Competitive Spatial Pricing for Urban Parking Systems: Network Structures and Asymmetric Information

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Abstract

Inspired by new technologies to monitor parking occupancy and process market signals, we aim to expand the application of demand-responsive pricing in the parking industry. Based on a graphical Hotelling model wherein each garage has information for its incoming parking demand, we consider a general competitive spatial pricing in parking systems under asymmetric information structure. We focus on the impact of urban network structure on the incentive of information sharing. Our analyses suggest that the garages are always better off in a circular-networked city, while they could be worse off in the suburbs of a star-networked city. Nevertheless, the overall revenue for garages is improved and the aggregate congestion is reduced under information sharing. Our results also suggest that information sharing helps garages further exploit the customers who in turn become worse-off. Therefore, policy-makers should carefully evaluate their transportation data policy since impacts on the service-providers and the customers are typically conflicting. Using the SFpark data, we empirically confirmed the value of information sharing. In particular, garages with higher price-demand elasticity and lower demand variance tend to enjoy larger benefits via information sharing. These insights support the joint design of parking rates structure and information systems.

Keywords: Smart parking; Information systems design; Bertrand competition; Game with incomplete information.

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1 Introduction

Parking is an important industry overlooked by the revenue management community. It is estimated that revenue from parking in the US alone will increase from $25 billion in 2017 to nearly $29 billion by 2018 (Zanona, 2016). The recent development in technologies enables new pricing instruments in the parking industry, which can potentially reshape the market and contribute to smart parking. Price instruments make it possible to shift parking demand spatially and temporally. Many city municipalities have pioneered the "demand-responsive pricing". New York City's PARK Smart varies parking prices depending on the time of day. San Francisco's SFpark program was launched in 2011. They coined the so-called "performance-based rates", in which parking prices increase or decrease depending on garages' occupancy. The complexity of this system is surpassed by Berkeley's GoBerkeley, in which parking prices vary by time of the day, location, and even duration.

The essence of these parking instruments is to comprehend "market signals" from available demand information. Motivated by this idea, we focus on the transmission and processing of market signals in the parking market. For example, a parking data analytics company, Smarking, has been building information systems for garage owners. Keeping track of occupancy data, Smarking helps garage owners predict parking demand by sharing and analyzing historical data. Since the success of Smarking relies crucially on garage owners' willingness to open their data, a natural question arises: what are garage owners' incentives for information sharing? Besides, how does such information sharing affect the parking industry in general?

In this paper, we establish quantitative methods to address these questions. We consider the pricing competition among parking garages and investigate how an information service provider can be a game-changer by coordinating their pricing strategies through information sharing. We extract the demand structure from the Hotelling model (Hotelling, 1929) and incorporate uncertainties. Since garages have different information accesses to the demand uncertainties, they engage in price competition with incomplete information.

Each garage chooses a parking rate to maximize her conditional expected payoff and they reach a Bayesian-Nash equilibrium. However, the equilibrium payoff is changed if they opt to share their private information in advance. Our analysis reveals that information sharing produces win-win outcomes for all participants in most cases.

The contributions of this paper are as follows. We expand the application of information management in the parking industry, which contributes to a crucial part of the general framework of smart cities. We establish a game-theoretic framework to analyze the influence of information sharing among competing urban garages. We focus on the impact of urban network structure on the incentive of information sharing: while the garages are always better off in a circular city, they could be worse off in the suburbs of a star city. Through the graph Laplacian matrix, we isolate the effect of network structure and analyze how the uncertainties in the intrinsic parking utility propagate. As a result of information sharing, the overall revenue for garages is improved and the aggregate congestion is reduced. However, our results also suggest that information sharing helps garages further exploit the customers who in turn become worse-off. Therefore, policy-makers should carefully evaluate their transportation data policy since impacts on the service-providers and the customers are typically conflicting. We empirically confirmed the value of information sharing through a case study using the SFpark data. In particular, garages with higher price-demand elasticity and lower demand variance tend to enjoy larger benefits via information sharing. These insights support the joint design of parking rates structure and information systems.

Section 2 reviews relevant literature. In Section 3, we introduce the Hotelling model setup and the parking price competition and derive its Bayesian-Nash equilibrium. In Section 4, we analyze the impact of information sharing under two typical graph structures. In Section 5, we apply our model on the SFpark data to show the potential benefit from information sharing in reality. Section 6 concludes the paper. In the online supplement materials, we present model extensions and mathematical proofs.

2 Literature review

The Hotelling model on networks has a long tradition in economics literature, e.g., Salop (1979) and Eiselt and Laporte (1993). Recent interests are emerging in extending Hotelling's model of price competition to a general graph, e.g., Heijnen and Soetevent (2018). Our model is most closely related to service pricing in Hotelling models. Xu et al. (2016) study the pricing in Hotelling queue with flexible customers. Yang et al. (2014) present a duopoly waiting-time competition model based on a hub-spoke Hotelling queueing network. Our model is also related to spatial pricing in OM literature, e.g., He et al. (2017) and Bimpikis et al. (2019).

In terms of information sharing, our paper is closest to Liao et al. (2019), wherein information sharing decisions are made in a Hotelling market. Our model is a mirrored version such that the producers are distributed on the Hotelling line therein, instead of the consumers in our model. The questions regarding the incentive of information sharing are known as the "endogenous information structure" problems in the economic theory. For example, Vives (1988) shows that the investments in information are strategic complements in certain economies. Zhou and Chen (2016) demonstrate the power of targeted information release. Information sharing is also studied in the context of supply chain management, e.g., in Lee et al. (2000). Our findings are consistent with theirs in the sense that information sharing is more beneficial given more correlated demands.

We learn from a long stream of literature in economics and operations research on price competition with incomplete information. Gal-Or (1985) and Raith (1996) study the incentives for information sharing in oligopoly. Morris and Shin (2002) consider the value of a public forecast in coordinating agents' actions. Colombo et al. (2014) model the interaction of public and private information, as a recent extension along this stream of research. We consider a class of equilibria wherein each agent's action depends linearly on its forecast and forms Bayes' estimator for others' forecasts (Radner, 1962). Our model adopts a price competition economy with general networked substitutions and an arbitrary demand covariance matrix.

