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Integrating transit systems with ride-sourcing services: A study on the system users' stochastic equilibrium problem

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ABSTRACT

Ride-sourcing services are continuously gaining popularity among urban travelers. Due to their flexibility and responsiveness, transit agencies spot chances of collaborating with Transportation Network Companies (TNCs) in order to improve service quality, promote transit usage, and meanwhile lower their operational cost. In this paper, we studied an integrated transit system where ride-sourcing services complement the transit service as an efficient and economical access mode. The users of the integrated system consist of not only transit riders but also drivers for TNCs. Riders maximize their utilities by choosing whether and how to use the integrated system, and drivers maximize their payoffs by deciding how to serve riders. More importantly, in this two-sided system, their decisions are unfolded to each other and mutually influence one another such that an equilibrium is reached where no one can improve their outcome. We modeled the stochastic mode choice of riders and zone choice of drivers and captured the interactive attributes in a fixed-point problem. The existence of such an equilibrium was proved, and an iterative solution algorithm that utilizes network algorithms was proposed to find an equilibrium pattern of the stochastic demand-supply problem. A real-world case study using the Twin Cities' data was carried out, and insights into the mechanism of the integrated transit system were developed. Various aspects of the system including user perception, demand estimation, pricing strategy, and subsidy strategy were investigated through the proposed model. These research outcomes contribute to a better comprehension of this promising mobility system and provide valuable knowledge on the planning and design of such systems.

1. Introduction and background

With the ubiquitous presence and use of mobile phones and electronic payment technology, the new taxi-hailing services, app-based ride-sourcing services, have been witnessed a noticeable emergence in the past decade. Companies providing this type of new transportation mobility services are generally referred to as Transportation Network Companies (TNCs) (The Public Utilities Commission of the State of California 2012). As an example of shared mobility, ride-sourcing services are continually gaining popularity across the world. Global leading TNCs such as Uber and DiDi, complete 15 million trips every day in over 600 cities (Uber, 2019) and 30 million rides in over 1000 cities (Iqbal, 2018) worldwide, respectively. Since the emergence, they have seized the ground transportation market. It shows that Uber and Lyft's combined market share had grown from around 25% in 2015 to 63% in 2017, which is well above rental car's and taxi's share that is 29% and 8% respectively (Richter, 2017). The dominance by the ride-sourcing

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services was also observed in New York City in terms of total daily ride (Bhuiyan, 2018).

However, transit systems, another bulk component of the ground transportation, seem to have a rather ambiguous interplay with the ride-sourcing services. Conclusions on whether ride-sourcing services supplement to or compete with transit services are not unanimous. The study commissioned by the Metropolitan Areas Planning Council revealed that in the Boston Region, riders tended to use ride-sourcing services instead of transit services (Gehrke, 2018). The explanation was that once riders' propensity for using ride-sourcing services is developed, it is difficult to attract them to the schedule-based services. It is claimed that about 42% of ride-sourcing rides would have otherwise taken place by transit. In addition, 15% of ride-sourcing rides during peak hours were shifted from the transit system. Another survey-based report conducted in US metropolitan areas showed that ride-sourcing service users reduced their transit use (Clewlow and Mishra, 2017). The most significant negative impacts were on bus services with a 6% ridership reduction. On the other hand, it was also revealed that the use of heavy rail system services increased by 3% for ride-sourcing riders. Therefore, it concluded that the role ride-sourcing services play in the presence of the transit system varies depending on how the transit system performs.

The prevailing belief of the impacts on the transit system, despite some negative evidence, is more complementary than substitutive. The positive role that ride-sourcing services can play in mitigating the first and last mile problem of the transit system is well acknowledged. A study shows that the more passengers use shared modes such as ride-sourcing services, the more they are inclined to take transit (SUMC, 2016). It concludes that there are more trips substituted by shared mobility services for automobile trips than transit trips, and shared modes complement the public transit. Evidence from San Francisco shows that ride-sourcing sometimes is likely to be a complement to the transit system, though competitions for some individual trips do exist (Rayle et al., 2016). Excitedly, many transit agencies view the partnership with TNCs as a win-win model and are actively incorporating private TNCs for providing better mobility services. There has been a slew of such cooperation: Dallas Area Rapid Transit (DART) incorporated Lyft into their paratransit services; Pierce Transit in Seattle started a pilot with Lyft that provided first/last mile connections between rider' home and transit stops; the city of Centennial, Colorado implemented on-demand service incorporation with several TNCs to provide connections to the light rail stations in Centennial (APTA, 2018). The recently released report by Lyft, a major TNC in the U.S., showed that 21% of their passengers "already use public transit more frequently because they can rely on Lyft to get the first or last mile service when their starting point or destination is not directly along a transit route" (Lyft, 2018).

The concept of transit and supplementary mobility services partnership is by no means new. Paratransit is one of the pioneers of such. It was originally designed to serve as flexible routes to connect public transit for general passengers and later was reserved to refer to the services provided to the elderly and people with disabilities. A more general form of partnership is known as Demand Adaptive System (DAS) (Malucelli et al., 1999; Li and Quadrifoglio, 2011; Errico et al., 2013; Chen and Nie, 2017; Maheo et al., 2017). It usually has semi-fixed routes where vehicles (feeder bus, shuttle bus, etc.) can deviate from their routes to pick up and drop off passengers from and/or to major transit line stops. Transit agencies can save operational costs in the low-demand catchment area by substituting regular transit services with DAS while passengers still get access to transit services. These services are oftentimes operated by transit agencies. Nowadays, connector/feeder services provided by TNCs play the role. Because of the flexibility of TNCs vehicles, the partnership complements the transit system to a great extent while being economically sustainable as well (Stiglic et al., 2018; Yan et al., 2018).

Despite the appealing concept of the transit and ride-sourcing service integration, little is known regarding how travelers react to the new travel mode and how drivers participate in providing services. It is vital to envisage the new integrated mobility service system for both transit agencies and TNCs. Transit agencies have the potentials in increasing the ridership by providing better access and reducing operational costs by replacing services in the low-demand area with mobility-on-demand services in collaboration with TNCs. TNCs can potentially enhance the market and increase profit by understanding and reacting to the interaction of demand and supply under the integration (Merkert et al., 2020). For them, the market is two-sided. On the demand side, potential riders hope to receive ride-sourcing services with low expenses; on the supply side, drivers hope to make more money from their services. Riders' decisions on whether to use, and how to use the ride-sourcing services affects drivers decisions on whether to participate and how to service, and vice versa. When making decisions, both sides have imperfect information about, for example, waiting time for mobility services, surge prices, and demand levels. Hidden in these factors that affect their decisions is the demand-supply stochasticity. The two sides, through their mutual impacts and interactions, will eventually stabilize at an equilibrium point when none can improve their benefits. This is a stochastic equilibrium problem.

Studying the equilibrium problem helps to understand the fundamental mechanism of the integrated system and supports the development of such a system in aspects as demand forecasting, infrastructure planning, and policy regulation-making. Though there is a quite rich body of literature digging into the design of the ride-sourcing (or ride-sharing) system itself (Masoud and Jayakrishnan, 2017; Masoud et al., 2017; Zha et al., 2018), studies on the integrated transit and ride-sourcing market equilibrium have been scarce. This paper, therefore, aims to fill the existing research gap in the equilibrium analysis from a system perspective, in order to provide more insights into this newly developed promising transportation mobility services.

The remainder of the paper is structured as follows. In Section 2, a literature review is provided in Section 2. Section 3 describes the problem of interest, and Section 4 derives fundamental functions involving demand and supply variables, followed by the mathematical formulation of the stochastic equilibrium model for the integrated system. A solution algorithm is given in Section 5. Computational results on a real network are provided in Section 6 and the paper closes with a summary of findings and future work in the last section.

2. Literature review and research contribution

The literature review summarizes the (i) studies on the interplay of transit services and ride-sourcing services, and (ii) demand and supply studies on the ride-sourcing market. It is noteworthy to distinguish the ride-sourcing with the ride-sharing. “Ride-sharing” describes the shared mobility mode that two or more travelers together finish their trips by sharing one vehicle and the primal motivation for the sharing is to reduce monetary costs. In contrast, “ride-sourcing” refers to the services that are provided by privately owned cars and drivers provide for-hire rides to make a profit.

2.1. Studies on the interplay of transit and ride-sourcing services

The dichotomy of the interactions between the public transit system and ride-sourcing services has already been seen in the introduction. Most of the research in this field is empirical. Stated preference and revealed preference are the main data source. To avoid redundancy, we summarize the common conclusions of this strand of research. An in-vehicle survey conducted for ride-sourcing users showed that ride-sourcing services were usually substitutes for transit (Gehrke, 2018); Clewlow and Mishra (2017) obtained similar conclusions and pointed out these negative effects were more obvious for bus and light rail services, but ride-sourcing services tended to supplement commuter rail; Schwieterman and Scott Smith (2018)’s paired-trip study on Chicago Transit Authority (CTA) transit and ride-sharing showed that the risk of losing ridership also depends on origins and destinations: commuters who start or end their trips in downtown are less likely to switch to ride-sourcing services. A study based on the survey conducted in San Francisco reported that 33% of the trips by ride-sourcing would have been realized by transit were ride-sourcing services not available (Rayle et al., 2016). What was also revealed is that the total travel time of the majority of ride-sourcing trips was 50% shorter than that by transit. However, SUMC (2016) found that ride-sourcing complemented the transit use since two systems tended to serve different types of trips by time and purpose. For example, ride-souring trips were more likely to happen during the late night for recreational purposes where transit services operate in low frequency or even unavailable. An extension of the report published later further explored the interactions (SUMC, 2018).

2.2. Supply and demand studies for ride-for-hire services

Ride-for-hire services have been around for a long history (Orr, 1969). The forms of ride-for-hire services include the conventional taxi services and the more novel app-based ride-sourcing (e-hailing) services. The two business models share a few affinities. For example, there is a designated driver for each trip, and passengers get direct services without transfers or intermediate stops. One distinction between the two is that ride-sourcing vehicles do not cruise on the streets to pick up riders. The only way a ride-sourcing vehicle services a rider is that they are matched through TNC’s online matching platform. In the literature, both business models received ample attention from researchers.

Early studies for the conventional taxi market are economic analyses on an aggregated level (Douglas, 1972; De Vany, 1975; Information for Regulating, 1983; Arnott, 1996; Cairns and Liston-Heyes, 1996; Çetin and Eryigit, 2011). As is pointed out by Yang and Wong (1998), aggregated level analysis neglects the road network configuration that has direct impacts on the demand pattern. Yang and Wong (1998) modeled the movement of vacant and occupied taxis on the network level by the balance of drivers and riders while ignoring the congestion and demand uncertainty. Wong et al. (2001, 2002) incorporated elastic demand and road congestion. The demand was modeled through a function of endogenous variables including waiting time, travel time, and monetary fare of taxi trips. Traffic congestion incurred by the number of drivers and riders in the context of ride-sharing was also studied (Huayu et al., 2015,b). They developed a mathematical model with equilibrium constraint capturing the relationship between supply and demand. To model the meeting and searching behaviors of taxi drivers and riders, a Cobb-Douglas matching function was later adopted (Yang et al., 2010b; Yang and Yang, 2011; Zha et al., 2016, 2017; Nourinejad and Ramezani, 2020). The function sets up the relationship between waiting times and demand and supply, and are oftentimes used to capture the equilibrium state of the two-sided market. Moreover, for-hire ride market drivers’ self-scheduling problem (Yang et al., 2005; Zha et al., 2017; Sun et al., 2019), pricing problems under equilibrium and non-equilibrium (Yang et al., 2010a; Wang et al., 2016; Nourinejad and Ramezani, 2020), and market regulation strategies (Yang et al., 2002; Zha et al., 2016, 2018) were also studied.

The above studies focus on the monopolistic ride-for-hire market, where only one type of service is available to passengers. Qian and Ukkusuri (2017) studied an equilibrium problem of the coexistence of the conventional taxi services and app-based ride-sourcing services. The equilibrium between the passengers and two types of drivers (conventional taxi’s and ride-sourcing’s) was modeled as a multiple-leader-follower game. Riders choose the service with the minimum travel costs and drivers make decisions on zones to service based on their perceived revenue. On the integration of public transit with ride-sourcing services, we mention the following studies. Salazar et al. (2018) developed a network flow model for determining the rider-vehicle-transit assignment for each request under the social welfare objective. A pricing scheme was also developed to align travelers’ private goal with the integrated system’s social welfare goal. Shen et al. (2018) developed a simulation framework for the integrated transit and shared autonomous vehicles (SAVs) system. The simulation study on Singapore for morning peak hours shows that the integrated system can improve service levels and are financially sustainable. Wen et al. (2018) also developed an agent-based simulation study on the subject but incorporated the demand estimation.

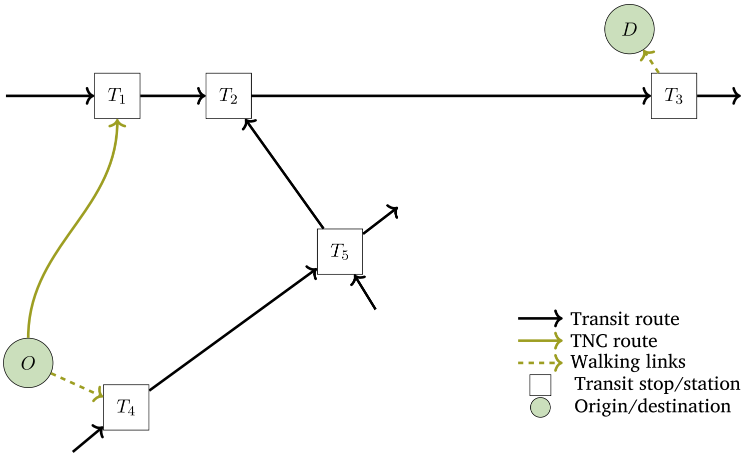


Fig. 1. Two alternative travel modes from *O* to *D*: using ride-sourcing services as an access mode to travel from O to the transfer station T_1, or taking local bus system to get to the transfer station T_2, which includes one extra transfer and possibly some extra walking trips.

