# ULTRA-HIGH-Q NANOMECHANICS THROUGH DISSIPATION DILUTION: TRENDS AND PERSPECTIVES

Nils J. Engelsen<sup>1</sup>, Aman R. Agrawal<sup>2</sup> and Dalziel J. Wilson<sup>2</sup>
<sup>1</sup>Ecole Polytechnique Fédérale de Lausanne, Lausanne, SWITZERLAND and
<sup>2</sup>Wyant College of Optical Sciences, University of Arizona, Tucson, AZ USA

## **ABSTRACT**

Strained nanomechanical resonators have recently achieved quality factors of 1 billion through the phenomenon of dissipation dilution. Remarkably, the potential of these devices seems unexhausted, exhibiting a scaling law of roughly one order of magnitude (in *Q* factor) every three years. This paper reviews advances which led to this point, including phononic crystal "soft-clamping," strain engineering, and a trend towards centimeter-scale devices with extreme aspect ratios. Recent trends include investigation of strained crystalline thin films, fractal-patterned supports, and machine-learning-optimized supports. New possibilities emerging from these advances range from cavity free quantum optomechanics to ultra-sensitive accelerometry.

## **KEYWORDS**

Nanomechanical resonator, dissipation dilution, quality factor, stress, silicon nitride, quantum optomechanics

### INTRODUCTION

Recent years have seen the emergence of a new class of ultra-high-Q nanomechanical resonators fashioned out of strained thin films [1-3]. The physical principle behind their performance is dissipation dilution, an effect whereby an elastic body is subjected to a conservative stress field, increasing its stiffness without adding loss [17]. The ability to achieve extreme dimensions and stresses at the nanoscale has enabled dilution factors (the ratio of final to initial Q) in excess of  $10^5$ , yielding Q factors as high as 109 for devices made of amorphous glass (in particular,  $Si_3N_4$ , for which the intrinsic loss tangent is  $Q_0^{-1} \sim 10^{-4}$ ) [2,3]. Here we review advances that have led to this point, and highlight a trend, shown in Fig. 1, which suggests the Q factor of strained nanomechanical resonators will continue to improve. We begin with a historical example that illustrates the importance of mode-shape on the dilution factor and conclude with a brief overview of applications.

## HISTORICAL EXAMPLE (STRING)

The history of dissipation dilution traces its roots to the gravitational wave community, where it was used to explain the anelastic behavior of thin wires used to suspend kg test masses [16]. It is useful to consider this example as it illustrates the basic concept of dissipation dilution and its dependence on resonator geometry. Towards this end, consider a beam of cross-sectional area (area moment) A(I), length L, and elastic modulus  $E_0(1+iQ_0^{-1})$  subject to tensile stress  $\sigma = mg/A$  due to mass loading (Fig. 2). The ratio of work done by the beam against conservative (tensile) and frictional (elastic) stress yields the dilution factor

$$\frac{Q}{Q_0} = 1 + \frac{W_{\sigma}(u_{\sigma})}{W_E(u_{\sigma})} \equiv D_Q \tag{1}$$

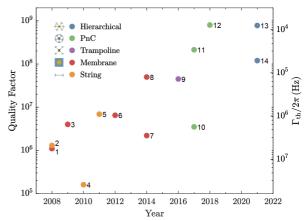


Figure 1: *Q* factor of strained nanomechanical resonators versus time. #1-10 correspond to [4-13]; #11-13 to [1-3]; #14 to [14].

where  $u_{\sigma}$  is the mode shape of the *tensioned* beam,

$$W_{\sigma}(u_{\sigma}) = \frac{1}{2} \int A(x)\sigma(x)u'_{\sigma}(x)^{2}dx \tag{2}$$

is the work done against gravity due to elongation, and

$$W_E(u_\sigma) = \frac{1}{2} E \int I(x) u_\sigma''(x)^2 dx \tag{3}$$

is the work against internal stress due to mode curvature.