Optimizing parking policies is one of the direction to reduce and/or eliminate cruising behavior, e.g., designing parking permits (Zhang et al., 2011; Wang et al., 2018), managing parking reservations (Chen et al., 2015; Wang and Wang, 2019), and applying dynamic pricing strategies (Qian and Rajagopal, 2014; Lei and Ouyang, 2017). One of the most well-known parking economic experiments is SFpark, in which demand-responsive pricing has been implemented by San Francisco municipalities. Transportation researchers analyze the SFpark experiment to study the impact on garage occupancy, price-elasticity, cruising reduction, and eventually social welfare (see, e.g., Millard-Ball et al. (2013); Chatman and Manville (2014)). On the other hand, Mackowski et al. (2015) seek a quantitative framework for optimal parking prices, taking into account the price-demand relationship. A theoretical counterpart has been conducted in Anderson and De Palma (2004), where they analyze the equilibrium distribution of customers on a single line. Arnott and Inci (2006) also use theoretical models to analyze the effects of parking fees on traffic congestion and social welfare. Different from existing literature, our model emphasizes two directions of parking pricing competition: (1) the price-demand relationship is multi-dimensional, which requires the idea of cross-elasticities; (2) we focus on the information asymmetry since the demand knowledge is private and uncertain.

3 Model setup

3.1 *A basic graphical Hotelling model*

We start with a generalization of the traditional Hotelling model of spatial price competition (Hotelling, 1929). We model the urban parking market via an undirected graph (N, E), where the vertex set N records garage locations and the edge set E documents the parking demand induced from the transportation network. We consider a single-period setting, which can be viewed as an arbitrary time period during a day. (A multi-period

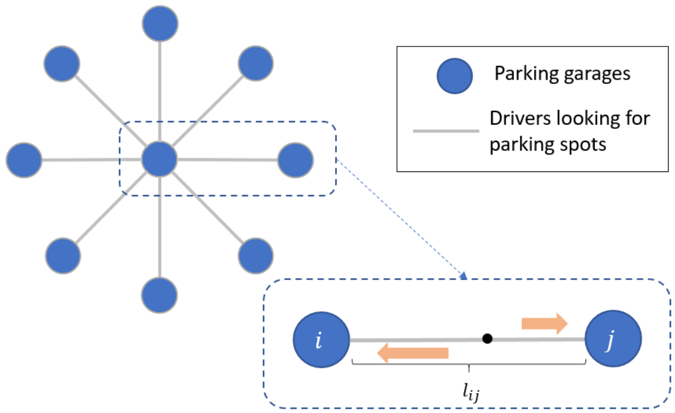

Figure 1: A graphical structure of the parking competition

extension is provided in the online supplement §A.1.) The garages, indexed by the vertex set $N = \{1, 2, \cdots, n\}$, engage in a price competition. Each garage i sets a parking price p_i, given a unit parking space cost c_i. In this paper, we assume every garage is controlled by a different entity, whereas, in the online supplement §A.3, we relax this assumption and analyze the competition among garage coalitions.

Figure 1 depicts a star-shaped city structure as an example. Specifically, each node denotes a parking garage in the competition. On each link, we have customers who are looking for parking spots in the area. We assume that each customer only chooses from two candidate garages. So the potential customers choosing from garage i and garage j form the cruising traffic represented by an undirected link ij. Suppose infinitesimal customers are evenly distributed on each link ij with density λ_{ij}. Let l_{ij} be the length of the link. Note that we do not model the intended destination of each customer. Instead, we assume customers are close to their destinations when they begin searching for parking spots. The behavior of these customers is modeled by the utility method introduced next.

Customers. Consider a customer on ij with her distance to garage i being x, and thus $l_{ij} - x$ to the garage j. She chooses to park at either garage i or garage j, based on the following utility evaluation. Her utility for parking at garage i is calculated as

$$u_{ij}(x) = \underbrace{v_i - p_i}_{\text{net intrinsic parking value}} - \underbrace{c_t \cdot x}_{\text{travel cost}} - \underbrace{c_w \cdot \lambda_{ij} \cdot x^2}_{\text{congestion cost}}. \quad (1)$$

Here, v_i is a heterogeneous intrinsic service value if parking at garage i. p_i is the price charged to every customer, leaving $v_i - p_i$ being the net intrinsic parking value for garage i. c_t is the cost coefficient for traveling a unit distance, and c_w is the cost coefficient for unit congestion. We assume a convex cost structure to characterize the negative externality of parking (e.g., Anderson and De Palma, 2004). The x-squared term, which is referred as the congestion cost, represents any cost growing superlinearly when more customers are directed to the same garage, e.g., extra time spent on searching for an available spot. Moreover, quadratic from produces tractable closed form equilibrium for further analytical discussion. Throughout the paper, we assume that v_i is high enough to maintain a highly competitive environment, which will be further discussed in the later section.

Alternatively, this customer can also travel a distance of $l_{ij} - x$ to park at garage j. Similarly, her utility for parking at garage j is symmetrically defined. Every customer chooses the garage which maximizes her payoff. Thus, the position of the marginal customer who is indifferent between garage i and j is given by solving x from

$$v_i - p_i - c_t \cdot x - c_w \cdot \lambda_{ij} x^2 = v_j - p_j - c_t \cdot (l_{ij} - x) - c_w \cdot \lambda_{ij}(l_{ij} - x)^2, \quad (2)$$

which results in

$$x_{ij} = \frac{l_{ij}}{2} + \frac{v_i - v_j - (p_i - p_j)}{2(c_t + c_w \lambda_{ij} l_{ij})}. \quad (3)$$

The position given by Equation (3) indicates that if a customer on link ij has a distance to garage i within $[0, x_{ij}]$, she chooses garage i; Otherwise, she goes to the alternative garage j. Here, we impose an assumption on all Hotelling model's parameters such that x_{ij} always lies in $[0, l_{ij}]$. This excludes the extreme case where a garage attracts every customer on a link from her competitor.