2.3. Research contribution

Equilibrium involving the ride-sourcing market has been extensively studied. However, when it comes to the choice behaviors of service providers, the proactivity of ride-sourcing service drivers has been neglected. As mobility service providers, drivers have full control over where and when they want to provide ride-sourcing services according to their experience and prediction. Drivers' self-scheduling problem answering the question of "when" has been studied in the literature (Yang et al., 2005; Zha et al., 2017; Sun et al., 2019). However, the "where" question—drivers' stochastic choice of service zones—has been underexplored. Current studies focus on the outcome of the matching between riders and drivers through TNC's platform while neglecting the drivers' self-positioning process (Zha et al., 2016, 2017; Nourinejad and Ramezani, 2020). Adopting Cobb-Douglas production functions to describe the matching and searching procedure between drivers and riders was their main approach, where driver's own service zone choice decisions have been overlooked and are described as deterministic processes dictated by central matching platforms. To improve this, we employed two mathematical optimization model to describe both riders' mode choice and drivers' stochastic zone choice behaviors. Besides, as trip length and cost affect riders' and drivers' behavior, the heterogeneity of trip length and cost is explicitly modeled. This can be viewed as a generalization of the existing models (Zha et al., 2016, 2017) in which trips are effectively undifferentiated. Lastly, in contrast to the optimization objective of maximizing TNC's total profit in the aforementioned studies, our model views the problem from both drivers' and riders' own perspective, and therefore the objective of the supply side is to maximize individual driver's payoff. Our model can hence be considered as an alternative approach to studying the two-sided (integrated) rides-sourcing system.

As a study aiming at understanding the mechanisms of the integrated system in which ride-sourcing services complement the transit system, this research also enriches the related studies in the following aspects:

1. Formulated the demand-supply equilibrium problem as a fixed-point problem and proved the existence of the equilibrium;
2. Proposed an iterative solution algorithm that adopts network algorithms for solving the stochastic equilibrium problem;
3. Developed insights into the mechanism of the system and gained theoretical knowledge about the system including user perceptions, demand prediction, the optimal fleet-size, pricing strategy, and subsidy strategy.

3. Problem statement

We describe the problem and define the integrated system that is comprised of the transit system and the ride-sourcing (TNC) system. With the assumptions stated, the scope of the work that we restrain research efforts within is also provided.

3.1. Problem description

Ride-sourcing services can be integrated into the transit system as a potential solution for the first/last mile problem. Let's consider the following two means of transportation for a passenger traveling from his/her origin(*O*) to the destination(*D*) (see Fig. 1):

- first taking TNC services as the access mode, then taking the rapid transit services at the connection station T_1 that can take the passenger to the destination *D* (possibly with some extra egress connections at T_3);
- first walking/cycling to a bus stop and taking a local/feeder bus (possibly with some transfers) from there to access the rapid transit system at the connections stations(T_2) where rapid transit services transport the passenger to his/her destination *D* (possibly with some extra egress connections at T_3).

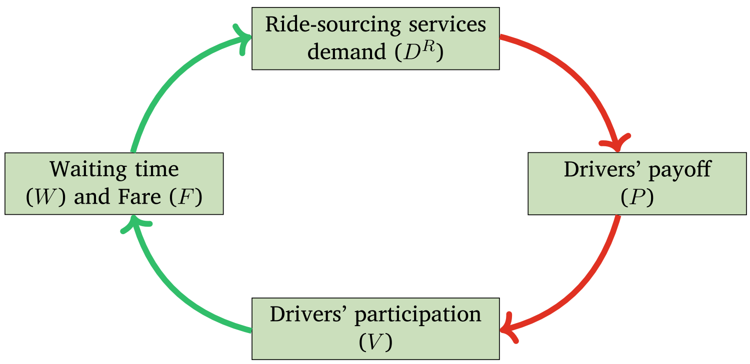


Fig. 2. Mutual effects between driver and rider participation. Rides-sourcing demand surge increases drivers' payoff, which stimulates higher participation of drivers. This will lead to reduced waiting time and fare and will again induce additional demand for the system. Effects between demand, payoff, and participation are positive, while negative between participation, waiting time and fare, and demand.

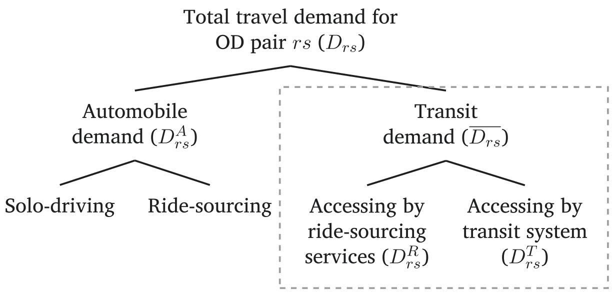


Fig. 3. Tree diagram of demand taxonomy.

The decision between the two travel modes generally depends on the comprehensive evaluation of the two modes by each traveler. A traveler may develop their own framework for calculating a comprehensive (generalized) cost for using each mode based on their perceived costs, which can be comprised of, just to name a few, waiting time, travel time, monetary cost (fare), etc.

For transit services, most routes are operated based on their schedules. Travel time and waiting time are relatively stable, especially for rapid transit systems such as metro and BRT systems. In contrast to this, the uncertainties of using ride-sourcing services are more significant. Waiting time for a TNC vehicle depends on the number of vacant vehicles. Travel time of ride-sourcing trips is subject to traffic conditions on the network. In addition, fares for using the ride-sourcing services also change much more frequently in response to the relative ratio of demand and supply and the real-time congestion. The existence of these uncertainties results in a stochastic equilibrium. High demand for ride-sourcing services stimulates more drivers to participate because of potentially higher profit for drivers. More participating drivers lead to reduced waiting time and fare, which, in turn, improves the attractiveness of ride-sourcing services and attract more riders. Fig. 2 depicts the mutual relationship between these variables within the ride-sourcing system. This looping process terminates when equilibria of riders' mode choice and drivers' participation decision are attained. This is the stochastic equilibrium problem and the following provides the equilibrium definition.

Definition. An equilibrium is reached in the integrated transit system when no riders can improve his/her random utilities by unilaterally changing decisions on whether and how to use the system, and no drivers can improve his/her payoffs by unilaterally altering zone choice decisions.

3.2. Assumptions

With the introduction of ride-sourcing services, in addition to using local transit services to access the main transit system, transit-dependent riders have an additional access/egress mode. As we analyze the integrated transit-ride-sourcing system, we restrict the analysis to the transit-riders' market, with the following assumptions regarding the settings of the stochastic equilibrium problem.

1. The equilibrium analysis is restricted to the transit-involved mode as is shown in the dashed box in Fig. 3. The independence of the demand of the automobile mode (the left branch) and the transit mode (the right branch) is assumed.

2. As ride-sourcing is used as an access mode, the congestion impact of the access trips with ride-sourcing services is neglected. Together with the first assumption of independence, constant travel times are assumed for ride-sourcing trips.
3. Total demand for using transit services is known a priori, and splitting between the two access modes is captured in the proposed model.
4. Public transit services operate with constant travel speed, fare, and service frequencies.

4. Fundamental functions and mathematical models

In this section, we describe basic variables including demand, waiting time, fare, and payoff, and functions that capture the relationship among these variables. Following this, we present the mathematical models of mode choice for riders and zone choice for drivers. We use the link-node approach to model the network and related notations are listed below.

r, s : index for origin and destination zones;
t : index for a transfer station that connects a ride-sourcing trip and a transit trip;
Z : set of origin and destination zones;
T, R : index for the two travel modes representing the transit access and ride-sourcing access, respectively;
$\overline{D_{rs}}$: total demand rate for using transit services for OD pair rs;
D_{rs}^R : demand rate for accessing transit by ride-sourcing mode for OD pair rs;
D_{rs}^T : demand rate for accessing transit by transit mode for OD pair rs;
V_r : ride-sourcing services supply (number of ride-sourcing service drivers) in zone r;
V : total number of available drivers in the system;
P_r : average payoff for a driver by participating and providing services in zone r;
W_r^R : rider's average waiting time for a pick-up in zone r;
F_{rt}^R : fare of using ride-sourcing services for traveling from origin r to the transfer station t;

4.1. Demand

Total travel demand between origin r and destination zone s,$\overline{D_{rs}}$, splits between two accessing modes. Since the two modes share the same component—transit, the major factor that distinguishes the two is the availability and attractiveness of ride-sourcing services provided at travelers' origin zones. Fare and waiting time are usually the main considerations from the perspective of ride-sourcing riders. Note that travel time and distance are reflected in the fare which also includes surge charge, base fees, and service fees.

Surge charge plays an important role in balancing demand and supply. It is used as a demand management means for regulating the high demand when there is not a sufficient supply of services by TNCs. The surge charge is closely related to the number of vehicles (supply) and the number of riders (demand) during a certain time period.

Let D_{rs}^R, with unit being riders per hour, be the demand rate for ride-sourcing access trips for the OD pair rs. Define N_r as the average number of available TNC vehicles during a time period in the origin zone r,

$$N_r = V_r - \sum_{s \in Z} D_{rs}^R t_{rs}^R, \quad \forall r \in Z, \tag{1}$$

where, t_{rs}^R, with unit being hour, is the travel time from the origin (r) to the connection station (s, the destination for the ride-sourcing trip segment) where riders transfer to the main transit services. Eq. (1) computes on average how many TNC vehicles are vacant at any instant in the study time period. For example, a 10-min ride-sourcing trip occupies a TNC vehicle for $1/6$ of its in-service time in an hour. Note also that the following condition has to hold as $N_r \geqslant 0$,

$$V_r \geqslant D_{rs}^R t_{rs}^R.$$

Waiting time at the origin (W_r^R) is readily available given the number of available TNC vehicles, N_r. Intuitively, waiting time is decreasing with N_r, that is,

$$W_r'^R\left(N_r\right) = \frac{dW_r^R}{dN_r} < 0, \quad \forall r \in Z. \tag{2}$$

Fare for the ride-sourcing services from the origin to the transfer point is calculated as:

$$F_{rs}^R(N_r) = \left(1 + S_r(N_r)\right)\left(\alpha t_{rs}^R + \beta L_{rs}^R\right) + B, \quad \forall rs \in Z^2. \tag{3}$$

where S_r is the function for calculating the surge charge multiplier, B is a fixed charge including base fee, service fee and other fixed charges, α and β are the coefficients set up by TNCs for time-and distance-based fees, and L_{rs}^R is the distance traveled by TNC vehicles. Surge charges should decrease with available TNC vehicles. That is, $\frac{dS_r}{dN_r} < 0$. Thus, with a predetermined route and constant travel time, the fare is a function of available TNC vehicles as well. In addition, it is also a decreasing function.

$$F'^{R}_{rs}\left(N_r\right) = \frac{dF^R_{rs}}{dN_r} < 0, \quad \forall rs \in Z^2. \tag{4}$$

Demand for using ride-sourcing services is dependent on its waiting time and fare. In what follows, we denote the demand function as $d_{rs}(\cdot)$.

$$D^R_{rs} = d_{rs}\left(F^R_{rs}, W^R_r\right) = d_{rs}\left(F^R_{rs}\left(N_r\right), W^R_r\left(N_r\right)\right), \quad \forall rs \in Z^2. \tag{5}$$

The demand function should satisfy the following two conditions, that is demand decreases with fare and waiting time, respectively. Mathematically,

$$\frac{\partial d_{rs}}{\partial F_{rs}} < 0, \quad \forall rs \in Z^2 \tag{6}$$

$$\frac{\partial d_{rs}}{\partial W^R_r} < 0, \quad \forall rs \in Z^2. \tag{7}$$

4.2. Properties of the demand function

The Eq. (5) is first rewritten as the following form where the expression for available TNC vehicles (1) is combined.

$$D^R_{rs} = d_{rs}\left(F_{rs}\left(V_r - \sum_{s\in Z} D^R_{rs} t_{rs}\right), W^R_r\left(V_r - \sum_{s\in Z} D^R_{rs} t_{rs}\right)\right), \quad \forall rs \in Z^2. \tag{8}$$

Let the vector $\mathbf{D}^R = [D^R_{11}, D^R_{12}, \dots, D^R_{rs}, \dots, D^R_{|Z||Z|}]$. The demand function in (8) can be rewritten as

$$D^R_{rs} = d_{rs}\left(F_{rs}\left(\mathbf{D}^R\right), W^R_r\left(\mathbf{D}^R\right)\right), \quad \forall rs \in Z^2. \tag{9}$$

Further define $d(\cdot)$,$F(\cdot)$, and $W(\cdot)$ as the corresponding vector version function for $d_{rs}(\cdot)$,$F_{rs}(\cdot)$, and $W^R_r(\cdot)$, the function can be expressed as the following compact form.

$$\mathbf{D}^R = d\left(F\left(\mathbf{D}^R\right), W\left(\mathbf{D}^R\right)\right) \tag{10}$$

Proposition 1. *If i) the mapping $\mathbf{F} = F(\mathbf{D}^R)$ and $\mathbf{W} = W(\mathbf{D}^R)$ is continuous and takes values in the non-empty, compact, and convex set F and W, respectively, ii) the demand map $\mathbf{D}^R = d(\mathbf{F}, \mathbf{W})$ is continuous, iii) the feasible set of ride-sourcing demand D is nonempty, convex, and compact, and iv) $\forall\, \mathbf{F} \in F$ and $\mathbf{W} \in W$, the map d takes values in the set D, then the fixed-point problem defined in (10) has at least one solution.*

Proof. We are going to check the conditions of the Brouwer's fixed-point existence theorem (Cantarella, 1997) to show the existence of an equilibrium solution.

- The demand function $d(F(\mathbf{D}^R), W(\mathbf{D}^R))$ is continuous in $\mathbf{D}^R$ because it is a composition of continuous functions.
- The feasible set of ride-sourcing demand D is nonempty, convex, and compact (assumption iii)).
- $\forall\, \mathbf{D}^R \in D, \quad d(F(\mathbf{D}^R), W(\mathbf{D}^R)) \in D$ (assumptions i and iv).