A key aspect of dissipation dilution is its mode-shape dependence, which depends on boundary conditions and tensile stress. For a doubly clamped rectangular beam of width w, thickness h and uniform stress  $\sigma$ , it can be shown that  $W_{\sigma} \propto wh\sigma/L$  and that  $W_{E}$  has separate components due to mode curvature at the clamps  $W_{E}^{(c)} \propto wh^{2}\sqrt{\sigma E}/L^{2}$  and antinodes  $W_{E}^{(a)} \propto wh^{3}E/L^{4}$ , respectively, yielding

$$D_Q \approx \frac{W_\sigma}{W_F^{(c)} + W_E^{(a)}} = \frac{1}{2\lambda + \pi n^2 \lambda^2}$$
 (4)

where n is the mode index and  $\lambda = (h/L)\sqrt{E/(12\sigma)}$  is a factor that parameterizes the distance  $L\lambda$  from the clamps over which mode curvature exceeds that of a sinusoid [2].

While specific to a uniform rectangular beam, Eqs. 1-4 reveal several key features of dissipation dilution that have informed recent developments in the field:

- 1.  $D_Q$  is higher for high aspect ratio (L/h) devices, motivating the use of long, thin strings and membranes.
- 2. The mode curvature at the clamps plays a critical role in determining the scaling of  $D_Q$  with resonator frequency  $f \sim \sqrt{\sigma}/L$ , scaling as  $Q \sim f^{-1}$  for normal beams and  $Q \sim f^{-2}$  if the beam is "soft-clamped".
- 3. Localizing motion in regions of high stress (so-called "strain-engineering") can yield higher  $D_0$ .

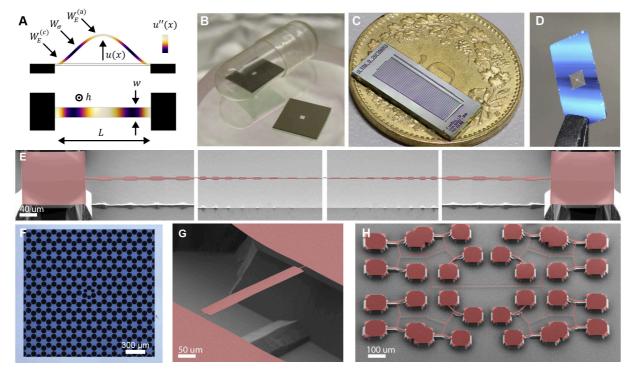


Figure 2: Mechanical resonator gallery. A: Toy model of dissipation dilution. B: Commercial (Norcada) 0.5 mm x 0.5 mm Si<sub>3</sub>N<sub>4</sub> membrane [43]. C: Array of twenty-five 7 mm Si<sub>3</sub>N<sub>4</sub> nanobeams [2]. D: 5 mm x 5 mm Si<sub>3</sub>N<sub>4</sub> trampoline [28]. E: Strain-engineered nanostring [2]. F: Soft-clamped membrane [41]. G: Clamp-tapered beam [24]. H: Binary tree beam resonator [3].

Access to extreme aspect ratios, high stress, and precise dimensions have allowed the above features to be exploited powerfully in nanomechanical resonators, particularly strings and membranes released from silicon nitride, a common semiconductor stressor film. Below we explore four examples that contribute to the trend shown in Fig. 1.

## Silicon nitride membranes, strings, & trampolines

Silicon nitride is a common nanomechanics material synonymous with dissipation dilution because of its deposited tensile stress. Specifically, when deposited by LPCVD on Si (at ~1000 K), stoichiometric Si<sub>3</sub>N<sub>4</sub> (E~300 GPa) relaxes to a tensile stress of  $\sigma$ ~1 GPa due to thermal mismatch. Common film thicknesses range from ~10 nm for capping layers to ~ 100 nm for optical waveguides. A combination of lithography and wet etching has been used to release strings and membranes with transverse dimensions varying from several microns to centimeters, enabling dissipation dilution factors as high as  $D_0$ ~10<sup>4</sup>.

Early studies of dissipation dilution in nanomechanical resonators employed both "low-stress" (PECVD,  $\sigma \sim 0.1$  GPa) and "high-stress" (LPCVD  $\sigma \sim 1$  GPa) SiN membranes and strings with  $L \sim 1$  mm dimensions and  $h \sim 50$ -300 nm thickness [5-11]. A meta-study by Villanueva and Schmid [18] found that for a wide set of experiments employing similar devices, the model given by Eq. 1 is valid and implies that the internal loss coefficient of SiN films is dominated by surface loss, scaling at room temperature as

$$Q_{0 \text{ SiN}}^{-1}(h \text{ [nm]}) \approx (70 h)^{-1} + (3 \times 10^4)^{-1}$$