Based on the above customer behavior, the aggregate demand to garage i is

$$d_i = \sum_{j:ij\in E} \lambda_{ij} \left[\frac{l_{ij}}{2} + \frac{v_i - v_j}{2(c_t + c_w\lambda_{ij}l_{ij})}\right] - p_i \cdot \sum_{j:ij\in E} \frac{\lambda_{ij}}{2(c_t + c_w\lambda_{ij}l_{ij})} + \sum_{j:ij\in E} \frac{\lambda_{ij}}{2(c_t + c_w\lambda_{ij}l_{ij})} \cdot p_j. \tag{4}$$

The demand to each garage takes a linear function of the parking prices. We further investigate the interactive behaviors of garages and customers through a multi-agent pricing game with uncertainty and information asymmetry.

3.2 *Pricing game with asymmetric information*

Garages. Recall from Equation (4) that the parking demand d_i for garage i is linearly decreasing in her own parking rate p_i, but increasing in p_j of other garages. Suppose the capacity of each garage is always sufficient. Then, all garages form a Bertrand price competition (Bertrand, 1883). Here, we extend (4) to consider a linear demand function,

$$d_i = \alpha_i - \beta_{ii}p_i + \sum_{j\neq i} -\beta_{ij}p_j + \theta_i, \forall i \in N, \tag{5}$$

or in a compact form, $d = \alpha - \boldsymbol{\beta} p + \theta$, where $\alpha, \theta, d, p \in \mathbb{R}^n$ and $\boldsymbol{\beta} \in \mathbb{R}^{n\times n}$.

Here, $\alpha_i > 0$ corresponds to the market potential, i.e., the baseline demand driven by dedicated customers. As we analyze one slice in time, we abstract away from its inter-temporal variability. $\beta_{ii} > 0$ is the price elasticity. Our market structure setup corresponds to a scenario wherein garages are not price-takers but enjoy the market power. $\beta_{ij} \leq 0$ is the cross-price elasticity which captures the fact that the parking demand increases as the price of her competitor increases. Finally, the parking demand exhibits random deviations θ_i for each garage. The deviations originate from the uncertain valuation of parking garages from the customer's perspective.

Relating (5) to the Hotelling demand (4), the price elasticities are analogs of the coefficients in front of prices in (4); the demand potentials together with the noise terms are related to the service value v with uncertainties. We will elaborate on their connections and link the pricing game model back to the Hotelling model in Section 4.

The payoff function of garage $i \in N$ can be expressed as

$$\pi_i = d_i \cdot (p_i - c_i) = \left(\alpha_i - \beta_{ii} p_i + \sum_{j \neq i} -\beta_{ij} p_j + \theta_i \right) (p_i - c_i), \tag{6}$$

which is the demand times the profit of providing a unit parking spot. The physical meaning of the cost c_i can be generic. This cost not only includes infrastructure and operating costs, but also reflects a shadow price for holding this spot for future release.

Information structure and sequence of events. We assume that $\theta := [\theta_1, \theta_2, ..., \theta_n]^\top$ is drawn from a multivariate normal distribution with a zero mean, i.e., $\theta \sim \mathcal{N}_n(\mathbf{0}, \mathbf{\Sigma})$. $\mathbf{\Sigma}$ is the covariance matrix. Without information sharing, garage i can only predict its own demand uncertainty θ_i but not the others'.

The sequence of events is as follows. First, each garage forecasts her demand uncertainty θ_i. For the base model, we assume the demand forecast is 100% accurate. That is to say that garage i observes θ_i without error. Extension for inaccurate demand forecast is in the online supplement §A.2. Meanwhile, they estimate the competitors' demand uncertainties based on the Bayesian estimator $\hat{\theta}_j = \mathbb{E}(\theta_j | \theta_i)$. Then, each garage i makes her pricing decision p_i. Finally, the market clears and demands d_i are determined; each garage receives a corresponding payoff π_i.

Pricing equilibrium. We briefly introduce the equilibrium concept. We focus exclusively on the *linear Bayesian-Nash equilibrium* when garage i chooses a parking rate p_i to maximize her payoff, i.e., $p_i = A_i + B_i \theta_i, \forall i \in N$ for some constant scalars A_i and B_i's. We can interpret A_i as the *baseline price*; B_i as the *response factor* with respect to the signal θ_i. If information is shared among all garages, then garage i can utilize all her known signals to set her parking rate, i.e., $p_i = A_i + \sum_j B_{ij} \theta_j$. In her calculation for the expected market price, she forms an expectation of the other garages' prices. Such equilibrium concept is commonly seen in the literature, e.g., Vives (1988) and Morris and Shin (2002).

Under a certain information structure, the baseline prices and response factors in the equilibrium pricing strategy are determined by the market structure. In Section 3.3, we present the analysis of two-garage symmetric model to explore the intuition. The equilibrium in a general model is provided in Section 3.4.

3.3 *Analysis of two-Garage models*

We begin with a stylized setup with two symmetric garages. $\beta_{11} = \beta_{22} = \hat{\beta}$, $\beta_{12} = \beta_{12} = -\beta$, and $c_1 = c_2 = c$:

$$\begin{cases} d_1 = \alpha - \hat{\beta} p_1 + \beta p_2 + \theta_1, \\ d_2 = \alpha + \beta p_1 - \hat{\beta} p_2 + \theta_2, \end{cases} \tag{7}$$

We assume that signals are drawn from symmetric bivariate normal distribution $(\theta_1, \theta_2)^\top \sim \mathcal{N}_2 \left(0, \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right)$. As a standard stability constraint, we require $0 < \beta < \hat{\beta}$.

Under the above symmetric setting, the equilibrium pricing under the private information structure of both garages is denoted as $p_i = A + B\theta_i$ where $A, B \in \mathbb{R}$.

Proposition 1. *The following comparative statics hold:*

1. *Baseline parking rates decrease in price elasticity, i.e.,* $\frac{\partial A}{\partial \hat{\beta}} < 0$, *and increase in cross-price elasticity, i.e.,* $\frac{\partial A}{\partial \beta} > 0$;
2. *Response factor toward private signals is always positive, i.e.,* $B > 0$, *while the response is more aggressive when private signals are more correlated, i.e.,* $\frac{\partial B}{\partial \rho} > 0$;
3. *Garages' payoffs decrease in private elasticity, i.e.,* $\frac{\partial \mathbb{E}\pi}{\partial \hat{\beta}} < 0$, *and increase in cross-price elasticity, i.e.,* $\frac{\partial \mathbb{E}\pi}{\partial \beta} > 0$;
4. *Garages' payoffs increase in the degree of demand correlation, i.e.,* $\frac{\partial \mathbb{E}\pi}{\partial \rho} > 0$.
5. *Information sharing is always preferred.*

The key to understanding this proposition is our assumption of the positive cross-price elasticity, which implies a strategic complement: an increase in one garage's rate will lead to a higher demand for another garage. The first three statements in the proposition are straightforward. Firstly, baseline rates decrease in price elasticity (due to the inverse relationship between price and demand) and increase in cross-price elasticity (due to the strategic complements). Consequently, garages' payoffs decrease in private elasticity and increase in cross-price elasticity.