All conditions of the Brouwer's fixed-point existence theorem are satisfied and therefore an equilibrium solution exists. □

In terms of the uniqueness of an equilibrium solution, it usually requires the monotonicity of the function $F(\mathbf{D}^R)$ and $W(\mathbf{D}^R)$ (Cantarella, 1997). However, this is generally not the case for our problem. The Jacobian matrix of $F(\mathbf{D}^R)$, for example, has entries

$$\frac{dF_{rs}}{dD^R_{rs}} = \frac{dF_{rs}}{dN_r}\frac{dN_r}{dD^R_{rs}} = -t_{rs}\frac{dF_{rs}}{dN_r} > 0$$

and

$$\frac{dF_{rs}}{dD^R_{rs'}} = \frac{dF_{rs}}{dN_r}\frac{dN_r}{dD^R_{rs'}} = -t_{rs'}\frac{dF_{rs}}{dN_r} > 0.$$

A matrix for one origin and three destinations, for instance, takes the following form:

$$\begin{bmatrix} -t_{11}\frac{dF_{11}}{dN_1} & -t_{12}\frac{dF_{11}}{dN_1} & -t_{13}\frac{dF_{11}}{dN_1} \\ -t_{11}\frac{dF_{12}}{dN_1} & -t_{12}\frac{dF_{12}}{dN_1} & -t_{13}\frac{dF_{12}}{dN_1} \\ -t_{11}\frac{dF_{13}}{dN_1} & -t_{12}\frac{dF_{13}}{dN_1} & -t_{13}\frac{dF_{13}}{dN_1} \end{bmatrix}.$$

Depending on the travel time between different ODs and $\frac{dF_{rs}}{dN_r}$, the Jacobian matrix is not always positive (semi) definite. Therefore, the uniqueness of an equilibrium solution is not guaranteed.

In what follows, we specify some involved functions and show the existence of equilibrium solution for the specified forms.

Surge charge function $S_r(\cdot)$ takes the reciprocal function form:

$$S_r\left(N_r\right) = \frac{\lambda_1}{\delta + N_r} \quad \forall r \in Z, \tag{11}$$

where δ is a small positive constant used to avoid numerical issues.

Waiting time function $W_r^R(\cdot)$ takes the reciprocal function form:

$$W_r^R\left(N_r\right) = \frac{\lambda_2}{\delta + N_r} \quad \forall r \in Z. \tag{12}$$

Demand function follows the multinomial logit model and the utility (u_{rs}^R) of the ride-sourcing trip is a linear combination of fare and waiting time. That is,

$$D_{rs}^R = \overline{D_{rs}}\frac{1}{1 + e^{-\theta\left(u_{rs}^T - u_{rs}^R\left(D_{rs}^R, V_r\right)\right)}}, \quad \forall rs \in Z^2, \tag{13}$$

where u_{rs}^T is the utility for taking transit services from origin r to destination s.

Proposition 2. *With functions taking the forms as shown in (1), (3), (11), (12), (13), all the assumptions in Proposition 1 are met, and the problem has at least one solution.*

Proof. First of all, when the logit demand function is used and with linear utility functions, conditions (6) and (7) are satisfied.

The continuity of the demand function is still maintained as it is a composition of continuous functions. The feasible demand set D, in this case, is

$$\left\{ d^R \mid 0 \leqslant d_{rs}^R \leqslant \overline{D_{rs}} \quad \forall rs \in Z^2 \right\}.$$

It is non-empty, compact, and convex. In addition, due to the function form (3), (11), and (12), the image set of the fare function is nonempty and bounded, and so is the image set of the waiting time function. We let the set F and W be the closed convex hull of the image set of the two functions, respectively. Therefore, F and W are nonempty, compact, and convex. Lastly, $\forall \mathbf{F} \in F$ and $\mathbf{W} \in W$, the logit demand function maps the demand $\mathbf{D}^R$ into the set D. Therefore, there exists at least one equilibrium pattern. □

4.3. Payoff and its properties

Earning money is the main incentive for drivers to provide ride-sourcing services. The money that a driver makes depends on how much on average riders pay for the service. Usually, drivers only obtain a portion of payments from riders as their payoffs. We have limited knowledge about the strategy that TNCs pay their drivers and practically it varies from trips to trips, from cities to cities, and from companies to companies. To simplify the problem, we use a percentage number as a proxy for payoff drivers receive. A coefficient η is defined as the percentage of payment that goes to drivers' own pockets.

The total payment paid by riders originated from zone r is $\sum_{\forall s} F_{rs}^R D_{rs}^R$. A portion of riders' payments goes to the drivers' participating in the same origin zone, and if an equal probability of picking up riders for each driver in the zone is assumed, the average payoff that one driver can make is:

$$P_r = P_r\left(F_{rs}^R, D_{rs}^R, V_r\right) = \eta\frac{\sum_{\forall s} F_{rs}^R D_{rs}^R}{V_r}, \quad \forall r \in Z. \tag{14}$$

It is obvious that the payoff function is a decreasing function of the number of participating drivers, given the demand and fare. Mathematically,

$$P_r' = \frac{dP_r}{dV_r} < 0, \quad \forall r \in Z. \tag{15}$$

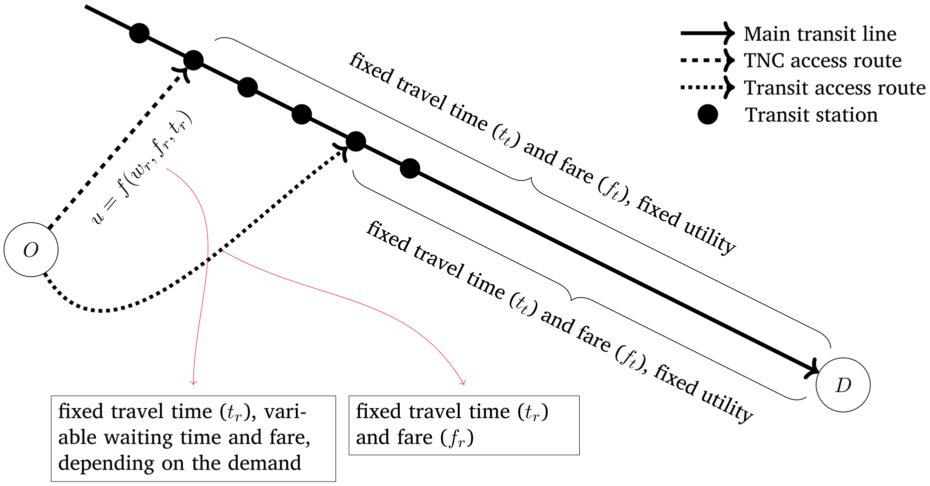


Fig. 4. Mode choice graphic representation.

4.4. Fixed-point problem and optimization models for riders and drivers

The problem presented in (13) shows the fixed-point problem presented in terms of demand. It also indicates that the utilities are not merely dependent on demand but also the number of participating drivers, V_r. In this double-sided market, potential drivers also make decisions on whether and how to participate in the system. Similar to riders' decision, we model drivers' stochastic decisions by the Logit model as well. Therefore, we can also derive a fixed-point equation for drivers' zone choice problem:

$$V_r = V\frac{e^{\theta P_r\left(V_r, \sum_{s\in Z} D^R_{rs}\right)}}{\sum_{k\in Z} e^{\theta P_k\left(V_k, \sum_{s\in Z} D^R_{ks}\right)}}, \quad \forall r \in Z. \tag{16}$$

We set up the fixed-point problem that captures the mutual influences illustrated in Fig. 2 from riders' and drivers' perspective. The two problems are equivalent and neither of them can be expressed in an explicit function form. Therefore, we propose the following two optimization models that separately deal with riders' mode choice problem and drivers' zone choice problem, and collectively exchange revealed decisions and further adjust decisions, the iterative scheme of which mimics the decision-making process of riders and drivers. In what follows, we first present the mathematical formulation for the riders' and drivers' problem and in the next section discuss how classic traffic assignment algorithms are adapted to solve them.

For riders, the split of demand is based on the individual's stochastic choices of which access mode to use and which transfer station to travel to. The riders' mode choice model is formulated below. A graph representation is shown in Fig. 4.

Riders' mode choice model

$$\min \sum_{ij\in\mathcal{A}} \int_0^{x_{ij}} u_{ij}\left(\omega\right) d\omega + \frac{1}{\theta} \sum_{rs\in Z^2} \sum_{\pi\in\Pi_{rs}} h^\pi Ln\left(h^\pi\right) \tag{17}$$

$$\text{s.t.} \quad \sum_{\pi\in\Pi^{rs}} h^\pi = \overline{D_{rs}} \tag{17a}$$

$$h^\pi \geqslant 0 \quad \forall \pi \in \Pi \tag{17b}$$

$$x_{ij} = \sum_{\pi\in\Pi_r} h^\pi \delta^\pi_{ij} \quad \forall ij \in \mathcal{A} \tag{17c}$$

Note that the model is presented with an abstract network where u_{ij} is the generalized cost(utility) of an equivalent transit link or ride-sourcing link. An equivalent link represents a sequence of links in a physical network. Π is the path set and its element is π. h^π represents path flow and x_{ij} is link flow which is related to the path flow by the 0–1 indicator δ^π_{ij}. The riders' mode choice model presented above is an adaption of the classic stochastic User Equilibrium (SUE) model (Fisk, 1980), where θ in the objective function is used as a general representation of perception error distribution parameter for riders, of which nested structure will be studied in numerical experiments. Constraint (17a) stipulates that all demand for the integrated transit system should be allocated to either transit access mode or ride-sourcing access mode.

In the drivers' zone choice model, the objective is analogous to that of riders' SUE problem. The payoff, P_r, is determined by the fare of trips from r to transfer stops s,F_{rs}, and the decision of which transfer station to use is made by riders. The first term in the objective maximizes the payoffs. Given a decreasing function as is shown in (15), the objective function, thus, is convex. Constraint (18a)

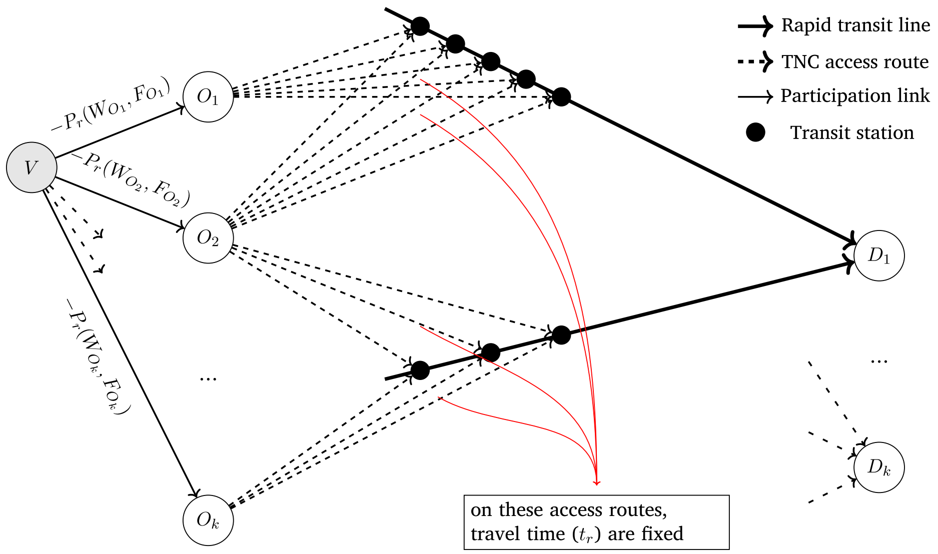


Fig. 5. Drivers' zone choice problem graphical representation.

stipulates that each of the participating drivers should be allocated to one service zone. Constraints (18b) are non-negativity constraints for decision variables V_r. The formulation is as follows.

Drivers' zone choice model

$$\min \sum_{r\in Z}\int_0^{V_r} -P_r\left(\omega\right)d\omega + \frac{1}{\theta_3}\sum_{r\in Z}V_r Ln\left(V_r\right) \tag{18}$$

$$\text{s.t.} \quad \sum_r V_r = V \tag{18a}$$

$$V_r \geqslant 0 \quad \forall r\in Z \tag{18b}$$

Proposition 3. *The drivers' zone choice behavior follows the Logit choice model.*

Proof. See Appendix A. □

The drivers' zone choice problem is graphically represented in Fig. 5. For example, there is a rider traveling from O_2 to D_1. There are several attractive integrated routes if the rider uses the integrated mode. The rider can either take ride-sourcing services to one station among those five on the upper main transit line or one among the four on the lower main transit line. This rider evaluates these potential transfer stations and makes the decision (modeled with nested logit with details in Appendix B). Once the decision is made, the origin and destination for the TNC trip are determined. All riders make their transfer stop choices, or in other words, make their destination choices for ride-sourcing trips. The average payoff, P_r, for serving each zone is updated and is perceived by drivers. Graphically, expected "disutility" for drivers zone choice links (which are artificial) get updated and drivers choose to travel to zones that are connected with them by participation links.

5. Solution algorithm

5.1. Solving riders' mode choice problem

The riders' mode choice problem is to determine the modal split between the two modes, i.e. transit access demand (D_{rs}^T) and ride-sourcing service access demand (D_{rs}^R). Given the routes for TNC vehicles and thus the transfer stations, the utilities for both modes can be calculated, and using the Logit model defined in (13) the demand for both modes can be computed. Note that the mode split function defined in (13) does not entail a closed-form solution. One can solve the system of nonlinear equations using the Newton's method (Bierlaire and Crittin, 2006). However, when the system is large computing the inverse of a matrix becomes cost-prohibitive. We propose to use the Method of Successive Averages (MSA). As is shown in Fig. 4, utilities on the ride-sourcing path and transit path are known given the solution from the problem (18). The algorithm works as follows:

Step 1 Compute the incumbent modal split using the Logit function (13) based on currently evaluated utilities;
Step 2 Update modal split by successive averaging with the diminishing step size;

Step 3 Compare the gap measure with the termination gap threshold.