Eq. 5 explains the trend prior to 2014 in Fig. 1, in which progressively larger uniform strings and membranes were developed. For these "hard-clamped" resonators, the predominant loss mechanism is curvature at the clamps

$$D_Q^{\rm HC} = \frac{1}{2\lambda}$$

yielding a thickness-independent quality factor of

$$Q^{\rm HC,SiN} = D_Q^{\rm HC} Q_{0,\rm SiN} \sim 10^7 \frac{L/n}{1~\rm mm} \sqrt{\frac{\sigma}{1~\rm GPa}}$$

More recently, string-like "trampoline" resonators with  $L \sim 10$  mm dimensions have been developed, yielding quality factors as high as  $10^8$  when appropriately tailored to minimize curvature loss at the clamps [12,14,27,28].

# Soft-clamped phononic crystal membranes

Phononic crystals (PnC) were first employed to combat recoil losses in millimeter-scale SiN membrane resonators, whose fundamental mode is notoriously sensitive to how the device chip is mounted [10]. While this strategy involves patterning a PnC into the chip, a more ingenious strategy is to pattern a PnC *into the membrane* around a central defect, creating a localized mode whose amplitude decays exponentially towards the clamps [1]. In addition to mitigating recoil losses, an ideal ``soft-clamped'' mode experiences quadratically enhanced dissipation dilution

$$D_Q^{SC} = \frac{1}{\pi^2 n^2 \lambda^2}$$

yielding a surface-loss-dominated quality factor of

$$Q^{\text{SC,SiN}} = D_Q^{\text{SC}} Q_{0,\text{SiN}} \sim 10^{10} \left(\frac{L/n}{1 \text{ mm}}\right)^2 \frac{\sigma}{1 \text{ GPa}} \frac{100 \text{ nm}}{h}$$

Notably, L/n here refers to the defect size, which must be

 $n \sim 10$  times smaller than the membrane to approximate perfect soft clamping. Also, soft-clamped modes are less susceptible to surface loss, manifesting in the inverse proportionality  $Q^{\rm SC} \sim h^{-1}$ . Using photolithography to achieve precise dimensional control over a large surface area, soft-clamped membranes with  $L/n \sim 100$  um defects and thickness as small as  $h \sim 20$  nm have been developed, yielding Q factors as high as 200 million at room temperature [1,19] and 1 billion at cryogenic temperatures [20]. These devices have been used in a series of groundbreaking quantum optomechanics experiments, including feedback cooling to the ground state and displacement measurement beyond the standard quantum limit [20]–[24].

# Strain-engineered phononic crystal nanobeams

The strain profile  $\epsilon(x) = E\sigma(x)$  of a beam can be tailored by changing its area A(x) (due to conservation of tension,  $T = \sigma(x)A(x)$ ). This effect renormalizes the modeshape u(x) and can enhance dissipation dilution when  $\epsilon(x)$  is co-localized with regions of high curvature  $u_{\sigma}''(x)$ . For example, by tapering the width of a uniform rectangular beam as it approaches the clamps (see Fig. 2C), the quality factor of its fundamental flexural mode can be increased by as much as  $\sqrt{\epsilon_{\text{yield}}/\epsilon_0} = \sqrt{\sigma_{\text{yield}}/\sigma_0}$  ( $\approx$  2.5 for high stress Si<sub>3</sub>N<sub>4</sub>) [24]. Similar effects have been reported for trampoline resonators [27,29], with the counter-intuitive finding that filleted tethers can enhance the Q factor relative to straight tethers [29].

While the effect of strain-engineering is only moderate in the case of high prestress, its combination with soft-clamping has yielded some of the highest Q nanomechanical resonators to date. Intuitively, the idea is to co-localize  $\epsilon$  within the defect, yielding enhancement of the soft-clamped Q by as much as as  $\sigma_{\rm yield}/\sigma_0$ . Ghadimi et. al. [2] implemented this strategy by patterning a PnC into a tapered rectangular beam (importantly, the unit cell must be tailored to fix the PnC bandgap). Applied to L=7 mm, h=20 nm thick beams with a defect length of  $L\sim 100~\mu m$  (Fig. 2A), Q factors of up to 800 million were recorded at room temperature (for f=1.3 MHz). To date this is the highest Q reported for a room temperature mechanical resonator of any size or frequency, despite the defect mode possessing an effective mass of several picograms.