The fourth statement provides an interesting result: garages' expected payoff is higher when demands are more positively correlated. The additional payoff stems from taking advantage of the demand deviation through known information. If demand signals are more positively correlated, due to demand cross-elasticity, the behavior of garage 2 amplifies the demand fluctuation of garage 1. Then, garage 1 reacts more aggressively on θ_1 and obtains a higher expected payoff. On the contrary, if demand is more negatively correlated, the behavior of garage 2 dampens the demand fluctuation of garage 1, resulting in a lower expected payoff. In other words, demand signals, even if being private, serve as a collusion instrument. A higher correlation will strengthen the belief in such positive feedback.

Finally, it is proven that both garages are always better off when sharing signals. This complies with the intuition that demand signals serve as a collusion instrument to exploit demand fluctuation. This effect is intensified now due to communication.

3.4 *General model*

In this subsection, we present the n-garage equilibrium under asymmetric information. We first clarify the algebraic notations and assumptions applied throughout the rest of the paper.

Notations. Recall $N = \{1, \cdots, n\}$ and suppose $S, S' \subset N$ are index sets. For a matrix (boldface) $\boldsymbol{G} \in \mathbb{R}^{n \times n}$, we define its submatrix $G_{SS'} \in \mathbb{R}^{|S| \times |S'|}$ which takes rows of $\boldsymbol{G}$ indexed by S and columns indexed by S'. In particular, let G_{ij} be the element at row-i and column-j. (Vectors is a specific matrix with one dimension, and therefore follow the same subscript scheme). When either S or S' takes N, we simply denote it by $\star$, i.e.,

$G_{S\star}$ is the submatrix by picking rows indexed by S from $\boldsymbol{G}$, while $G_{\star j}$ stands for the jth column vector of $\boldsymbol{G}$. We denote $\boldsymbol{O}$ as the zero-matrix, $\boldsymbol{I}$ as the identity matrix, and e as the column all-ones vector such that $ee^\top$ is the all-ones matrix.

Assumption 1. *$\beta_{ii} \geq 0$, $\beta_{ij} \leq 0, \forall j \neq i$, and $\boldsymbol{\beta}$ is positive semidefinite; $\boldsymbol{\Sigma}$ is symmetric positive semidefinite.*

Information structure. We define the information structure as a boolean matrix $\boldsymbol{M} \in \{0,1\}^{n\times n}$: $M_{ij} = 1$ if garage i knows θ_j; otherwise, $M_{ij} = 0$. Let $N_i := \{j \in N : M_{ij} = 1\}$ be the set of garages whose information is known by garage i. Garage i sets her parking rate $p_i = A_i + \sum_{j\in N_i} B_{ij}\theta_j$ based on her known signals. By enforcing $B_{ij} = 0, \forall j \notin N_i$, i.e., garage i cannot utilize her unknown information θ_j, we have an $n \times n$ matrix $\boldsymbol{B}$ representing all the pricing coefficients on the signals. Then, the pricing policies is compactly written as $p = A + \boldsymbol{B}\theta$. The following proposition solves the Bayesian-Nash equilibrium given an arbitrary $\boldsymbol{M}$.

Proposition 2. *Under the information structure $\boldsymbol{M}$, the equilibrium pricing strategy is $p_i = A_i + \sum_{j\in N} B_{ij}\theta_j, \forall i \in N$, where parameters A and $\boldsymbol{B}$ are determined by the following.*

$$A = [A_1, A_2, ..., A_n]^\top = \boldsymbol{Q}^{-1}\left[\alpha_1 + \beta_{11}c_1, \alpha_2 + \beta_{22}c_2, ..., \alpha_n + \beta_{nn}c_n\right]^\top, \tag{8}$$

$$\begin{cases} Q_{i\star}\boldsymbol{B}\Sigma_{\star N_i} - \Sigma_{iN_i} = \boldsymbol{0}, \forall i \in N, \\ B_{ij} = 0, \forall M_{ij} = 0. \end{cases} \tag{9}$$

Here, $\boldsymbol{Q} := \boldsymbol{\beta} + diag[\boldsymbol{\beta}]$ is the elasticity matrix with its diagonal entries doubled and $N_i := \{j \in N : M_{ij} = 1\}$ is the information index set of garage i.

The equilibrium payoff is given by

$$\mathbb{E}\left[\pi_i\right] = \beta_{ii}\left(A_i - c_i\right)^2 + \beta_{ii}B_{i\star}\Sigma B_{i\star}^\top. \tag{10}$$

For each garage i, her pricing policy $p_i = A_i + \sum_{j\in N} B_{ij}\theta_j$ is a combination of two parts. First, A_i represents a baseline parking rate. Second, $\{B_{ij} : j \in N_i\}$ are response factors that form a rate adjustment based on the garage i's known information θ_{N_i}. The baseline price A is determined by the demand structure but irrelevant to the information structure $\boldsymbol{M}$, while the response factor $\boldsymbol{B}$ is crucially affected by $\boldsymbol{M}$. Utilizing this property, we isolate the effect of the information structure.

Information value. We define the information value as the second term in (10),

$$V_i := \beta_{ii} B_{i\star} \boldsymbol{\Sigma} B_{i\star}^\top. \tag{11}$$

Note that if we remove the demand signals θ from the model, garages engage in a standard Bertrand competition which has equilibrium prices $p_i = A_i$ and (expected) payoff $\pi_i = \beta_{ii} (A_i - c_i)^2$. This can be verified by taking $\boldsymbol{M} = \boldsymbol{O}$ or $\boldsymbol{\Sigma} = \boldsymbol{O}$ in Proposition 2. Thus, the information structure $\boldsymbol{M}$ results in an additional expected payoff V_i. The minimum possible information value is achieved at $\boldsymbol{M} = \boldsymbol{O}$. Private information structure corresponds to $\boldsymbol{M} = \boldsymbol{I}$. $\boldsymbol{M} = ee^\top$ is the sharing (complete) information structure. In this paper, we mainly focus on the comparison between the private information structure and the sharing information structure. We use **value of information sharing** to denote the increase in V_i from the $\boldsymbol{M} = \boldsymbol{I}$ case to the $\boldsymbol{M} = ee^\top$ case. In the online supplement §A.4 and §A.5, we further discuss garages' endogenous preference on other information structures.

