Dynamic Pricing and Learning under the Bass Model

SHIPRA AGRAWAL, Industrial Engineering and Operations Research, Columbia University
STEVEN YIN, Industrial Engineering and Operations Research, Columbia University
ASSAF ZEEVI, Graduate School of Business, Columbia University

Most of the dynamic pricing and learning literature has focused on a relatively simple setting where given current pricing decision, demand is independent of past actions and demand values. With the evolution of online platforms and marketplaces, the focus on such homogeneous modeling environments is becoming increasingly less realistic. For example, platforms now rely more and more on online reviews and ratings to inform and guide consumers. Product quality information is also increasingly available on online blogs, discussion forums, and social networks, that create further word-of-mouth effects. One clear implication on the dynamic pricing and learning problem is that the demand environment can no longer be assumed to be static; for example, in the context of online reviews, sales of the product trigger reviews/ratings, and these in turn influence subsequent demand behavior etc.

To that end, product diffusion models, such as the popular Bass model [1, 2], are known to be extremely robust and parsimonious, capturing aforementioned word-of-mouth and imitation effects on the growth in sales of a new product. The Bass model describes the process by which new products get adopted as an interaction between existing users and potential new users. It creates a state-dependent evolution of market response which is well aligned with the impact of recent technological developments, such as online review platforms, on the customer purchase behavior.

Motivated by these observations, the objective of this paper is to investigate a novel formulation of the the dynamic pricing and demand learning problem, where the evolution of demand is governed by the Bass diffusion model, and where the parameters of this model are unknown a priori and need to be learned from repeated interactions with the market.

The Bass diffusion model [1, 2] has two parameters: the "coefficient of innovation" representing external influence or advertising effect; and the "coefficient of imitation" representing internal influence or word-of-mouth effect. Let m be the number of potential buyers, i.e., the market size, and let X_t be fraction of customers who has already adopted the product until time t. Then, mX_t represents the cumulative sales (aka adoptions) up until time t, and $m(1 - X_t)$ is the size of remaining market yet to be captured. The instantaneous sales at time t, $m\frac{dX_t}{dt}$ can then be expressed as

$$m\frac{dX_t}{dt} = \underbrace{m\alpha(1 - X_t)}_{\text{sales due to external influence}} + \underbrace{m\beta X_t(1 - X_t)}_{\text{sales due to internal influence or imitation}} \tag{1}$$

A generalization of the Bass diffusion model that can be harnessed for the dynamic pricing context was proposed by Robinson and Lakhani [4]. In the latter model, p_t denotes the price posted at time t. Then, given previous adoption level X_t the number of new adoptions at time instant t is given by

$$m\frac{dX_t}{dt} = me^{-p_t}(\alpha + \beta X_t)(1 - X_t). \tag{2}$$

Thus, the current price affects not only the immediate new adoptions and revenue, but also future adoptions due to its dependence on the adoption level X_t, which essentially forms the state at time t in this stateful demand model. Finding a revenue-maximizing optimal pricing trajectory in such a dynamic demand model

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is non-trivial even when the model parameters are known, e.g., see [3] for some characterizations. In this paper, we consider a stochastic variant of the above Bass model, where customers arrive sequentially and the number of customer arrivals (aka adoptions) d_t until time t is a non-homogeneous Poisson process with rate λ_t given by the right hand side of (2).

Our main contribution is the development of an algorithm that satisfies a high probability regret guarantee of the order $m^{2/3}\log(T)$; where the market size m and planning horizon T are known a priori. Moreover, we show that no algorithm can incur smaller order of loss by deriving a matching lower bound. A key insight that we derive and utilize in our algorithm design is the *concavity* of the optimal value in the deterministic Bass model setting. Using this concavity property, we can show that the optimal value in the deterministic Bass model is always higher than in the stochastic model, and therefore can be used as a stronger benchmark to compete with. Based on this insight, we design an algorithm that alternates between using a low "exploratory" price aimed at observing demand in order to improve the estimates of model parameters, and following (a lower confidence bound on) the deterministic optimal prices for the estimated market parameters.

To obtain the lower bound, we make the key observation that when the adoption level is low, the coefficient of imitation parameter cannot be estimated accurately. This makes intuitive sense because when the number of adopters is small, we do not expect to be able to measure the word of mouth effect accurately. Secondly, we show that the optimal prices differ significantly under different imitation parameters. Finally, we also derive novel dynamic-programming based inequalities that allow for lower bounding the loss in long-term revenue in terms of instantaneous pricing errors made by any non-anticipating algorithm. Therefore, any algorithm will make significant pricing errors when the adoption number is low, and in turn incurring large regret early on in the planning horizon. We believe that our analysis techniques could be useful in other finite horizon MDP problems as well.

An interesting and perhaps surprising aspect of our upper and lower bounds is the role of the horizon T vs. market size m. Our upper bound depends sublinearly on m but only logarithmically on the horizon T. And in fact our lower bound indicates that for any fixed α, β the most interesting (and challenging) instances of this problem are characterized by T which is of constant order, and large m. This insight highlights the distinct nature of pricing under stateful models like the Bass model when compared to the independent demand models and multi-armed bandit based formulations where asymptotics with respect to T form the main focus of the analysis. Interesting directions for future research include investigation of other stateful demand models where the concavity property and other new dynamic programming based insights derived here may be useful.

CCS Concepts: • **Theory of computation → Online learning algorithms**.

Additional Key Words and Phrases: Dynamic Pricing, Diffusion Models

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