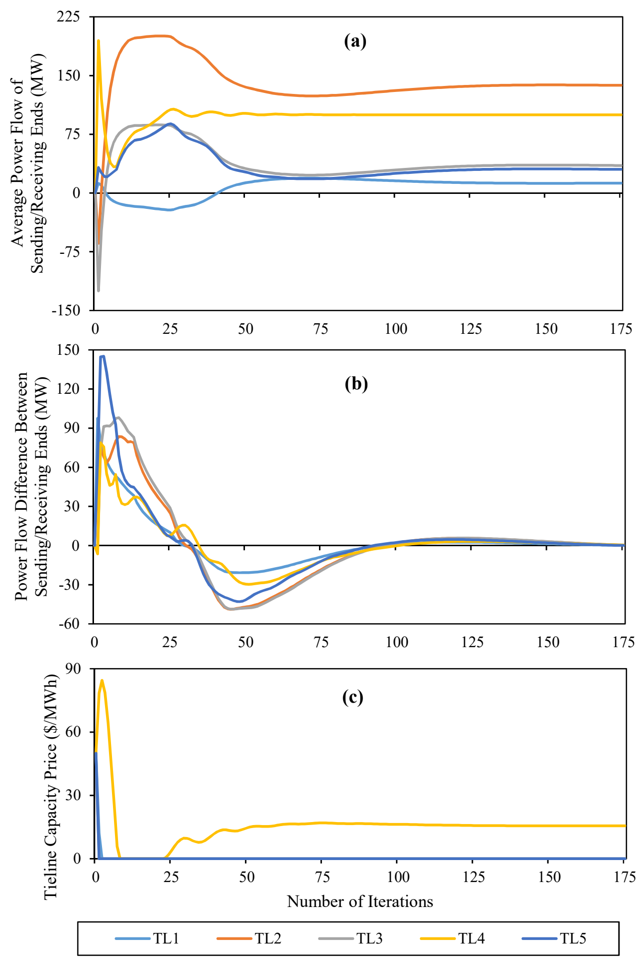

Fig. 3. (a) Average value of power flows calculated at sending and receiving ends, $\frac{T^k_{a,(i,j)} - T^k_{a',(j,i)}}{2}$, (b) Differences between power flows calculated at the sending and receiving ends, $T^k_{a,(i,j)} + T^k_{a',(j,i)}$, (c) Tieline capacity prices, $\mu^k_{a,(i,j)}$, ($/MW)

Figures 3-(a) and (b) respectively represent the average of and difference between tieline power flows calculated at the sending and receiving ends for each iteration, while Fig. 3-(c) shows the capacity price of the associated tielines. The initial values of tieline power flows are all zero, meaning that the initial state corresponds to the independent market clearing of all areas. For the first few iterations, as shown in Fig. 3-(a), we observe abrupt variations and erratic changes in tieline power flows. After few iterations, the convex weighted updates in (15) take effect and instill smoother changes until the power flow trajectories start to settle and the areas start to reach consensus on the amount of power flows at the sending and receiving ends (please see Fig. 3-(b)). The capacity prices in Fig. 3-(c) are all initialized at $50 per MWh, which is slightly greater than the highest incremental cost rate of the most expensive generator. The power flow of tieline number 4 converges to its maximum limit, 100MW, and bears a capacity price of $15.6 per MWh, while the power flows of other tielines remain within their operating limits and all the associated capacity prices converge to zero. We present the locational marginal prices at tieline incident buses, or indeed the import/export quotes of the areas, in Fig. 4 for all iterations. Further, we provide the optimal tieline power flows, capacity prices, and the locational marginal prices at incident buses in Table IV.

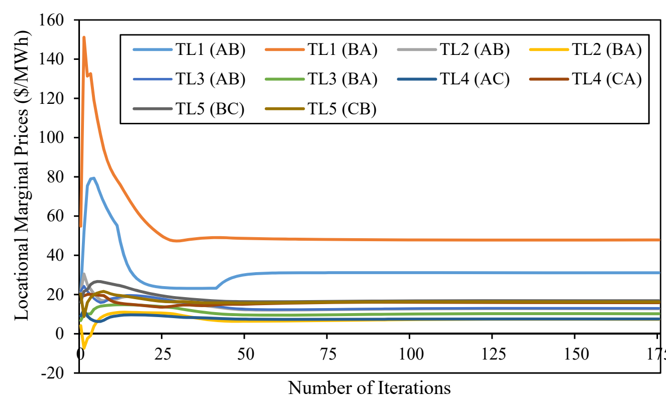

Fig. 4. Locational marginal prices of tieline incident buses ($/MWh)

B. Incentive Transfers Analysis

Here we first provide the results of truthful reporting and next consider the untruthful reporting.

1) Truthful Reporting: We provide the operation costs of areas in Table V for the full coupling and independent operation of areas. As expected, any coupling leads to a reduction in total operation cost of the system compared to independent operation of areas, and the full coupling aggregate cost reduction is $\$3,866$,

With a participation fee $R = 2,578$, the incentive transfers are as shown in Table VI. Area B receives the highest reward due to its marginal contribution to the coupling. Areas A and C come second and third in rewards, respectively (Table VI). The net transfer (i.e. marginal contribution to coupling minus participation fee) is also displayed. In the end, markets A and C are charged $732 and $1253 respectively and market B is compensated $732 + 1253 = \$1985$ for a zero net budget.