Process iteratively until the convergence criterion is attained. Details are presented in Algorithm 1.

5.2. Solving stochastic zone choice problem for drivers

Given the fare and demand for each origin zone, the drivers' zone choice problem (18) can be solved through the combination of Frank-Wolfe algorithm and the Logit assignment algorithm.

Were no entropy term presented in the objective function (18), the problem became a deterministic zone choice problem. In solving a problem whose objective is the following with the same feasibility region:

$$\min \quad F\left(V\right) = \sum_{r\in Z}\int_0^{V_r} -P_r\left(\omega\right)d\omega,$$

notice that the objective in the above program is strictly convex, since the function P_r is monotonically decreasing, and thus the problem is a convex problem with equality constraint. One can adopt the conditional gradient method (Frank-Wolfe algorithm) as a solution algorithm. The key idea of the Frank-Wolfe algorithm is to iteratively approximate the objective function using a linear function. In this case, a linear approximation of the objective function around V^k is

$$F\left(V\right) \approx F\left(V^k\right) + \nabla F\left(V^k\right)^T\left(V - V^k\right). \tag{19}$$

Minimizing the function (19) reduces to minimize $\nabla F(V^k)^T V$, and in this case, it is equivalent to the following program:

$$\min \quad \sum_{r\in Z} -P_r\left(V_r^k\right)V_r \quad \text{s.t.} \quad \left(18\text{a}\right),\left(18\text{b}\right) \tag{20}$$

Notice that $P_r(V_r)$ is always positive, thus the solution to the above linear program is obvious, that is assigning V to the zone r with the largest value of profit P_r. This is comparable to an all-or-nothing assignment in the traffic assignment problem (TAP). The descending and feasible direction is found, then we shall decide the step size. Exact line search or the diminishing step-size can be used, which corresponds to the Frank-Wolfe and MSA in TAP, respectively. To calculate the step-size using the exact line search, the following problem of α is to be solved.

$$\min_{\alpha\in[0,1)} \quad F\left(V^k + \alpha\left(V^* - V^k\right)\right) \tag{21}$$

To accommodate the stochastic feature of drivers' zone choice behaviors, however, we need to modify the above algorithm. Instead of applying an "all-or-nothing" type of driver assignment strategy, drivers' zone choices are calculated by a Logit-type assignment with dispersion parameter θ_3. That is,

$$V_r = V\frac{e^{\theta_3 P_r}}{\sum_{r\in Z} e^{\theta_3 P_r}}, \quad \forall r \in Z. \tag{22}$$

After obtaining driver-zone assignment, step-size can be calculated in the same way as is shown in Eq. (21).

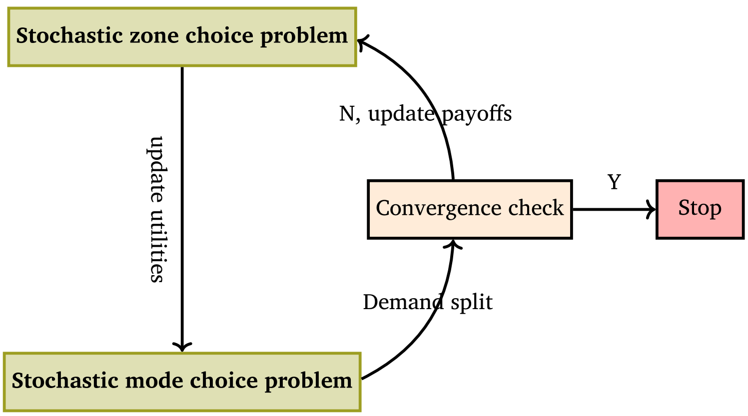


Fig. 6. Iterative solution algorithm framework.

Table 1
Numerical experiment input and default parameter settings.

Input/Parameter	Description	Value
$\overline{D_{rs}}$	Total transit riders (scaled number)	21344
λ_1	Surge charge coefficient	5.0
λ_2	Waiting time coefficient	6.6
α	Time-based TNC fare coefficient	0.26 ($/min)[a]
β	Distance-based TNC fare coefficient	0.85 ($/mile)[a]
B	Base, service, and other fixed service fee	5.0 ($) [a]
η	Ride-sourcing income reduction factor	0.5
ϵ_1	Outer loop relative error threshold	0.005
ϵ_2	Mode choice relative error threshold	0.01
ϵ_3	Zone choice relative error threshold	0.05

[a] UBER ride price estimator https://www.uber.com/us/en/ride

Table 2
Perception dispersion parameters.

Parameter	Description
θ_1	Perception dispersion parameter of riders for transfer station choice
θ_2	Perception dispersion parameter of riders for mode choice
θ_3	Perception dispersion parameter of drivers for zone choice

What is discussed above builds on the base with a certain demand distribution and fare realization. However, the assignment obtained by solving a drivers' problem will affect riders' choice and result in different demand and fare for each OD pair. Thus, the demand and fare should be updated according to the solution from the problem (18). Therefore, it is natural to have a solution algorithm that iterates between solving the riders' mode choice problem and the drivers' zone choice problem

5.3. *Complete solution algorithm framework*

Fig. 6 shows the overall idea of how the complete problem is approached. Iteratively solve the two problems and update the payoff evaluation in drivers' zone choice problem, and utilities in riders' mode choice problem until convergence. The complete solution algorithm for solving the stochastic equilibrium of the integrated system is given in Appendix C.

6. Numerical experiments

In this section, we present numerical experiments using the data of the Minneapolis-Saint Paul metropolitan area, with the purpose of 1) showing the convergence of the proposed solution algorithm; 2) showcasing the interaction and equilibrium of the demand and supply; 3) showing the efficiency gains of the integrated transit system; 4) shedding light on the partnership between public transit agencies and private TNCs; and 5) computationally exploring equilibrium uniqueness.

6.1. *Experiment input*

Three main data sets used in the study are the transportation network data, general transit feed specification (GTFS) data and the activity-based travel demand model data. For a traveler originating from zone r, his/her travel time in transit system was obtained by the scheduled-based shortest path algorithm (Khani, 2017) using the GTFS data. The ride-sourcing trip's travel time and distance were obtained by running the shortest path algorithm using the road network data, and the monetary cost was calculated based on both travel time and travel distance. For the convenience of analysis, traffic analysis zone (TAZ) was used as analysis unit. There are around 3,000 TAZs in the metro area which about half of them are covered by transit services. The demand in each TAZ was obtained from the activity-based travel demand model.

The numerical experiments were carried out with multiple origins and a single destination, which can be easily extended to cases with multiple destinations. The destination was selected to be the Minneapolis-St. Paul airport (MSP), which on average had a daily passenger volume of more than 100,000 in 2017, and around 60% of them were originated locally (MSP, 2018). Besides, there is a light rail system connecting the downtown area and the airport providing a reliable and fast public transit service. In the following tests, we focus on the *access* to the transit system.

The following assumptions were adopted for numerical tests, and the default parameter settings used throughout the study are as shown in Table 1 if not otherwise stated.

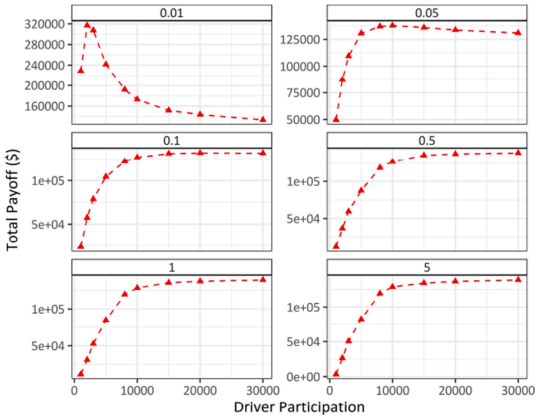


Fig. 7. Total payoff for drivers with varied mode choice perception dispersion parameter θ_2 and total participating drivers.

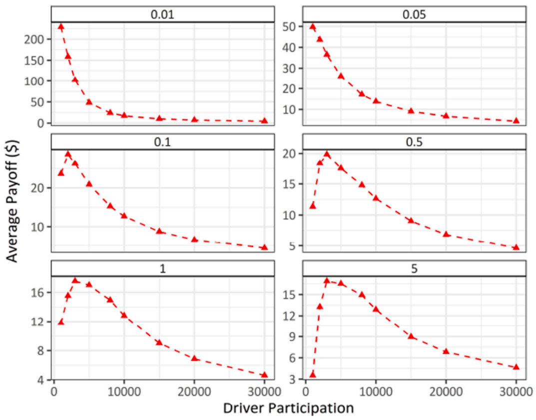


Fig. 8. Average payoff for drivers with varied mode choice perception dispersion parameter θ_2 and total participating drivers.

- Transit riders use the light rail as their main transit mode, and they access it by either other transit services or by using ride-sourcing services;
- TAZ demand for the airport is scaled up to better reflect mechanism of the system.

6.2. Perception dispersion parameter analysis

This set of tests was designed for understanding how varied perceptions among the system users (riders and drivers) affect system performance. There is a lack of related studies in understanding travelers' behavioral reactions to the system, and we hope to gain some insights into the issue for developing some policy rules for the integrated system.

Transit and TNC service riders perceive travel time and monetary cost; TNC drivers perceive payoffs for serving different zones. The stochasticity of system users' perception was reflected through the dispersion parameter in the logit model. Table 2 summarizes these perception dispersion parameters. The first two are related to riders' mode and destination (i.e. transfer stop) choice and the last is for drivers' zone choice. In modeling riders' decisions, we adopted a nested logit model as mentioned earlier. A rider decides both access modes and transfer stations if necessary: one evaluates the ride-sourcing access mode comprehensively based on various potential combinations of transfer stations and succeeding transit trips, and determines between the two access modes. See Appendix B for details for the nested-logit model.

6.2.1. Sensitivity analyses on riders' mode choice perception dispersion parameter θ_2

The following tests were conducted with fixed riders' perception parameter θ_1 and drivers' perception parameter θ_3, the values of which is 0.05 and 0.1, respectively. The total payoff, average payoff, number of ride-sourcing riders, and workload (rider-driver ratio) were compared based off varied θ_2 and the total number of participating drivers V.

The total payoff is defined as $\sum_{r\in Z} P_r V_r$, which is a system-wide measure of revenue that all participating drivers in service make. Note that every single plotted point in Fig. 7 is an equilibrium under the corresponding conditions. A set of total participating drivers (labor pool size) were chosen to be {1000, 2000, 3000, 5000, 8000, 10000, 15000, 20000, 30000} as to include both over-supplying

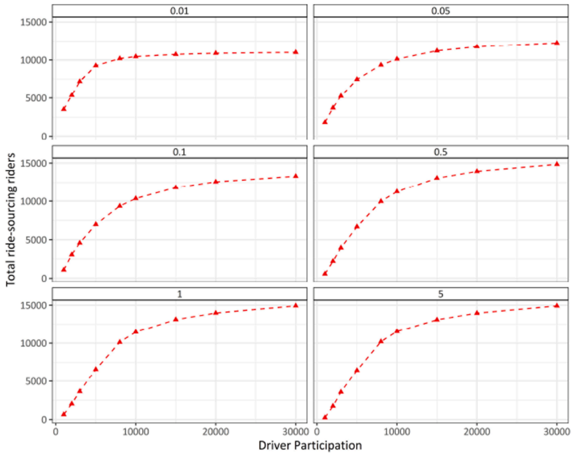


Fig. 9. Total number of riders of the integrated transit system with varied mode choice perception dispersion parameter θ_2 and total participating drivers.

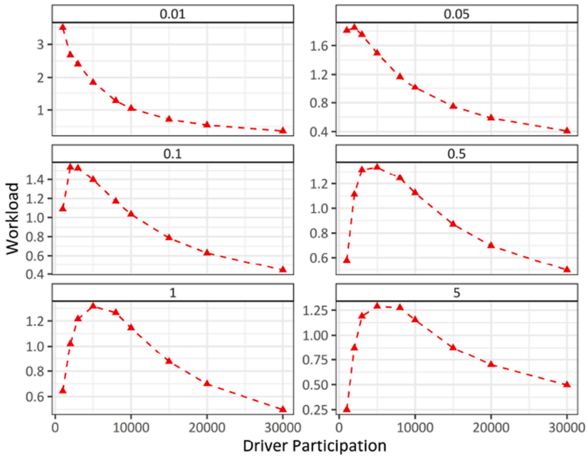


Fig. 10. Rider driver ratio (demand supply ratio) with varied mode choice perception dispersion parameter θ_2 and total participation drivers

and under-supplying scenarios. Firstly, in the subplot where $\theta_2 = 0.01$, the total payoff curve exhibits a skewed bell shape that reaches the maximum total payoff of around $320,000 at $V = 2,000$. The total payoff increases first and then decreases. In the second subplot ($\theta_2 = 0.05$), a similar pattern is observed with the difference that the maximum total payoff is lowered to $140,000 at $V = 10,000$; further increasing θ_2 to 0.1, with the optimal point being (20000, 132000), the optimal labor pool moves further to the right, which is already close to the total demand. In the subsequent tests, the total payoff becomes monotone with respect to V. In these cases(when θ_2 is 0.5 or 1.0 or 5.0), the system is apparently not efficient since the optimal labor pool size is even greater than the total demand. To sum up, an increase of mode choice perception dispersion parameter θ_2 leads to larger optimal labor pool size and smaller optimal total payoff when a TNC wants to maximize its total system payoff. From the standpoint of maximizing the system total payoff for drivers, the optimal number of TNC drivers (labor pool size) does exist when the mode choice perception dispersion parameter is below the threshold 0.1 in our case. *When riders become more rational, more drivers are needed for reaching the optimal total payoff. In addition, optimal total payoff decreases when riders become more rational.*

From the perspective of maximizing individual drivers' average payoff, however, it shows a rather distinct pattern. In Fig. 8, we recorded the results for the average payoff. When θ_2 takes small values, say 0.01 or 0.05, the average payoff defined as $\frac{1}{V}\sum_{r\in Z}\sum_s P_{rs}D_{rs}^R$ decreases monotonically with the number of participating drivers, whereas the total payoff curve exhibits a peak. Larger θ_2, however, exhibits a bell curve shape. In other words, in terms of the average payoff, there is an optimal labor pool size for cases where θ_2 is above a threshold. For example, when $\theta_1 = 0.1$, the optimal average payoff is about $30 and the corresponding optimal labor pool size is 2,000; when $\theta_2 = 0.5$, the optimal average payoff is about $20 with corresponding optimal labor pool size being 3,000. *Riders who are more rational lead to less average payoff as is reflected by the decreasing optimal average payoff.*

Comparing Fig. 7 and Fig. 8, we see that the fleet-size rendering the largest total payoff does not necessarily equal that of the largest average payoff. To see why this is the case, we further compared the demand for using the integrated system in Fig. 9 and the workload

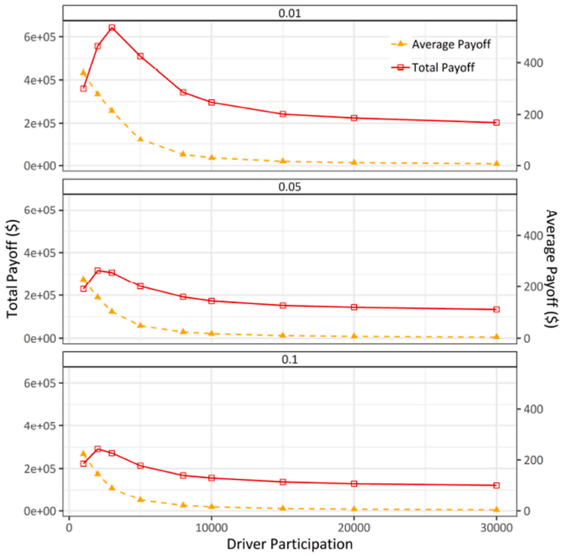


Fig. 11. Total payoff and average payoff curves with varied riders' transfer station choice perception dispersion parameter $\theta_1 \in \{0.01, 0.05, 0.1\}$.

