VEHICLE DISPATCHING IN MODULAR TRANSIT NETWORKS: A NONLINEAR MIXED-INTEGER PROGRAMMING MODEL

ABSTRACT

Modular vehicle (MV) technology offers the possibility of flexibly adjusting vehicle capacity by docking/undocking modular pods into vehicles of different sizes en route to meet passenger demand. Based on the MV technology, a modular transit network system (MTNS) concept is proposed to overcome the mismatch between fixed vehicle capacity and spatially varying travel demand in traditional public transportation systems. To achieve the optimal MTNS design, a mixed-integer nonlinear programming model is developed to balance the tradeoff between the vehicle operation cost and the passenger trip time cost. The nonlinear model is reformulated into a computationally tractable linear model. The linear model solves lower and upper bounds to the original nonlinear model, thus producing a near-optimal solution to the MTNS design. This reformulated linear model can be solved with off-the-shelf commercial solvers (e.g., Gurobi). Two numerical examples in different contexts are used to demonstrate the proposed model's applicability and its effectiveness in reducing system costs.

Keywords: public transit; modular vehicle; operational design; mixed-integer nonlinear programming

1. Introduction

Most current public transportation systems (e.g., mass transit) adopt vehicles with fixed capacities that cannot adapt to temporal and spatial variations in travel demand. This mismatch between the vehicle capacity and travel demand causes either excessive passenger waiting (e.g., in areas with a high demand relative to the vehicle capacity) or low vehicle occupancy (e.g., in areas with a high vehicle capacity relative to the demand).

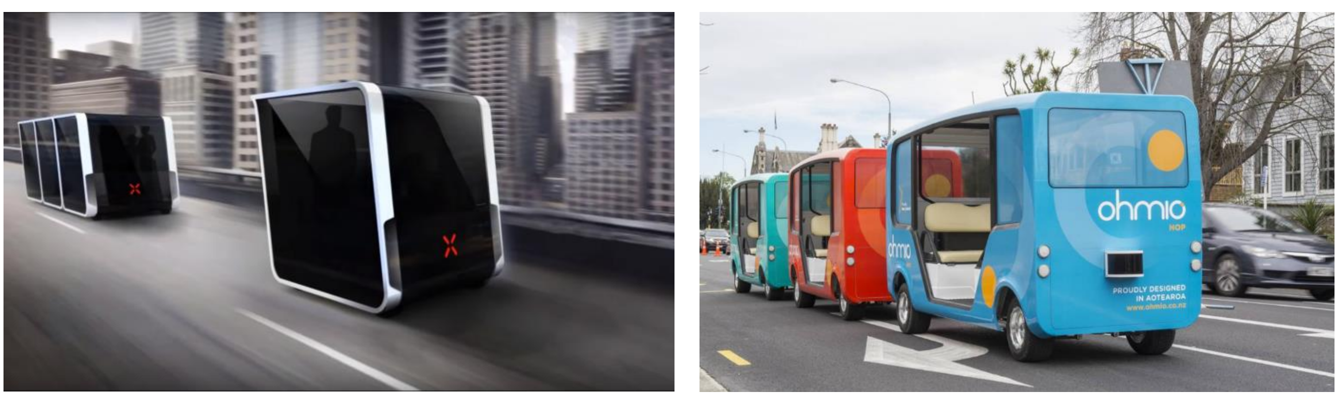
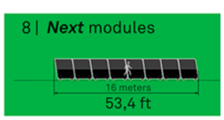
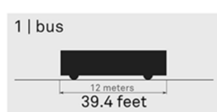
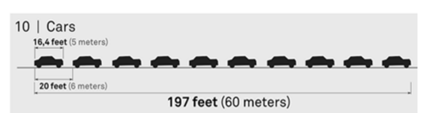


Figure 1 MV concepts proposed by (a) NEXT (source: https://www.next-future-mobility.com) and (b) Ohmio LIFT (source: https://ohmio.com).

Emerging modular vehicle (MV) technology holds the promise of overcoming these issues. The MV technology allows modular pods to be dynamically docked/undocked into vehicles of different sizes en route (Figure 1; Chen et al., 2019, 2020). This technology have been tested by multiple companies, such as NEXT (Next Future Transport, 2019) and Ohmio LIFT (Ohmio, 2018). We propose a modular transit network system (MTNS) that uses the MV technology. In the MTNS system, MVs operating in a transportation network can be quickly reassembled at nodes (or stations) to obtain different capacities that suit downstream travel demand. With flexible vehicle capacity adjustment, the MTNS can effectively reduce passenger waiting time (by forming long MV chains) and improve vehicle occupancy (by forming short MV chains), thereby overcoming the limitations of traditional public transportation systems. According to the economies of scale in urban mass transportation, the travel cost of a vehicle is usually a concave function over the number of modular pods in it (Chen et al., 2020). As a result, this MTNS has potentials in reduce the system operation cost.

To better understand the MTNS, we compare it with two benchmark systems: the fixed-capacity shuttle bus system (FSBS) and the passenger car system (PCS). The FSBS can be viewed as a special case of the MTNS, where each vehicle has a fixed capacity and provides transportation service without intermediate stops. The FSBS is used mostly in areas where bus stops are sparse and scattered; in these areas, there is a low economic incentive for intermediate stops because of the long detour distance. Since the vehicles' capacities are fixed, performance of the FSBS may be limited if passenger demand exhibits considerable spatial variation. Specifically, the FSBS may not fully utilize vehicle capacity in places with low demand, and it may not be able to serve all passengers in places with intensive demand. By contrast, the vehicle capacity in a general MTNS is adjustable to passenger demand. Therefore, the MTNS better fit varying passenger demand by dynamically adjusting vehicle capacity. Further, vehicles in a PCS are private passenger cars (or taxis) with a small fixed capacity. The advantage of a PCS is service convenience for individual travelers (e.g., no waiting and transfer times, and direct door-to-door service). However, a PCS may be the most expensive system since more vehicles are needed to serve the same demand. A detailed comparison of the three systems is provided in Table 1.

Table 1 Comparison of alternative systems

	MTNS	**FSBS**	**PCS**
Overall cost	Operation cost; Waiting time cost; Riding time cost	Operation cost; Waiting time cost; Riding time cost	Operation cost; Riding time cost
Transfer cost	Considered	Considered	No
Transfer mode	Walk	In-vehicle transferred	No
Vehicle type	Flexible capacity	Fixed capacity	Fixed capacity
Occupancy	6 passengers/pod	36~48 passengers/bus	1~4 passengers/car
Vehicle length (48 passengers)	 Flexible; small per passenger	 Fixed; small per passenger	 Fixed; long per passenger

Note: Data and figure source: https://www.next-future-mobility.com

A class of related studies has focused on designing a transit system to serve a transportation network. However, few studies have investigated the design of MTNSs in the literature. Most current studies have focused on transportation network design to provide comprehensive services to an urban area (Almasi et al., 2018; Cepeda et al., 2006; Daganzo, 2010; Fan et al., 2018; Guo et al., 2017; Nourbakhsh and Ouyang, 2012; Tong and Wong, 1999; Wu et al., 2016). The goal is to minimize the system cost, which includes the operation costs (Alshalalfah and Shalaby, 2012; Diana et al., 2006; Nourbakhsh and Ouyang, 2012;

Pei et al., 2019a; Quadrifoglio et al., 2006, 2007, 2008) and trip time costs (Niu et al., 2015; Pei et al., 2019b; Quadrifoglio et al., 2008), accessibility (Nassir et al., 2016; Owen and Levinson, 2015), and so on. For example, Ortega and Wolsey (2003) investigated an incapacitated fixed-charge network flow problem to minimize network design costs and passenger flow costs. Daganzo (2010) analyzed the structure of urban transit networks to increase accessibility. Ouyang et al. (2014) used the continuum approximation technique to design bus networks for cities where the travel demand varies gradually in space. The authors proposed heterogeneous route configurations to reduce the costs for both bus users and the operating agency. Tong et al. (2015) developed an urban transit network design model to maximize the number of accessible activity locations in a space-time network within a given travel time budget. Despite these fruitful developments, most existing transit network design studies have only considered vehicles with fixed capacity.

A handful of recent studies have investigated MV operations in transit systems. Table 2 provides a brief summary of these studies. Most of these studies propose a variable-capacity operation approach with modular transits based on the MV concept. Guo et al. (2018) proposed a simulation-based model for designing a many-to-one (M-to-1) system in the MV context. (Chen et al., 2019, 2020)proposed both discrete and continuous models for designing an MV shuttle system under oversaturated traffic conditions. Rau et al. (2019) propose a dynamic autonomous road transit system by varying the number of modular pods in each vehicle. Zhang et al.(2020) mathematically modeled a MV transit system with a time-expanded network, thereby reducing the size of the optimization problem. Shi et al. (2020) proposed a variable-capacity operation approach for two corridors sharing a portion of stations. Caros and Chow (2020) proposed a two-sided day-to-day learning framework to simulate the performance of a mobility service using modular autonomous vehicles capable of en-route passenger transfers. Dai et al., (2020) proposed a joint design of bus capacity and dispatch headway in a mixed traffic environment consisting of both human-driven vehicles and MVs. Despite these pioneering explorations, most studies have only considered a shuttle system or a transit line, vehicle dispatching for the proposed MTNS have not been well studied. While one may be easily tempted by the idea that we can solve the MTNS design with existing methods since we just only need to design the service frequency and capacity of each shuttle routes jointly, the problem being investigated is much more complicated for two reasons. First, a network consists of multiple lines so there are interactions between different lines (e.g., transfer). These interactions are not modeled in transit line/shuttle studies. Thus, existing methods cannot be used directly. Second, the problem of designing one transit line is NP-hard (Liu and Ceder, 2017; Sayarshad and Chow, 2015; Wang and Qu, 2015). Thus, most studies have simply proposed heuristics to solve near-optimal solutions. A network model would be a harder NP-hard problem due to the many more decision variables (since there are more lines and transfer decisions) and constraints (since we need to add constraints to describe interactions between different lines). Thus, most existing solution algorithms for transit network design likely fail due to the dramatic increase in the solution space.