TABLE IV
OPTIMAL COUPLING OUTCOMES FOR THE THREE-AREA IEEE-RTS

Tieline Number	Power Flow (MW)	Capacity Price ($/MWh)	Sending End LMP ($/MWh)	Receiving End LMP ($/MWh)
TL1	13.1	0	31.1	47.8
TL2	-136.5	0	12.7	7.3
TL3	-34.3	0	13.0	10.2
TL4	-100.0	15.6	15.9	7.5
TL5	-29.3	0	16.3	16.8

TABLE V
COUPLING SCENARIOS FOR THE THREE-AREA IEEE-RTS

	Coupling Scenarios	
	(A+B+C)	(A,B,C)
Operation Cost of A	29,822	31,843
Operation Cost of B	34,062	33,366
Operation Cost of C	25,995	28,537
Total Operation Cost	89,879	93,745

TABLE VI
SUMMARY OF NET TRANSFER FROM TRUTHFUL REPORTING

Participation Fee R ($)	Marginal Contribution to Coupling ($)			*Net* Coupling Transfer $M_i - R$ ($)			Mechanism's Net Budget ($)
	M_A	M_B	M_C	A	B	C	
2,578	1,846	4,563	1,325	-732	1985	-1253	0

2) Deviation Reporting: In this part we consider three deviation reporting cases. In each case deviations only come from one area at a time. The deviation cost functions are equal to the given cost functions scaled by a factor of $1+r$, meaning that, the areas A, B, and C respectively use the cost functions $C^D_{i,g}(P_{A,g}) = (1+r)C_{i,g}(P_{A,g}), g \in \mathcal{G}_i,\ i \in \{A,B,C\}$ and $r \in \{-0.1, +0.1\}$. We have provided the net cost reductions of areas in Table VII.

TABLE VII
NET COST REDUCTIONS OF AREAS FROM TRUTHFUL REPORTING AND REPORTING COSTS WITH $r \in \{-0.1, +0.1\}$

	Net Cost Reduction ($)		
	Truthful	$r = +0.1$	$r = -0.1$
Market A	$1,289$	$1,252$	$1,279$
Market B	$1,289$	$1,279$	$1,223$
Market C	$1,289$	$1,283$	$1,284$

None of the areas manage to improve their net cost reduction through deviation reporting, and all the attempts lead to lower cost reductions compared to the truthful reporting counterpart (Table VII). Since the agent attempting a manipulation receives a fraction of the net cost reduction resulting from coupling, there is no incentive to manipulate the supply function in the original internal market equilibrium.

C. Discussion on Computation Time

The proposed iterative market mechanism for coupling is solved using CPLEX 12.6.2 on a desktop computer with a 2.7GHz $i7$ processor and 16GB of RAM. Each area takes a fraction of a second (0.1s-0.2s) to run their DC-OPF (in parallel with other areas). Thus, the maximum time elapsed for each iteration is equal to the greatest simulation time of areas ($\leq$ 0.2 s), plus the time required for tieline capacity price calculation and data exchange/storage (≤ 0.1 s). Given the following convergence bounds for tieline power flows and capacity prices:

$$T^k_{a,(i,j)} + T^k_{a',(i,j)} \leq 0.5,\ \forall (i,j) \in \mathcal{T}_a, \quad (27)$$

$$\mu^k_{(i,j)} - \mu^{k-1}_{(i,j)} \leq 0.1,\ \forall (i,j) \in \mathcal{T}_a, \quad (28)$$

the iterative market coupling converges in 175 iterations. Thus, the total simulation time is $\leq 175 \times 0.3 = 52.5$s, which is favorable for hour-ahead scheduling.

In order to further highlight the scalability of the proposed model, two more test cases are simulated:

- Case 1: IEEE-RTS + three extra tielines
- Case 2: IEEE-RTS + IEEE-RTS + three tielines connecting the corresponding mirrored areas

The tieline data for cases 1 and 2 are available in Appendix X-G. For the same convergence bounds as in (27) and (28), cases 1 and 2 converge respectively in 230 and 178 iterations showcasing the scalability of the proposed model for highly interconnected areas, as well as greater number of participating areas.

VIII. CONCLUSIONS

We have introduced and analyzed a market design for scheduling and pricing power flows between interconnected electricity markets. Each area operator participates (on behalf of agents active in its market) by iteratively submitting bids for trading energy across interties and the prices for interconnection capacity are adjusted as a function of excess demand. The proposed design allows individual area operators to retain local control and can be easily implemented *after* existing short-run markets (e.g. hour ahead) have cleared. We introduce *monetary (incentive) transfers* and show that the market coupling design does not alter the structure of incentives in each internal market, i.e. any internal market equilibrium will remain so (approximately) after coupling is implemented. In day-ahead markets, interconnection lines with adjacent markets can be used for procuring reserves. In our future work, we plan to extend the market coupling mechanism to enable trading of reserves across interconnection lines.

IX. Acknowledgements

Alfredo Garcia gratefully acknowledges partial support from NSF grant #1829552 and the Texas A&M Energy Institute. Ceyhun Eksin and Furkan Sezer were, in part, supported by grants from the National Science Foundation (NSF CCF-2008855 and NSF ECCS-1953694).

X. Appendix

A. Proof of Lemma 1

The first order conditions with respect to $\theta_{a,i}$, $\theta_{a',j}$, and $T_{a,(i,j)}$ for tieline $(i,j) \in \mathcal{T}_a$ in the centralized DC-OPF problem (2)-(8) are:

$$\sum_{j|(i,j)\in\mathcal{F}_a} \Big[\frac{1}{x_{a,(i,j)}}(\alpha^*_{a,i} - \alpha^*_{a,j} + \eta^*_{a,(i,j)} - \kappa^*_{a,(i,j)}) - \frac{1}{\bar{x}_{a,(i,j)}}\xi^*_{a,(i,j)}\Big] = 0 \quad (29)$$

$$\sum_{j|(i,j)\in\mathcal{F}_{a'}} \Big[\frac{1}{x_{a',(i,j)}}(\alpha^*_{a',i} - \alpha^*_{a',j} + \eta^*_{a',(i,j)} - \kappa^*_{a',(i,j)}) + \frac{1}{\bar{x}_{a,(i,j)}}\xi^*_{a,(i,j)}\Big] = 0 \quad (30)$$

$$\alpha^*_{a,i} - \alpha^*_{a',j} + \bar{\eta}^*_{a,(i,j)} - \bar{\kappa}^*_{a,(i,j)} + \xi^*_{a,(i,j)} = 0 \quad (31)$$