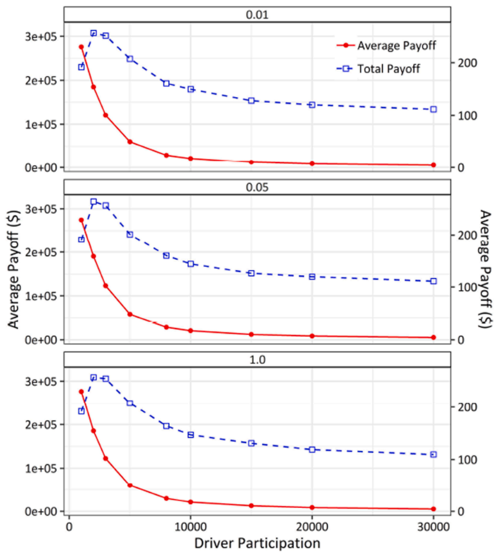


Fig. 12. Total payoff and average payoff curves with varied driver zone choice perception dispersion parameter $\theta_3 \in \{0.01, 0.05, 1.0\}$.

in Fig. 10.

In Fig. 9, there are some common features across all testing cases. First, the number of riders using the integrated system monotonically increases. Second, the curve is concave. That is, the demand for the integrated system increases when the supply increases. However, the demand increasing rate decreases. This shows the "decreasing returns to scale" property that the demand does not increase at the same rate as the supply does (Yang and Yang, 2011). Intuitively, more participating drivers in the system who provide TNC services should attract more riders to the integrated system. With more drivers in the system, however, the marginal utility for using the integrated system is decreasing because waiting time and fare surge reduction extent get smaller and smaller. From the drivers' side, when the labor pool size is small, drivers are complements and they attract more riders; Whereas when the total number of drivers is large, drivers are more likely to compete and act as substitutes which only induce a limited number of additional demand. This is consistent with the analysis of Benjaafar et al. (2018).

On the magnitude of the number the integrated system riders, when riders are not rational they are more likely to choose modes in a completely random way, which results in an equal probability of choosing the two alternative modes. This is reflected in the first subplot ($\theta_1 = 0.01$) that when the total participating number is getting larger, the number of riders of the integrated system becomes about half of the total demand $\overline{D}$. In contrast, when riders are rational, having more drivers attracts more riders, but having fewer drivers loses riders drastically. This indicates that rational riders are more sensitive to the supply level.

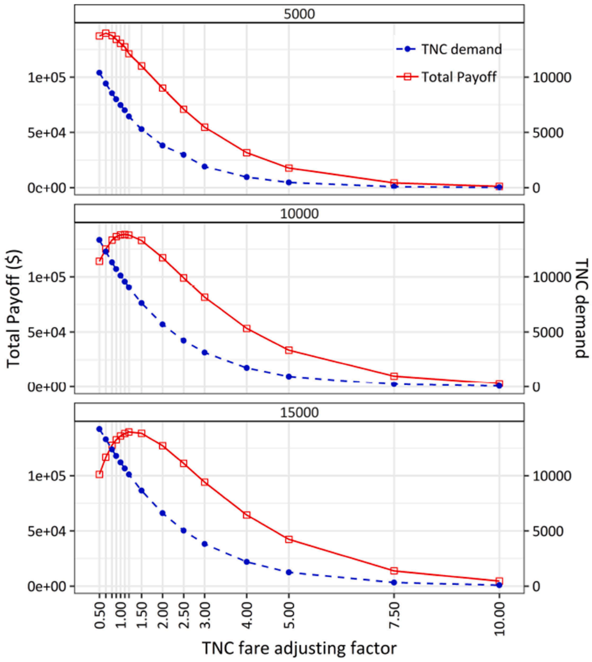


Fig. 13. Total payoff for drivers and demand for using the integrated system with varying total number of drivers $V \in \{5000, 10000, 15000\}$ and fare adjusting factor $\mu \in \{0.5, 0.65, 0.80, 1.0, 1.2, 1.5, 2.0, 2.5, 3.0, 4.0, 5.0, 7.5, 10.0\}$.

In Fig. 10, we defined a workload parameter to reflect how busy on average a driver can be. It was calculated by the quotient of the total number of TNC service riders and the total number of drivers at equilibrium. The pattern displayed simulates the pattern of average payoff shown in Fig. 8. When the ride-sourcing system is either over-supplied or under-supplied, the average workload is low and idle time is longer, and only when the labor pool size is kept within an appropriate range can drivers become busy and more profitable in terms of the average payoff.

To summarize the tests carried out for mode choice parameter θ_2, we can draw the following conclusions:

1. Demand for using the integrated system exhibits the property of "decreasing returns to scale";
2. There exist optimal labor pool sizes in terms of maximum average payoff for drivers when riders are rational; and
3. Rational riders result in lower maximum total payoff and average payoff for drivers.

6.2.2. Sensitivity analyses on riders' transfer station choice perception dispersion parameter θ_1 and drivers' zone choice perception dispersion parameter θ_3

The first group of tests are designed to understand impacts of riders' transfer station choice behavior, thus we fixed θ_2 and θ_3 to be 0.01 and 0.1 respectively, and tested on varied θ_1. Only the results of average payoff and total payoff were presented for the sake of brevity (see Fig. 11).

In Fig. 11, all three average payoff curves exhibit the monotonically decreasing pattern, while total payoff curves peak at some optimal labor pool sizes. Similar to the results of θ_2, when riders are less rational, the maximum total payoff for drivers is higher than that of rational riders'. This also applies to the average payoff for drivers. One reason behind this is as mentioned previously. It could also be attributed to that when a rider is rational, it is more likely for him/her to take a TNC vehicle to the transfer stop that is closest to his/her origin. That is, a rational transit rider is less likely to take a long ride-sourcing trip if there is a shorter one. Thus, shorter distance ride-sourcing trips generally result in less fare and therefore less total payoff for drivers.

The second group of tests were carried out with fixed θ_1 (0.5) and θ_2 (0.05) and varied θ_3 and V for the sensitivity analysis on drivers' perception dispersion parameter (see Fig. 12).

Unlike riders' perception dispersion parameters, the performance of the integrated system is not very sensitive to drivers' perception dispersion parameter θ_3. It can be seen from the three subplots in Fig. 12, ranging θ_3 from 0.01 to 1.0 does not lead to significant changes in total payoff and average payoff. We conclude that the integrated system is more sensitive to riders' perception and their choice behaviors rather than drivers'.

In summary, we draw the following conclusions:

1. Riders who are more rational in selecting transfer stations result in lower payoff to drivers, which is consistent with mode choice parameter results;
2. Drivers' zone choice decisions are not as influential as riders' choice decisions in terms of system performance.

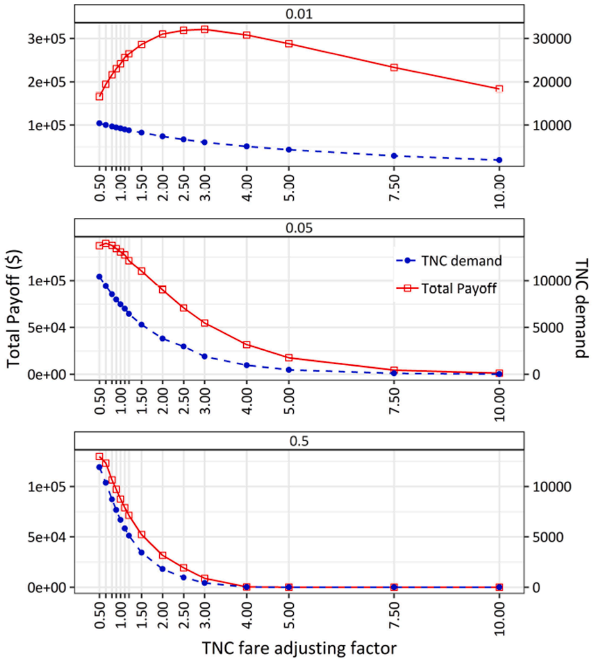


Fig. 14. Total payoff for drivers and demand for using the integrated system with 5000 drivers, varied fare adjusting factor $\mu \in \{0.5, 0.65, 0.80, 1.0, 1.2, 1.5, 2.0, 2.5, 3.0, 4.0, 5.0, 7.5, 10.0\}$ and with varied $\theta_2 \in \{0.01, 0.05, 0.5\}$.

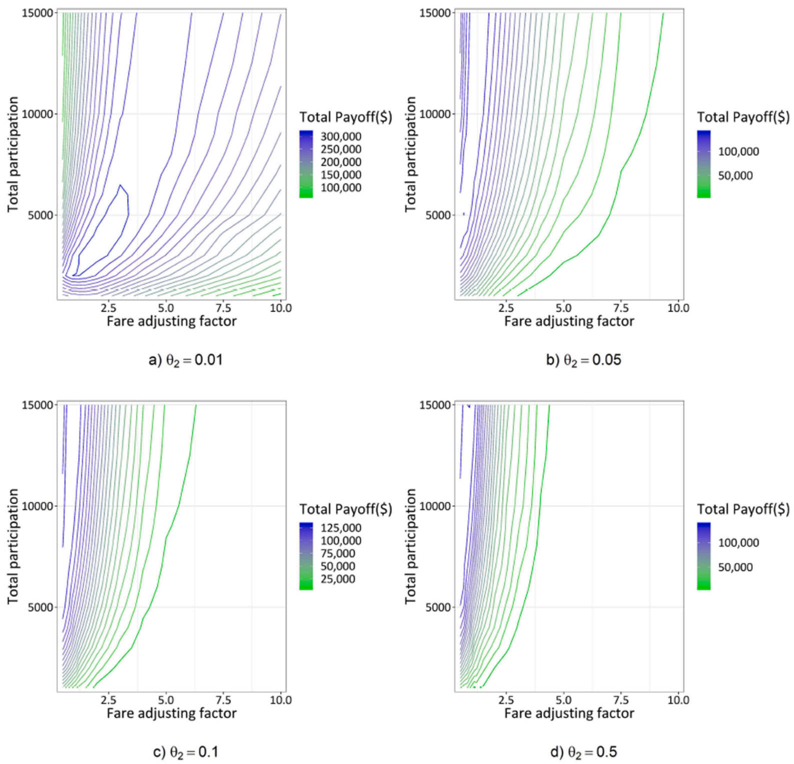
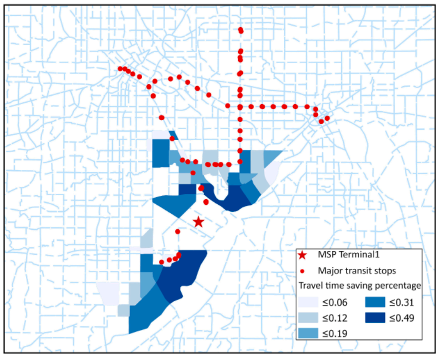
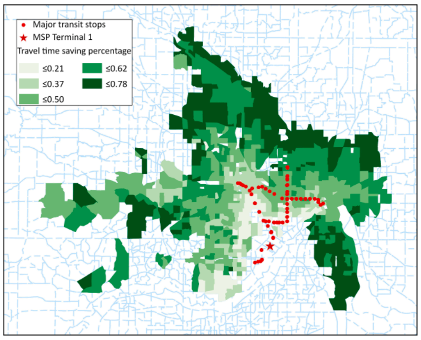


Fig. 15. Contour lines of total payoff for drivers and demand for using the integrated system with varied number of drivers, varied fare adjusting factor $\mu \in \{0.5, 0.65, 0.80, 1.0, 1.2, 1.5, 2.0, 2.5, 3.0, 4.0, 5.0, 7.5, 10.0\}$ and with varied $\theta_2 \in \{0.01, 0.05, 0.1, 0.5\}$.



(a) Traffic analysis zones whose travel time is shorter by using the transit system exclusively



(b) Traffic analysis zones whose travel time is shorter by using the integrated system

Fig. 16. Traffic analysis zones with travel time comparison. Tests were carried out with $\theta_1 = 0.01, \theta_2 = 0.01, \theta_3 = 0.1, V = 5000$. The destination MSP airport is indicated with a pentagram on the map. Round dots show the light rail system and a BRT line currently in operation in the metropolitan area.