Table 2 Comparison of current related models and the proposed model

Paper	**Objective function**	**Decision variable(s)**	**Model type[*]**	**Constraint type**	**Vehicle type**	**Vehicle rebalance**	**System topology**	**Model approach**
Niu et al. (2015)	Passenger waiting time	Timetable, dwelling time, and speed profile	MINLP	Linear constraints	Fixed-capacity vehicle	No	Corridor	Mathematical programming
Chen et al., (2020, 2019)	Operation cost; passenger	Timetable and vehicle types	MILP & CA	Linear constraints	MV	No	Shuttle	Mathematical programming & analytical

	waiting time							model
Guo et al. (2018)	Myopic policy cost	Switching of transit service	-	Linear constraints	MV	No	M-to-1 network	Simulation
Rau et al.,(2019)	Effective use of capacity	Adaptive Fleet Size	-	-	MV	No	Network	Simulation
Caros and Chow(2020)	operator cost and user cost	En-route transfer	MILP	-	MV	No	Hub-and-spoke	Simulation/ insertion heuristic
Zhang et al.(2020)	Number of served requests	Timetable; vehicle types; module match	MILP	Linear constraints	MV	No	Network	Mathematical programming
Shi et al. (2020)	Operation cost; passenger waiting time	Timetable; vehicle types	MILP	Nonlinear constraints	MV	No	Corridor	Mathematical programming
Dai et al., (2020)	Operation costs; waiting time	scheduling and capacity	MINLP	Linear constraints	MV	No	Corridor	Mathematical programming
Our model	Operation cost; total time cost	Transfer strategies; vehicle types	MINLP	Linear constraints	MV	Considered	Network	Mathematical programming

Note: MINLP=mixed-integer nonlinear programming; MILP=mixed-integer linear programming; CA=continuum approximation

To bridge these gaps and achieve the vision of MTNSs, this paper proposes a mathematical approach to describe MTNS operations and determine the optimal MTNS design. The contributions of this paper are follows.

First, a new modular transit network system (MTNS) is proposed for serving passenger travel demands across a general road network with the emerging modular vehicle technology. It can optimally allocate and schedule a MV fleet over a general transportation network and reach a balance between the operation cost and passenger trip time cost. The strategy decisions include the dispatch frequency and vehicle capacity for each dispatch. Second, we formulate this problem as a mixed-integer nonlinear programming (MINLP) model that captures detailed traveler waiting time costs with nonlinear functions related to vehicle schedules. This model has a complex biconvex function and mixed-integer decision variables and is thus difficult to solve directly. To facilitate the approach and to obtain the optimal solution, we mathematically revise the formula to produce a computationally tractable linear model and solve both lower and upper bounds to the original nonlinear model, thus yielding a near-optimal solution. This revised mixed-integer linear programming (MILP) model can be solved using off-the-shelf commercial solvers (e.g., Gurobi). Third, two numerical examples show that the MTNS is more effective than the classic transit bus and passenger car systems in both urban and freeway settings.

The remainder of this paper is organized as follows. Section 2 introduces the operation characteristics, notation, concept, and assumptions of the proposed MTNS. Section 3 presents the MTNS model with alternative systems. Section 4 tests the proposed model with two numerical examples in China and conducts sensitivity analyses. Finally, Section 5 provides conclusions and recommends future research directions.

2. MTNS operation description

This section introduces the MTNS and the underlying assumptions. For the convenience of readers, notation used throughout the paper is summarized in Table 3.

Table 3 Notation

Sets	
$\mathcal{I}$	Set of service stations (nodes), $\mathcal{I} := \{1, \ldots, I\}$
$\mathcal{S}$	Set of MV types, $\mathcal{S} := \{1, \cdots, S\}$
$\mathcal{M}$	Set of a sequence of numbers, $\mathcal{M} := \{1, \ldots, M\}$
Parameters	
i, j, k, l	Service station index, $i, j, k, l \in \mathcal{I}$
s	MV type index, $s \in \mathcal{S}$
m	Sequence number index, $m \in \mathcal{M}$
n	Capacity of a single MV pod
ij	Link ij index for a link starting from station i ending at station $j, i, j \in \mathcal{I}$
d_{ij}	Distance of link ij, km
q_{ij}	Passenger demand from origin $i \in \mathcal{I}$ and destination $j \in \mathcal{I}$
G_{ijs}	Traffic capacity (i.e., the maximum rate of passing vehicles) on link ij specific for type-s MVs
G_{ij}	Traffic capacity on link ij, $G_{kl} = \max_{s \in \mathcal{S}} G_{kls}$
C_s	Unit-distance operation cost for type-s MV, $s \in \mathcal{S}$, \$/km
C_t	Value of time per passenger, \$/h
C_{FSBS}	Unit-distance operation cost of the fixed-capacity shuttle bus system (FSBS), \$/km
C_{PCS}	Unit-distance operation cost of the passenger car system (PCS), \$/km
v	Constant MV operating speed, km/h
β	Transfer cost per passenger, \$/h
τ_m	Waiting time of the m^{th} segment in the linearized model, $m \in \mathcal{M}$, h
F_{MTS}	System cost of the MTNS (\$)
F_{FSBS}	System cost of the FSBS (\$)
F_{PCS}	System cost of the PCS (\$)
Decision variables	
x_{kls}	Continuous variable; type-s MV dispatch rate from stations k to l, $x_{kls} \in \mathbb{R}^+, k, l \in \mathcal{I}, s \in \mathcal{S}$
e_{kls}	Binary variable; $e_{kls} = 1$ if MVs from station k to station l are type s; otherwise, $e_{kls} = 0$. $k, l \in \mathcal{I}$, and $s \in \mathcal{S}$
y_{ijkl}	Continuous variable; Number of passengers traveling from stations i and j use MVs from stations k to l along their routes; $y_{ijkl} \in \mathbb{R}^+$, and $i, j, k, l \in \mathcal{I}$.
z_{klm}	Binary variable; $z_{klm} \in \{0,1\}$, $z_{klm} = 1$ if the waiting time of MVs from stations k to l is in the range segment m; otherwise, $z_{klm} = 0$. $k, l \in \mathcal{I}$, and $m \in \mathcal{M}$
w_{ijkl}	Continuous variable; total waiting time of passengers traveling from stations i to j that use MVs from stations k to l along their routes; $w_{ijkl} = y_{ijkl} \sum_{m \in \mathcal{M}} z_{klm} \tau_m \in \mathbb{R}^+$, and $i, j, k, l \in \mathcal{I}$.
x_{kl}^{F}	Continuous variable; shuttle bus dispatch rate from stations k to l; $x_{kl}^{\mathrm{F}} \in \mathbb{R}^+$, and $k, l \in \mathcal{I}$.
x_{ij}^{P}	Continuous variable; passenger car flow rate from stations i to j; $x_{ij}^{\mathrm{P}} \in \mathbb{R}^+$, and $i, j \in \mathcal{I}$.

Note: $\mathbb{R}^+$ denotes the set of nonnegative real numbers.

As Figure 2 shows, the MTNS operation is a three-step process: collecting travel requests, optimizing dispatch strategies, and providing services. First, passengers send their travel requests with their origins and destinations to a central processing system. Second, the integrated requests are fed into an optimization model (which will be presented in the next section) to solve the optimal dispatch strategy (i.e., the dispatch headway and MV types) and the passenger itineraries (i.e., the MVs a passenger has to ride on to travel from the origin to the destination). Next, the optimal dispatch strategy is used to instruct system operations, and the passenger itineraries are sent to passengers. Passengers will travel according to the optimized itineraries. Different from existing transit systems, the proposed MTNS adopts a fully automated passenger transfer process. Before a MV reaching a transfer station, passengers heading a destination station will be informed to walk to a modular pod that will eventually travel to that station if there is one. Thus, passengers do not have to all alight the vehicle for transferring, which is expected to lower the transfer hassle.