Consider the first order conditions with respect to $\theta_{a,i}$ and $T_{a,(i,j)}$ in the decentralized DC-OPF problem (9)-(14) for area $a \in \mathcal{A}$:

$$\sum_{j|(i,j)\in\mathcal{F}_a} \Big[\frac{1}{x_{a,(i,j)}}(\hat{\alpha}_{a,i} - \hat{\alpha}_{a,j} + \hat{\eta}_{a,(i,j)} - \hat{\kappa}_{a,(i,j)}) - \frac{1}{\bar{x}_{a,(i,j)}}\hat{\xi}_{a,(i,j)}\Big] = 0 \quad (32)$$

$$\hat{\alpha}_{a,i} + \hat{\xi}_{a,(i,j)} = \alpha^*_{a',j} - \frac{\mu^*_{(i,j)}}{2}\text{sign}(\hat{T}_{a,(i,j)}) \quad (33)$$

where $\hat{T}_{a,(i,j)}$ is the optimal transfer flow for area a and $\text{sign}(x) = 1$ if $x \geq 0$ and $\text{sign}(x) = -1$ if $x < 0$.

We check now that the centralized DC-OPF solution is a solution to each decentralized DC-OPF problem, namely that conditions (17)-(18) are satisfied with $\hat{\alpha}_{a,i} = \alpha^*_{a,i}$, $\hat{\xi}_{a,(i,j)} = \xi^*_{a,(i,j)}$ and

$$\hat{T}_{a,(i,j)} = \begin{cases} T^*_{a,(i,j)} & T^*_{a,(i,j)} \geq 0 \\ -T^*_{a,(i,j)} & T^*_{a,(i,j)} < 0 \end{cases}$$

Thus, it can be seen that equation (8) is equivalent to (17). Similarly, equation (18) corresponds to equation (10) above. The remaining optimality conditions related to internal operation of area a match those of the centralized OPF problem.

B. Proof of Proposition 1

We prove by contradiction. Suppose that there exists an optimal solution such that for a tieline $(i^*,j^*) \in \mathcal{T}_a$, we have $T^\epsilon_{a,(i^*,j^*)} = \epsilon \neq 0$ for any capacity price $\mu_{(i^*,j^*)} > 0$. Denote the power generation and tieline flows of this optimal solution with $P^\epsilon_{a,g}$ and $T^\epsilon_{a,(i,j)}$, respectively. In this case, the optimal objective value in (9) is given by

$$\sum_{g\in\mathcal{G}_a} C_{a,g}(P^\epsilon_{a,g}) - \epsilon\alpha_{a',j^*} + \frac{\mu_{(i^*,j^*)}}{2}(|\epsilon| - \bar{T}_{a,(i*,j*)}) - \sum_{(i,j)\in\mathcal{T}_a\setminus(i^*,j^*)} \frac{\mu_{(i,j)}}{2}\bar{T}_{a,(i,j)}. \quad (34)$$

Consider a feasible solution with $T_{a,(i,j)} = 0$ for all $(i,j) \in \mathcal{T}_a$. We denote the power generation of the feasible solution with zero tieline flows by $P^0_{a,g}$. Given the feasible solution, the objective value is given by $\sum_{g\in\mathcal{G}_a} C_{a,g}(P^0_{a,g}) - \sum_{(i,j)\in\mathcal{T}_a} \frac{\mu_{(i,j)}}{2}\bar{T}_{a,(i,j)}$.

Considering the difference between the objective value in (34) with that of the feasible solution when $T_{a,(i^*,j^*)} = 0$, we have

$$\sum_{g\in\mathcal{G}_a} C_{a,g}(P^\epsilon_{a,g}) - \epsilon\alpha_{a',j^*} + \frac{\mu_{(i^*,j^*)}}{2}|\epsilon| - \sum_{g\in\mathcal{G}_a} C_{a,g}(P^0_{a,g}) < 0 \quad (35)$$

which must be negative by the assumption that $(P^\epsilon_{a,g}, T^\epsilon_{a,(i,j)})$ is optimal. Observe now that there exists a capacity price value $\bar{\mu}_{(i^*,j^*)} > 0$ such that the difference in objective values (35) becomes positive for any $\epsilon \neq 0$. Thus, for each intertie (i,j), there exists a price $\bar{\mu}_{(i,j)} > 0$ such that for $\mu_{(i,j)} \geq \bar{\mu}_{(i,j)}$ the optimal solution will have no flow along the intertie, i.e. $T_{a,(i,j)} = 0$.

C. Proof of Lemma 2

Suppose the sequence $\{\mu^k_{(i,j)} : k > 0\}$ does not converge. So either *(i)*, $\mu^k_{(i,j)} \to +\infty$ or *(ii)*, it oscillates with $\liminf \mu^k_{(i,j)} < \limsup \mu^k_{(i,j)}$. Case *(i)* is easily discarded since from the assumption on maximum intertie capacity price:

$$\mu^k_{(i,j)} \to +\infty \quad \Rightarrow \quad \left|T^k_{a,(i,j)}\right| + \left|T^k_{a',(i,j)}\right| \to 0$$

which is a contradiction. Now let us consider case *(ii)*. Capacity price oscillation implies that for some $\epsilon > 0$

$$\limsup_k\Big\{\frac{\left|T^k_{a,(i,j)}\right| + \left|T^k_{a',(i,j)}\right|}{2} - \bar{T}_{a,(i,j)}\Big\} \geq \epsilon > 0$$
$$\geq \liminf_k\Big\{\frac{\left|T^k_{a,(i,j)}\right| + \left|T^k_{a',(i,j)}\right|}{2} - \bar{T}_{a,(i,j)}\Big\}$$