6.3. TNCs' pricing strategy exploration

In this subsection, we explore how pricing strategy should be designed from the perspective of the TNCs to maximize its drivers' total payoff. A new parameter μ named as "TNC fare adjusting factor" was defined. As is mentioned, the current pricing strategy adopted by most TNCs are both distance- and time-based, which takes the following form:

$$F_{rs}^{R} = \left(1 + S_r\left(N_r\right)\right)\left(\alpha t_{rs}^{R} + \beta L_{rs}^{R}\right) + B.$$

The new parameter is used as a scaling factor operating on the entire baseline fare structure. That is, an adjusted new fare for an OD pair (r, s) is

$$\widetilde{F}_{rs}^{R} = \mu F_{rs}^{R}.$$

If $\mu < 1$, the new fare is discounted; if $\mu > 1$, it rises in price. Figs. 13 and 14 summarizes the results for the varied total number of drivers and perception dispersion parameter θ_2, respectively.

The results shown in Fig. 13 were obtained by fixing θ_1, θ_2, and θ_3 to 0.01, 0.05 and 0.1, respectively and varying the total number of drivers. The decreasing pattern in demand is straightforward given that fare for the same trip increases with a higher TNC fare adjusting factor. In addition, increasing the number of drivers, the number of riders increases for each fare adjusting factor, which was consistent with the results shown and explained in the previous section. Curves of total payoff, however, show that some certain fare adjusting factors render the best total payoff for the same level of supply. For example, when there are 5,000 drivers, a pricing strategy for the TNC is to lower their price to 0.65 of the current fare level; when there are 15,000 drivers in the system, a slight increase to 1.20 of the current fare level maximizes total payoff; and when drivers are around 10,000, total payoffs resulted from applying any adjusting factor within the range $[1.0, 1.2]$ are very comparable. It is also obvious when the adjusting factor is greater than a certain threshold, say 2.0, both the demand and the total payoff go down drastically. When it becomes extremely expensive to use the TNC services, the demand and total payoff converge to zero. The results indicate that a generally small fleet-size should come with a relatively low pricing level, otherwise high price makes the attractiveness of ride-sourcing services very limited and loses the demand significantly.

We also tested the change of total payoff and the demand with respect to the mode choice perception parameter θ_2 by fixing the total number of drivers to 5,000 and θ_1 and θ_3 to 0.01 and 0.1, respectively (see Fig. 14). The decreasing and convergence toward zero trends of the demand go without saying. What is also consistent with previous test results is that a combination of more rational riders and lower fare attracts more demand. When it comes to the total payoff curve, the three curves show quite different shapes. For the case where $\theta_2 = 0.01$ and riders are less rational, the best total payoff a system with 5,000 drivers can achieve is when the fare is 3 times as expensive as the current baseline case. For more rational riders, the peak of the total payoff curve shifts to the left, which gives rise to the optimal total payoff when the current fare is reduced. In addition, when riders become more rational, the magnitude of the maximum total payoff shrinks to about half of the less rational case. This is again in agreement with our previous finding that it is more difficult for TNCs and their drivers to make a profit when the riders are more rational.

Two sets of tests together show that a TNC's optimal pricing strategy should not only take into account the perception error of users, but also the labor pool size. To this end, we conducted a few more similar tests and recorded the results in Fig. 15. The derivative contour plot that takes as input a total number of participating drivers V, fare adjusting factor μ, and perception dispersion parameter θ_2, and output the total payoff for each combination of the three parameters. The results should provide a complete view of the impacts of the three input parameters on the total payoff and can give some insight into the optimal pricing strategy for TNCs.

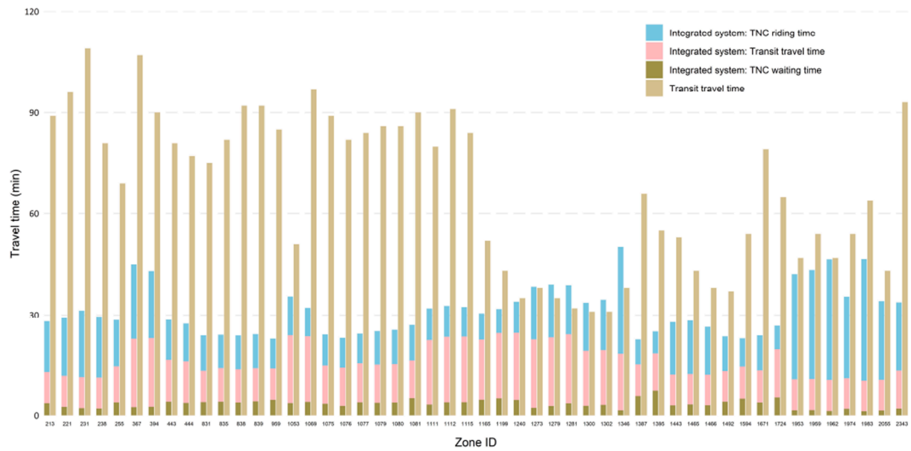


Fig. 17. Total travel time comparison between the integrated system and transit system solely for the top 50 TNC demand TAZs. Tests were carried out with $\theta_1 = 0.01, \theta_2 = 0.01, \theta_3 = 0.1, V = 5000$.

Table 3
Total travel time saving and total extra paid TNC fare.

θ_2	V	$D^R_{rs}/\overline{D_{rs}}$	Total travel time saving ΔT (h)	Total Extra fare paid ΔM ($)	Average cost per hour travel time saving ($/h)
0.01	5000	0.43	9709.85	523333	53.89
	10000	0.49	11174.32	356246	31.88
0.05	5000	0.35	10190.90	276295	27.11
	10000	0.47	12715.27	284771	22.40
0.50	5000	0.31	10878.85	184944	17.00
	10000	0.53	15914.62	260966	16.39
1.00	5000	0.31	10767.03	179509	16.67
	10000	0.54	16069.13	264618	16.47

Reading Fig. 15, we see the following common features. Fixing the total number of drivers, there exists a fare adjusting factor that maximizes the total payoff, and this adjusting factor occurs mostly at values less than 5.0. A subplot of Fig. 13 is such a cross-sectional analysis. Fixing the fare adjusting factor, there exists a labor pool size that maximizes the total payoff which occurs mostly with a relatively large number of drivers. A larger labor pool size makes a TNC able to have a relatively high pricing level so as to maximize the drivers' total payoff. This is reflected that the ridge of the contour goes in a southwest-northeast direction on the graph. Besides, a slight increase in fare requires a drastic increase in labor pool size in order to achieve the best total payoff. In short, to maintain the largest total payoff, a higher pricing level should come with a larger labor pool size, and vice versa. The fact that with a higher value of θ_2 contour lines become denser indicates that rational riders are more sensitive to fare change and the total payoff shrinks as riders become more rational.

6.4. *Travel time comparison*

This subsection serves to show the efficiency gains of the proposed integrated system where (potential) transit users have an additional access mode which is the ride-sourcing services. The ride-sourcing services can bring them to trunk lines of the mass transit system in a shorter time or even provide access to the transit system for those who would not otherwise be accessible to the transit system. In exchange for a shorter travel time trip (travel time saving), a traveler using the integrated system need to pay extra money for the ride-sourcing services. Therefore, two measures including travel time saving and extra cost were used in these tests to show whether or not and how efficient the integrated system is.

Recall that experiments were conducted with a single destination, the MSP airport that is connected with a light rail system (see Fig. 16 for details). Travel times for the complete trip using either transit or TNC services as an access mode were compared based on travelers' origin zones. Fig. 16a displays geographic locations of the zones whose travel time is shorter by using transit as an access mode. The travel time saving percentage was also plotted for comparison. We see that zones in the vicinity of the destination have shorter transit travel time, and are the zones that are mostly accessible to the light rail system. In addition, the closer zones are to the destination, the more travel time saving is by using the transit system exclusively. Among these zones, travel time saving by the transit system (exclusively) can be as large as about 50%. Fig. 16b shows the zones with shorter travel time by the integrated system and the extent to which travel time saving is. It is intuitive that travel time saving along the light rail system is relatively low compared to zones that are far away from the light rail system. Zones that are in the peripheral regions of the light rail system generally obtain a travel time saving of less than 20%. Travel time saving becomes more significant for zones that are not covered by good transit services and

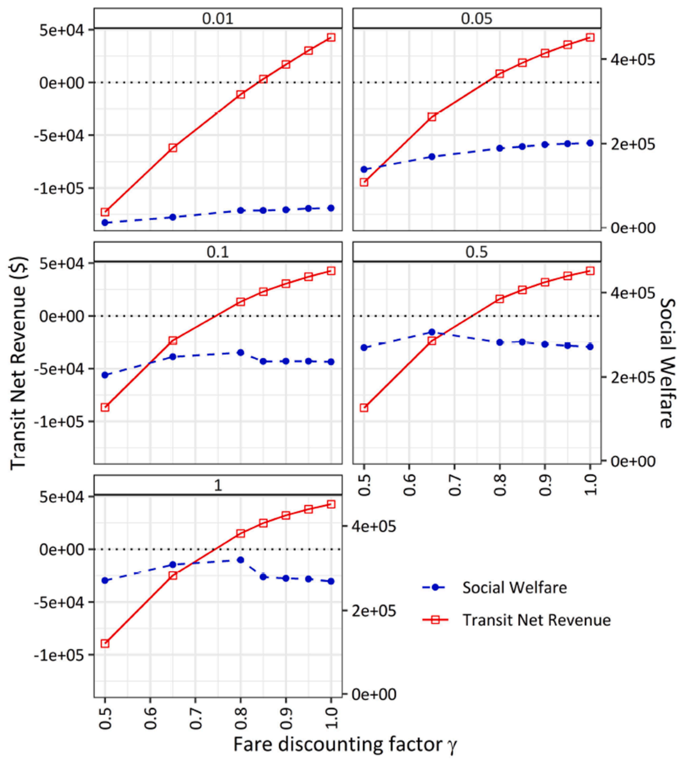


Fig. 18. Transit system revenue and total social welfare for varied subsidy discounting factor and θ_2 with $V = 5000, \theta_1 = 0.05, \theta_3 = 0.1$.

are far from the light rail system. The travel time saving by using the integrated system is 57% on average and can be as high as 78%.

Fig. 17 selected the top 50 TAZs in terms of the integrated system demand and compares the travel time for both modes. Travel time for the integrated system is broken down into three main components: ride-souring waiting time, ride-sourcing riding time, and transit travel time. For most of these zones, the integrated system's travel time is significantly shorter than its counterpart. For the integrated system, either ride-sourcing riding time or transit travel time dominates the entire travel time. Zones 1240, 1273, 1279, 1281, 1300, 1302 and 1346 locate in downtown Minneapolis and are also close to the light rail system, whcih explains why the using transit system exclusively takes shorter time compared to the integrated mode. Using the integrated system does save travelers a significant amount of travel time but with the expense of paying more for their trips. Table 3 compares the travel time saving and extra fare paid.

We included both total travel time saving and total extra fare paid by the integrated system user. Total travel time saving (ΔT) was calculated by summing across TAZs travel time saving weighted by the corresponding demand, and total extra fare paid (ΔM) was calculated by summing the extra fare weighted by the corresponding demand for each TAZ. Therefore, the two number measures the total extra monetary cost induced by taking ride-sourcing services that shorten the travel time by the total travel time saving. In the last column, the quotient of the two measures was computed to reflect the monetary cost travelers paid on average for a unit of travel time saving. For riders that were more irrational, they behaved to have a rather high value of time (VOT) mainly because their choices between the modes were random with very limited rationality. This results in a large extra total paid fare without benefit much from the integrated system. In contrast, in the cases of more rational riders, the total extra fare paid by them was much smaller with slightly longer travel time saving. This results in a lower observed VOT for rational riders. Besides, rational riders' observed VOTs were relatively stable given the varied total number of drivers, while this was not the case for irrational riders.

6.5. *Transit agency subsidy strategy*

Some current practice adopted by transit agencies is to subsidize the ride-sourcing travelers if a portion of their trip is completed with transit services. To shed some light on how transit agencies should subsidize ride-sourcing trips for those transit riders who use them as an access mode to public transit, we conducted this set of experiments. In the following tests, for simplicity, we adopted a fare discounting factor $\gamma(\in [0, 1])$ that applies to riders' ride-sourcing trip fare so that riders only pay discounted fares equal to γ of the regular fare, and the discounted portion $(1 - \gamma)$ is paid to TNCs by transit agencies. The following assumptions are made for these experiments:

- Equal and flat transit fares τ (not distance-based, free transfer within 2 hours and we assume all trips using transit can be finished in 2 hours).

- There is no service change in the transit system, thus operational and capital cost for the transit system remains constant in all scenarios.

In Fig. 18, transit net revenue and social welfare are used. Transit net revenue is defined as follows:

$$\text{Transit net revenue}\left(\$\right) = \text{transit farebox revenue}\left(\tau\sum_{rs\in Z^2}\overline{D_{rs}}\right) - \text{transit agency's subsidized monetary cost}\left(\left(1-\gamma\right)\sum_{rs\in Z^2}F_{rs}^{R}D_{rs}^{R}\right).$$

Social welfare is defined as the consumer surplus (maximum price travelers are willing to pay less the actually price paid for the travel-time saving), total payoff for drivers, and transit net revenue:

$$\text{Social welfare}(\$) = \text{Travelers' consumer surplus}(\rho\Delta T - \gamma\Delta M) + \text{Drivers' total payoff} + \text{Transit net revenue},$$

where ρ is the VOT parameter and for simplicity we take the universal value of 20($/h) for all tests.