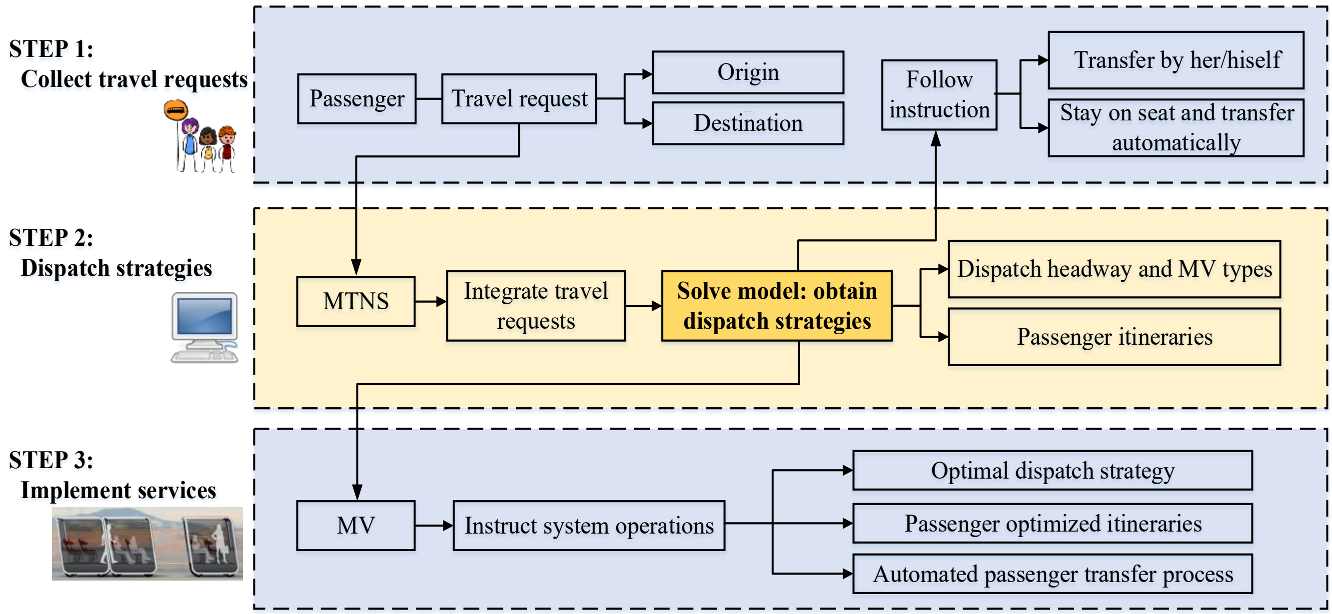


Figure 2 Operation process in the MTNS

The MTNS operates in a transportation network consisting of a set of stations (or nodes) $\mathcal{I}$, indexed as $i \in \mathcal{I}$, distributed in space and a set of links connecting pairs of stations. We denote a link starting from station $i \in \mathcal{I}$ and ending at station $j \in \mathcal{I}$ as (i, j) and its length as d_{ij}. Let q_{ij} denote the passenger demand from origin $i \in \mathcal{I}$ to destination $j \in \mathcal{I}$, and we assume that this demand stays constant throughout the period investigated in this problem. We denote the set of MV types that can be dispatched to serve the passengers as $\mathcal{S} := \{1, \cdots, S\}$, indexed as $s \in \mathcal{S}$. A type-s MV has s modular pods and thus a capacity of sn, where n denote the capacity of a single pod. During the operation, MVs flexibly adjust the vehicle capacity via docking/undocking to meet passenger demand. This process can be controlled manually or, in the future, automatically.

To better understand the potential benefits of the MTNS, let us consider a simple illustrative example. Figure 3 shows an example with five service stations ($\mathcal{I} = \{1, \dots, 5\}$) and six types of MVs ($\mathcal{S} = \{1, \cdots, 6\}$). In this figure, on each link between two stations, the arrows of different colors represent different MV types, and the line weights represent the MV dispatch frequencies. The OD pairs and sampled demands associated with station 4 are listed in Table 4. The optimal operation strategy of station 4 is also presented in Table 4. We see that some passengers take direct MVs without transfers (e.g., $1 \to 4, 2 \to 4, 5 \to 4, 4 \to 1$, and $4 \to 5$), and other passengers make multiple transfers to complete the trip (e.g., $3 \to 5 \to 4, 4 \to 5 \to 1, 4 \to 5 \to 2, 4 \to 5 \to 3$). Moreover, passengers with the same origins and destinations may take multiple routes. For example, for OD $4 \to 1$, 7.79% of passengers take route $4 \to 5 \to 1$, with an average waiting time of 0.154 h for the first segment and 0.22 h for the second segment, and 92.21% of passengers instead take route $4 \to 1$, with an average waiting time of 0.154 h.

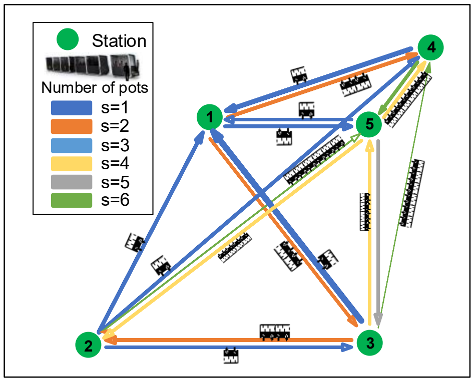


Figure 3 Illustration of optimal MTNS operations

Table 4 Optimal operation strategies for station 4

OD pairs	Demands (q_{ij})	Optimal operation strategies	Optimal vehicle type (s)	Average waiting time ($\frac{1}{2*x_{kls}}$)
1 → 4	22	1 → 4	2	0.198
2 → 4	15	2 → 4	1	0.2
3 → 4	10	3 → 5 → 4	4; 4	0.42; 0.22
5 → 4	23	5 → 4	4	0.22
4 → 1	21	4 → 1	1	0.154
		4 → 5 → 1	6; 4	0.154; 0.22
4 → 2	45	4 → 5 → 2	6; 4	0.154; 0.218
4 → 3	11	4 → 5 → 3	6; 5	0.154; 0.211
4 → 5	22	4 → 5	6	0.22

Finally, to facilitate the model formulation, we introduce the following assumptions in the investigated problem.

Assumption 1. The demands are stationary over the investigated time period. The assumption of static traffic flow patterns is commonly adopted in transportation network modeling. Moreover, the passenger arrival follows the random distribution. Thus, the average passenger waiting time is a half of the headway, which has been widely used in waiting time cost estimation (Ansari Esfeh et al., 2020).

Assumption 2. All passengers waiting at a station follow the transfer policy specified in the MTNS. Additionally, each link ij has a traffic capacity (i.e., the maximum rate of passing vehicles) G_{ijs} specific to each type-s MVs. Since different types of MVs have different lengths, MVs may have type-specific traffic capacities on the same link).

Assumption 3. Only one type of MVs can operate on a link. This assumption is made to ensure the computational tractability of the model. Additionally, this assumption is reasonable since stationary traffic flow on a link is likely associated with one optimal MV configuration.

Assumption 4. Each station has sufficient space to store the reserved pods to off-set local demand perturbations at the station. This assumption ensures that each station always dispatch MVs on schedule according to the optimal dispatch frequency, even with local demand perturbations, and ensures that the pods are sufficient. Thus, we do not pose a fleet size constraint on the system operation and can dispatch as many vehicles as we need. We also don't have to consider the vehicle dwell time and the cost for vehicle purchase and maintenance. While the fleet planning problem is relevant, it belongs to the planning stage and can be separated from the operational problem. The optimal fleet size can be determined after the operational plan is solved.

Assumption 5. The congestion is not considered since only a small portion of the demand takes the proposed service, which would little impact the road network congestion patterns. Thus, the system design would not affect each link's travel time.

3. Methodology

This section provides model formulations for the investigated and related benchmark systems.

3.1 Model formulation for the MTNS system

3.1.1 Original model formulation

The investigated problem involves optimizing the vehicle dispatch strategy (specified by x_{kls} and e_{kls}) and passenger itineraries (specified by y_{ijkl}) to minimize the total system cost. We first introduce the following decision variables in the MTNS:

- x_{kls}: Continuous variable x_{kls} denotes the dispatch rate of type-s MVs from stations k to l. We assume that the traffic demand on any link is much higher than the capacity of an MV, and thus, without much loss of generality, x_{kls} is set as a continuous decision variable.
- y_{ijkl}: Continuous variable y_{ijkl} denotes the rate of passengers that traveling from stations i and j use MVs from stations k to l along their routes. This flexible notation allows a passenger to transfer across multiple MV links to complete a trip if it is favorable for the passenger.
- e_{kls}: Binary variable e_{kls} denotes whether the MVs from stations k to l are type s. If yes, $e_{kls} = 1$, and otherwise $e_{kls} = 0$.

Objective function

The objective function formulated in Equation (1) aims to minimize the overall system cost, which consists of two components: the operation cost and passenger trip time cost. The passenger trip time cost can be further calculated by the passenger waiting cost, the riding time cost (in-vehicle travel cost), and the transfer penalties. Let C_s denote the operation cost of each type-s MV per unit distance; the operation cost includes MV depreciation, maintenance, infrastructure investment, electricity, and fuel costs. With the unit-distance operation cost C_s, the unit-time operation cost for all type-s MVs in the system is simply a product of C_s and the total travel distance per unit time, $\sum_{k\in\mathcal{I},l\in\mathcal{I}} x_{kls}d_{kl}$. This operation yields the system operation cost as $\sum_{k\in\mathcal{I},l\in\mathcal{I},s\in\mathcal{S}} C_s x_{kls}d_{kl}$, as the first term of Equation (1) specifies. Let C_t denote value of time per passenger. With this, the passenger trip time cost, including the passenger waiting time and riding time, is formulated as the second term of Equation (2). Specifically, the average waiting time of a passenger riding an MV on link kl is $\frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}}$, where $\sum_{s\in\mathcal{S}} x_{kls}$ is the MV dispatch frequency on link kl (Assumption 1). For mathematical convenience, we define the formula $\frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}}$ as a large value when the value of $\sum_{s\in\mathcal{S}} x_{kls}$ to 0. Next, with a constant MV operating speed v, the riding time for a passenger riding an MV on link kl is $\frac{d_{kl}}{v}$. Furthermore, let β denote the extra cost for a passenger to make one additional transfer to capture hassles during the transfer process. Note that the extra cost due to splitting and reassembling operations of MVs can be included in parameter β. Thus, the total transfer cost for the passenger throughout the trip is the product of β and the total number of transfers. Since during a trip a passenger makes exactly one additional transfer at each leg except the first leg, the total number of transfers throughout the passenger's trip is $\sum_{i\in\mathcal{I},j\in\mathcal{I},k\neq i\in\mathcal{I},l\in\mathcal{I}} y_{ijkl}$. This approach yields to transfer penalty formulated as the third term in Equation (1). We want to note that although the en-route transfer operations of MV may reduce the transfer hassle, passengers may still have to wait before the vehicle leaves the transfer station because of the asynchronous. Thus, objective function (1) incorporates costs related to the transfer process. These include the transfer cost caused by the transfer time (which is incorporated in waiting time cost) and transfer inconvenience cost (which is formulated as the transfer penalty component in the objective function). These components also quantify the tradeoff between serving passengers with more direct services (i.e., more vehicles) and with more transfers (i.e., less vehicles) in the system.

$$\min_{x_{kls},y_{ijkl},e_{kls}} F_{MTS} := \sum_{k\in\mathcal{I},l\in\mathcal{I},s\in\mathcal{S}} C_s x_{kls} d_{kl} + \sum_{i\in\mathcal{I},j\in\mathcal{I},k\in\mathcal{I},l\in\mathcal{I}} C_t y_{ijkl}\left(\frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}} + \frac{d_{kl}}{v}\right) + \sum_{i\in\mathcal{I},j\in\mathcal{I},k\neq i\in\mathcal{I},l\in\mathcal{I}} \beta y_{ijkl} \tag{1}$$

Constraints

We consider four groups of constraints in the MTNS. These include vehicle capacity Constraint (2), pod conservation Constraint (3), passenger flow conservation Constraints (4)-(6), and unique MV type Constraints (7)-(8), as follows.