4 Urban network structures

In this section, we examine the effect of information sharing on Hotelling networks with homogeneous links. To analytically demonstrate the benefit of information sharing, we studied two specific network structures — circular city, where identical garages are evenly distributed on a circle; and star city, where a central garage is surrounded by $m = n - 1$ peripheral ones. All traffic parameters l_{ij} and λ_{ij} are assumed to be identical on every link. Then, we derive the pricing equilibrium with or without information sharing. The results suggest that information sharing not only improves the aggregate expected profit for garages but also reduces the overall congestion cost for customers on the links. However, information sharing helps garages to exploit total customer welfare.

First, we established the transformation from the Hotelling model to the linear price-demand formulation, with some homogeneity assumptions and parameter definitions. The homogeneity assumptions are made to retain tractability of our model so that we can highlight the impact of network topology and information sharing.

Link homogeneity. Assume $\lambda_{ij} = \lambda$ and $l_{ij} = l, \forall ij \in E$. This leads to identical cross-price elasticities $-\beta_{ij} = \frac{\lambda}{2(c_t + c_w \lambda l)} =: \beta_0, \forall ij \in E$.

Service value with homogeneous uncertainties. Assume that v follows a multivariate normal distribution with identical variances $\mathrm{var}\left[v_i\right] = \sigma_v^2, \forall i$, and identical correlations $\mathrm{corr}\left(v_i, v_j\right) = \rho_v, \forall i \neq j$. Namely, $v \sim \mathcal{N}\left(\bar{v}, \sigma_v^2\left(\left(1-\rho_v\right)\boldsymbol{I} + \rho_v ee^\top\right)\right)$ where $\bar{v}$ is the exogenous mean service value vector. The variability of v_i represents the fluctuation in the internal "attractability" of garage i. For example, an on-sale period at a mall potentially increases the value of parking at its nearby garage. The random outcome of v_i is private knowledge to garage i while the entire distribution is common knowledge to all garages. The uncertainty in the service value v results in fluctuation in demand potentials.

In the next lemma, we will show that the Hotelling model can be reduced to a Bertrand price competition. The key message is concerning how the uncertainties in the intrinsic parking utility propagate to shape the covariance in the demand function. The key to unfold this propagation and isolate the effect of network structure is through the graph Laplacian matrix.

Definition 1. (Laplacian Matrix) For a given simple graph (N, E), its Laplacian matrix $\boldsymbol{L}$ is an $n \times n$ matrix defined as:

$$L_{ij} = \begin{cases} \text{degree of node } i, & \text{if } i = j, \\ -1, & \text{if } ij \in E, \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

Lemma 1. *Given a homogeneous Hotelling network with Laplacian matrix* $\boldsymbol{L}$*, the demand function* (4) *is equivalent to* $d = \alpha - \boldsymbol{\beta} p + \theta$*, wherein deterministic coefficients* $\alpha = \frac{\lambda l}{2}\mathrm{diag}\boldsymbol{L} +$

$\beta_0 \boldsymbol{L}\bar{v}$, *and* $\boldsymbol{\beta} = \beta_0 \boldsymbol{L}$. *The noise term* $\theta \sim \mathcal{N}_n(\mathbf{0}, \boldsymbol{\Sigma})$ *where*

$$\boldsymbol{\Sigma} = \beta_0^2 \boldsymbol{L} \operatorname{var}[v] \boldsymbol{L}^\top = \beta_0^2 \sigma_v^2 (1 - \rho_v) \boldsymbol{L}\boldsymbol{L}^\top. \tag{13}$$

We mainly focus on the price elasticity matrix $\boldsymbol{\beta}$ and demand covariance $\boldsymbol{\Sigma}$ as these terms are the ones affecting information sharing. Given the homogeneities, the cross-price elasticity is $\beta_0 > 0$ on each link and 0 between unconnected garages. The price elasticities increase as the customers' costs c_t, c_w go down or the population density λ goes up. For each garage i, the sum of cross-price elasticity equals the self-price elasticity in absolute value. This means that \$1 increase in price p_i and \$1 decrease in all i's neighbors result in the same amount of demand reduction on i. This is nevertheless a stylized assumption due to the reduction from the networked Hotelling model, while at the same time automatically satisfied the stability condition in the Bertrand price competition. This assumption is not crucial.

Due to the spatial substitution in the parking demand, a higher correlation coefficient reduces overall demand uncertainty. Equation (13) shows that the uniform covariance $\rho_v = \operatorname{corr}(v_i, v_j)$ only proportionally changes each component of $\boldsymbol{\Sigma}$. Since varying ρ_v does not essentially affect the demand covariance structure, we restrict to $\sigma_v^2 = 1$ and $\rho_v = 0$ in the following discussions. This symmetry in the covariance is again due to the reduction from the networked Hotelling model. By abstracting away from the correlation in the intrinsic parking valuation, we focus on the network structure, as the covariance depends crucially on the Laplacian matrix.

Customer welfare and customer congestion. We define customer welfare as the expected total utility of customers under equilibrium garage pricing. The customer welfare on link ij is

$$U_{ij} = \mathbb{E}\left[\int_0^{x_{ij}} \left(v_i - p_i - c_t x - c_w \lambda x^2\right) \lambda dx + \int_0^{l - x_{ij}} \left(v_j - p_j - c_t x - c_w \lambda x^2\right) \lambda dx\right]. \tag{14}$$

Here, the expectation is taken with respect to the randomness of intrinsic values v. The the equilibrium prices p rely on v according to Proposition 2. Furthermore, x_{ij} is implicitly determined by v and p through Equation (3). Similarly, we define the aggregate cost of all customer on link ij,

$$C_{ij} := \mathbb{E}\left[\int_0^{x_{ij}} \left(c_t x + c_w \lambda x^2\right) \lambda dx + \int_0^{l-x_{ij}} \left(c_t x + c_w \lambda x^2\right) \lambda dx\right]. \tag{15}$$

We identify C_{ij} as a metric of congestion on link ij since it stems from the travel cost and congestion cost of individual customers.

