Variable-Capacity Operations with Modular Transits for Shared-Use Corridors

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Word Count: 5,347 words + 6 tables = 6,847 words

Submitted [07/30/2019]

ABSTRACT

Since passenger demand in urban transit systems is asymmetrically distributed across different periods in a day and different geographic locations across the cities, the tradeoff between vehicle operating cost and service quality has been a persistent problem in transit operational design. The emerging modular vehicle technology offers us a new perspective to solve this problem. Based on this concept, we propose a variable-capacity operation approach with modular transits for shared-use corridors, in which both dispatch headway and vehicle capacity are decision variables. This problem is rigorously formulated as a mixed integer linear programming model that aims to minimize the overall system cost, including passenger waiting time cost and vehicle operating cost. Because the proposed model is linear, the state-of-the-art commercial solvers (e.g. Gurobi) can be used to obtain the optimal solution of the investigated problem. With numerical experiments, we demonstrate the feasibility of the mathematical model, verify the effectiveness of the proposed model in reducing overall system cost in transit systems, as well as the robustness of the proposed model with different parameter settings.

Keywords: Transit, Modular Vehicle, Variable-Capacity Operation, Shared-Use Corridors

INTRODUCTION

A characteristic in many urban transit (UT) networks is that several routes share an overlapping section, also known as shared-use corridors. Because of the limited resources and safety considerations in UT systems, transit vehicles are dispatched at discrete time points separated with a minimum dispatch headway while passengers arrive at each station continuously. As a result, vehicles are usually overcrowded in shared-use corridors (with high demand) during peak hours and relatively empty in non-overlapping segments (with low demand) during off-peak hours. Furthermore, it is almost inevitable for passengers to wait at transfer stations for boarding another vehicle due to the synchronization issue between different routes. These issues result in a substantial increase in the vehicle operational cost (e.g. energy cost) and passenger waiting cost in many UT systems (*1*).

This paper is interested in investigating an innovative solution to the aforementioned issues with the emerging modular vehicle (MV) technology. MVs (see Figure 1), currently being developed and tested by several for-profit organizations such as NEXT (*2*), Ohmio LIFT (*3*), feature a flexible adjustment of vehicle capacity through dynamically assembling multiple MVs into one or splitting one into multiple. With the MV technology, vehicles that are operating separately on different routes can be concatenated together into one longer vehicle on the shared-use corridor. When reaching a transfer station, the combined vehicle can be detached into multiple shorter ones, each of which heads to the destination of a route in the non-overlapping segment. This seemingly simple operation paradigm can introduce substantial benefits for UT systems with overlapping routes. Specifically, the operational cost can be reduced due to its sub-additive nature (*4*) and a lower possibility of moving empty vehicle units in non-overlapping segments. Also, the passenger waiting cost may be reduced because of the larger transportation capacity in the shared-use corridor and possibly more frequent dispatches in non-overlapping segments. Further, the MV technology also allows en-route transfer in the future. Specifically, passengers heading to different destinations can be assigned to modular units that will ultimately take them to their final destinations. This en-route transfer operation could release passengers from the additional waiting time and inconvenience caused by transferring.

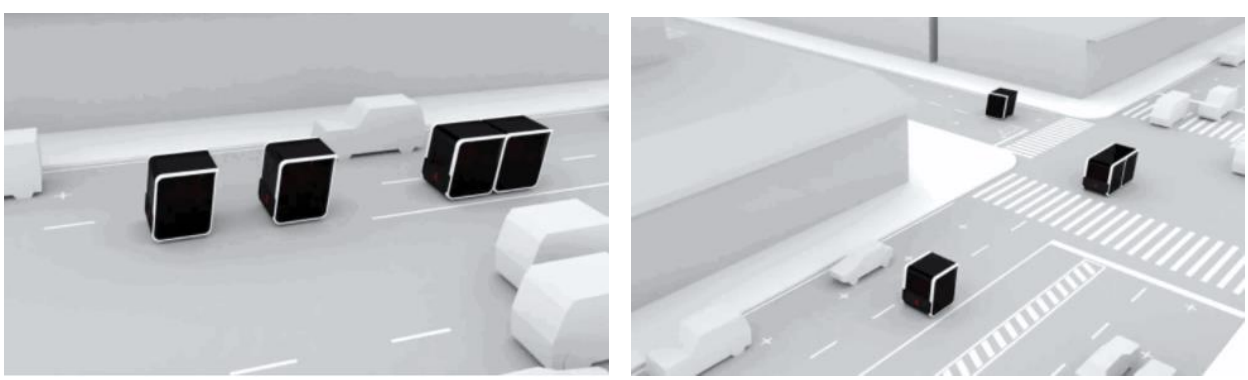
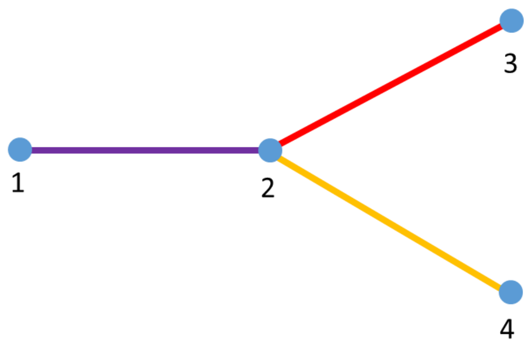


Figure 1 MV concept proposed by NEXT (*2*). Vehicles in black represent MVs while others are conventional vehicles with a fixed capacity. The number of modular units in a MV is adjusted according to the number of passengers onboard. MVs with more passengers contain more units (e.g., 2 in the figure) while those with fewer passengers consist of fewer units (e.g., 1 in the figure).

To better understand the potential benefits of MV-based operation paradigm, let us consider a simple illustrative example as shown in Figure 2. There are two routes in the network, with one of them traveling through stations 1, 2, 3 named by route 1 and the other stations 1, 2, 4 named by route 2. The distance between each two linked stations is 1. Let the dispatch cost of a vehicle with l modular units be $10l^{0.7}$ and the unit-time passenger waiting cost be 2. Note that here the dispatch cost of a vehicle is assumed to be a concave function over the number of modular units in it to reflect the economics of scale in urban mass transportation; this assumption has been applied in various studies (*4*, *5*). Suppose that in the exiting practice, only vehicles with six units can be dispatched. Then the optimal solution is to dispatch vehicles with 6 units on both route 1 and 2 at time 2, resulting in a passenger waiting cost of 14 and a vehicle dispatch cost of 140. Yet, if the proposed operation paradigm is introduced, a solution with two dispatches can be found. At time 1, dispatch a vehicle with 6 units to travel from stations 1 to 2 and then it splits into two vehicles with 3 units running from stations 2 to 3 and from stations 2 to 4, respectively. At time 2, dispatch a vehicle with 3 units to travel from stations 1 to 2 and then it splits into a vehicle with 1 unit running from stations 2 to 3 and another vehicle with 2 units from stations 2 to 4. This solution produces a passenger waiting cost of 0 and a vehicle dispatch cost of 126. Therefore, the proposed MV-based operation paradigm can reduce both the passenger waiting cost and vehicle dispatch cost in this case.