## Hierarchal (binary tree) resonators

A disadvantage of PnC-based soft-clamped resonators is that the defect mode is of high order  $(n \gg 1)$ , leading to intermodulation noise [41] and requiring relatively large resonator dimensions compared to uniform structures with the same mode frequency. Recently a new type of soft-clamped resonator has been proposed which allows the *fundamental mode* (n = 1) to be soft-clamped, based on hierarchical (fractal-like) micropatterning [25].

An example of a two-branch (binary tree) hierarchal resonator is shown in Fig. 2D. The basic concept emerges from the fact that by splitting a tensile-stressed string into two symmetric branches, its mode gradient u'(x) is reduced by a factor of  $\cos \theta$ , where  $\theta$  is the branching angle. By creating a binary tree shape through cascaded branching, the fundamental mode of the full structure can thus be

soft clamped [3, 25], substantially improving the quality factor over devices of similar dimensions.

Initial demonstrations of binary tree resonators have yielded room temperature Q factors approaching 1 billion for transverse dimensions  $L \sim 1$  mm, rivaling the performance of PnC soft-clamped membranes and strings with 10-fold larger dimensions. For example, a device similar to that in Fig. 2D, approximately 2 mm long and 20 nm thick, exhibited a fundamental Q factor of 800 million at room temperature and 1.1 billion at 6 K [3]. Comparable performance was achieved with by branching the tethers of a L $\sim$ 2 mm trampoline resonator. Intriguingly, a similar strategy was realized by topology optimization of membranes [14].

## **APPLICATIONS**

Development of ultra-high-Q nanomechanical resonators has largely been driven by two pursuits: force sensing, in which the ratio of quality factor to mass m regulates the magnitude of the thermal force noise [15],

$$F_{\rm th} = \sqrt{8\pi k_B T m f/Q}$$

and quantum experiments [30], in which the quality factor dictates the magnitude of the thermal decoherence rate

$$\Gamma_{\rm th} = \frac{k_B T}{\hbar Q}$$

and the zero-point displacement spectral density (the displacement sensitivity required to operate at the SQL)

$$x_{\rm ZP} = \frac{\hbar Q}{m(2\pi f)^2}.$$

To highlight the potential of UHQ nanomechanics, it is interesting to consider defect modes of the strain-engineered 1D PnC in Figure 2A ( $m \sim pg$ ,  $Q \sim 10^9$ ,  $f \sim MHz$ ), which at room temperature have a thermal force sensitivity of  $F_{\rm th} \sim {\rm aN}/{\sqrt{\rm Hz}}$ , a decoherence rate of  $\Gamma_{\rm th}/2\pi \sim 10$  kHz, and a zero-point displacement spectral density of  $x_{ZP}$  ~  $pm/\sqrt{Hz}$  [2]. The force sensitivity rivals that of typical atomic force microscope (AFM) cantilevers operating at He-4 temperatures (<10 K) and acoustic frequencies  $(f\sim 1 \text{ kHz})$ , suggesting applications like high-speed magnetic resonance force imaging, which is currently being pursued with PnC membranes [26]. The decoherence rate is two orders of magnitude smaller than the mechanical frequency, implying that if the defect mode were prepared in its ground state, it would undergo 100 coherent oscillations, sufficient for consideration as a quantum memory. Finally, owing to its large zero-point motion, the ground state should be accessible by measurement-based feedback with a readout sensitivity  $x_{\rm ZP}/\sqrt{n_{th}}\sim 1~{\rm fm}/\sqrt{{\rm Hz}}$  , where  $n_{\rm th} = k_B T/hf$  is the thermal phonon application [30]. This sensitivity is well within reach of cavity-enhanced optical interferometers and is in principle accessible with a Michelson interferometer with milliwatts of probe power [29]. Recognition of these opportunities has spurred a new generation of quantum optomechanics experiments focusing

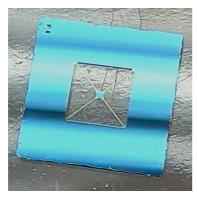


Figure 3: A mass-loaded trampoline for acceleration sensing.

on measurement-based control [19,30] and room temperature operation [27,31], as well as proposals for "cavity-free" quantum optomechanics based on single-pass optical readout techniques [29,42].