Thus the sequence $\{\frac{|T^k_{a,(i,j)}|+|T^k_{a',(i,j)}|}{2} - \bar{T}_{a,(i,j)}\}$ exhibits infinitely many upcrossings of $\frac{\epsilon}{4}$ and $\frac{3\epsilon}{4}$. Let $k > 0$ denote an index for an iteration in which an upcrossing of $\frac{\epsilon}{4}$ takes place, i.e.:

$$\frac{\left|T^k_{a,(i,j)}\right| + \left|T^k_{a',(i,j)}\right|}{2} - \bar{T}_{a,(i,j)} \geq \frac{\epsilon}{4}, \text{ and}$$
$$\frac{\left|T^{k-1}_{a,(i,j)}\right| + \left|T^{k-1}_{a',(i,j)}\right|}{2} - \bar{T}_{a,(i,j)} < \frac{\epsilon}{4}. \quad (36)$$

Since $T^{k+1}_{a,(i,j)} - T^{k}_{a,(i,j)} = \rho_k(\hat{T}^k_a - T^k_a)$ it follows that

$$\sum_{\ell=1}^{\tau(k)}(T^{k+\ell}_{a,(i,j)} - T^{k+\ell-1}_{a,(i,j)}) = \sum_{\ell=1}^{\tau(k)}\rho_{k+\ell}(\hat{T}^{k+\ell}_a - T^{k+\ell}_a) \leq 2\bar{T}\sum_{\ell=1}^{\tau(k)}\rho_{k+\ell} \quad (37)$$

where $\bar{T} < \infty$ us a uniform upper bound on unconstrained intertie flows. This allows us to obtain a lower bound on number of iterations $\tau(k) > 0$ needed to upcross $\frac{3\epsilon}{4}$ from $\frac{\epsilon}{4}$:

$$2\bar{T}\sum_{\ell=1}^{\tau(k)}\rho_{k+\ell} > \frac{3\epsilon}{4} - \frac{\epsilon}{4} = \frac{\epsilon}{2}$$

Since $\tau(k)\rho_k > \sum_{\ell=1}^{\tau(k)}\rho_{k+\ell}$, it follows that $\tau(k) > \frac{2\bar{T}}{\rho_k} \to +\infty$ since $\rho_k \to 0^+$. We also have $\mu^{k+\tau}_{(i,j)} \geq \mu^{k}_{(i,j)} + \beta\tau\frac{\epsilon}{4}$ for all $0 < \tau \leq \tau(k)$. Since $\sum_{\ell=1}^{\tau(k)}\rho_{k+\ell} \to 0^+$, it follows that there exists $\tau > 0$ such that $\mu^{k+\ell}_{(i,j)} > \bar{\mu}_{(i,j)}$ and $\hat{T}^{k+\ell}_a = 0$ for all $\tau < \ell \leq \tau(k)$. Hence, $\left|T^{k+\ell}_{a,(i,j)}\right| \leq \left|T^{k+\tau}_{a,(i,j)}\right|$ for all $\tau < \ell \leq \tau(k)$ which is a contradiction to upcrossing $\frac{3\epsilon}{4}$. The non-negativity of $\mu^{\infty}_{(i,j)}$ follows by the max operation in (18).

D. Proof of Theorem 1

We start by stating (without proof) a useful result.

Proposition 2 *Let $y_k \geq 0$ and $y_{k+1} \leq (1-\rho_k)y_k + b_k$ with $\rho_k \in (0,1)$ and $b_k > 0$ such that*

$$\textstyle\sum_k \rho_k = \infty \quad \sum_k b_k < \infty$$

It holds that $y_k \to 0$ and $\sum_k \rho_k y_k < \infty$.

The proof has two parts. In the first part, we show that under diminishing step size updates, the sequence $\mathbf{x}^k_a$ is Cauchy, and thus $\mathbf{x}^k_a$ converges to $\mathbf{x}^\infty_a$. In the second part, we use this convergence along with the convergence of capacity prices (Lemma 2) to show that the KKT conditions of the centralized and decentralized DC-OPF are equivalent, concluding that $\boldsymbol{x}^\infty_a$ and $\mu^\infty_{(i,j)}$ correspond to the centralized optimal solution.
By definition of the update rule, it holds that

$$\begin{aligned}&\boldsymbol{x}^{k+1}_a - \boldsymbol{x}^k_a \\ &= (1-\rho_{k+1})\boldsymbol{x}^k_a + \rho_{k+1}\hat{\boldsymbol{x}}^{k+1}_a - (1-\rho_k)\boldsymbol{x}^{k-1}_a - \rho_k\hat{\boldsymbol{x}}^k_a \\ &= (1-\rho_k)(\boldsymbol{x}^k_a - \boldsymbol{x}^{k-1}_a) + \rho_k(\hat{\boldsymbol{x}}^{k+1}_a - \hat{\boldsymbol{x}}^k_a) \\ &\qquad + (\rho_{k+1}-\rho_k)(\hat{\boldsymbol{x}}^{k+1}_a - \boldsymbol{x}^k_a) \quad (38)\end{aligned}$$