OD	Demand at time 1	Demand at time 2
(1,2)	1	1
(1,3)	3	1
(1,4)	2	1
(2,4)	1	1

Figure 2 An illustrative example

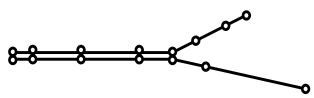
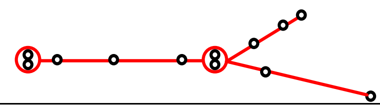
In light of these potential benefits, this paper focuses on designing operational plans for UT systems with the variable capacity design and shared-use corridor such that the total cost in the systems can be minimized. The operational decisions in the investigated problem include the time and number of modular units for each dispatch. The contributions of this paper are twofold. First, we propose an innovative MV-based operation paradigm for UT systems with shared-use corridor and formulate the operational decision problem as a rigorous mathematical programming model. The model is linear in its nature and therefore can be solved to optimality with state-of-the-art commercial solvers, e.g. Gurobi. Second, we conduct a case study based on real-world passenger demand data collected from Pinellas Suncoast Transit Authority (PSTA), the public transit provider for Pinellas County, FL. Results demonstrate the feasibility of the mathematical model, verify the advantage of the MV-based operation paradigm, and shed other interesting managerial insights. Overall, this paper provides for UT operators valuable insights on future integration of MVs into conventional transit services and offers a numerical method for designing optimal operational plans for this integrated system.

The remainder of this paper is organized as follows. Section 2 provides a brief review of related studies. Section 3 formally introduces the investigated problem. In Section 4, a mixed integer linear programming formulation of the investigated problem is presented, which allows the solution approach of commercial solvers. Section 5 presents results from the case study. Finally, Section 6 briefly summarizes the paper and discusses future research directions.

LITERATURE REVIEW

Shared-use corridor is a common feature in many UT networks in areas with high passenger demand. In the operation, because the dispatched vehicle will share not only the corridors but also the dispatch headway, the crowded situation will inevitably happen even when vehicles are dispatched extremely frequently (*6*). To solve this problem, a plenty of research has been conducted based on adjusting timetable and dispatch plan (*7–15*). Though excellent works have been done in these studies, the improvement of the proposed solutions still are limited. One possible reason is the current operation mode restricts the further development of the shared-use corridor systems. However, to the best of the research team's knowledge, few of existing studies have considered the innovative operation mode proposed in this study. MV operation is an innovative concept that is rapidly developed in recent years. The outstanding performance of this technology has already been approved by several research in UT systems (*1*, *16*). Following these studies, we aims to propose a new operation mode that can further extend this emerging technology to the UT system with overlapping routes. TABLE 1 compares the model for the proposed operation mode with the recent related studies in terms of characteristics and solution approaches.

TABLE 1 Comparison between existing models and our model

	Existing models	Existing MV models	Our model
Structure			
Objective function	Transfer cost, operating cost, passenger waiting time	Operating cost, passenger waiting time	Operating cost, passenger waiting time
Decision Variable	Timetable, dwelling time and speed profile	Timetable and vehicle types	Timetable, vehicle types and vehicle concatenated and detached operation process
Model	Linear, nonlinear and simulation-based	Linear and simulation-based	Linear
Vehicle type	Fixed capacity vehicle	MV	MV
Overlapping route operation	Considered	No	Considered
Solution approaches	Optimization, heuristic algorithms and simulation techniques as state in TABLE 2.	Optimization, simulation techniques	Optimization
Publication	(*5–14*)	Chen et al.(*1*); Guo et al. (*16*)	-

Existing studies have revealed the difficulty in handling the real-world transit scheduling problem due to its large problem scale and complicated formulation structure (*15*). Three types of approach have been proposed to tackle this problem, including optimization method, heuristic algorithms, and simulation techniques. TABLE 2 summarizes these three approaches in the recent literature. It can be observed that optimization methods and heuristics algorithms are the most widely adopted solution methods for the transit scheduling problem (*7*, *12*, *17–20*). Optimization methods aim to obtain the exact optimal or near optimal solutions while the heuristic algorithms usually will be stuck in local optimum. Thus, if the computation resources are available, the priority should be given to optimization methods. As a result, comprehensively taking into account several kinds of factors (e.g. resources occupancy, performance and computation time), this study focuses on developing an optimization method for the investigated problem.

TABLE 2 Summary of previous studies on timetable algorithms

Solution methods	Classifications	References

Optimization methods	Existing commercial solvers	CPLEX	Sun et al., 2014 (*21*)
		GAMS	Niu et al., 2015 (*7*); Yang et al., 2016(*17*)
		Gurobi	Chen et al., 2019 (*1*)
	Customized algorithms	Branch and bound	Albrecht, 2009 (*18*)
		Branch and cut	Barrena et al., 2014 (*12*)
		Branch and price	Lin and Kwan, 2016 (*19*)
		Lagrangian decomposition	Zhou and Teng, 2016
Heuristics algorithms	Tabu search		Paquette et al., 2013(*22*); Kirchler and Wolfler Calvo, 2013(*23*); Detti et al., 2017(*24*)
	Simulated annealing		Reinhardt et al., 2013(*25*); Braekers et al. 2014(*26*)
	Neighborhood search		Braekers and Kovacs, 2016(*27*); Masmoudi et al., 2016(*28*); Detti et al., 2017(*24*); Molenbruch et al., 2017(*29*)
	Genetic algorithms		Núñez et al., 2014(*30*); Muñoz-Carpintero et al., 2015(*31*)
	Hybrid algorithms		Molenbruch et al., 2017(*29*); Pimenta et al., 2017(*32*); Lim et al., 2017(*33*); Schönberger, 2017(*34*)
Simulation techniques			Adamski and Turnau (1988) (*35*); Yang et al. (2016) (*36*)

PROBLEM DESCRIPTION

For the convenience of the readers, the key notation used throughout the paper is summarized in TABLE 3.