## **OUTLOOK**

This paper has briefly reviewed the state of the art in high Q strained nanomechanics, highlighting techniques such as soft-clamping and strain-engineering which have recently produced megahertz frequency nanomechanical resonators with room temperature quality factors as high as  $10^9$ . Looking forward, the trend in Fig. 1 suggests that higher quality factors are achievable. PnC and hierarchal based soft-clamping are still in their infancy, for example, and improved designs are emerging based on a combination of physical intuition, numerical modeling [44,45], and machine-learning based optimization [14].

Another route to higher Q is new materials.  $\mathrm{Si}_3\mathrm{N}_4$ , like most amorphous glasses, has bulk intrinsic  $Q_0 \sim 10^4$  for temperatures between 1-300 K. By contrast, crystalline films can have intrinsic quality factors of  $Q_0 > 10^6$  below 10 K, and can be epitaxially grown with tunable stress due to lattice mismatch. Strained nanomechanical resonators made of InGaP [32], GaAs [33], and SiC [34] are being pursued, with a focus on understanding interface defects which increase loss and constrain aspect ratios (and in anticipation of quantum technology based on strain-coupled electronic defects [35]). A particularly promising material is strained silicon-on-insulator, which is commercially available and compatible with silicon nanophotonics. Preliminary results have shown  $Q \sim 10^9$  for Si PnC nanostrings at 10 K, on par with the best  $\mathrm{Si}_3\mathrm{N}_4$  [36].

Finally, while the contemporary history of UHQ nanomechanics dovetails with the pursuit of high Q/m factors for *contact* force sensing, an alternative approach is to mass-load a nanomechanical resonator, thereby realizing a high  $Q \times m$  test mass for *inertial* force sensing. Mass-loaded membranes and trampolines under development for optomechanical accelerometry can have thermal sensitivities of  $a_{\rm th} = F_{\rm th}/m \sim ng_0/\sqrt{\rm Hz}$  with bandwidths of  $f=10~\rm kHz$  [37]; however, these devices have yet to be studied with a systematic dissipation dilution approach. Realizing UHQ mass-loaded nanomechanical resonators (c.f. Fig. 3) would give access to an array of fundamental weak forces ranging from non-Newtonian gravity [38], to spontaneous waveform collapse [39], to ultralight dark matter [40].

### **ACKNOWLEDGEMENTS**

The authors thank Mohammad Bereyhi and Alberto Beccari for assistance with figures in the manuscript. N.J.E. acknowledges support from the Swiss National Science Foundation under grant no. 185870 (Ambizione). D.J.W. acknowledges support from the National Science Foundation under grant no. 1945832.