In each subplot of Fig. 18, with the increase of fare discounting factor, γ, the portion total subsidy a transit agency needs to pay gets smaller. Note also that the transit fare income is a constant, thus these curves present the increasing trend. A horizontal line of zero transit revenue was plotted as a reference line, which represents the point where transit fare income exactly cancels out the subsidy expenditures. Under current settings, if a transit agency pays no more than 15% of the TNC fare, then the transit system is profitable when neglecting operating and capital costs. In terms of social welfare curves, when θ_2 is below 0.05, social welfare is maximized with no subsidy; when riders are more rational, the social welfare curve peaks at some discounting factor ranging from 0.65 to 0.8. For example, when $\theta_2 = 0.1$, social welfare is maximized when $\gamma = 0.80$ with the optimal value being around $260,000 and the transit system is profitable; whereas when $\theta_2 = 0.5$, social welfare is maximized when $\gamma = 0.65$ but the subsidy outweighs the total fare collection of the transit system.

The results of this subsection are by no means perfect to reflect the complexity of the integrated system where subsidy exists. There are other aspects where the system can be improved not only from the planning level that how the transit system should be designed, how frequent local bus routes should be operated or replaced with other on-demand services but also from the policy-making level that how the subsidies should be devised. However, as a pilot study exploring the potential partnership between the public and private transportation providers, we still feel the importance of these tests and the derived insights into the issue.

6.6. Experiments on the solution uniqueness

As is discussed in Section 4.2, the equilibrium uniqueness of such a system is generally not guaranteed. It is of interest to test through computational experiments to see if the equilibrium solution of riders' and drivers' choices is unique or not. To show this, we conducted a series of experiments with varied initial conditions and key parameter settings. Specifically, besides the driver participation initialization presented in line 7 of Algorithm 1, we tested another initialization method: assigning drivers proportionally to the potential demand in each TAZ. Mathematically, it is $V_r = V\frac{\sum_{s\in Z}\overline{D_{rs}}}{\sum_{r\in Z}\sum_{s\in Z}\overline{D_{rs}}}\quad \forall r\in Z$. We refer to the former and the latter initialization method as equal participation initialization and proportional participation initialization. In addition to participation initialization, we also tested a different demand profile with randomized total potential demand at each zone, which in the sequel is referred to as demand profile B. These tests are tested with total participation V and the key mode dispersion parameter θ_2 taking discrete values in their respective range, with other parameters held constant.

The computational results are presented in Table D.4 and D.5. We documented the total demand for the integrated services (D_{rs}^{R}) and the total payoff. To compare the equilibrium solutions with the two initial conditions, four metrics are computed. In the table, the four metrics from left to right is the relative difference in total integrated service demand (⑤ = (|① −③|)/(0.5(① + ③))), the relative difference in the total payoff (⑥ = (|② −④|)/(0.5(② + ④))), the average difference of integrated-service demand across TAZs, and the average difference of driver participation across TAZs. The last two are computed by taking the average of the absolute value of the difference in integrated-service demand of each zone, and the difference in the number of participating drivers of each zone. They present a difference value by comparing equilibrium solution zone-by-zone.

The results reveal the following key information: 1) Even with very distinct initial conditions, the final equilibrium solutions obtained are very similar to each other by either the aggregated relative difference (columns ⑤and ⑥) or the disaggregated absolute difference (columns ⑦ and ⑧) standards. The majority of the relative difference is less than 0.5%. 2) The difference between the two obtained equilibrium solutions increases with the decrease of the total number of participating drivers and the increase of parameter θ_2. Still, we are unable to draw any conclusion on the uniqueness. The solution algorithm is not exact and the difference may be due to computational errors. It could also be the case that there are multiple equilibrium solutions, but their values are similar. Due to these reasons, we still cannot conclude on the uniqueness. However, from a practical perspective, ending in a different equilibrium state (assuming non-uniqueness) may not change the system performance (such as total payoff) drastically.

6.7. Experimental test summary and managerial implications

We set out from the initial state with zero TNC vehicles. Increasing the fleet-size, ride-sourcing service's waiting time is significantly reduced compared to the initial state, making it increasingly attractive. Travelers start to use the ride-sourcing services, but as

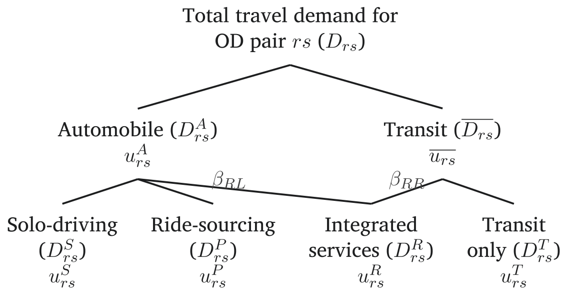


Fig. 19. Tree diagram of nested logit model.

more drivers join, waiting time reduction ceases to play a dominant role. That is, with limited improvements in the waiting time, the marginal demand decreases. When the system is over-supplied, the total payoff for TNCs only increases marginally but the competition between drivers becomes more intense, leading to a drastic drop in their average payoff. Therefore, TNCs should manage their fleet-size to a slightly under-supplied state to guarantee the participation of drivers while still earning a good amount of revenue. When it comes to the TNC pricing strategies, it still comes down to the trade-off between the waiting time reduction and fare increase. With a relatively smaller fleet-size, the price should be set lower to compensate for the waiting time increase; with a relatively larger fleet-size, the price could be set higher while riders benefit from lowered waiting time.

It has been shown that riders that are more rational make it hard for the ride-sourcing to make higher revenue. Besides, riders' decisions (especially the mode choice decisions) affect the performance of the ride-sourcing system more than drivers' decisions. Therefore, TNCs should encourage more ride-sourcing services towards transit-deficient areas as rational travelers living in good transit service areas are less likely to take ride-sourcing services. Complementing transit services by serving areas with poor transit services is indeed the original motivation for the integration. High-level economic analysis indicate that there exist an optimal range for subsidizing integrated trips, with potential insight on how agencies should allocate budget to achieve the highest social welfare.

6.8. Model limitation and implications

As is presented in Fig. 3, our current analysis is restricted in the transit branch with constant total demand of the transit and integrated mode ($\overline{D_{rs}}$). To model its demand elasticity, riders' stochastic choice between the automobile and transit branches, one unavoidably needs to incorporate the traffic congestion effect as it is a crucial utility determinant to the automobile branch. In addition, the demand for the integrated mode can affect the road network congestion level as it involves the use of ride-sourcing services. Moreover, transit services can also be affected by road congestion, especially when local bus services are considered. Besides congestion, the supply of ride-sourcing services affecting service waiting times also contributes to the demand elasticity. One, however, may not use a function such as

$$\overline{D_{rs}} = D_{rs} \exp\Big(-V_t\Big), \tag{23}$$

where D_{rs} is the total potential demand and V_t represents the overall utility function of the transit branch, to compute the demand. It is because changes in the supply of ride-sourcing services affect the utility of the integrated mode in the transit branch and also change the utility of the ride-sourcing services in the automobile branch, making the assumption suggested by (23) that the transit branch demand $\overline{D_{rs}}$ is monotone in utility V_t inappropriate. In short, when studying demand elasticity both the congestion effect and the supply of ride-sourcing services requires the inclusion of the automobile branch.

When both of the branches are included in the discrete choice model, a cross-nested logit model, shown in Fig. 19, can better reflect the taxonomy of all modes of interest. In the figure, the utility of each mode and branch are indicated by suitable notations of u_{rs}. We conceptually present their utility functions and nested logit probability functions. Parameter β_{RL} and β_{RR} represents the degree of membership of the integrated mode to the automobile (left) branch and the transit (right) branch, respectively.

$$u^R_{rs} = u^R_{rs}\left(D^R_{rs}, W_r\right) \tag{24}$$

$$u^P_{rs} = u^P_{rs}\left(D^P_{rs}, W_r\right) \tag{25}$$

$$IV_R = e^{-\theta_R u^T_{rs}} + \beta_{RR}^{\delta_R} e^{-\theta_R u^R_{rs}} \tag{26}$$

$$IV_L = e^{-\theta_L u^A_{rs}} + e^{-\theta_L u^P_{rs}} + \beta_{RL}^{\delta_L} e^{-\theta_L u^R_{rs}} \tag{27}$$

$$D^R_{rs} = \overline{D_{rs}} \frac{{\beta_{RR}}^{\delta_R} e^{-\theta u^R_{rs}}}{IV_R} \tag{28}$$

$$\overline{D_{rs}} = D_{rs}\frac{IV_R^{\frac{1}{\delta_R}}}{IV_L^{\frac{1}{\delta_L}} + IV_R^{\frac{1}{\delta_R}}} \tag{29}$$

Demand split between the two branches and multiple modes is selectively presented by (24)–(29). (24) and (25) emphasize that the utilities are functions of corresponding demand (affecting congestion) and waiting time W_r (note it is the same waiting time for both utilities). (26) and (27) computes the inclusive value of the right and left branches, respectively. δ_R and δ_L are nesting parameters, and θ_R and θ_L are dispersion parameters for the two branches. (28) computes the demand for the integrated demand, which involves the conditional probability of choosing the integrated mode in the transit nest. (29) gives the transit branch demand, which involves the probability of choosing the transit nest between the two nests. With the cross-nested logit model, the interrelations between the two nests are described in detail, with which the demand elasticity can be better studied.

When it comes to the modeling of traffic congestion, a traffic assignment model that is capable of dealing with non-separable link cost is needed as congestion exists in multiple parts of the complete system and involving modes both contribute to and are affected by road network congestion. The optimization problem like (17) can no longer be employed, and neither is its solution algorithm Algorithm 1. In terms of network representations, one may need to employ an extended network representation where each mode is represented by one duplicated layer of the original physical road network, and the integrated mode and other combinations of modes are represented by allowing transferring between layers. With this representation, each mode maintains its unique utility function. The congestion effect on the original physical link can be represented as a function of traffic flows on all the extended links duplicated from it. In terms of modeling approaches, one can adopt Variational Inequalities or Complementarity formulations to study the non-separable traffic assignment problem.

Without a specific cross-nested logit model and a traffic assignment model capturing congestion effects, anticipating changes in the computational results can be difficult considering demand elasticity. However, were we to ignore the congestion effect, we could discuss an extreme case that the integrated services are perceived more as ride-sourcing services than transit services, that is, $\beta_{RL} \gg \beta_{RR}$. The integrated mode approximately belongs to the automobile nest only. With more supply of ride-sourcing drivers, the overall utility for the automobile nest increases more significantly, and thus the demand for the transit nest ($\overline{D_{rs}}$) is likely to decrease. On the other hand, when $\beta_{RL} \ll \beta_{RR}$, the integrated services are more perceived as transit services. The demand change is again unclear as utilities of both nests rise. Nonetheless, the analysis does imply that only when the integrated services are planned and used as an *auxiliary* mode for accessing/egressing transit, can it potentially enhance transit service quality. This is indeed what the integrated services originally expected and planned for. Another important implication from our tests that TNCs should focus on transit-deficient areas to provide services should hold as those areas can see significant travel experience improvement with the integrated services, regardless of considering demand elasticity or not. As to the sensitivity of logit dispersion and nesting parameters, pricing strategies, and subsidy strategies, computational tests are needed before stating any conclusions.

7. Conclusion

The growth trend of the app-based ride-sourcing service market is of no doubt. Potentials exist in integrating these on-demand services with the transit system in improving urban transportation mobility, as the partnership of the public and private transportation sectors has been practiced. We studied a stochastic equilibrium problem of the integrated system where uncertainties over decisions of both riders and drivers and their mutual impacts were modeled. In order to learn about the system mechanism, we first developed functions that build the connections of demand and supply by intermediate variables such as fare, waiting time, and payoff. Properties of these individual functions were explored and a fixed-point problem was set up that reflects the decision of both riders and drivers. To cope with the implicit fixed-point problem, we proposed two optimization problems that capture the riders' mode and destination choice and drivers' zone choice decision processes. An iterative framework that exchanges and updates the unfolded decisions of each other was proposed as the solution algorithm to the stochastic equilibrium problem. The algorithm iterates between solving a classic SUE problem for riders with a Nested Logit model embedded, and solving a stochastic zone choice problem for drivers. Numerical experiments using the Minneapolis-Saint Paul metropolitan area's real data were carried out. Experiments were carried out for testing perception error dispersion parameter sensitivity, fare structure parameter sensitivity, the efficiency of the integrated system, and transit agency subsidy strategies.

From the experimental results, we gain some insights into the system mechanism and provide some managerial suggestions. The total payoff and average payoff exhibit different patterns with varied peaks and optimal labor-pool sizes, but due to the "decreasing returns to scale" property of the ride-sourcing demand, an over-supplied ride-sourcing system can only attract a limited number of travelers. A TNC is thus suggested to maintain a relatively under-supplied fleet-size to guarantee the maximum average payoff for its drivers. In terms of the efficiency of the system, travelers save travel time at the expense of paying more. Rational riders make the best use of the integrated system and on average the population saves travel time up to 57%. The integrated system is more sensitive to riders' mode choice perception error dispersion parameter among the three perception dispersion parameters of both riders and drivers. Therefore, a TNC should encourage services toward transit-deficient areas as people with access to sufficient and quality transit services are less likely to use ride-sourcing. It was also explored how transit agencies should subsidize travelers if they access the transit system by ride-sourcing services. The results are encouraging that in some cases, 10–15% of TNC fare subsidy from public transit agency yields the maximized social welfare and positive transit net revenue. Lastly, even though the equilibrium uniqueness is not guaranteed theoretically, our computational tests show that solutions obtained with distinct initial conditions are in close proximity to each other.

As the first equilibrium study on the newly developed and promising integrated transit system, the research outcomes offer valuable insights into the system. We believe the related results are useful for future infrastructure planning, policy-making, and system design and development. For future work, some assumptions can be relaxed for a deeper understanding of the system. In the current setting, traffic congestion on ride-sourcing trips is ignored, and therefore an extension on congestive networks for TNC-trips well deserves more efforts. Related to this, a system considering more travel modes such as park-and-ride can be included where elastic demand and interactions of automobile network and transit network can be examined too.

CRediT authorship contribution statement

Yufeng Zhang: Methodology, Software, Data curation, Writing - original draft, Visualization, Validation. **Alireza Khani:** Conceptualization, Methodology, Supervision, Validation, Writing - review & editing.