TABLE 3 Notation

Parameters	
$\mathcal{I}_1$	Set of main route stations from station 1 to n. $\mathcal{I}_1 = \{1,2,\cdots,n\}$
$\mathcal{I}_2$	Set of branch route 1 stations. $\mathcal{I}_2 = \{n+1, n+2, \cdots, n+2p\}$
$\mathcal{I}_3$	Set of branch route 2 stations. $\mathcal{I}_3 = \{n+2p+1, n+2p+2, \cdots, n+2p+2q\}$
$\mathcal{I}_4$	Set of main route stations from direction n to 1. $\mathcal{I}_4 = \{n+2p+2q+1, \cdots, 2n+2p+2q\}$
$\mathcal{I}$	Set of stations. $\mathcal{I} = \{\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_4\} = \{1,2,\cdots,2n+2p+2q\}$
$\mathcal{I}_i^+$	Set of stations behind of station i. $\mathcal{I}_i^+ = \{i+1, i+2, \cdots, I\}$ $i \in \mathcal{I} \backslash \{I\}$
$\mathcal{I}_i^-$	Set of stations in front of station i. $\mathcal{I}_i^- = \{1,2,\cdots,i-1\}$ $i \in \mathcal{I} \backslash \{1\}$
$\mathcal{T}$	Set of time intervals. $\mathcal{T} = \{1,2,\cdots,T\}$
$\mathcal{L}$	Set of units of vehicle. $\mathcal{L} = \{1,2,\cdots,L\}$
δ	Length of one time interval.
$p_{ijt'}$	Number of passengers arriving at station i at time interval $[t'-1, t']$ destined to station j. $\forall i \in \mathcal{I}, j \in \mathcal{I}_i^+, t' \in \mathcal{T}$
C	Capacity of one single unit.
H	Minimum headway.
e_l	General dispatch cost of a vehicle with l modular unites. $l \in \mathcal{L}$
w	Coefficient to convert the waiting time to waiting cost.
Decision variables	
y_{lt}	=1, if a vehicle dispatched at time t with l units. =0, otherwise. $l \in \mathcal{L}, t \in \mathcal{T}$
r_{lt}	=1, if a vehicle dispatched at time t goes to branch route 1 with l units. =0, otherwise. $l \in \mathcal{L}, t \in \mathcal{T}$
o_{lt}	=1, if a vehicle dispatched at time t goes to branch route 2 with l units. =0, otherwise. $l \in \mathcal{L}, t \in \mathcal{T}$

u_{lt}	=1, if a vehicle dispatched at time t leaves station $n+2p+2q+1$ with l units. =0, otherwise. $l \in \mathcal{L}, t \in \mathcal{T}$
c_{it}	Capacity for vehicle that dispatches at time t arrives at station i
$b_{ijt't}$	Boarding passengers at station i destined to station j arrived at time t' board vehicle dispatched at time t
$z_{ijt't}$	Waiting passengers at station i destined to station j arrived at time t' when vehicle dispatched at time t coming.

We consider a UT system consisting of two bi-directional routes with an overlapping segment and two non-overlapping segments, as shown in Figure 3. We name the overlapping segment as the main route and the non-overlapping segments branch route 1 and branch route 2, respectively. The number of stations on the main route, branch route 1 and branch route 2 are n, p, and q, respectively. Note that because this paper considers both directions of each route, we assume that there are two stations corresponding to opposite directions at the same physical station. Therefore, the set of stations of the main routes from direction 1 to n and that from direction n to 1 are denoted as $\mathcal{I}_1 \coloneqq [1,2,\cdots,n]$ and $\mathcal{I}_4 \coloneqq [n+2p+2q+1,\cdots,2n+2p+2q]$, respectively. Likewise, the set of stations on branch route 1 is denoted as $\mathcal{I}_2 \coloneqq \{n+1,n+2,\cdots,n+p\} \cup \{n+p+1,\cdots,n+2p\}$ and the set of stations on branch route 2 is denoted as $\mathcal{I}_3 \coloneqq \{n+2p+1,n+2p+2,\cdots,n+2p+q\} \cup \{n+2p+q+1,\cdots,2n+2p+2q\}$. Thus, the set of stations in the UT systems can be defined as $\mathcal{I} \coloneqq \mathcal{I}_1 \cup \mathcal{I}_2 \cup \mathcal{I}_3 \cup \mathcal{I}_4$, indexed as $i,j \in \mathcal{I}$. Further, we denote the set of downstream and upstream stations of station i as $\mathcal{I}_i^+ \coloneqq \{i+1,i+2,\cdots,I\}, \forall i \in \mathcal{I}\backslash\{I\}$ and $\mathcal{I}_i^- \coloneqq \{1,2,\cdots,i-1\}, \forall i \in \mathcal{I}\backslash\{1\}$, respectively, where $I := 2n+2p+2q$ is the number of stations in the UT system.