#### REFERENCES

- [1] Y. Tsaturyan, A. Barg, E. S. Polzik, and A. Schliesser, "Ultracoherent nanomechanical resonators via soft clamping and dissipation dilution," *Nat Nano*, vol. 12, no. 8, pp. 776–783, Aug. 2017,
- [2] A. H. Ghadimi *et al.*, "Elastic strain engineering for ultralow mechanical dissipation," *Science*, vol. 360, no. 6390, pp. 764–768, May 2018,
- [3] A. Beccari *et al.*, "Hierarchical tensile structures with ultralow mechanical dissipation," *arXiv:2103.09785*
- [4] J. D. Thompson, B. M. Zwickl, A. M. Jayich, F. Marquardt, S. M. Girvin, and J. G. E. Harris, "Strong dispersive coupling of a high-finesse cavity to a micromechanical membrane," *Nature*, vol. 452, no. 7183, pp. 72–75, Mar. 2008
- [5] S. S. Verbridge, R. Ilic, H. Craighead, and J. M. Parpia, "Size and frequency dependent gas damping of nanomechanical resonators," *App. Phys. Lett.*, vol. 93, no. 1, p. 013101, 2008.
- [6] D. J. Wilson, C. A. Regal, S. B. Papp, and H. J. Kimble, "Cavity Optomechanics with Stoichiometric SiN Films," *Phys. Rev. Lett.*, vol. 103, no. 20, p. 207204, Nov. 2009.
- [7] Q. P. Unterreithmeier, T. Faust, and J. P. Kotthaus, "Damping of nanomechanical resonators," *Phys. Rev. Lett.*, vol. 105, no. 2, p. 027205, 2010.
- [8] S. Schmid, K. D. Jensen, K. H. Nielsen, and A. Boisen, "Damping mechanisms in high-\$Q\$ micro and nanomechanical string resonators," *Phys. Rev. B*, vol. 84, no. 16, p. 165307, Oct. 2011.
- [9] P.-L. Yu, T. Purdy, and C. Regal, "Control of material damping in high-Q membrane microresonators," *Phys. Rev. Lett.*, vol. 108, no. 8, p. 083603, 2012.
- [10] Y. Tsaturyan *et al.*, "Demonstration of suppressed phonon tunneling losses in phononic bandgap shielded membrane resonators for high-Q optomechanics," *Opt. Exp.*, vol. 22, no. 6, pp. 6810–6821, 2014.
- [11] S. Chakram, Y. S. Patil, L. Chang, and M. Vengalattore, "Dissipation in Ultrahigh Quality Factor SiN Membrane Resonators," *Phys. Rev. Lett.*, vol. 112, no. 12, p. 127201, Mar. 2014.
- [12] C. Reinhardt, T. Müller, A. Bourassa, and J. C. Sankey, "Ultralow-noise SiN trampoline resonators for sensing and optomechanics," *Physical Review X*, vol. 6, no. 2, p. 021001, 2016.
- [13] A. H. Ghadimi, D. J. Wilson, and T. J. Kippenberg, "Radiation and Internal Loss Engineering of High-Stress Silicon Nitride Nanobeams," *Nano Lett.*, vol. 17, no. 6, pp. 3501–3505, Jun. 2017.
- [14] D. Høj, F. Wang, W. Gao, U. B. Hoff, O. Sigmund, and U. L. Andersen, "Ultra-coherent nanomechanical resonators based on inverse design.".
- [15] P. R. Saulson, "Thermal noise in mechanical experiments," *Phys. Rev. D*, vol. 42, no. 8, p. 2437, 1990,
- [16] G. I. González and P. R. Saulson, "Brownian motion of a mass suspended by an anelastic wire," *J. Acoust. Soc. Am.*, vol. 96, no. 1, pp. 207–212, 1994.
- [17] S. A. Fedorov *et al.*, "Generalized dissipation dilution in strained mechanical resonators," *Phys. Rev. B*, vol. 99, no. 5, p. 054107, Feb. 2019.

- [18] Villanueva, L.G and S, Schmid. "Evidence of surface loss as ubiquitous limiting damping mechanism in SiN microand nanomechanical resonators." *Phys. Rev. Lett.* 113.22 (2014): 227201.
- [18] S. A. Fedorov, A. Beccari, A. Arabmoheghi, D. J. Wilson, N. J. Engelsen, and T. J. Kippenberg, "Thermal intermodulation noise in cavity-based measurements," *Optica, OPTICA*, vol. 7, no. 11, pp. 1609–1616, Nov. 2020.
- [19] M. Rossi, D. Mason, J. Chen, Y. Tsaturyan, and A. Schliesser, "Measurement-based quantum control of mechanical motion," *Nature*, vol. 563, no. 7729, p. 53, Nov. 2018.
- [20] D. Mason, J. Chen, M. Rossi, Y. Tsaturyan, and A. Schliesser, "Continuous force and displacement measurement below the standard quantum limit," *Nature Physics*, vol. 15, no. 8, Art. no. 8, Aug. 2019.
- [21] M. Rossi, D. Mason, J. Chen, and A. Schliesser, "Observing and Verifying the Quantum Trajectory of a Mechanical Resonator," Dec. 2018.
- [22] M. Rossi, L. Mancino, G. T. Landi, M. Paternostro, A. Schliesser, and A. Belenchia, "Experimental Assessment of Entropy Production in a Continuously Measured Mechanical Resonator," *Phys. Rev. Lett.*, vol. 125, no. 8, p. 080601, Aug. 2020.
- [23] J. Chen, M. Rossi, D. Mason, and A. Schliesser, "Entanglement of propagating optical modes via a mechanical interface," *Nature Communications*, vol. 11, no. 1, Art. no. 1, Feb. 2020.
- [24] Mohammad. J. Bereyhi et al., "Clamp-Tapering Increases the Quality Factor of Stressed Nanobeams," Nano Lett., vol. 19, no. 4, pp. 2329–2333, Apr. 2019.
- [25] S. A. Fedorov, A. Beccari, N. J. Engelsen, and T. J. Kippenberg, "Fractal-like Mechanical Resonators with a Soft-Clamped Fundamental Mode," *Phys. Rev. Lett.*, vol. 124, no. 2, p. 025502, Jan. 2020,
- [26] D. Hälg et al., "Membrane-Based Scanning Force Microscopy," Phys. Rev. Applied, vol. 15, no. 2, p. L021001, Feb. 2021.
- [27] Norte, R. A., Moura, J. P., & Gröblacher, S. (2016). "Mechanical resonators for quantum optomechanics experiments at room temperature". *Phys. Rev. Lett.*, 116(14), 147202
- [28] Pluchar, C. M., Agrawal, A. R., Schenk, E., & Wilson, D. J. (2020). "Towards cavity-free ground-state cooling of an acoustic-frequency silicon nitride membrane." *Applied Optics*, 59(22), G107-G111.
- [29] Sadeghi, P., Tanzer, M., Christensen, S. L., & Schmid, S. (2019). "Influence of clamp-widening on the quality factor of nanomechanical silicon nitride resonators." *J. App. Phys.*, 126(16), 165108.
- [30] Wilson, D. J., Sudhir, V., Piro, N., Schilling, R., Ghadimi, A., & Kippenberg, T. J. (2015). "Measurement-based control of a mechanical oscillator at its thermal decoherence rate." *Nature*, 524(7565), 325-329.
- [31] Serra, E., Borrielli, A., Marin, F., Marino, F., Malossi, N., Morana, B., Piergentili, P., Prodi, G.A., Sarro, L., Vezio, P. and Vitali, D., 2021. "Silicon-nitride nanosensors toward room temperature quantum optomechanics." arXiv preprint arXiv:2104.14302.
- [32] Bückle, M., Hauber, V.C., Cole, G.D., Gärtner, C., Zeimer, U., Grenzer, J. and Weig, E.M., 2018. "Stress control of tensile-strained In1– x Ga x P nanomechanical string resonators." *App. Phys. Lett.*, 113(20), p.201903.
- [33] Yamaguchi, H.. "GaAs-based micro/nanomechanical resonators." Semiconductor Science and Technology 32, no. 10 (2017): 103003.
- [34] Romero, E., et al. "Engineering the dissipation of crystalline micromechanical resonators" *Phys. Rev. App.* 13.4 (2020): 044007.