$\sum_{i\in\mathcal{I},j\in\mathcal{I}} y_{ijkl} \leq \sum_{s\in\mathcal{S}} x_{kls} sn$	$\forall k,l \in \mathcal{I}$	Vehicle capacity constraint	(2)
$\sum_{k\in\mathcal{I}\setminus\{l\},s\in\mathcal{S}} x_{kls} s = \sum_{k\in\mathcal{I}\setminus\{l\},s\in\mathcal{S}} x_{lks} s$	$\forall\, l \in \mathcal{I}$	Pod conservation constraint	(3)
$\sum_{l\in\mathcal{I}\setminus\{i\}} y_{ijil} = q_{ij}$	$\forall i,j \in \mathcal{I}$	Passenger flow conservation constraint	(4)
$\sum_{k\in\mathcal{I}\setminus\{j\}} y_{ijkj} = q_{ij}$	$\forall i,j \in \mathcal{I}$	Passenger flow conservation constraint	(5)
$\sum_{k\in\mathcal{I}} y_{ijkl} = \sum_{k\in\mathcal{I}} y_{ijlk}$	$\forall\, l \in \mathcal{I}\setminus\{i,j\}, i,j \in \mathcal{I}$	Passenger flow conservation constraint	(6)
$\sum_{s\in\mathcal{S}} e_{kls} = 1$	$\forall k,l \in \mathcal{I}$	Unique MV type constraint	(7)
$x_{kls} \leq e_{kls} G_{kls}$	$\forall k,l \in \mathcal{I}, s \in \mathcal{S}$	Unique MV type constraint	(8)
$x_{kls} \in \mathbb{R}^+ \cup \{0\}$	$\forall k,l \in \mathcal{I}$, and $s \in \mathcal{S}$	Variable domain	(9)
$y_{ijkl} \in \mathbb{R}^+ \cup \{0\}$	$\forall i,j,k,l \in \mathcal{I}$	Variable domain	(10)
$e_{kls} \in \mathbb{B}$	$\forall k,l \in \mathcal{I}$, and $s \in \mathcal{S}$	Variable domain	(11)

Constraint (2) is the vehicle capacity constraint, which mandates that for each link kl, the total MV capacity (shown in the right-hand side, abbr. RHS) is sufficient to serve all the passengers using this link (shown in the left-hand side, abbr. LHS). Constraint (3) involves the conservation of the MV pods circulating in the system; i.e., the number of the total MV pods arriving at each station l (LHS) is identical to the number of total MV pods departing from station l per unit time (RHS). This pod conservation constraint ensures that vehicles are balanced; i.e., the number of the total modular pods in the system remains constant throughout the operation. However, the vehicle balance does not necessarily equate to passenger flow balance, and thus the model allows imbalanced passenger flows at a station. Constraints (4), (5), and (6) are related to the conservation of the passenger flows. Constraint (4) requires that passengers traveling between origin i and destination j must all leave origin i. Likewise, Constraint (5) imposes that passengers traveling between origin i and destination j must all arrive at destination j. Constraint (6) means that for each station l, the number of passengers arriving at station l (LHS) must equal that leaving station l (RHS). Constraints (7) are proposed to limit only one type of MV to serve link kl. Further, Constraint (8) specifies that the MV flow on each link should not exceed the traffic capacity. Constraints (9), (10) and (11) are the variable domains.

3.1.2 Linearization approximation

In objective function (1), the waiting time cost term $\sum_{i\in\mathcal{I},j\in\mathcal{I},k\in\mathcal{I},l\in\mathcal{I}} C_t y_{ijkl} \frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}}$ is biconvex since both $\sum_{i\in\mathcal{I},j\in\mathcal{I},k\in\mathcal{I},l\in\mathcal{I}} y_{ijkl}$ and $\sum_{k\in\mathcal{I},l\in\mathcal{I}} \frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}}$ are convex functions of the corresponding decision variables. It is commonly known that mathematical programming problems with biconvex terms are difficult to solve directly (Gorski et al., 2007; Liberti and Pantelides, 2006). To facilitate the solution approach, this section reformulates the waiting time cost component as a linear term via the following two steps.

Step 1: Bilinear model reformulation

We first reformulate the waiting time cost term as a bilinear term by dividing the feasible region of the waiting time into M segments. We construct a arithmetic sequence $\tau_1, \tau_2, \dots, \tau_m, \dots, \tau_M$ satisfying $\tau_1 \le \frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}} < \tau_M$. This sequence can be dynamically changed to reach a lower approximation error (i.e., the difference between the approximated and original objective values). Then we introduce binary variables $z_{klm} \coloneqq \{0,1\}, k, l \in \mathcal{I}, m \in \mathcal{M}$ to denote whether the waiting time of MVs on link kl is in the range of the m^{th} segment. That is, we set $z_{klm} = 1$ if $\exists m \in \mathcal{M}\backslash\{M\}, s.t. \tau_m \le \frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}} < \tau_{m+1}$; otherwise, $z_{klm} = 0$. With this, $\frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}}$ is then linearized to $\sum_{m\in\mathcal{M}} z_{klm}\tau_m$, and $\sum_{i\in\mathcal{I},j\in\mathcal{I},k\in\mathcal{I},l\in\mathcal{I}} C_t y_{ijkl} \frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}}$ in the original objective function is reformulated to a bilinear component $\sum_{i\in\mathcal{I},j\in\mathcal{I},k\in\mathcal{I},l\in\mathcal{I}} C_t y_{ijkl} \sum_{m\in\mathcal{M}} z_{klm}\tau_m$ subject to linearization Constraints (12)-(15). Constraint (12) postulates that the waiting time can occupy one and only one segment of the time intervals with $z_{klm} = 1$ (e.g., $[\tau_m, \tau_{m+1})$). Let G_{kl} denotes the traffic capacity on link ij ($G_{kl} = \max_{s\in\mathcal{S}} G_{kls}$). as it shows in Assumption 2, each link kl has a traffic capacity G_{kls} (i.e., the maximum rate of passing vehicles) specific to type s. Constraints (13) and (14) specify that the value of $2\sum_{s\in\mathcal{S}} x_{kls}$ falls above $1/\tau_{m+1}$ and below (inclusive) $1/\tau_m$ to be consistent with $\tau_m \le \frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}} < \tau_{m+1}$. Note that Constraints (13) and (14) are activated only when $z_{klm} = 1$ to ensure that z_{klm} indicates the right time interval segment. Constraint (15) specifies z_{klm} as a binary variable.

$$\sum_{m\in\mathcal{M}} z_{klm} = 1 \qquad \forall k, l \in \mathcal{I} \tag{12}$$

$$2\sum_{s\in\mathcal{S}} x_{kls} > 2G_{kl}(z_{klm} - 1) + \frac{1}{\tau_{m+1}} \qquad \forall k, l \in \mathcal{I}, m \in \mathcal{M} \tag{13}$$

$$2\sum_{s\in\mathcal{S}} x_{kls} \le \frac{1}{\tau_m} + 2G_{kl}(1 - z_{klm}) \qquad \forall k, l \in \mathcal{I}, m \in \mathcal{M} \tag{14}$$

$$z_{klm} \in \{0,1\} \qquad \forall k, l \in \mathcal{I}, m \in \mathcal{M} \tag{15}$$

Step 2: Linear model reformulation

Since the waiting time component in *step 1*, $\sum_{i\in\mathcal{I},j\in\mathcal{I},k\in\mathcal{I},l\in\mathcal{I}} y_{ijkl} \sum_{m\in\mathcal{M}} z_{klm}\tau_m$, is a bilinear term that remains challenging to solve, we further linearize this term. Here, we introduce continuous variables $w_{ijkl} \in \mathbb{R}^+, i, j, k, l \in \mathcal{I}$. With this, we revise the bilinear term to $\sum_{i\in\mathcal{I},j\in\mathcal{I},k\in\mathcal{I},l\in\mathcal{I}} C_t w_{ijkl}$ as a linear term with the following constraints. Constraints (16) and (17) ensure that the value of w_{ijkl} is identical to $\sum_{m\in\mathcal{M}} z_{klm} y_{ijkl}\tau_m$. This is because when $z_{klm} = 0$, Constraints (16) and (17) always hold for all feasible values of w_{ijkl} allowed by the demand and thus are not activated; only when $z_{klm} = 1$ does Constraint (16) yield $y_{ijkl}\tau_m \le w_{ijkl}$ and Constraint (17) yield $w_{ijkl} \le y_{ijkl}\tau_m$, and thus $w_{ijkl} = y_{ijkl}\tau_m$. Constraint (18) specifies each w_{ijkl} as a nonnegative continuous variable.

$$y_{ijkl}\tau_m - q_{ij}\tau_m(1 - z_{klm}) \le w_{ijkl} \qquad \forall i, j, k, l \in \mathcal{I}, m \in \mathcal{M} \tag{16}$$

$$w_{ijkl} \le y_{ijkl}\tau_m + q_{ij}\tau_m(1 - z_{klm}) \qquad \forall i,j,k,l \in \mathcal{I}, m \in \mathcal{M} \qquad (17)$$

$$w_{ijkl} \in \mathbb{R}^+ \cup \{0\} \qquad \forall i,j,k,l \in \mathcal{I} \qquad (18)$$