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Appendix A. Proof for Proposition 3

Construct the Lagrangian function of (18):

$$L\left(V_r,\kappa\right)=\sum_{r\in Z}\int_0^{V_r}-P_r\left(\omega\right)d\omega+\frac{1}{\theta_3}\sum_{r\in Z}V_rLn\left(V_r\right)+\kappa\left(\sum_{r\in Z}V_r-V\right), \tag{A.1}$$

where κ is the Lagrangian multiplier associated with the equality constraint (18a). Now we derive the KKT conditions as follows:

$$\frac{\partial L(V_r,\kappa)}{\partial V_r}=-P_r\left(V_r\right)+\frac{1}{\theta_3}\left(1+Ln\left(V_r\right)\right)+\kappa\geqslant 0,\quad \forall r\in Z \tag{A.2}$$

$$V_r\frac{\partial L(V_r,\kappa)}{\partial V_r}=V_r\left(-P_r\left(V_r\right)+\frac{1}{\theta_3}\left(1+Ln\left(V_r\right)\right)+\kappa\right)=0,\quad \forall r\in Z \tag{A.3}$$

$$\frac{\partial L(V_r,\kappa)}{\partial \kappa}=\sum_{r\in Z}V_r-V=0 \tag{A.4}$$

$$V_r\geqslant 0,\quad \forall r\in Z. \tag{A.5}$$

For zones with positive number of drivers, according to (A.3), we have that:

$$\kappa=-P_r\left(V_r\right)+\frac{1}{\theta_3}\left(1+Ln\left(V_r\right)\right),\quad \forall r\in Z \tag{A.6}$$

and take the quotient of two equations defined in (A.6), we easily get the following:

$$\frac{V_k}{V_r}=e^{\theta_3(P_k(V_k)-P_r(V_r))},\quad \forall k\in Z. \tag{A.7}$$

The following is obvious by summing the above equation over all zones. Hence, we proved the Logit choice equivalence of the drivers' stochastic zone choice problem in (18).

$$\begin{aligned}&\frac{\sum_{k\in Z}V_k}{V_r}=\frac{\sum_{k\in Z}e^{\theta_3P_k(V_k)}}{e^{\theta_3P_r(V_r)}}\Rightarrow\\&V_r=V\frac{e^{\theta_3P_r(V_r)}}{\sum_{k\in Z}e^{\theta_3P_k(V_k)}}\end{aligned} \tag{A.8}$$

Appendix B. Riders' nested logit choice model

The Nested Logit model is used in modeling travelers' joint choice of travel mode and transfer stops/stations (destination choice).

Before traveling, travelers calculate utilities for the exclusive transit system and the integrated transit system. For the integrated travel mode, a traveler also decides where to make the transfer between the ride-sourcing services and public transit services. In these computational experiments, we assume that for those integrated travel system riders, they each have a subset of transit stations that are appealing to them and among which one station will be chosen. Let u_s be the (dis) utility of transferring at transit station $s \in S$, then the probability of choosing station s as a transfer station conditional on choosing the integrated system is:

$$P\left(s|R\right) = \frac{e^{-\theta_1 u_s}}{\sum_{k\in S} e^{-\theta_1 u_k}}, \tag{B.1}$$

where θ_1 is the perception error dispersion parameter for transfer station choices of riders (or destination choice in terms of the first leg of the trip—the ride-sourcing access trip). Riders then perceive the overall utilities for using the integrated mode by calculating a measure called "inclusive value", which is the log of the denominator of (B.1) and is usually referred to as "logsum". Rescaled inclusive values enter the mode choice model as a measure of overall utility for the integrated system. In the literature, it is sometimes referred to as "composite cost" (Williams, 1977; Ortúzar, 2001; Watling et al., 2015), which takes the following form:

$$CC_R = -\frac{1}{\theta_1} ln\left\{\sum_{k\in S} e^{-\theta_1 u_k}\right\}. \tag{B.2}$$

Then, the choice between the integrated transit system and exclusive transit system follows another Logit choice model with the mode choice perception dispersion parameter θ_2. Therefore, the probability of choosing the integrated transit system is:

$$P\left(R\right) = \frac{e^{-\theta_2 CC_R}}{e^{-\theta_2 CC_R} + e^{-\theta_2 u_T}}. \tag{B.3}$$

Appendix C. Solution algorithm

We use a superscript (I) to index variables and solutions obtained from iteration I. The solution algorithm contains details such as convergence metric calculation and nested Logit choice calculation. The gap measure for the riders' and drivers' problem is $\sum_{rs\in Z^2}\left|D_{rs}^{R(I+1)} - D_{rs}^{R(I)}\right|$ and $\frac{\left|\sum_{r\in Z} -V_r^{(I+1)}P_r + \sum_{r\in Z} V_r^{(*)}P_r\right|}{\left|\sum_{r\in Z} -V_r^{(*)}P_r\right|}$ respectively, where $V_r^{(*)}$ denotes the optimal solution for (22).

Algorithm 1. Algorithm for the equilibrium problem of the integrated transit-ride-sourcing system

Input Total transit demand ($\overline{D}$), total number of drivers (V), convergence threshold $\epsilon_1, \epsilon_2, \epsilon_3$, perception dispersion parameter $\theta_1, \theta_2, \theta_3$, maximum candidate transfer station number N
Output Optimal zone choice and mode choice decisions

procedure INITIALIZATION
  Set initial gaps: $G_o = Inf, G_d = Inf, G_r = Inf$
  Set iteration index: $I_o = 0$
while $G_o \geqslant \epsilon_1$ **do**
  procedure RIDERS MODE CHOICE PROBLEM
    if $I_o = 0$ **then**
      Initialize participation distribution: $V_r = V/|V|$
    Set $I = 0$
    while $G_r \geqslant \epsilon_2$ **do**
      for $rs = 1$ **to** $\left|Z^2\right|$ **do**
        Generate a set of attractive transfer stations $S = \{t_i \mid i = 1, 2, \ldots N\}$
        Compute utilities of using transit as access mode u_{rs}^T
        Compute utilities of using TNC as access mode $u_{rts}^R(\theta_1)$
        Compute the inclusive value: $u_{rs}^R = log(\sum_{t\in S} u_{rts}^R)$
        Determine modal split: $D_{rs}^{R(*)} = \overline{D}_{rs} e^{\frac{\theta_2}{\theta_1} u_{rs}^R} / \left(e^{\frac{\theta_2}{\theta_1} u_{rs}^R} + e^{-\theta_2 u_{rs}^T}\right)$
      Update demand split: $D_{rs}^{R(I+1)} = \frac{1}{I+1} D_{rs}^{R(*)} + \left(1 - \frac{1}{I+1}\right) D_{rs}^{R(I)}$
      Calculate gap: $G_r = \sum_{rs\in Z^2}\left|D_{rs}^{R(I+1)} - D_{rs}^{R(I)}\right|$
      Set $I = I + 1$
    Set $D_{rs}^{R(I_o)} = D_{rs}^{R(I)}$
  procedure DRIVERS ZONE CHOICE PROBLEM
    if $I_o = 0$ **then**
      Initialize participation distribution: $V_r = V/|V|$

(continued on next page)

(*continued*)

Set $I = 0$
while $G_d \geqslant \epsilon_3$ **do**
  Update payoffs for serving each zones P_r
  Calculate optimal driver assignment using (22), denote the solution as $V_r^{(*)}$
  Solve (21) for the step-size α
  Update driver assignment: $V_r^{(I+1)} = \alpha V_r^{(*)} + (1-\alpha)V_r^{(I)}$
  Calculate gap: $G_d = \dfrac{\left|\sum_{r\in Z} - V_r^{(I+1)}P_r + \sum_{r\in Z} V_r^{(*)}P_r\right|}{\left|\sum_{r\in Z} - V_r^{(*)}P_r\right|}$
Set $I = I + 1$
Set $V_r^{(I_{out})} = V_r^{(I)}$
Set $I_o = I_o + 1$
Calculate gap: $G_o = min\left\{\dfrac{\sum_{rs\in Z^2}\left|D_{rs}^{R(I_o)} - D_{rs}^{R(I_o-1)}\right|}{\overline{D}}, \dfrac{\sum_{r\in Z}\left|V_r^{(I_o)} - V_r^{(I_o-1)}\right|}{V}\right\}$

Appendix D. Equilibrium uniqueness experiment results

Tables D.4 and D.5.

Table D.4
Equilibrium solution comparison for the original demand profile

θ_2	V	Equal Participation Initialization		Proportional Participation Initialization		Difference			
		① Demand (D_{rs}^R)	② Total Payoff ($)	③ Demand (D_{rs}^R)	④Total Payoff ($)	⑤Rel Total Demand Diff (%)	⑥Rel Total Revenue Diff (%)	⑦Avg TAZ Demand Abs Diff	⑧Avg TAZ Paricipation Abs Diff
0.01	30000	10997	133339	11000	133354	0.0	0.0	0.01	0.03
	20000	10882	143493	10890	143627	0.1	0.1	0.02	0.01
	10000	10440	172773	10403	173012	0.4	0.1	0.05	0.02
	5000	9254	241552	9269	241893	0.2	0.1	0.03	0.02
	3000	7200	307439	7238	308107	0.5	0.2	0.06	0.1
	2000	5345	316746	5356	316820	0.2	0.0	0.05	0.03
0.05	30000	12257	130914	12267	131013	0.1	0.1	0.03	0.11
	20000	11738	133696	11749	133780	0.1	0.1	0.02	0.04
	10000	10118	137906	10137	138109	0.2	0.1	0.07	0.1
	5000	7474	130586	7518	130968	0.6	0.3	0.12	0.03
	3000	5251	109457	5256	110270	0.1	0.7	0.13	0.05
	2000	3695	87251	3763	90161	1.8	3.3	0.31	0.56
0.1	30000	13289	131838	13289	131942	0.0	0.1	0.02	0.01
	20000	12558	132016	12594	132108	0.3	0.1	0.03	0.03
	10000	10336	126839	10317	126573	0.2	0.2	0.02	0.02
	5000	6985	104388	7002	104329	0.2	0.1	0.01	0.04
	3000	4574	78783	4635	79922	1.3	1.4	0.22	0.41
	2000	3067	57539	3148	58234	2.6	1.2	0.53	0.4
1.0	30000	14911	138988	14964	139414	0.4	0.3	0.06	0.03
	20000	13977	137244	14087	137836	0.8	0.4	0.07	0.08
	10000	11450	128204	11492	128392	0.4	0.1	0.05	0.05
	5000	6573	84954	6588	85291	0.2	0.4	0.05	0.09
	3000	3645	52617	3741	52996	2.6	0.7	0.25	0.17
	2000	2039	31078	2100	31857	2.9	2.5	0.29	0.45
5.0	30000	14901	138701	15035	138934	0.9	0.2	0.12	0.1
	20000	13960	136614	14018	137390	0.4	0.6	0.06	0.08
	10000	11530	128786	11626	128993	0.8	0.2	0.16	0.04
	5000	6442	82565	6529	83134	1.3	0.7	0.36	0.15
	3000	3570	50659	3629	51367	1.6	1.4	0.39	0.4
	2000	1744	26495	1832	26939	4.9	1.7	0.52	0.43

Table D.5
Equilibrium solution comparison for the demand profile B

θ_2	V	Equal Participation Initialization		Proportional Participation Initialization		Difference			
		① Demand (D^R_{rs})	② Total Payoff ($)	③ Demand (D^R_{rs})	④Total Payoff ($)	⑤Rel Total Demand Diff (%)	⑥Rel Total Revenue Diff (%)	⑦Avg TAZ Demand Abs Diff	⑧Avg TAZ Paricipation Abs Diff
0.01	30000	13798	140151	13799	140207	0.0	0.0	0.02	0.05
	20000	13683	153353	13707	153685	0.2	0.2	0.04	0.08
	10000	12784	179124	12854	180241	0.5	0.6	0.14	0.05
	5000	10695	211823	10755	211876	0.6	0.0	0.16	0.04
	3000	8681	245322	8701	244412	0.2	0.4	0.06	0.1
	2000	7038	286523	7053	286766	0.2	0.1	0.05	0.03
0.05	30000	17325	175582	17328	175879	0.0	0.2	0.02	0.11
	20000	16736	186770	16748	187023	0.1	0.1	0.05	0.02
	10000	14427	201336	14523	203198	0.7	0.9	0.32	0.04
	5000	9471	171950	9574	173498	1.1	0.9	0.24	0.05
	3000	6054	131105	6025	130186	0.5	0.7	0.15	0.05
	2000	3797	93976	3948	96818	3.9	3.0	0.57	0.07
0.1	30000	20245	205638	20250	205331	0.0	0.1	0.03	0.02
	20000	19235	214121	19212	213503	0.1	0.3	0.05	0.1
	10000	15585	215431	15608	215862	0.1	0.2	0.04	0.08
	5000	8985	152001	8955	151559	0.3	0.3	0.1	0.12
	3000	4925	90662	4978	91436	1.1	0.9	0.21	0.04
	2000	2652	49433	2715	51927	2.3	4.9	0.32	0.09
1.0	30000	24753	248999	24771	249369	0.1	0.1	0.04	0.08
	20000	23678	261981	23709	262054	0.1	0.0	0.05	0.1
	10000	17378	232322	17296	230191	0.5	0.9	0.14	0.05
	5000	8137	124961	8192	123884	0.7	0.9	0.18	0.05
	3000	3675	56875	3654	58298	0.6	2.5	0.36	0.11
	2000	1386	21856	1379	21304	0.5	2.6	0.12	0.03
5.0	30000	24786	249350	24882	250770	0.4	0.6	0.08	0.12
	20000	23749	262021	23861	263516	0.5	0.6	0.14	0.07
	10000	17448	232076	17448	232076	0.0	0.0	0.03	0.02
	5000	8155	124058	8231	126790	0.9	2.2	0.2	0.14
	3000	3462	53346	3577	55729	3.3	4.4	0.51	0.14
	2000	1268	19765	1312	20385	3.4	3.1	0.16	0.03

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