Since the optimal solution and Lagrange multipliers for each area $a \in \mathcal{A}$ are Lipschitz continuous (see e.g. [24]), it follows that

$$\begin{aligned}\left\|\hat{\boldsymbol{x}}^{k+1}_a - \hat{\boldsymbol{x}}^k_a\right\| &\leq \sum_{a'\in\mathcal{A}(a)} L_a\left\|\boldsymbol{x}^{k+1}_{a'} - \boldsymbol{x}^k_{a'}\right\| + L_a\|\boldsymbol{\mu}^k_a - \boldsymbol{\mu}^{k-1}_a\| \\ &= \mathcal{O}(\rho_k) + L_a\|\boldsymbol{\mu}^k_a - \boldsymbol{\mu}^{k-1}_a\|\end{aligned}$$

for some $L_a > 0$. Thus, $\sum_k \rho_k\left\|\hat{\boldsymbol{x}}^{k+1}_a - \hat{\boldsymbol{x}}^k_a\right\| < \infty$. Note also that $\sum_k|\rho_{k+1}-\rho_k| < \infty$. Hence, we invoke Proposition 2 with $y_{k+1} := \|\boldsymbol{x}^{k+1}_a - \boldsymbol{x}^k_a\|$ and

$$b_k := \rho_k\left\|\hat{\boldsymbol{x}}^{k+1}_a - \hat{\boldsymbol{x}}^k_a\right\| + |\rho_{k+1}-\rho_k|\|\hat{\boldsymbol{x}}^{k+1}_a - \boldsymbol{x}^k_a\|$$

to show $y_k \to 0, \sum_k \rho_k y_k < \infty$. In a similar manner to (38) it can be shown that:

$$\begin{aligned}\boldsymbol{x}^{k+T}_a - \boldsymbol{x}^k_a &= (1-\rho_k)(\boldsymbol{x}^{k+T-1}_a - \boldsymbol{x}^{k-1}_a) \\ &+ \sum_{\ell=1}^{T}\rho_k(\hat{\boldsymbol{x}}^{k+\ell}_a - \hat{\boldsymbol{x}}^{k+\ell-1}_a) \\ &+ \sum_{\ell=1}^{T-1}(\rho_{k+\ell}-\rho_{k+\ell-1})(\hat{\boldsymbol{x}}^{k+\ell}_a - \boldsymbol{x}^{k+\ell-1}_a) \\ &+ \sum_{\ell=1}^{T-1}(\rho_k-\rho_{k+\ell})(\boldsymbol{x}^{k+\ell}_a - \boldsymbol{x}^{k+\ell-1}_a)\end{aligned}$$

for $T < \infty$. Hence, as before with $y_{k+T} := \|\boldsymbol{x}^{k+T}_a - \boldsymbol{x}^k_a\|$ it follows by Proposition 2 that $y_k \to 0$. As similar argument is used to prove $y_{k+T} := \|\boldsymbol{x}^{k+T}_a - \boldsymbol{x}^k_a\| \to 0$. Thus the sequence $\{\boldsymbol{x}^k_a : k > 0\}$ is Cauchy and converges to say $\boldsymbol{x}^\infty_a$. Since $\boldsymbol{x}^k_{a'} \to \boldsymbol{x}^\infty_{a'}$ for all adjacent areas $a' \in \mathcal{A}$, by continuity of optimal solutions and Lagrange multipliers it follows that the sequence $\{\hat{\boldsymbol{x}}^k_a : k > 0\}$ has a limit. By construction, $\boldsymbol{x}^k_a$ is a weighted average of $\{\hat{\boldsymbol{x}}^\ell_a\}^k_{\ell=0}$, hence $\|\hat{\boldsymbol{x}}^k_a - \boldsymbol{x}^k_a\| \to 0$ and $\hat{\boldsymbol{x}}^k_a \to \boldsymbol{x}^\infty_a$.

In the second part of the proof we show that the iterative mechanism satisfies the KKT conditions of the centralized OPF problem. We begin by showing that $\{\boldsymbol{x}^\infty\}_{a\in\mathcal{A}}$ is a feasible solution of the centralized OPF problem. Note that a decentralized DC-OPF solution that is feasible for all areas satisfies all the conditions of the centralized DC-OPF except (7). The convergence of the sequence $\mathbf{x}^k$ to $\mathbf{x}^\infty$ implies that $T^\infty_{a,(i,j)} = -T^\infty_{a',(i,j)}$ for $(a,a') \in \mathcal{T}$. We consider two cases of the capacity price: $\mu_{(i,j)} = 0$ and $\mu_{(i,j)} > 0$ to argue that inter-tie capacity limits are obeyed. If $\mu^\infty_{(i,j)} = 0$, then it must be that $T^\infty_{a,(i,j)} \leq \bar{T}_{a,(i,j)}$ because of the capacity price updates in (18). If $\mu^\infty_{(i,j)} > 0$, then $T^\infty_{a,(i,j)} = \bar{T}_{a,(i,j)}$ by the condition in (19). Thus, the intertie capacity limits are satisfied by $\boldsymbol{x}^\infty_a$. The dual feasibility and the complementary slackness conditions hold by Lemma 2.
Next we show that the first order stationarity conditions of the centralized problem are satisfied by $\mathbf{T}^\infty_a$ and $\boldsymbol{\theta}^\infty_a$. In the limit as $k \to \infty$, let $\mathcal{L}_A(\boldsymbol{x}_A, \boldsymbol{x}^\infty_B, \alpha^\infty_B, \mu^\infty)$ and $\mathcal{L}_B(\boldsymbol{x}_B, \boldsymbol{x}^\infty_A, \alpha^\infty_A, \mu^\infty)$ denote the Lagrangian of the decentralized DC-OPF problem for areas A and B, respectively. The terms $\boldsymbol{x}^\infty_B, \alpha^\infty_B$, and μ^∞ in $\mathcal{L}_A(\cdot)$ represent the terms in the objective (9) and tie-flow constraint (13) of Area A coming from the decoupling of constraints in the centralized DC-OPF. We note that $\frac{\partial\mathcal{L}_A(\cdot)}{\partial\mathbf{P}_A} = \frac{\partial\mathcal{L}(\cdot)}{\partial\mathbf{P}_A}$ and $\frac{\partial\mathcal{L}_A(\cdot)}{\partial\theta_{A,i}} = \frac{\partial\mathcal{L}(\cdot)}{\partial\theta_{A,i}}\ \forall(i,j) \in \mathcal{F}_a$ where $\mathcal{L}(\cdot)$ is the Lagrangian of the centralized DC-OPF.
Next we consider the first order stationarity conditions with respect to $\theta_{A,i}$, $\theta_{B,j}$ and $T_{A,(i,j)}$ for tieline $(i,j) \in \mathcal{T}_A$ in the decentralized DC-OPF problem and show their equivalence to the corresponding first order conditions of the centralized DC-OPF in (29)-(31).