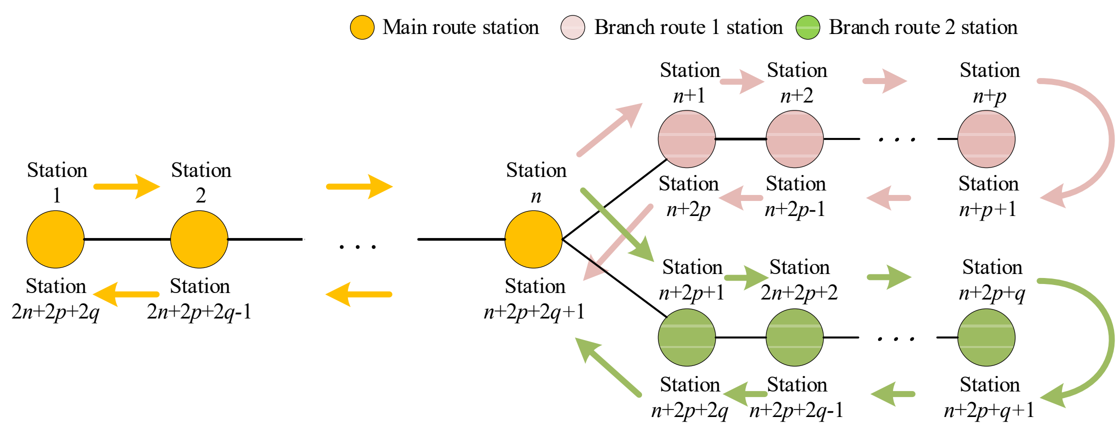


Figure 3 The investigated UT system

The operational horizon is divided into T intervals with an equal length of δ by a set of discrete time points $\mathcal{T} \coloneqq \{1,2,\cdots,T\}$. During the operational horizon, passengers arrive at each station continuously and we denote the number of passengers arriving at station i at time interval $[t'-1,t']$ destined to station j as $p_{ijt'}, \forall i \in \mathcal{I}, j \in \mathcal{I}_i^+, t' \in \mathcal{T}$. For the convenience of the notation, define $p_{ij0} = 0, \forall i \in \mathcal{I}, j \in \mathcal{I}_i^+$. To serve these passengers, vehicles will be dispatched to serve the passengers with a minimum headway H. The number of modular units in each vehicle can be selected from a set of available units $\mathcal{L} \coloneqq [1,2,\cdots L]$, indexed as $l \in \mathcal{L}$, where L is the maximum number of units in a vehicle. The capacity of a vehicle with l modular units are lC, with C being the capacity of a single modular unit. In the proposed MV-based operation paradigm, each dispatched vehicle starts its journey at station $1 \in \mathcal{I}$ and detaches into two vehicles at station $n \in \mathcal{I}$. Afterwards, one of the detached vehicles commences a round trip starting from station $n \in \mathcal{I}$ and visits all stations in branch route 1 (i.e. stations in $\mathcal{I}_2$) sequentially. Likewise, the other vehicle commences a round trip starting from station $n \in \mathcal{I}$ and visits all stations in branch route 2 (i.e. stations in $\mathcal{I}_3$) sequentially. When these two detached vehicles come back to station n, they will be concatenated together as one vehicle and then heads back to station $1 \in \mathcal{I}$.

The purpose of this paper is to find an optimal operation plan (including the time and number of modular units for each dispatch) for the investigated UT system such that the system cost can be minimized. Following previous studies (e.g. (*1*, *37*)), this study considers two cost components for the investigated UT systems. First, we consider the general cost spent on dispatching vehicles, which can include energy cost, driver cost, crew salary, and so on. We denote the general dispatch cost of a vehicle with l modular units as $e_l, \forall l \in \mathcal{L}$. The other cost component we consider is the passenger waiting cost, a measure commonly adopted to evaluate the service quality of UT systems. To this end, we introduce a coefficient w to convert the waiting time to waiting cost.

Finally, to facilitate the model formulation, we introduce the following assumptions in the investigated problem. These assumptions have been used in other studies on operational design for UT systems.

Assumption 1. First, we assume that oversaturated situation is not permitted at each station $i \in \mathcal{I}$. That is, all passengers waiting at a station can board the first vehicle after the arrival (*7*, *8*). Interested readers can refer to (*38*, *39*) for transit operation research under oversaturated situation, which will shed insights into adapting the proposed model to address the case where oversaturated traffic is present. Additionally, there are also plenty of routes with unsaturated passenger demand at each station in the real world, at least during the off-peak hours if not the entire day. Hence, the unsaturated study proposed here remains important.

Assumption 2. Further, we assume a constant dwelling time at stations and running time between two consecutive stations for all dispatches (*21*).

Assumption 3. Finally, we assume that the vehicle stock is always sufficient at station 1 and station $n \in \mathcal{I}$. This way, there are always vehicles with any feasible numbers of modular units available for dispatch (*1*, *5*, *9*). While the fleet planning problem is relevant, but it belongs to the planning stage and can be separated from the operational problem. Thus, we do not pose a fleet size constraint on the system operation. Note that after solving the optimization model, the optimal fleet size, i.e., the number of modular units, can be determined.

MATHEMATICAL MODEL

To mathematically formulate the investigated problem, we consider three groups of constraints in the system, i.e., constraints on vehicle operation, constraints on passenger behavior, and constraints on the feasible range of decision variables, as follows.

Original formulation

Constraints on vehicle operation

Different from conventional transit operations, vehicles concatenation and detachment may happen at station 1 and n in the proposed UT system. To formulate this operation process in this system, we first introduce the following decision variables:

y_{lt}: Equals 1 if a vehicle with l units is dispatched at time t at station 1; otherwise 0.

r_{lt}: Equals 1 if a vehicle dispatched at time t goes to branch route 1 with l units; otherwise 0.

o_{lt}: Equals 1 if a vehicle dispatched at time t goes to branch route 2 with l units; otherwise 0.

u_{lt}: Equals 1 if a vehicle dispatched at time t will travel back to station 1 with l units; otherwise 0.

With these, we formulate the vehicle operation constraints as follows.

$$\sum_{l \in \mathcal{L}} \sum_{t=t'}^{t'+H} y_{lt} \leq 1 \ \ \forall t' \in \{1,2,\cdots,T-H\} \tag{1}$$

$$\sum_{l\in\mathcal{L}} lr_{lt} + \sum_{l\in\mathcal{L}} lo_{lt} = \sum_{l\in\mathcal{L}} ly_{lt} \quad \forall t \in \mathcal{T} \tag{2}$$

$$c_{(2n+2p+2q+1)t} = \sum_{l\in\mathcal{L}} lu_{lt}C \quad \forall t \in \mathcal{T} \tag{3}$$

$$\sum_{l\in\mathcal{L}} u_{lt} \leq 1 \quad \forall t \in \mathcal{T} \tag{4}$$

$$\sum_{l\in\mathcal{L}} r_{lt} \leq 1 \quad \forall t \in \mathcal{T} \tag{5}$$

$$\sum_{l\in\mathcal{L}} o_{lt} \leq 1 \quad \forall t \in \mathcal{T} \tag{6}$$

Due to the limited units in stock and safety considerations in UT system, Constraint (1) suggests that the vehicle dispatch headway between two consecutive vehicles cannot be less than the minimum dispatch headway (i.e. H).Constraint (2) and (3) are related to the vehicle concatenation and detachment operation. Constraint (2) is the conservation requirement on modular units; i.e., the sum of modular units assigned to each branch route must equal that in the vehicle running on the main route. Since the cycle time for transit system usually is not short, to improve the robustness of the proposed UT system, Constraint (3) allows the vehicle to adjust its capacity at station $n + 2p + 2q + 1$ after the vehicles from branch route concatenate with each other. Constraint (4) - (6) ensures that only one vehicle can be dispatched at an arbitrary time interval.

Constraints on passenger behavior

This set of constraints considers the passenger behavior in the proposed UT system. To formulate this passenger behavior in this system, we introduce the following decision variables.

$b_{ijt't}$: Number of passengers arriving during interval $[t'-1, t']$ at station i destined to station j that board the vehicle dispatched at time t.

$z_{ijt't}$: Number of passengers arriving during interval $[t'-1, t']$ at station i destined to station j that are waiting when vehicle dispatched at time t is arriving at station i.

c_{it}: Capacity of vehicle dispatched at time t arrives at station i.