With the above linearization steps, the investigated MTNS problem is reformulated as the following MILP model with objective (19), subject to vehicle capacity Constraint (2), pod conservation Constraint (3), passenger flow conservation Constraints (4)-(6), unique MV type Constraints (7)-(8), linearization Constraints (12)-(14) and (16)-(17), and variable domain Constraints (9)-(11), (15), and (18):

$$\begin{aligned}\min_{x_{kls}, y_{ijkl}, e_{kls}, z_{klm}, w_{ijkl}} F_{MTS} &:= \sum_{k\in\mathcal{I}, l\in\mathcal{I}, s\in\mathcal{S}} C_s x_{kls} d_{kl} \\ &+ C_t\left(\sum_{i\in\mathcal{I}, j\in\mathcal{I}, k\in\mathcal{I}, l\in\mathcal{I}} w_{ijkl} + \sum_{i\in\mathcal{I}, j\in\mathcal{I}, k\in\mathcal{I}, l\in\mathcal{I}} y_{ijkl}\frac{d_{kl}}{v}\right) \qquad (19) \\ &+ \sum_{i\in\mathcal{I}, j\in\mathcal{I}, k\neq i\in\mathcal{I}, l\in\mathcal{I}} \beta y_{ijkl}\end{aligned}$$

$$s.t.\, constraints\ (2) - (18)$$

This above process successfully revises the original nonlinear model (NLM) to a linear model (LM), reducing the solution complexity and enabling the model to be solved with a mixed linear integer programming solver. However, an approximation error ensues from the revision of the waiting time cost term in the LM. The following theoretical properties of the relationship between the NLM and the LM solutions are shown to quantify the approximation error.

Theorem 1. The optimal objective value of the LM is a lower bound to that of the NLM.

Proof. For the NLM, we denote the optimal solution to variables $\{x_{kls}, y_{ijkl}, e_{kls}\}$ as $\{x^*_{kls}, y^*_{ijkl}, e^*_{kls}\}$ and the associated optimal objective value as F^*_{NLM}, which is the value of Equation (1) after plugging in $\{x^*_{kls}, y^*_{ijkl}, e^*_{kls}\}$.

By plugging the dispatch solution $\{x^*_{kls}, y^*_{ijkl}, e^*_{kls}\}$ into Constraints (12) and (18), we can solve the corresponding $\{z_{klm}, w_{ijkl}\}$ values, denoted as $\{z^*_{klm}, w^*_{ijkl}\}$. Obviously, $\{x^*_{kls}, y^*_{ijkl}, e^*_{kls}, z^*_{klm}, w^*_{ijkl}\}$ is a feasible solution to the LM, and we denote the corresponding objective value, i.e., the value of Equation (19) after plugging in $\{x^*_{kls}, y^*_{ijkl}, e^*_{kls}, z^*_{klm}, w^*_{ijkl}\}$, as F_{LM}.

Then, we obtain

$$F^*_{\text{NLM}} - F_{\text{LM}} = C_t * \sum_{i\in\mathcal{I}, j\in\mathcal{I}, k\in\mathcal{I}, l\in\mathcal{I}} y_{ijkl}\frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}} - \sum_{i\in\mathcal{I}, j\in\mathcal{I}, k\in\mathcal{I}, l\in\mathcal{I}} w_{ijkl}.$$

Since Constraints (16) and (18) ensure that the value of w_{ijkl} is identical to $\sum_{m\in\mathcal{M}} z_{klm} y_{ijkl}\tau_m$, $F^*_{\text{NLM}} - F_L$ can be reformulated as follows:

$$F^*_{\text{NLM}} - F_{\text{LM}} = C_t * \sum_{i\in\mathcal{I}, j\in\mathcal{I}, k\in\mathcal{I}, l\in\mathcal{I}} y_{ijkl}\left(\frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}} - \sum_{m\in\mathcal{M}} z_{klm}\tau_m\right).$$

Note that Constraints (12)-(15) ensure that the value of $\frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}}$ falls between τ_m and τ_{m+1} for the m value with $z_{klm} = 1$. It obviously indicates that $\frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}} \ge \sum_{m\in\mathcal{M}} z_{klm}\tau_m$ and consequentially $F^*_{\text{NLM}} \ge F_{\text{LM}}$.

Further, by definition, the objective value F_{LM} corresponding to any feasible solution to the LM is not less than its optimal objective value, denoted as F^*_{LM}. This yields $F^*_{\text{NLM}} \geq F^*_{\text{LM}}$. This completes the proof.

Theorem 2. Let $\{x'_{kls}, y'_{ijkl}, e'_{kls}, z'_{klm}, w'_{ijkl}\}$ denote the optimal solution to the LM. Then, $\{x'_{kls}, y'_{ijkl}, e'_{kls}\}$ is a feasible solution to the NLM, and the corresponding objective value, i.e., the value of Equation (1) after plugging in $\{x'_{kls}, y'_{ijkl}, e'_{kls}\}$, denoted by F'_{NLM}, constitutes an upper bound to the optimal objective value of the NLM, F^*_{NLM}.

Proof. The linearization process successfully revises the original NLM to a LM by reformulating the nonlinear component to a linear term and adding a series of new linear Constraints (12)-(18). Since the LM and NLM share the same Constraints (2)-(11), the solution $\{x'_{kls}, y'_{ijkl}, e'_{kls}\}$, which is optimal and thus feasible in the LM, is also feasible in the NLM. Then, F'_{NLM} is a feasible objective value of the NLM, and thus $F'_{\text{NLM}} \geq F^*_{\text{NLM}}$. Here, we complete the proof.

With the above theoretical properties, by solving the optimal solution to the LM, we obtain a set of near-optimal solutions to the NLM (i.e., $\{x'_{kls}, y'_{ijkl}, e'_{kls}\}$ with objective value F'_{NLM}) and a lower bound of the optimal objective value (i.e., F^*_{LM}). The optimality gap of the near-optimal solution can be evaluated as $(F'_{\text{NLM}} - F^*_{\text{LM}})/\, F^*_{\text{LM}}$. Note that the approximation error between the NLM and LM is determined by the sizes of the intervals $\{[\tau_m, \tau_{m+1}]\}$ that contain the corresponding $\left\{\frac{1}{2\sum_{s\in\mathcal{S}} x_{kls}}\right\}$ values. Thus, to reduce the approximation error, we may redistribute the $\{\tau_m\}$ values according to the obtained $\{x_{kls}\}$ solutions as follows.

a) Evenly divide $[0, \tau_M]$ into M intervals to solve the LM. This step produces the LM solution and system cost. Then, plug the LM solution into Equation (1) to calculate the corresponding NLM objective value.
b) Gather the values of $\{x_{kls}\}$ in the LM solution into k clusters, and then redistribute the $\{\tau_m\}$ values with a higher density around each cluster.
c) Then, solve the LM again with the new $\{\tau_m\}$ values.

This process can be repeated until the approximation error is acceptable. Note that while these three steps update the values of $\{\tau_m\}$, they do not affect the validity of Theorems 1 and 2 since the theorems take $\{\tau_m\}$ as a set of input parameters that can be given any values.

3.2 Alternative systems

To compare with the proposed MTNS, this section describes two benchmark systems: the fixed-capacity shuttle bus system (FSBS) and the passenger car system (PCS). We adapt the above MTNS model to specify the FSBS and PCS as flows based on the related characteristics. Note that there are many different possible benchmark systems to compare the proposed MTNS with. However, it is not possible to enumerate each of them in one study since solving the optimal design for each of these systems is a very challenging task. Here we simply select two existing benchmark systems to reveal the benefits of the flexible capacity operations in the MTNS. More studies are needed to fully understand its advantages over other systems.

In the FSBS, each vehicle has a fixed capacity of n^{F} and provides direct point-to-point transportation without intermediate stops. Let C_{FSBS} denote the FSBS operation cost per distance. The FSBS model can be obtained by replacing x_{kls} with x^{F}_{kl} (denoting the shuttle bus dispatch rate on link kl) in objective function (20), vehicle capacity Constraint (21), pod conservation Constraint (22), and other Constraints (3)-(6), (9), and (12)-(18) as follows:

$$\min_{x_{kl}^{\mathrm{F}},y_{ijkl},e_{kls}} F_{FSBS} := \sum_{k\in\mathcal{I},l\in\mathcal{I}} C_{FSBS}x_{kl}^{\mathrm{F}}d_{kl} + \sum_{i\in\mathcal{I},j\in\mathcal{I},k\in\mathcal{I},l\in\mathcal{I}} C_t y_{ijkl}\left(\frac{1}{2x_{kl}^{\mathrm{F}}}+\frac{d_{kl}}{v}\right) + \sum_{i\in\mathcal{I},j\in\mathcal{I},k\neq i\in\mathcal{I},l\in\mathcal{I}} \beta y_{ijkl} \tag{20}$$

$s.t.\ constraints$ (3) − (6), (9), (12) − (18)

$$\sum_{i\in\mathcal{I},j\in\mathcal{I}} y_{ijkl} \le x_{kl}^{\mathrm{F}} n^F \qquad \forall k\in\mathcal{I}, l\in\mathcal{I} \tag{21}$$

$$\sum_{k\in\mathcal{I}\setminus\{l\}} x_{kl}^{\mathrm{F}} = \sum_{k\in\mathcal{I}\setminus\{l\}} x_{lk}^{\mathrm{F}} \qquad \forall\, l\in\mathcal{I} \tag{22}$$

$$x_{kl}^{\mathrm{F}} \in \mathbb{R}^+ \cup \{0\} \qquad \forall k\in\mathcal{I}, l\in\mathcal{I} \tag{23}$$