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRS.2021.3100782, IEEE Transactions on Power Systems

The first order conditions with respect to $T_{a,(i,j)}$ for areas $a \in \{A, B\}$, i.e., $\frac{\partial \mathcal{L}_A(\boldsymbol{x}_A, \boldsymbol{x}_B^k, \alpha_B^k, \mu^k)}{\partial T_{A,(i,j)}}$ and $\frac{\partial \mathcal{L}_B(\boldsymbol{x}_B, \boldsymbol{x}_A^k, \alpha_A^k, \mu^k)}{\partial T_{B,(i,j)}}$, are respectively as follows:

$$\hat{\alpha}_{A,i}^{k+1} + \hat{\xi}_{A,(i,j)}^{k+1} = \alpha_{B,j}^k - \frac{\mu_{(i,j)}^k}{2}\text{sign}(\hat{T}_{A,(i,j)}^{k+1}),$$
$$\hat{\alpha}_{B,j}^{k+1} + \hat{\xi}_{B,(i,j)}^{k+1} = \alpha_{A,i}^k - \frac{\mu_{(i,j)}^k}{2}\text{sign}(\hat{T}_{B,(i,j)}^{k+1}).$$

Recall $\hat{\boldsymbol{x}}_a^k \to \boldsymbol{x}_a^\infty$. Hence,

$$\hat{\alpha}_{A,i}^{\infty} + \hat{\xi}_{A,(i,j)}^{\infty} = \alpha_{B,j}^\infty - \frac{\mu_{(i,j)}^\infty}{2}\text{sign}(\hat{T}_{A,(i,j)}^{\infty}), \quad (39)$$
$$\hat{\alpha}_{B,j}^{\infty} + \hat{\xi}_{B,(i,j)}^{\infty} = \alpha_{A,i}^\infty - \frac{\mu_{(i,j)}^\infty}{2}\text{sign}(\hat{T}_{B,(i,j)}^{\infty}). \quad (40)$$

Since $\hat{T}_{B,(i,j)}^\infty = -\hat{T}_{A,(i,j)}^\infty$ adding (39) and (40) yields $\hat{\xi}_{B,(i,j)}^\infty = -\hat{\xi}_{A,(i,j)}^\infty$. Hence the KKT conditions for intertie flows in the centralized problem are satisfied.

E. Proof of Lemma 3

The first order term in a Taylor expansion of $V_{a.k}$ is:

$$\Delta V_{a,k} \triangleq \sum_{(i,j)\in\mathcal{T}_a} \Big[\frac{\partial V_{a,k}}{\partial \alpha_{a',j}}(\alpha_{a',j}^k - \alpha_{a',j}^{k-1}) + \frac{\partial V_{a,k}}{\partial \mu_{(i,j)}}(\mu_{(i,j)}^k - \mu_{(i,j)}^{k-1}) + \frac{\partial V_{a,k}}{\partial \theta_{a',j}}(\theta_{a',j}^k - \theta_{a',j}^{k-1})\Big].$$

By the envelope theorem, we have

$$\frac{\partial V_{a,k}}{\partial \alpha_{a',j}} = -\hat{T}_{a,(i,j)}^k, \quad \frac{\partial V_{a,k}}{\partial \mu_{(i,j)}} = \frac{1}{2}\left|\hat{T}_{a,(i,j)}^k\right|, \quad \frac{\partial V_{a,k}}{\partial \theta_{a',j}} = \frac{\hat{\xi}_{a,(i.j)}^k}{\bar{x}_{a,(i.j)}}.$$

Equation (26) is the first order optimality condition for flow across intertie (i,j) for market $a \in \mathcal{A}$.

F. Proof of Theorem 2

Consider area $a \in \mathcal{A}$ reporting information consistent with strictly convex differentiable cost functions $C_{a,g}^D(\cdot)$ that differs from the reported supply cost functions for internal market clearing $C_{a,g}(\cdot)$, i.e. $C_{a,g}^D(\cdot) \neq C_{a,g}(\cdot)$. Such deviation yields a mechanism output denoted by $\hat{\boldsymbol{x}}^{D,k} = (\hat{\mathbf{T}}^{D,k}, \hat{\boldsymbol{\theta}}^{D,k}, \hat{\boldsymbol{\alpha}}^{D,k})$. Let us denote by $C_{a,k}^F$ the cost for area a after k iterations and $r_{a,k}^F$ the intertie flow payments with respect to reported supply costs for internal market clearing,

$$C_{a,k}^F := \sum_{g\in\mathcal{G}_a} C_{a,g}(\hat{P}_{a,g}^{D,k})$$

$$r_{a,k}^F := \sum_{(i,j)\in\mathcal{T}_a} \hat{\alpha}_{a',j}^{D,k-1}\hat{T}_{a,(i,j)}^{D,k} - \sum_{(i,j)\in\mathcal{T}_a} \frac{\hat{\mu}_{(i,j)}^{D,k-1}}{2}(\left|\hat{T}_{a,(i,j)}^{D,k}\right| - \bar{T}_{a,(i,j)}).$$

The net cost for area a after k iterations with respect to equilibrium supply costs for internal market clearing is, $V_{a,k}^F = C_{a,k}^F - r_{a,k}^F$. We also define $C_{-a,k}^F$ as the total cost of all areas except area a at iteration k when area a deviates from the equilibrium supply function for internal market clearing.