With these three variables, we formulate the passenger behavior as follows.

$$b_{ijt't} = \sum_{l\in\mathcal{L}} z_{ijt't} y_{lt} \quad \forall i \in \mathcal{I}, j > i \in \mathcal{I}, t' \leq t \in \mathcal{T}, t \in \mathcal{T} \tag{7}$$

$$z_{ijt't} = p_{ijt'} - \sum_{t''=1}^{t-1} b_{ijt't''} \quad \forall i \in \mathcal{I}, j > i \in \mathcal{I}, t' \leq t \in \mathcal{T}, t \in \mathcal{T} \tag{8}$$

$$\sum_{t \geq t' \in \mathcal{T}} b_{ijt't} = p_{ijt'} \quad \forall i \in \mathcal{I}, j > i \in \mathcal{I}, t' \in \mathcal{T} \tag{9}$$

$$c_{(i+1)t} = c_{it} - \sum_{j\in\mathcal{I}_i^+, t' \leq t\in\mathcal{T}, t\in\mathcal{T}} b_{ijt't} + \sum_{j\in\mathcal{I}_i^-, t' \leq t\in\mathcal{T}, t\in\mathcal{T}} b_{jit't} \quad \forall i \in \mathcal{I}\backslash\{1, n, n+2p, n+2p+2q\} \tag{10}$$

$$c_{1t} = \sum_{l\in\mathcal{L}} ly_{lt}C \quad \forall t \in \mathcal{T} \tag{11}$$

$$c_{(n+1)t} = \sum_{l\in\mathcal{L}} lr_{lt}C - \sum_{i\in\mathcal{I}_1, j\in\mathcal{I}_2, t' \leq t\in\mathcal{T}} b_{ijt't} \quad \forall t \in \mathcal{T} \tag{12}$$

$$c_{(n+2p+1)t} = \sum_{l\in\mathcal{L}} lo_{lt}C - \sum_{i\in\mathcal{I}_1, j\in\mathcal{I}_3, t' \leq t\in\mathcal{T}} b_{ijt't} \quad \forall t \in \mathcal{T} \tag{13}$$

$$c_{(n+2p+2q+1)t} = \sum_{l \in \mathcal{L}} l u_{lt} C - \sum_{i \in \mathcal{I}_2, j \in \mathcal{I}_4, t' \leq t \in \mathcal{T}} b_{ijt't} - \sum_{i \in \mathcal{I}_3, j \in \mathcal{I}_4, t' \leq t \in \mathcal{T}} b_{ijt't} \quad \forall t \in \mathcal{T} \tag{14}$$

Since oversaturated situation is not permitted in this paper, Constraint (7) is imposed to ensure that all waiting passengers can board the first arriving vehicle. Constraint (8) presents the relationship between passenger demand (i.e. $p_{ijt'}$), waiting passengers (i.e. $z_{ijt't}$) and boarded passengers (i.e. $\sum_{t''=1}^{t-1} b_{ijt't''}$). Constraint (9) suggests that all passengers waiting at all stations must be served at the end of the operational horizon.

Constraints (10) - (14) are related to passenger boarding and alighting behavior. Constraint (10) indicates that the available capacity for a vehicle at station $i+1$ equals to the available capacity for this vehicle at station i minus the boarding passengers at station i and plus the alighting passengers at station $i+1$. Due to vehicle concatenation or detachment, this available capacity calculation method is not suitable for all the stations. Therefore, for these specific stations, station $1, n, n+2p, n+2p+2q$, we propose constraints (11) - (14) to calculate the available capacity.

Variable domains

The following constraints define feasible region of each decision variable.

$$y_{lt}, r_{lt}, o_{lt}, u_{lt} \in \mathbb{B} \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{15}$$

$$c_{it}, b_{ijt't}, z_{ijt't} \in \mathbb{N} \quad \forall i \in \mathcal{I}, j > i \in \mathcal{I}, t' \leq t \in \mathcal{T}, t \in \mathcal{T} \tag{16}$$

Constraint (15) is a set of domain constraints that are related to vehicle operation states. Since the left capacity of the vehicle, boarding passengers and waiting passengers should always be nonnegative integer number, here we propose constraint (16) to achieve it.

Objective function

$$\min_{y_{lt}, r_{lt}, o_{lt}, u_{lt}, c_{it}, b_{ijt't}, z_{ijt't}} w \sum_{i \in \mathcal{I}, j > i \in \mathcal{I}, t' \leq t \in \mathcal{T}, t \in \mathcal{T}} \delta b_{ijt't}(t - t') + \sum_{l \in \mathcal{L}, t \in \mathcal{T}} y_{lt} e_l \tag{17}$$

The objective function aims to minimize the total passenger waiting time and operation consumption cost.

Revised formulation

It can be seen, all the proposed constraints except for Constraint (7) and objective function are linear in the original formulation. The right-hand side in Constraint (7) is a bi-linear term involving the multiplication of two decision variables. To simplify the model formulation and thus enable the solution approach of existing commercial solvers for integer linear programs (e.g. Gurobi), here we linearize Constraint (7). Specifically, we replace Constraint (7) with the following Constraints (18) - (21).

$$b_{ijt't} \leq M \sum_{l \in \mathcal{L}} y_{lt} \quad \forall i \in \mathcal{I}, j > i \in \mathcal{I}, t' \leq t \in \mathcal{T}, t \in \mathcal{T} \tag{18}$$

$$b_{ijt't} \leq z_{ijt't} \quad \forall i \in \mathcal{I}, j > i \in \mathcal{I}, t' \leq t \in \mathcal{T}, t \in \mathcal{T} \tag{19}$$

$$b_{ijt't} \geq z_{ijt't} - M\left(1 - \sum_{l \in \mathcal{L}} y_{lt}\right) \quad \forall i \in \mathcal{I}, j > i \in \mathcal{I}, t' \leq t \in \mathcal{T}, t \in \mathcal{T} \tag{20}$$

$$b_{ijt't} \geq 0 \quad \forall i \in \mathcal{I}, j > i \in \mathcal{I}, t' \leq t \in \mathcal{T}, t \in \mathcal{T} \tag{21}$$

where M is a given large positive number.

Therefore, the investigated problem can be formulated as a mixed integer linear programming model as follows.

$$\min_{y_{lt}, r_{lt}, o_{lt}, u_{lt}, c_{it}, b_{ijt't}, z_{ijt't}} w \sum_{i \in \mathcal{I}, j > i \in \mathcal{I}, t' \leq t \in \mathcal{T}, t \in \mathcal{T}} \delta b_{ijt't}(t - t') + \sum_{l \in \mathcal{L}, t \in \mathcal{T}} y_{lt} e_l \tag{22}$$

$$\text{s.t. } (1) - (6), (8) - (14), (15) - (16), (18) - (21)$$

NUMERICAL EXAMPLES

This section presents a set of numerical experiments with real-world travel data to assess the feasibility of the proposed model and the effectiveness of the proposed MV operational paradigm. The data were collected from Pinellas Suncoast Transit Authority (PSTA), the public transit provider for Pinellas County, FL. In this section, we use the state-of-the-art solver, Gurobi, to solve the proposed model. The numerical tests were performed on a PC with Windows 7 platform, Intel(R) Core(TM) i7-2130 with 2.7 GHz CPU and 8.00 GB memory. The code was implemented in MATLAB 2017.