In the PCS, where private passenger cars and taxis dominate, each vehicle has a small average occupancy of n^{P}. Let C_{PCS} denote the PCS operation cost per distance. The PCS considers an idealized situation where taxis and private vehicles transport passengers from their origins directly to their destinations without transfers, eliminating the need for waiting at the origins or transfer points. Thus, the system cost includes only the operation cost and passenger riding time cost from the origin to destination. With this approach, the PCS model can be obtained by replacing x_{kls} with x_{ij}^{P} (denoting the passenger car flow rate on link ij) in objective (24), pod conservation Constraint (25), passenger flow conservation Constraint (27), and variable domain Constraints (28)-(29) as follows:

$$\min_{x_{ij}^{\mathrm{P}},y_{ijkl},e_{kls}} F_{PCS} := \sum_{i\in\mathcal{I},j\in\mathcal{I}} C_{PCS}x_{ij}^{\mathrm{P}}d_{kl} + \sum_{i\in\mathcal{I},j\in\mathcal{I}} C_t y_{ijij}\frac{d_{ij}}{v} \tag{24}$$

$s.t.$

$$y_{ijij} \le x_{ij}^{\mathrm{P}} n_P \qquad \forall i\in\mathcal{I}, j\in\mathcal{I} \tag{25}$$

$$\sum_{i\in\mathcal{I}\setminus\{j\}} x_{ij}^{\mathrm{P}} = \sum_{j\in\mathcal{I}\setminus\{i\}} x_{ji}^{\mathrm{P}} \qquad \forall j\in\mathcal{I} \tag{26}$$

$$y_{ijij} = q_{ij} \qquad \forall i\in\mathcal{I}, j\in\mathcal{I} \tag{27}$$

$$x_{ij}^{\mathrm{P}} \in \mathbb{R}^+ \cup \{0\} \qquad \forall i\in\mathcal{I}, j\in\mathcal{I} \tag{28}$$

$$y_{ijij} \in \mathbb{R}^+ \cup \{0\} \qquad \forall i\in\mathcal{I}, j\in\mathcal{I} \tag{29}$$

4. Numerical example

To illustrate the application of the proposed MTNS model, this section explores two numerical examples with different network sizes. All experiments were performed on an Intel® Core™ i7-8550U 1.99 GHz CPU with 24 GB of RAM. The code was implemented in MATLAB 2019a and called a commercial MILP solver Gurobi (Cochran et al., 2011; Fuentes et al., 2019; Zhang et al., 2019) to solve the linearized model. The default parameter values are set as follows.

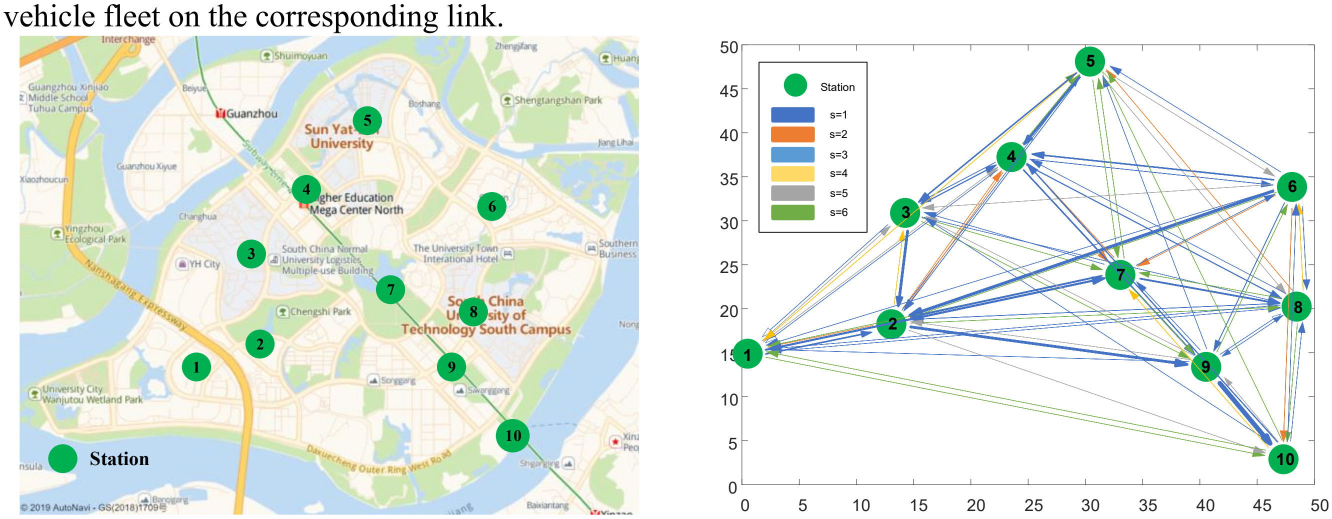
Table 5 Default parameter settings

Parameter	Value	Data source
$\mathcal{S}$	[1, 2, 3, 4, 5, 6]	NEXT website (https://www.next-future-mobility.com)
n	6 passengers	NEXT website (https://www.next-future-mobility.com)
n^{F}	36 passengers	Guangzhou No. 3 Bus Company (http://www.bus3.cn/sitecn/msg.aspx)

n^{P}	1.5 passengers	Freeway operation report of Guangdong Province, China (http://data.eastmoney.com/notices/detail/)
C_s	[0.143, 0.257, 0.347, 0.417, 0.471, 0.514] $/km	Operation cost is not a linear function of the number of dispatched pods; NEXT website (https://www.next-future-mobility.com)
C_{FFBS}	0.514 $/km	Guangzhou No. 3 Bus Company
C_{PTS}	0.143 $/km	Guangzhou Taxi Company
C_t	2.86 $/h in Guangzhou	Guangzhou Municipal Human Resources and Social Security Bureau reports in 2019 (http://gzrsj.hrssgz.gov.cn/english/)
β	0.142 $/passenger	Passenger transfer cost are determined by referring to the average income per capita from Guangzhou Municipal Human Resources and Social Security Bureau reports in 2019.
v	31.85 km/h	Operating speed of MVs on city roads (case 1) (http://www.gzjt.gov.cn/gzjt/)
	60.32 km/h	Operating speed of MVs on the freeway (case 2) (http://www.0512s.com/lukuang/)

4.1 Ten-station example

Example 1 is a public transit system in the Guangzhou Higher Education Mega Center, China. As shown in Figure 4 (a), we selected ten critical bus stops and collected the real-world travel demand data for each stop to investigate this example. The bus stops in this system are sparse and scattered, and it is uneconomic to form a transit corridor because of the long deviation cost. The travel demand data and road distance data were obtained from the Communications Commission of Guangzhou Municipality. In addition to the default parameter values specified in Table 1, we set $M = 20$ and $\tau =$[0.02: 0.01: 0.1, 0.2: 0.1: 1, 500, 1000] after extensive experiments. The optimal solution (exact solution) of the proposed model is obtained within 0.12 h. The optimal MTNS service strategy is shown in Figure 4(b). Different colors represent different MV types, and the thickness of each arrow illustrates the frequency of the modular vehicle fleet on the corresponding link.



(a) MTNS service station information

(b) Optimal routing result

Figure 4 Numerical example from Guangzhou, China

Cost comparisons

We compared the MTNS solutions with those from the FSBS and PCS alternatives. The number of modular pods in the FSBS vehicles is set to the optimal value of $S = 6$ (see Figure 5(e) for why this is optimal). In this experiment, we used the system cost (which includes the operation cost, waiting time cost, riding time cost, and transfer cost) as the criterion to evaluate the performance of different systems.

The results are shown in Table 6, where the percentage of cost reduction is calculated as $\frac{F_{FSBS}-F_{MTS}}{F_{MTS}} * 100\%$ and $\frac{F_{PTS}-F_{MTS}}{F_{MTS}} * 100\%$.

We found that the total system cost in the MTNS is less than those in the FSBS and in the PCS. The MTNS reduces the system cost by 7.10% compared with the FSBS and by 28.96% compared with the PCS. The cost savings are more pronounced when we remove the fixed free-flow travel time independent of the optimal decisions. That is, the revised system cost reduction becomes 31.39% compared with the FSB and 128.03% compared with the PCS. Note that the comparison between the MTNS and the FSBS is to show that a flexible capacity transit system performs better than a fixed system. The benefits of the MNTS may not be as evident when we compare it with a transit network system where different lines operate with vehicles of different sizes (e.g., vans, minibuses). Such a comparison is not offered here since it requires us to solve another optimal system design problem with the capacity of different lines as decision variables. This problem is non-trivial and is out of the scope of this paper. Thus, the results here only offer an upper bound to the benefits of the proposed MNTS with existing transit operations.

Regarding the system cost components, the reduction in the operation cost (33.63%) is the highest when comparing the MTNS with the FSBS. This is because the flexible vehicle capacity in the MTNS promotes frequent dispatching of small vehicles. In contrast, the FSBS can only dispatch vehicles with a stationary capacity, and lead to a frequent rate. The improvements in the riding costs are relatively minor, likely because the waiting time at the origin and transfer points is much less than the in-vehicle travel time overall, which is nearly by 77% in this example. While the in-vehicle travel time cost does not change dramatically, the bulk of the riding time cost is dominated by the travel distance and thus is independent of the transportation system. If we take away the fixed free-flow travel time, we see a much more significant improvement in the variable riding time cost affected by the transportation system settings. For reference, Table 6 provides the revised riding time cost, which is reduced by 1983.78%. Additionally, compared with the MTNS, the PCS has no waiting time and a shorter riding time because of the direct service without transfers. However, the PCS operation cost is higher than the MTNS operation cost by 290.86% due to much lower vehicle occupancies in the PCS.

Further, based on Theorems 1 and 2, we obtain a lower bound objective $F_{\text{LM}}^{*} = 1457.91$ and an upper bound objective $F_{\text{NLM}}' = 1465.60$. This yields an optimality gap of 0.52%, which is on a lower order of magnitude than the cost component improvements in Table 6 and thus is acceptable.