- [35] Wang, H., and Ignas L.. "Coupling spins to nanomechanical resonators: Toward quantum spin-mechanics." App. Phys. Lett. 117.23 (2020): 230501.
- [36] Beccari, A., Engelsen N. J., Fedorov S., Bereyhi, M. and Kippenberg. T.J., "Strained Silicon Nanomechanics." Bulletin of the American Physical Society (2021).
- [37] Zhou, F., Bao, Y., Madugani, R., Long, D.A., Gorman, J.J. and LeBrun, T.W., 2021. "Broadband thermomechanically limited sensing with an optomechanical accelerometer." *Optica*, 8(3), pp.350-356.
- [38] Westphal, T., Hepach, H., Pfaff, J. and Aspelmeyer, M., 2021. "Measurement of gravitational coupling between millimetre-sized masses." *Nature*, 591(7849), pp.225-228.
- [39] Nimmrichter, S., Hornberger, K. and Hammerer, K., 2014. "Optomechanical sensing of spontaneous wavefunction collapse." *Phys. Rev. Lett.*, 113(2), p.020405.
- [40] Manley, J., Chowdhury, M.D., Grin, D., Singh, S. and Wilson, D.J., 2021. "Searching for vector dark matter with an optomechanical accelerometer." *Phys. Rev. Lett.* 126(6), p.061301.
- [41] Fedorov, S.., et al. "Thermal intermodulation noise in cavity-based measurements." *Optica* 7.11 (2020): 1609-1616.
- [42] Hao, S., Singh, R., Zhang, J. and Purdy, T.P., 2020, September. "Cavity-less Quantum Optomechanics with Nanostring Mechanical Resonators." In *Frontiers in Optics* (pp. FW5C-4). Optical Society of America.
- [43] Wilson, D.J., 2012. "Cavity optomechanics with highstress silicon nitride films." California Institute of Technology.
- [44] Reetz, C. et. al. "Analysis of membrane phononic crystals with wide band gaps and low-mass defects." Phys. Rev. App. 12, no. 4 (2019): 044027.
- [45] Ivanov, E. et. al. "Edge mode engineering for optimal ultracoherent silicon nitride membranes." App. Phys. Lett. 117.25 (2020): 254102.