Experiment description and parameter settings

To test the effectiveness of the proposed variable-capacity operation approach, two routes from the PSTA system, Route 18 and Route CAT, are selected for the numerical experiments. Route 18 and route CAT are both bi-directional UT routes and they share an overlapping segment, i.e., the segment between station 1 and 2 in Figure 4 (a). Each of these two routes have hundreds of stations but in practice, PSTA operators only consider several stations with intensive demand when designing their timetable. Thus, following their practice, in this study we also use these stations for the operational design. Specifically, 8 and 4 stations are selected for Route 18 and CAT, respectively. The passenger demand data used in the experiments are obtained from the historic passenger count data from PSTA during weekdays, and the average passenger demand over all stations along the investigated routes during one weekday is exhibited in Figure 4 (b). It can be observed that the passenger demand distinctly fluctuates across different periods, which renders these two lines an ideal testbed for the proposed operation paradigm. In addition, default values of all other parameters in the numerical experiments are summarized in TABLE 4.

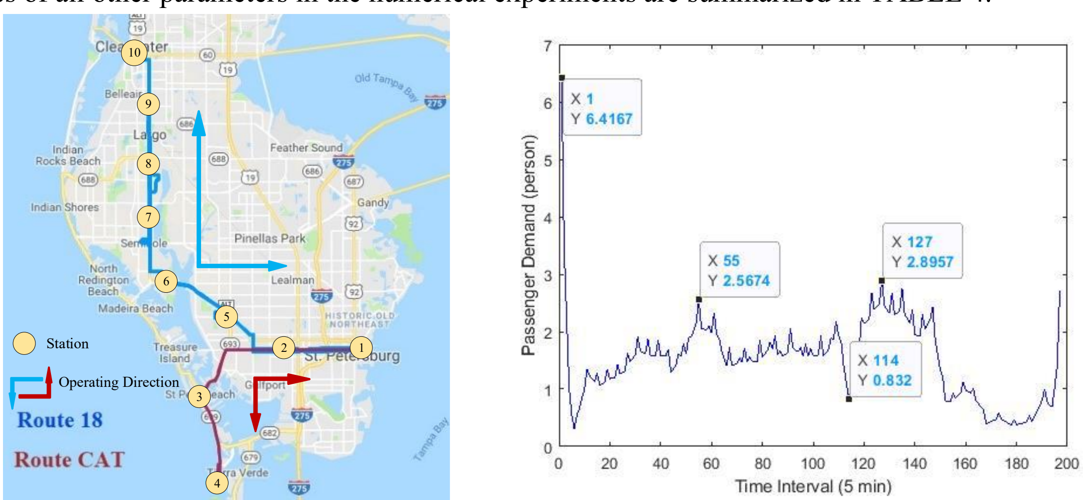


Figure 4 (a) Route 18 and CAT in PSTA (b) Time-varying passenger demand of Route 18 and Route CAT (X represents the time interval index, and Y represents the corresponding passenger demand)

TABLE 4 Default parameter settings

Parameter	Value	Note
$\mathcal{L}$	[1,2,3,4]	
C	40 pax/unit	Information from *Overview of Transit Vehicles* (*40*).
H	10 min	Minimum headway from PSTA existing operation schedule.
w	0.28 $/min	Based on average household income in Tampa in 2019 (https://www.deptofnumbers.com)
e_l	$12\ \$/mile * l * d$	d denotes the distance. For convenience, here we assume that the operating cost is a linear function of the number of dispatched units.
δ	5 min	

Computational results

With the aforementioned input data, we used Gurobi to solve the proposed model. After running for nearly 2 hrs, Gurobi produced an optimal operation schedule for the proposed operation mode as shown in Figure 5 (a)-(d). To evaluate the proposed operation schedule, here we set the real-world operation of these two routes as a benchmark, as shown in TABLE 5 (https://www.psta.net/). In this experiment, we use the number of served passengers, the number of dispatched units, passenger Average Waiting Time (AWT), passenger Waiting Time Cost (WTC), Vehicle Operating Cost (VOC) and Overall System Cost (OSC) as the criteria to evaluate the performance of the operation mode. The final results are shown in TABLE 6.

TABLE 5 Existing dispatch headway of Route 18 and Route CAT on weekdays

Route 18		Route CAT	
Period	**Dispatch Headway**	**Period**	**Dispatch Headway**
5:25-9:00	20 min	6:00-19:00	20 min
9:00-15:00	30 min	19:00-22:00	30 min
15:00-19:00	20 min		
19:00-22:00	60 min		