By (23), upon stopping after T-iterations the total change in cost inclusive of incentive transfers for area a can be written as follows is

$$\sum_{k=0}^{T}\left[C_{a,k+1}^F - r_{a,k+1}^F - (C_{a,k}^F - r_{a,k}^F) + \Delta\pi_{a,k}^F\right] = C_{a,T+1}^F - C_{a,0} + \sum_{a'\in\mathcal{A}\backslash a}[C_{a',T+1}^F - C_{a',0}] = C_a^F - C_{a,0} + \sum_{a'\in\mathcal{A}\backslash a}[C_{a'}^F - C_{a',0}] + \epsilon_{a,T}^F \quad (41)$$

where

$$\epsilon_{a,T}^F := \sum_{a\in\mathcal{A}}[C_{T+1,a}^F - C_a^F]$$

Similarly, we can write the total change in cost from reporting consistent with internal market clearing as follows:

$$\sum_{k=0}^{T}\left[C_{a,k+1} - r_{a,k+1} - (C_{a,k} - r_{a,k}) + \Delta\pi_{a,k}\right] = C_{a,T+1} - C_{a,0} + \sum_{a'\in\mathcal{A}\backslash a}[C_{a',T+1} - C_{a',0}] = C_a^* - C_{a,0} + \sum_{a'\in\mathcal{A}\backslash a}[C_{a'}^* - C_{a',0}] + \epsilon_{a,T} \quad (42)$$

where

$$\epsilon_{a,T} := \sum_{a\in\mathcal{A}}[C_{a,T+1} - C_a^*]$$

Hence by (41) and (42),

$$\sum_{k=0}^{T}[C_{a,k+1}^F - r_{a,k+1}^F - (C_{a,k}^F - r_{a,k}^F) + \Delta\pi_{a,k}^F] - \sum_{k=0}^{T}[C_{a,k+1} - r_{a,k+1} - (C_{a,k} - r_{a,k}) + \Delta\pi_{a,k}] = C^F - C^* + \sum_{a\in\mathcal{A}}[\epsilon_{a,T}^F - \epsilon_{a,T}] \geq \epsilon_T$$

The inequality follows from Theorem 1 (optimality of coupling markets in internal equilibrium) and

$$\epsilon_T := \inf_{C_{a,g}^F(\cdot)} \sum_{a\in\mathcal{A}}[\epsilon_{a,T}^F - \epsilon_{a,T}]$$

Here again, by convergence of the mechanism's output it follows that $\epsilon_T \to 0$ as $T \to \infty$.

G. Tieline Data for Enhanced Test Systems

TABLE VIII
TIELINE DATA FOR THE ADDITIONAL TIELINES OF CASE 1

Tieline Number	Sending End (Area-Bus)	Receiving End (Area-Bus)	Reactance (pu)	Capacity (MW)
TL6	A-14	B-16	0.161	500
TL7	A-12	C-17	0.097	500
TL8	B-2	C-1	0.104	500

TABLE IX
TIELINE DATA FOR THE ADDITIONAL TIELINES OF CASE 2 (A', B', AND C' REFER TO THE ADDITIONAL AREAS)

Tieline Number	Sending End (Area-Bus)	Receiving End (Area-Bus)	Reactance (pu)	Capacity (MW)
TL11	A-1	A'-8	0.11	500
TL12	B-2	B'-9	0.1	500
TL13	C-1	C'-7	0.105	500

REFERENCES

[1] S. Meyn, T. Samad, I. Hiskens, and J. Stoustrup, Eds., *Energy Markets and Responsive Grids: Modeling, Control, and Optimization*. Springer, 2018.

[2] D. K. Molzahn, F. Dörfler, H. Sandberg, S. H. Low, S. Chakrabarti, R. Baldick, and J. Lavaei, "A Survey of Distributed Optimization and Control Algorithms for Electric Power Systems," *IEEE Transactions on Smart Grid*, vol. 8, no. 6, pp. 2941–2962, 2017.

[3] A. G. Bakirtzis and P. N. Biskas, "A Decentralized Solution to the DC-OPF of Interconnected Power Systems," *IEEE Transactions on Power Systems*, vol. 18, no. 3, pp. 1007–1013, 2003.

[4] F. Zhao, E. Litvinov, and T. Zheng, "A marginal equivalent decomposition method and its application to multi-area optimal power flow problems," *IEEE Transactions on Power Systems*, vol. 29, no. 1, pp. 53–61, 2015.

[5] Z. Li, W. Wu, B. Zhang, and B. Wang, "Decentralized Multi-area Dynamic Economic Dispatch using Modified Generalized Benders Decomposition," *IEEE Transactions on Power Systems*, vol. 99, pp. 1–13, 2015.

[6] Y. Guo, L. Tong, W. Wu, B. Zhang, and H. Sun, "Coordinated Multi-Area Economic Dispatch via Critical Region Projection," *IEEE Transactions on Power Systems*, vol. 32, no. 5, pp. 3736–3746, 2017.

[7] A. Conejo and J. Aguado, "Multi-area Coordinated Decentralized DC Optimal Power Flow," *IEEE Transactions on Power Systems*, vol. 13, no. 4, pp. 1272–1278, 1998.

[8] G. Binetti, A. Davoudi, F. L. Lewis, D. Naso, and B. Turchiano, "Distributed Consensus-based Economic Dispatch with Transmission Losses," *IEEE Transactions on Power Systems*, vol. 29, no. 4, pp. 1711–1720, 2014.

[9] T. Erseghe, "Distributed Optimal Power Flow using ADMM," *IEEE transactions on Power Systems*, vol. 29, no. 5, pp. 2370–2380, 2014.