In particular, we are interested in the change in the customer welfare U_{ij} and congestion C_{ij} when the garage information structure alters. The following lemma indicates that the change in U_{ij} and C_{ij} can be measured by the change in the variance of the position of the marginal customer x_{ij}.

Lemma 2. *Suppose garages always set their prices according to the equilibrium strategy given by Proposition 2. When information structure is altered, we use Δ to denote the incremental change in a certain quantity. Then,*

$$\Delta C_{ij} = \Delta U_{ij} = \frac{\lambda^2}{2\beta_0} \Delta \left\{\mathrm{var}\left[x_{ij}\right]\right\}. \tag{16}$$

Moreover, the total change is

$$\Delta \left\{\sum_{ij \in E} C_{ij}\right\} = \Delta \left\{\sum_{ij \in E} U_{ij}\right\} = \frac{\beta_0}{2} \Delta \left\{\langle \boldsymbol{L}, \mathrm{var}\left[v - p\right]\rangle\right\}. \tag{17}$$

wherein the $\langle \cdot, \cdot \rangle$ term denotes the Frobenius inner product of the two $n \times n$ matrix, graph Laplacian $\boldsymbol{L}$ and covariance matrix of net value vector $v - p$.

Lemma (2) shows that the change in customers' aggregate cost depends crucially on the variance of the position of the marginal customer. Therefore, the information structure that minimizes the variance of x_{ij} will result in the least transportation cost on link ij. Intuitively, this means that customers are more steadily allocated among garages in the long run. However, the change in their net payoffs is twice as much so that the increase in customer utility is identical to that in the aggregate cost. As we reduce the congest cost, customer welfare will decrease as well. A social welfare planner would be interested in the trade-off between garage welfare and customer welfare. In this paper, we mainly focus on the analysis of information value and road congestion. To measure the aggregate cost on the graph, (17) provides a calculation method through the graph Laplacian and the customer net payoff vector $v - p$.

In the remaining part of this section, we utilize the relations established in Lemmas 1 and 2 to compare the system's performance between information sharing and information asymmetry, under different urban structures. Two types of representative graph structure are studied. A circular city (or many-sided polygons) is the simplest symmetric network possible. It represents the parking problem along a "ring road". On a larger scale, there are city designs with urban circumferential routes, such as Berlin, Bangkok, and Beijing). On a smaller scale, this structure models the parking problem around a large center area (e.g., a crowded CBD region). It is representative of some multi-centered metropolitans (such as the San Francisco Bay Area).

As a comparison to the circular city structure, we then investigate the star city structure where a central garage is surrounded by peripheral ones. Some European cities have a clear radial design, such as Coevoerden and Palmanova. In general, the star city is representative of the vast majority of American cities with downtown and suburbs.

For these two specific structures, we show that information sharing improves garages' aggregated payoff and alleviates traffic congestion. Table 1 lists the set of relative value metrics that will be used. Note that the numerical difference between the relative value metric and its original definition is a constant term that is invariant under all information structures.

4.1 *Circular city structure*

Consider a symmetric circular city based on a variant of the circular Hotelling model (Lerner and Singer, 1937). This stylized setup eliminates the "corner" difficulties of the

Table 1: Relative Value Metrics

Quantities	Definitions	Relative Value Metrics
Garage Profit	$\sum_{i \in N} \mathbb{E}[\pi_i]$	$\sum_{i \in N} V_i$ (= total information value)
Congestion	$\sum_{ij \in E} C_{ij}$	$\langle \boldsymbol{L}, \text{var}[v-p] \rangle$ (= aggregate cost)
Customer Welfare	$\sum_{ij \in E} U_{ij}$	$\langle \boldsymbol{L}, \text{var}[v-p] \rangle$
Social Welfare	= Garage Profit + Customer Welfare	

original Hotelling, and allows a focus on the essential interactions of service providers (Salop, 1979). Suppose n garages locate on the vertices of a regular n-sided polygon with a fixed side length l. We label all garages from 1 to n in a circular way. Then each garage i is linked to its neighbor $i-1$ and $i+1$ (mod n). Assuming the total number of garages is $n \geq 4$, the graph Laplacian is given by

$$\boldsymbol{L} := \begin{bmatrix} 2 & -1 & & -1 \\ -1 & 2 & \ddots & \\ & \ddots & \ddots & -1 \\ -1 & & -1 & 2 \end{bmatrix} = 2\boldsymbol{I}_n - \begin{bmatrix} & \boldsymbol{I}_n & 1 \\ 1 & & \end{bmatrix} - \begin{bmatrix} & & 1 \\ \boldsymbol{I}_n & 1 & \end{bmatrix}, \tag{18}$$

where $\boldsymbol{I}_n$ is the identity matrix of size $n \times n$.

In the symmetric circular model, all garages are identical. They maintain the same pricing strategy and earn the same information value. The following proposition summarizes the effect of information sharing under circular city scenario.

Proposition 3. *In the context of symmetric circular city with $n \geq 4$, information sharing improves the expected payoff to each garage and reduces congestion cost. The corresponding information value (per garage) and aggregate cost (per link) are listed as in Table 2.*

In this case, information sharing increases garages' information value and reduces the aggregate congestion cost. The effect is even more significant for instances with smaller n. Information sharing strictly increases the expected payoff to each garage. From the

Table 2: Circular city: information value and aggregate cost

	Private Information		Information Sharing
Information value	$27/64 \cdot \beta_0$	$<$	$\frac{2}{3(y_n-1)}\left(\frac{1}{3}\left(4y_{n-1}+y_n\right)+n-3\right)\cdot\beta_0$
Aggregate cost	$29/128 \cdot \beta_0$	$>$	$\frac{1}{3(y_n-1)}\left(\frac{\bar{y}_{n-1}-\bar{y}_n}{\bar{y}_1}+y_n-n\right)\cdot\beta_0$