Table 6 Cost comparisons of different operating systems (case 1)

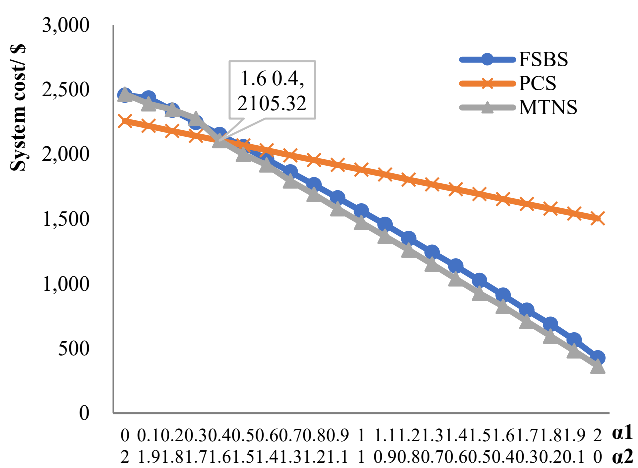
	MTNS	FSBS		PCS	
	Value	Value	% reduction	Value	% reduction
• System cost	$1,457.91	$1,561.43	7.10%	$1,880.17	28.96%
Revised system cost*	$329.81	$433.33	31.39%	$752.07	128.03%
• Operation cost	$192.41	$257.11	33.63%	$752.07	290.86%
• Waiting time cost	$135.34	$144.23	6.57%	-	-
• Riding time cost	$1,128.47	$1,135.81	0.65%	$1,128.10	-0.03%
Revised riding time cost*	$0.37	$7.71	1983.78%	-	-
• Transfer cost	$1.71	$24.29	1316.67%	-	-

Note: The revised system cost and revised riding time cost are calculated by removing the fixed free-flow travel time (equal to the riding time cost in the PCS) that is independent of the optimal decisions.

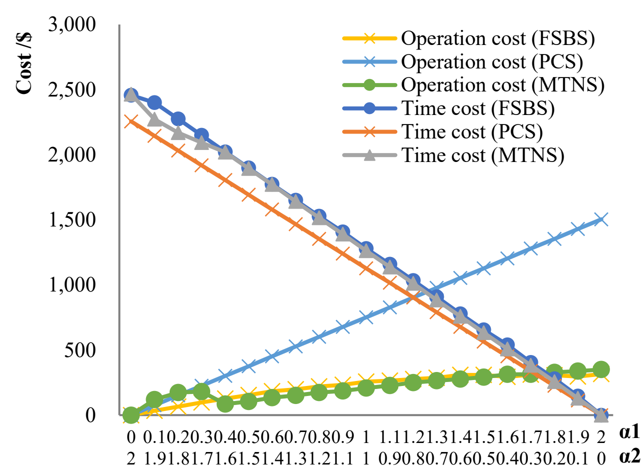
Sensitivity analysis

This section analyzes the sensitivity of cost components to critical parameters the in all three systems. Only one parameter is varied in each instance, and the other parameters remain at their default values. To evaluate the performance for different cases, we compared the overall system cost, operation cost, riding time cost, waiting time cost, and transfer cost. To simplify the sensitivity analysis for c_s and c_t, we introduced two rates to adjust the values as $c_s' = \alpha_1 * c_s$ and $c_t' = \alpha_2 * c_t$. Rates α_1 and α_2 are varied subject to $\alpha_1 + \alpha_2 = 2$, where $\alpha_1, \alpha_2 \in \mathbb{N}^+$. The results are plotted in Figure 5. The findings of parameters α_1, α_2, S, and β are briefly summarized as follows.

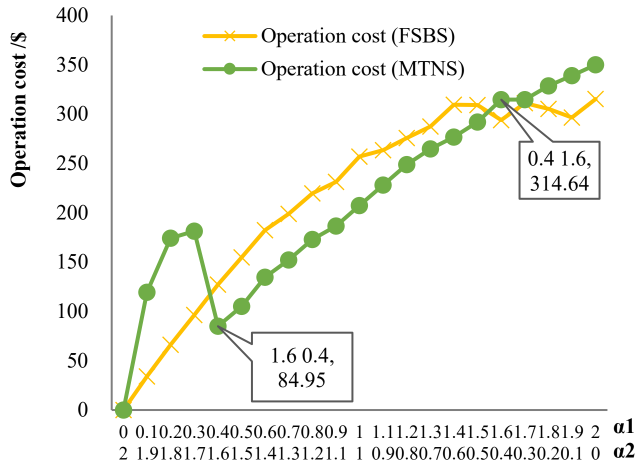
1. Figure 5(a) and (b) show that the proposed MTNS model effectively reduces the system cost. Compared with the FSBS, the MTNS always performs better (e.g., with a lower system costs) at all α_1 and α_2 values. Figure 5(c) plots the operation cost with varying α_1 and α_2 values. The operation cost of the MTNS becomes higher when the trip time cost dominates ($\alpha_1 \leq 0.4, \alpha_2 \geq 1.6$). Compared with the PCS, the system cost of the MTNS is lower when the operation cost rate is relatively high over the trip time cost range ($\alpha_1 \geq 0.4, \alpha_2 \leq 1.6$). However, when the time cost dominates ($\alpha_1 \leq 0.4, \alpha_2 \geq 1.6$), the PCS may work better than the MTNS due to its time savings from direct service.
2. Figure 5(d) plots the transfer cost with varying α_1 and α_2 values. We see that the transfer cost in the FSBS increases significantly with the increase in the operation cost, while that of the MTNS does not vary much with changes in α_1 and α_2.
3. The system cost decreases as the number of MV types (or S) increases, as shown in Figure 5(e). This result is evident since more MV types provide more flexible vehicle capacities. The system cost in the FSBS is a U-shaped curve with an optimal value of $S = 6$ (i.e., $n^F = 36$), which is the default parameter value we selected in the numerical example.
4. Figure 5(f) shows the sensitivity of the system cost to transfer cost β. We see that the system cost increases as the transfer cost rate per passenger increases. However, the transfer cost shares a rather small percentage of the system cost in the MTNS, the magnitude of the increases system cost is not substantial. While the transfer cost per passenger increases by 900% (i.e., from 0.07 to 0.71), the total system cost only increases by 0.3% (i.e., from 1456$ to 1461$).



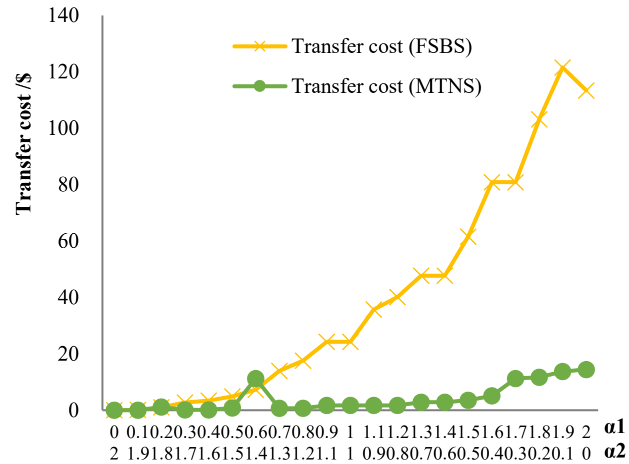
(a) System cost performance with α_1, α_2



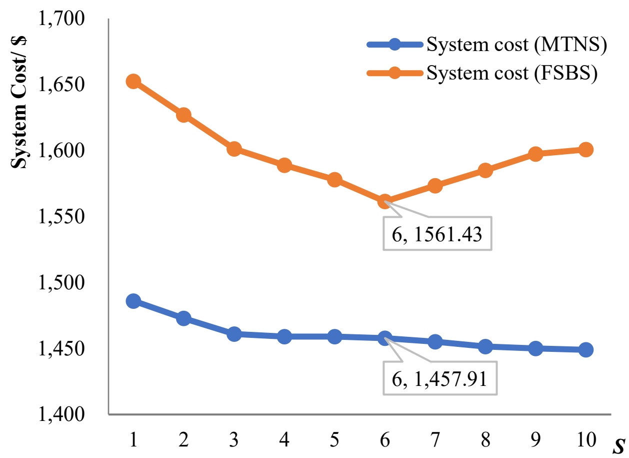
(b) Cost performance with α_1, α_2



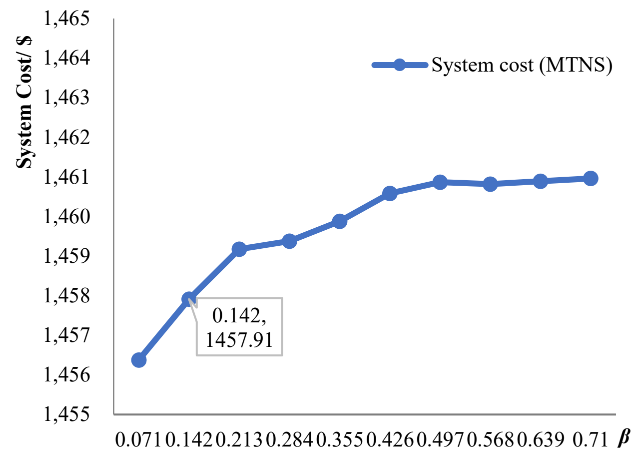
(c) Operation cost performance with α_1, α_2



(d) Transfer cost performance with α_1, α_2



(e) System cost performance with MV type S for the MTNS and n_F for the FSBS, $n^F = S * 6$



(f) System cost performance with transfer cost β

Figure 5 Sensitivity analysis of the criterion with different input parameters

Note that the above obtained optimal solutions to the LM may not be the exact optima to the original NLM. To investigate the approximation errors, we employ Theorems 1 and 2 to quantify the corresponding approximation gaps for a set of selected instances with different parameter settings, as shown in Table 7. We see that the approximation gaps are less than 4% for all of these instances, with an average of 1.66%, which is acceptable for engineering practice. Additionally, if we further refine the linearization approximation intervals, we would expect the gaps to continue to decrease (though more computational resources are needed).