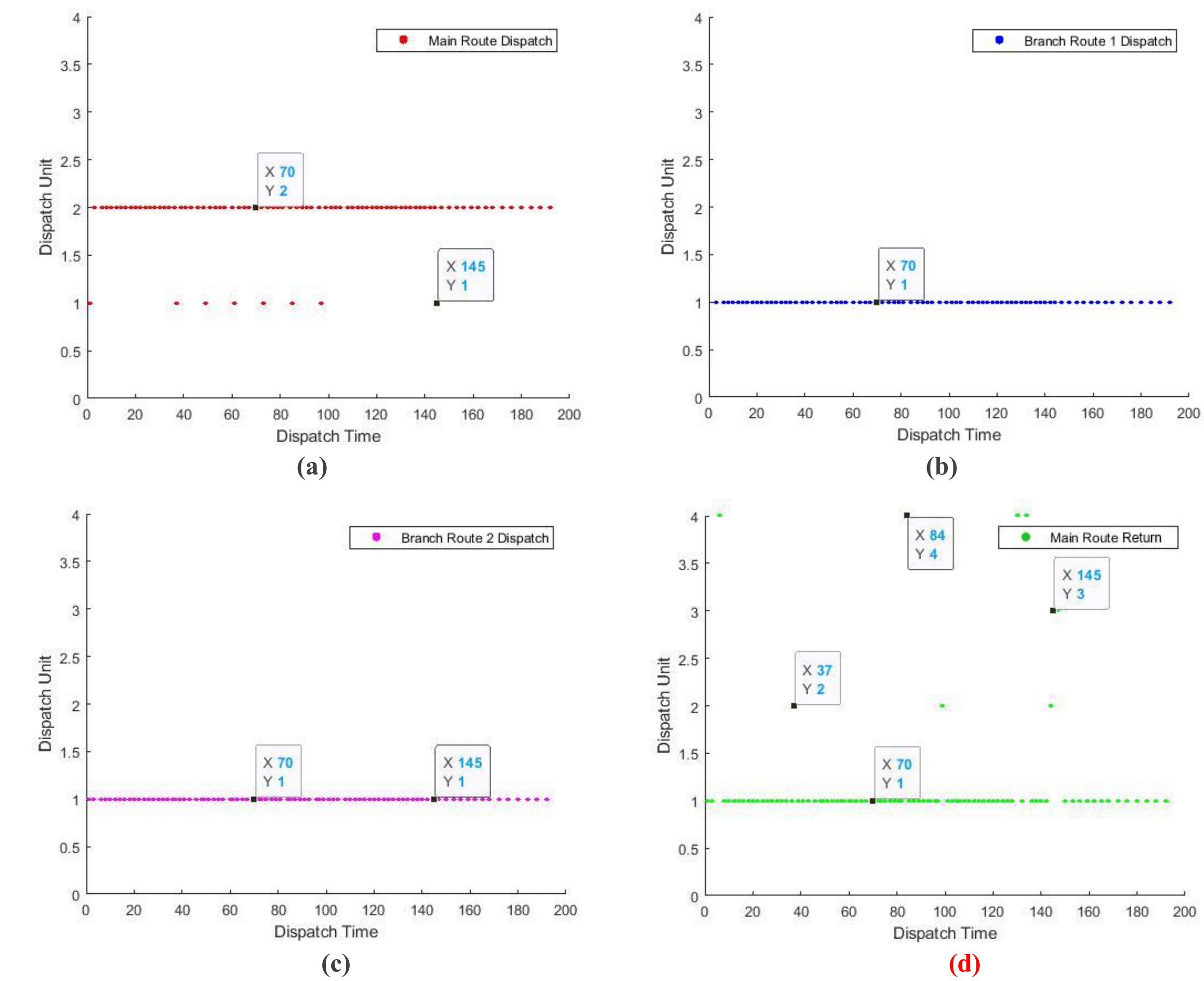


Figure 5 Optimal operation schedule for Route 18 and Route CAT

TABLE 6 Comparison between real-world operation and proposed operation mode

	Served Passengers	Dispatched Units	AWT (min)	WTC ($)	VOC ($)	OSC ($)

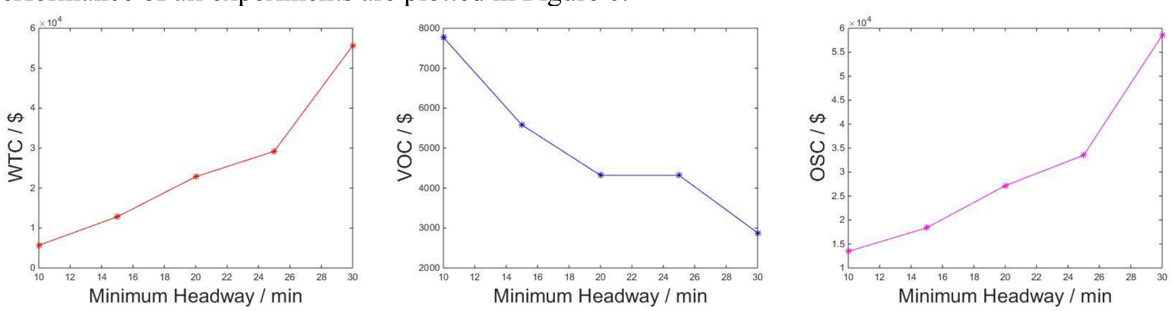
Route 18	3,193	32	7.66	6,930	2,304	9,235
Route CAT	3,434	45	7.42	7,220	1,620	8,841
Routes 18 and CAT	6,627	77	7.54	14,150	3,924	18,076
Proposed Mode	6,627	86	3.03	5,689	7,722	13,411

Figure 5 (a) - (d) present the operation schedule for Route 18 and Route CAT with the proposed operation mode. Since there are plenty of dispatches during the operational horizon, for illustrative purpose, we randomly select two dispatches, i.e. vehicles dispatched at time interval 70 and 145, as instances to explain the operation schedule. It can be observe in Figure 5 (a) that vehicle dispatched at time interval 70 consists of 2 units at Station 1. At Station 2, this vehicle will detach to two vehicles and one goes to Route 18 with 1 unit and the other goes to Route CAT with 1 unit (see Figure 5 (b) and (c)). After these two vehicles accomplish their own operation tasks in these branch routes, they return to Station 2 again. Based on the passenger demand on the main route in this specific scenario, the new formed vehicle is comprised of only one unit as shown in Figure 5 (d). For the vehicle dispatched at time interval 145, it leaves Station 1 with only 1 unit due to the low passenger demand, as can be seen from Figure 5 (a). At Station 2, this vehicle goes to Route CAT and no vehicle is sent to Route 18 due to sparse passenger demand on this route, as shown in Figure 5 (b) and (c). When this vehicle returns to Station 2, according to Figure 5 (d), 2 units are added to the vehicle to serve the high passenger demand on the main route. Therefore, a new formed vehicle with 3 units travels back to Station 1 through the main route. Moreover, since we allow the new formed vehicle to change its number of units at Station 2 according to the passenger demand on the main route, the proposed operation mode performs flexible capability adjustment at this station as well, which can be observed from the different kinds of dispatches as shown in Figure 5 (d).

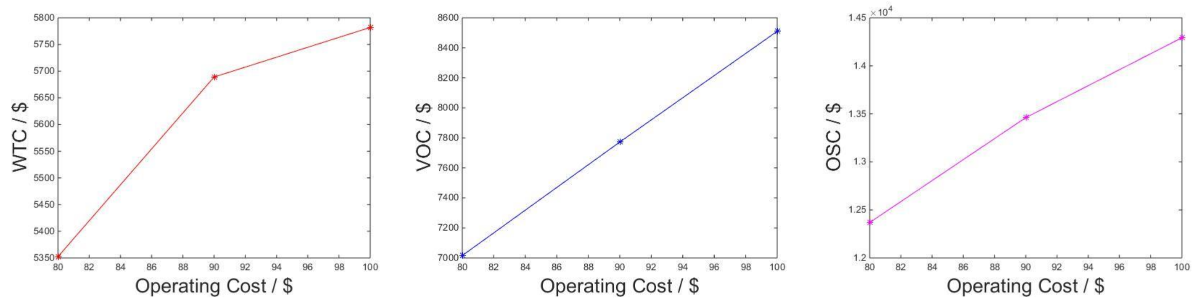
Then we move to the comparison between real-world operation and the proposed operation mode. As can be seen from TABLE 6, both the existing operation mode and proposed variable-capacity mode can successfully serve all the passenger demand. Note that for the existing mode, the values of the served passengers, dispatched units, WTC, VOC, and OSC are the summations of the corresponding values for Route 18 and Route CAT. The AWT for the existing mode is the average AWT for Route 18 and Route CAT. Compared with the existing operation, the proposed operation dispatches 11.68% more units, which naturally results in an increase of VOC. However, the more frequent dispatches with variable capacity produces a substantial decrease in AWT (by 59.81%) and accordingly WTC (by 59.80%). As a result, the proposed operation mode is able to reduce OSC in the investigated system by 25.81%. These results demonstrate the effectiveness of the proposed operation mode.