Here $y_k := \cosh\left(k\ln\left(2+\sqrt{3}\right)\right)$, and $\bar{y}_k := \sinh\left(k\ln\left(2+\sqrt{3}\right)\right)$

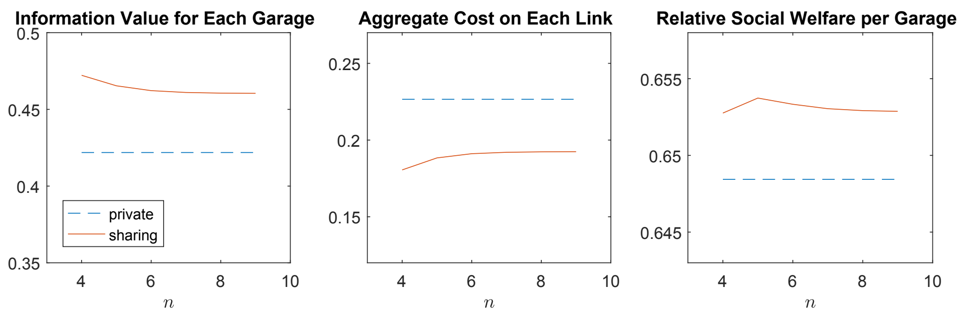

Figure 2: Circular city: relative value metrics (in β_0) under private/sharing information

garages' perspective, everyone is better off under information sharing. Thus, they have the incentives to exchange their demand information.

Figure 2 plots the comparison for the circular city model. The solid curves stand for the results under information sharing while the dashed curves are the results under private information. The increment in information value decreases asymptotically to a constant. For the circular model, a smaller n implies more intense interaction among garages. Therefore, information sharing exhibit greater benefit. As the number of garages increases, the model approaches an infinite line (a circle with an infinite perimeter). The payoff under information sharing approaches a limiting value, still dominating the outcome when garages stick to their private information. Note that these revenue-maximizing garages intend to set higher prices under higher demand (i.e, lager parking value v_i's). Information sharing helps their exploitation of demand. This follows from the two-player model as we have discussed: the demand signals serve as a collusion instrument for the garages to set higher rates with higher payoffs even without communication.

For a large n, information sharing reduces the aggregate cost. To understand the impact of information sharing on the aggregate cost, we need to examine the position of the marginal customer x_{ij}. Since x_{ij} is determined by comparing the relative value of the intrinsic parking utility $v-p$, a more informed pricing strategy will offset higher uncertainty v and thus more predictable market sizes (lower variations in x_{ij}). As we discussed in Lemma 2, lower variance in the threshold point x_{ij} indicates lower aggregate cost for the customers. Thus, information sharing among garages finally results in congestion cost reduction on the roads.

In the circular city model, social welfare marginally increases under information sharing. In particular, the garage welfare increases while the customer welfare decreases. This is not surprising as information sharing shifts the competitive pricing closer toward collusive pricing. Therefore, information sharing helps garages further exploit the customers.

Here, as the length of link l is fixed, the size of the city is expanding when n increases. On the other hand, if we consider a circle with a fixed perimeter such that garages become denser as n increases (i.e., substituting l by $\frac{l}{n}$ only affects the common factor β_0 in Table 2), similar results and intuitions hold. In addition, it is more convenient to compare the circular city with the star city of the same link length, to isolate other effects of the network structure.

4.2 *Star city structure*

As a comparison to the circular city structure, we then investigate the star city structure where traffic mainly falls on a spoke-hub network. Consider a symmetric star city with a central garage i and m identical peripheral garages. The graph Laplacian takes the

Table 3: Star city: information value and aggregate cost

	Private Information		Information Sharing
Information value (center)	$\left(\frac{2m}{7m-1}\right)^2 (m+1) \cdot \beta_0$	$<$	$\frac{1}{9}(m+1) \cdot \beta_0$
Information value (peripheral)	$2\left(\frac{3m-1}{7m-1}\right)^2 \cdot \beta_0$	$>$	$\frac{13m-5}{36m} \cdot \beta_0$
Total information value	$\frac{2m\left(11m^2-4m+1\right)}{(7m-1)^2} \cdot \beta_0$	$<$	$\frac{17m-1}{36} \cdot \beta_0$
Aggregate cost on each link	$\frac{2m(5m-3)}{(7m-1)^2} \cdot \beta_0$	$>$	$\frac{13m-5}{72m} \cdot \beta_0$

following form.

$$\boldsymbol{L} = \begin{bmatrix} m & -1 & \cdots & -1 \\ -1 & 1 & & \\ \vdots & & \ddots & \\ -1 & & & 1 \end{bmatrix} = \begin{bmatrix} m & -e^\top \\ -e & \boldsymbol{I_m} \end{bmatrix} \tag{19}$$

The first row and first column stand for the central garage, while the rest are peripheral garages.

We summarize the analytical result for star city in Proposition 4.

Proposition 4. *In the context of symmetric star city with $m \geq 3$, information sharing improves the expected payoff to the central garage but reduces that to the peripheral garages. Their total information value increases and the congestion cost is reduced. The corresponding information value (per garage) and aggregate cost (per link) are listed as in Table 3.*

Figure 3 illustrates the comparison of the quantities in Table 3 with the number of peripheral garages varying. If information sharing is applied, the center garage benefits the most, especially when the number of garages is large. The peripheral garages are subject to loss in their payoff. However, the payoff gain from the central garage dominate the payoff loss from the peripheral garages. Thus, their aggregate (or average) payoff increases under information sharing.