Table 7 Sensitivity analysis of the approximation gap with various parameter combinations

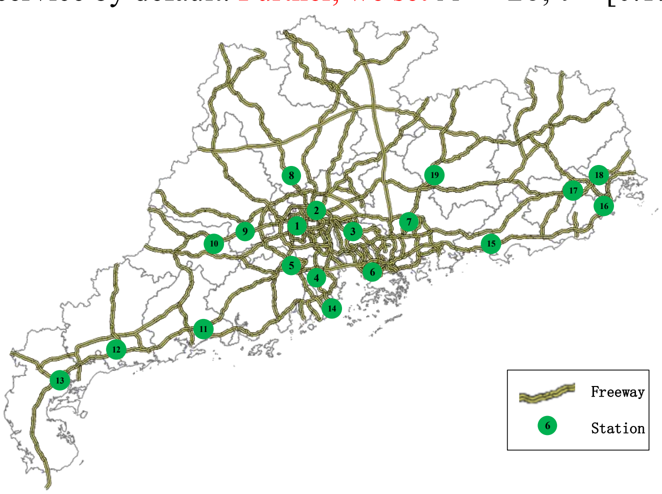
Instance number	α_1	α_2	S	β	F^*_{LM}	F'_{NLM}	Approximation gap
1	0	2	6	0.142	$2,464.89	$2,520.91	2.22%
2	0.5	1.5	6	0.142	$2,000.41	$2,054.86	2.65%
3	1	1	6	0.142	$1,457.91	$1,465.60	0.52%
4	1.5	0.5	6	0.142	$928.03	$957.41	3.07%
5	2	0	6	0.142	$364.40	$379.30	3.93%

6	1	1	2	0.142	$1,472.78	$1,483.56	0.73%
7	1	1	4	0.142	$1,459.00	$1,480.25	1.44%
8	1	1	8	0.142	$1,451.39	$1,464.36	0.89%
9	1	1	10	0.142	$1,449.00	$1,462.30	0.91%
10	1	1	6	0.071	$1,456.38	$1,475.15	1.27%
11	1	1	6	0.213	$1,459.17	$1,475.63	1.12%
12	1	1	6	0.284	$1,459.38	$1,475.74	1.11%
Average							1.66%

4.2 Nineteen-station example

To examine the model performance over different network topologies, we present another example with more stations (19 stations) and a larger network (the Guangdong Province freeway network). At this province-level spatial scale, the operations involve a new shared-mobility freeway system where travelers travel from one freeway station to another freeway station with shared MVs instead of driving their cars. For example, each traveler chooses a urban transportation service (e.g., bus, metro, BRT, taxi, shared bike, or MV) from their origin (e.g., home or office) to the nearest shared MTNS freeway service station. An MV then transports the passenger to the MTNS freeway service station nearest to the passenger destination. Finally, from this service station, the passenger transfers to another urban transportation service to reach their destination. This study assumes that local transportation decisions are exogenous to MTNS decisions, and thus, local transportation costs are not considered. The advantages of this proposed new system are associated with pooling riders in MVs with high occupancy (as opposed to the low occupancy in the PCS) and the flexible capacity (as opposed to the fixed capacity in the FSBS).

The input data include 295876 records of vehicles passing through 19 key toll stations in Guangdong, China (see Figure 6(a)), from 10:00-11:00 in May 2019. With an estimated average occupancy of 1.5 passengers per vehicle (Chow et al., 2010; Johnston and Ceerla, 1996; Siuhi and Mussa, 2007), we obtain the passenger OD demands as shown in Figure 6(b). We assume 3% of the passengers use the MTNS service by default. Further, we set $M = 20$, $\tau =[0.1: 0.025: 0.45, 0.5, 1, 500, 1000]$.



(a) 19 key toll stations in Guangdong Province, China

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0	7355	4418	886	1487	1754	1406	2541	864	245	137	68	79	674	86	61	103	19	352
2	6825	0	855	1821	2190	395	208	181	2597	534	162	61	84	403	14	11	14	5	35
3	4490	933	0	270	632	6277	1930	258	171	100	41	36	31	209	59	25	35	11	244
4	576	1808	197	0	1409	117	40	37	117	108	204	53	61	634	4	1	3	2	3
5	1392	1786	571	1423	0	313	89	13	65	83	75	28	39	1261	6	7	11	4	20
6	1850	437	6839	141	442	0	3317	82	85	52	19	33	13	199	191	46	87	14	284
7	1265	145	1595	42	66	2643	0	51	44	26	4	5	6	33	234	25	76	24	484
8	2047	159	241	18	10	71	53	0	107	14	7	2	2	14	3	1	0	2	10
9	674	2192	148	97	51	42	42	88	0	223	13	3	3	39	4	3	5	3	14
10	213	470	125	103	73	40	15	13	269	0	130	10	16	35	2	1	2	6	6
11	105	142	42	244	66	33	7	9	5	133	0	154	86	41	0	0	0	1	0
12	71	112	50	80	39	25	12	2	6	38	190	0	421	14	0	0	1	1	4
13	117	135	43	138	54	35	9	4	6	20	121	410	0	32	0	1	2	0	1
14	471	260	199	644	1169	176	35	10	52	31	61	21	17	0	11	1	5	2	8
15	107	4	50	3	8	222	253	5	3	2	1	1	7	4	0	36	149	20	6
16	60	11	26	1	5	59	42	3	4	2	0	1	0	6	49	0	1094	739	19
17	114	13	48	1	9	93	76	4	5	2	1	1	3	9	137	892	0	303	38
18	30	8	13	3	1	16	18	0	3	0	0	0	0	1	14	595	260	0	8
19	338	38	185	9	15	216	448	18	18	13	1	1	2	4	2	3	20	9	0

(b) OD demand data

Figure 6 The toll station information and OD demand data in Guangdong Province, China

In this case, the optimal results of the three systems are shown in Table 8. Compared with the FSBS, the MTNS performs well in reducing the system cost (by 4.62%). Again, this result is further improved (to by 15.98%) when we omit the free-flow travel time cost, which is a constant component in this system. If we decompose the total system cost into different components, we see significant improvements in the critical cost components. Specifically, the operation cost and waiting time cost are reduced by 2.29% and

9.76%, respectively. This reduction indicates the advantages of the proposed MTNS over the traditional FSBS. While the riding time cost does not change dramatically, it is dominated by the travel distance (range of approximately 100 km to 800 km in this case) and thus is not much affected by the transportation system. If we remove the free-flow travel time cost, we see a much more significant improvement in the variable riding time cost (by 111.67%). Moreover, the passenger transfer cost is also optimized in the MTNS. Additionally, compared with the PCS, the MTNS yields dramatic savings in the operation cost and system cost, but the time costs slightly increase due to added waiting and transfers. Finally, the MTNS model produces a lower bound $F^*_{\text{LM}} = 23{,}082.86$ and an upper bound $F'_{\text{NLM}} = 23{,}351.55$, which yields an optimality gap of 1.15% with a computational time of 0.3 h.

Table 8 Cost comparison of different operation systems (case 2)

	MTNS	FSBS		PCS	
	Value	Value	% reduction	Value	% reduction
• System cost	$23,082.86	$24,148.43	4.62%	$49,026.99	112.40%
Revised system cost*	$6,667.71	$7,733.29	15.98%	$32,611.84	389.10%
• Operation cost	$6,042.37	$6,632.29	9.76%	$32,611.84	439.72%
• Waiting time cost	$263.56	$269.59	2.29%	-	-
• Riding time cost	$16,748.82	$17,121.43	2.22%	$16,415.14	-1.99%
Revised riding time cost*	$333.68	$706.29	111.67%	-	-
• Transfer cost	$28.10	$125.13	345.27%	-	-

Note: The revised system cost and revised riding time cost are calculated by removing the fixed free-flow travel time (equal to the riding time cost of the PCS) that is independent of the optimal decisions.

5. Conclusion

By taking advantage of emerging MV technology, this paper proposes an approach to determine the optimal MTNS design (i.e., the allocation and scheduling of MV fleets over a general transportation network) to minimize the operation cost and passenger trip time cost. We formulate this problem into an MINLP model that captures detailed traveler waiting time costs with nonlinear vehicle scheduling functions. To facilitate the solution approach, we mathematically revise the MINLP model to produce a computationally tractable mixed-integer linear programming (MILP) model. This linear model solves both lower and upper bounds to the original nonlinear model, thus yielding a near-optimal solution with an optimality gap. This revised MILP model can be solved by using off-the-shelf commercial solvers (e.g., Gurobi) to obtain the exact solution. We explore two numerical examples to illustrate the applications of this model and compare it with alternative systems (i.e., the FSBS and PCS). The MTNS is shown to be more effective than the alternatives in both suburban (reduced by 7.10%, 33.63%, and 6.57% in the system cost, operation cost, and waiting time cost, respectively, compared with those in the FSBS; and by 290.86% and 28.96% in the operation cost and system cost, respectively, compared with those in the PCS) and freeway settings (reduced by 4.62% and 9.76% in the system cost and operation cost compared to those in the FSBS; and by 439.72% and 112.40% in the system cost and operation cost compared with those of the PCS, respectively). To further explore the robustness of the proposed model with different input parameters, a sensitivity analysis shows how the MTNS performance is affected by crucial parameter values and approximation gaps.

Since MV transit network system design is a novel research topic, the proposed model provides a foundation that may be extended in several directions. The proposed model is formulated as a mixed-integer linear programming problem and is solved with a commercial solver (i.e., Gurobi) in this study. Future work may focus on designing customized algorithms to further improve solution efficiency. Additional research is needed to explore dynamic and stochastic demands, en route link transfers, and associated vehicle coordination when operating a mixed fleet on the link. Moreover, the proposed model can be extended to consider the intercedence between system design decisions and traffic congestion patterns, and heterogeneous passenger behaviors (e.g., preferences regarding time windows, the service level, the willingness to pay, and the MV type). Moreover, it may be interesting to examine the impact of combinations of autonomous and electric MVs in future transportation modes.

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