Sensitivity analysis

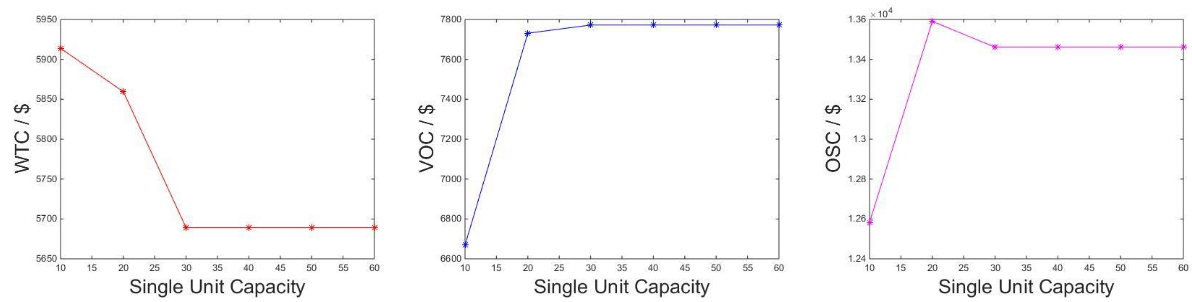
To further explore whether the proposed model can still achieve the expected performance in other transit systems when the system parameters may not be the same as the default values, this section carries out sensitivity analysis on several input parameters. In each experiment, only one operation-correlated parameter is varied and the other parameters keep the same as default value. To evaluate the performance of different experiments, again, here we use WTC, VOC and overall system cost as the criteria. The performance of all experiments are plotted in Figure 6.



(a) Performance of the criteria with varying minimum headway.



(b) Performance of the criteria with varying operating cost



(c) Performance of the criteria with varying single unit capacity

Figure 6 Sensitivity analysis of the criteria with different input parameters

Figure 6 (a) shows that as the minimum headway increases, both WTC and OSC will increase as well, but VOC will keep a decreasing trend. This is because the longer the minimum headway is, the lower the dispatch frequency will be, which intuitively will increase the WTC and OSC. On the other hand, the VOC will obviously be reduced because of the lower dispatch frequency. However, the VOC will be unchanged finally even the minimum headway keep increasing because that all passenger demands must be satisfied.

Figure 6 (b) indicates that as the operating cost per unit per mile increases, WTC, VOC and OSC all appear an increasing trend. The operating cost per unit per mile is positively correlated with VOC, so it is easy to understand why the curve of VOC and OSC goes upward as shown in Figure 6 (b). The reason for the increasing WTC cannot be identified without an analysis of the model structure. In the formulated model, the objective is to minimize the OSC, i.e., the summation of WTC and VOC. Once the operating cost per unit per mile increases, to minimize the OSC, our model will make a tradeoff between WTC and VOC. If VOC of dispatching a vehicle is higher than WTC, our model prefers to cancel this dispatch to achieve the minimum OSC. Thus, increasing the operating cost per unit per mile will make WTC increases.

Figure 6 (c) describes the trend of WTC, VOC and OSC as the single unit capacity increases. It can be observed that VOC and OSC experience an increasing trend while the WTC curve a decreasing trend at the beginning. Yet, when the single unit capacity is larger than 30, all three curves remain relatively stable regardless of the change in the single unit capacity. This trend is easy to understand since the passenger demand during a specific dispatch period is fixed, when the current dispatched vehicle already can accommodate all the waiting passengers, further increasing the capacity of the dispatched vehicle can affect neither WTC nor VOC. The decreasing trend of WTC and increasing trend of VOC at the beginning also can be explained by the tradeoff of our model.

In conclusion, the experiments show that the proposed model can effectively work with different input parameters, suggesting the relatively strong robustness of the proposed model.

CONCLUSIONS

This paper proposes a new solution to address the asymmetric demand in UT systems with the emerging MV technology. To solve the joint design problem of dispatch headway and vehicle capacity for UT systems with shared-use corridors, a new mathematical model is proposed. This model is essentially a mixed integer linear programming model that can be solved by state-of-the-art commercial solvers (e.g., Gurobi). Numerical experiments based on real-world data collected from Route 18 and Route CAT in the PSTA system are conducted to evaluate the effectiveness of the proposed model. Experiment results indicate that the proposed operation method can not only effectively reduce the overall system cost but also decrease the passenger average waiting time. To further explore the robustness of the proposed model with different input parameters, sensitivity analysis is performed, indicating that the proposed model presents relatively stable performance with different parameter settings.

Since the proposed model in this paper is a linear programming problem, we simply used a commercial solver for integer programming (i.e. Gurobi) to solve the investigated problem. Future works can focus on designing customized algorithms to further improve the solution efficiency. Besides, in the current model setting, one MV must wait for another to make a concatenation. In the future study, whether a MV should wait for another to concatenate can be included as a decision variable in the optimization model. Additionally, this study demonstrates potential benefits of the MV-based operation on a shared-use corridor. The emerging on-demand transit services (e.g., microtransits) may receive similar benefits should they incorporate a flexible capacity adjustment in response to the fluctuating passenger demand in their operational design. The proposed model can serve as a starting point for developing more sophisticated models to analyze the application of MVs in on-demand transit service, which will be an interesting future research direction. Finally, this paper especially focuses on studying the modular transit operation with predicted passenger demand, and stochastic passenger demand can be taken into consideration in the future studies.

ACKNOWLEDGMENTS

Comments and data provided by Mr. Labutka and Mrs. Borchers from Pinellas Suncoast Transit Authority and specialists from ITE annual meeting are gratefully appreciated.

AUTHOR CONTRIBUTIONS STATEMENT

The authors made the following contributions to the paper: study conception and design: Xiaowei Shi, Zhiwei Chen, Mingyang Pei and Xiaopeng Li; data collection: Mingyang Pei, Zhiwei Chen; analysis and interpretation of results: Xiaowei Shi; draft manuscript preparation: Xiaowei Shi, Zhiwei Chen, Mingyang Pei. All authors reviewed the results and approved the final version of the manuscript.

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