[10] Y. Guo, Y. Ji, and L. Tong, "Generalized Coordinated Transaction Scheduling: A Market Approach to Seamless Interfaces," *IEEE Transactions on Power Systems*, vol. 33, no. 5, p. 4683–4693, 2018.

[11] Y. Ji and L. Tong, "Multi-Area Interchange Scheduling Under Uncertainty," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1659–1669, 2018.

[12] A. Kargarian, J. Mohammadi, J. Guo, S. Chakrabarti, M. Barati, G. Hug, S. Kar, and R. Baldick, "Toward Distributed/Decentralized DC Optimal Power Flow Implementation in Future Electric Power Systems," *IEEE Transactions on Smart Grid*, vol. 9, no. 4, pp. 2574–2594, 2016.

[13] A. Cherukuri and J. Cortes, "Initialization-free distributed coordi- nation for economic dispatch under varying loads and generator commitment," *Automatica*, vol. 74, pp. 183–193, 2016.

[14] ——, "Iterative bidding in electricity markets: rationality and robustness ," *IEEE Transactions on Network Science and Engineering*, vol. 7, no. 3, pp. 1265–1281, 2020.

[15] S. Kar and G. Hug, "Distributed Robust Economic Dispatch in Power Systems: A Consensus+ Innovations Approach," in *2012 IEEE Power and Energy Society General Meeting*. IEEE, 2012, pp. 1–8.

[16] A. D. Dominguez-Garcia, S. T. Cady, and C. N. Hadjicostis, "Decentralized Optimal Dispatch of Distributed Energy Resources," in *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)*. IEEE, 2012, pp. 3688–3693.

[17] S. Yang, S. Tan, and J.-X. Xu, "Consensus based Approach for Economic Dispatch Problem in a Smart Grid," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4416–4426, 2013.

[18] F. Dörfler, J. W. Simpson-Porco, and F. Bullo, "Breaking the Hierarchy: Distributed Control and Economic Optimality in Microgrids," *IEEE Transactions on Control of Network Systems*, vol. 3, no. 3, pp. 241–253, 2015.

[19] A. Cherukuri and J. Cortés, "Distributed Generator Coordination for Initialization and Anytime Optimization in Economic Dispatch," *IEEE Transactions on Control of Network Systems*, vol. 2, no. 3, pp. 226–237, 2015.

[20] J. Barrera and A. Garcia, "Dynamic Incentives for Congestion Control," *IEEE Transactions on Automatic Control*, vol. 60, no. 2, pp. 299–310, 2015.

[21] F. Farhadi, S. Golestani, and D. Teneketzis, "A Surrogate Optimization-Based Mechanism for Resource Allocation and Routing in Networks With Strategic Agents," *IEEE Transactions on Automatic Control*, vol. 64, no. 2, pp. 464–479, 2019.

[22] A. Sinha and A. Anastasopoulos, "Distributed Mechanism Design With Learning Guarantees for Private and Public Goods Problems," *IEEE Transactions on Automatic Control*, vol. 65, no. 10, pp. 4106–4121, 2020.

[23] C. Grigg, P. Wong, P. Albrecht, R. Allan, M. Bhavaraju, R. Billinton, Q. Chen, C. Fong, S. Haddad, S. Kuruganty *et al.*, "The ieee reliability test system-1996. a report prepared by the reliability test system task force of the application of probability methods subcommittee," *IEEE Transactions on power systems*, vol. 14, no. 3, pp. 1010–1020, 1999.

[24] S. M. Robinson, "Strongly Regular Generalized Equations," *Mathematics of Operations Research*, vol. 5, pp. 43–62, 1980.

Alfredo Garcia received B.Sc in Electrical Engineering from the Universidad de los Andes, Colombia in 1991, D.E.A in Automatique et Informatique Industrielle from the Université Paul Sabatier in Toulouse, France in 1992, and Ph.D. in Industrial and Operations Engineering from the University of Michigan in 1997. From 1998 to 2000, he served as Commissioner in the Colombian Energy Regulatory Commission, and from 2001 to 2017 he was a member of the faculty at the University of Virginia and the University of Florida. He is currently a professor with Industrial & Systems Engineering Department, Texas A&M University. His research interests include game theory and dynamic optimization, with applications in electricity and communication networks.

Roohallah Khatami received the B.S. degree from Iran University of Science and Technology, Tehran, Iran, in 2007, M.S. degree from Amirkabir University of Technology, Tehran, Iran, in 2013, and the Ph.D. degree from the University of Utah, UT, USA, in 2019, all in Electrical Engineering. He was a postdoctoral research associate at the Department of Industrial & Systems Engineering, Texas A&M University, TX, USA, and currently a postdoctoral research fellow at the Department of Electrical & Computer Engineering, University of British Columbia, BC, Canada. His research interests include power systems operation and electricity markets.

Ceyhun Eksin received the B.Sc. degree in control engineering from Istanbul Technical University, Turkey, in 2005, the M.S. degree in industrial engineering from the Boğaziçi University, Turkey, in 2008, the M.A. degree in statistics from Wharton School in 2015, and the Ph.D. degree in electrical and systems engineering from the Department of Electrical and Systems Engineering, University of Pennsylvania, PA, in 2015. He was a postdoctoral researcher jointly affiliated with the Schools of Biological Sciences, and Electrical and Computer Engineering, Georgia Institute of Technology, GA. He is currently an assistant professor with the department of Industrial & Systems Engineering, Texas A&M University, TX, USA. His research interests focus on strategic learning and decentralized optimization in networked multiagent systems with applications to autonomous teams, epidemics, and energy systems.

Furkan Sezer received B.Sc. degree in industrial engineering from Boğaziçi University, Istanbul, Turkey, in 2019. He is currently pursuing the Ph.D degree in Industrial & Systems Engineering at Texas A&M University, College Station, TX USA. His research interests include distributed optimization, game theory, information design, and electricity markets.