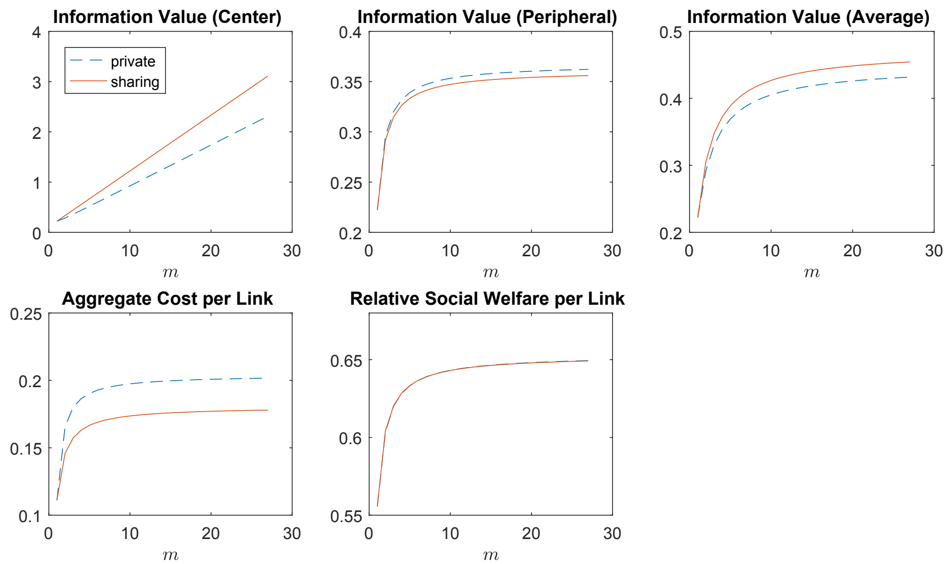

Figure 3: Star city: relative value metrics (in β_0) under private/sharing information

The central garage stands in a pivotal position in the network and the demand uncertainties from all peripheral garages propagate to its demand. Thus, the central garage is willing to acquire peripheral demand signals in addition to her own forecast. Once the information is shared, it benefits the most from leverage on the prices in response to the peripheral demand uncertainties. Therefore, it has the highest incentive to acquire private demand information from others.

For the peripheral ones, it turns out that they prefer keeping their information. If all signals are revealed, peripheral garages' prices under equilibrium become less correlated with the demand fluctuation. This can be interpreted as overreacting to the demand signals as they compete under complete information. They respond to all demand signals at a more intensive rate than they do in the private information scenario, resulting in a less efficient pricing outcome. Even though this phenomenon suggests that all garage may not spontaneously agree to information sharing, they still can achieve information sharing through some kind of payoff transfer contract. Because information sharing increases their aggregate expected payoff.

On the customers' side, similar to the results from the circular city, information sharing reduces the congestion as well as the customer welfare. The aggregate cost reduction on each link is increasing in the number of garages but asymptotically constant. Different from the circular city, the total social welfare slightly decreases after information sharing. But the difference is still marginal.

Unlike the circular model, the benefits of information sharing (information value enhancement and aggregate cost reduction) are greater as the scale of the star graph increases. This implies that information sharing is more welcomed when more peripheral garages are linked to the hub garage. This outcome is consistent with the general insight that information sharing provides greater improvement when garages are more closely and intensely correlated. As the number of peripheral garages increases, the center garage interacts with more opponents. Thus, more complete demand information is more appealing to her. However, for peripheral garages, they maintain direct competition only with the center garage. Thus, the scaling effect to peripheral ones is minor. Cost reduction is also more significant for a "denser" city structure. Therefore, information sharing improves both supply side and demand side simultaneously and it reveals greater power in cities that are more intensely connected and congested.

5 Case study

So far, we have shown the benefit of information sharing from symmetric demand structures. The goal of this case study is to further investigate information sharing under an empirical demand structure.

Since 2011, SFpark in the city of San Francisco has been using demand-responsive pricing to regulate parking availability among garages. SFpark adopts technologies such as parking sensors and smart meters to keep track of garages' real-time occupancy and to implement data-driven parking pricing. A parking rate is set for each garage (or street

Table 4: Price-occupancy data sample ($ per hour for rates)

Period	2011-08		2011-10		...	2016-06	
Street Block	Rate	Occupancy	Rate	Occupancy	...	Rate	Occupancy
01ST ST 200	3.5	56%	3.25	62%	...	2.5	62%
02ND ST 300	3.5	75%	3.5	66%	...	3.5	60%
...	...	...	...	...	...	...	...
VALENCIA ST 900	2	57%	1.75	54%	...	1.5	64%

parking block) during a time slot (by weekday/weekend and morning/afternoon/evening). After each period (usually 2 to 4 months), the parking rate is adjusted based on the recent performance. A garage with a relatively high (low) occupancy will increase (decrease) its rate in the next period. SFpark records the rates and average occupancies by garage and time slot. Table 4 is a sample of the price-occupancy data of a specific time slot (weekday mornings). The data source and more detailed adjustment rules are available at SFpark (2017).

We utilize their data records to learn an empirical demand structure, especially the price elasticities and demand contrivances. The original dataset contains over 200 parking garages and 18 periods of price adjustments from Oct 2011 to Jun 2016. We filter out garages that have 1) a complete historical data record over all periods, 2) a capacity no less than 10, and 3) a relatively significant price-elasticity. 28 garages are selected to run the experiment. Then, we extract a demand structure (5) from the price-occupancy data through linear regression. We assume the demand is represented by the occupancy data so that the demand value is normalized. Although capacities are not equal (typically, ranging from 10 to 30), adjusting for capacity only proportionally affects the price elasticities and the information value. This is further explained after we present the regression model. Besides, we ignore the unobserved demand due to capacity limits. For those most occupied garages, the SFpark rate adjustment policy tends to drive down the period average occupancy below 80%. Therefore, we assume excessive demand is negligible.

Regression Learning Model. Let $\left(p^{(t)}, d^{(t)}\right)$ be the price-occupancy data in period

$t = 1, 2, \cdots, T$. Denote $p^{(t)} = \left(p_1^{(t)}, p_2^{(t)}, \cdots, p_n^{(t)}\right)^\top$ as the parking rate vector in period t and $d^{(t)} = \left(d_1^{(t)}, d_2^{(t)}, \cdots, d_n^{(t)}\right)^\top$ as the occupancy vector. We apply the following high-dimensional regression method to estimate the linear coefficients α and $\boldsymbol{\beta}$ in (5) and the signal covariance $\boldsymbol{\Sigma}$. They are calculated by solving a constrained least square optimization problem.

$$\hat{\boldsymbol{\Sigma}} := \frac{1}{n}\left(d - \left(\hat{\alpha} - \hat{\boldsymbol{\beta}}p\right)\right)\left(d - \left(\hat{\alpha} - \hat{\boldsymbol{\beta}}p\right)\right)^\top \tag{20}$$

$$\hat{\alpha}, \hat{\boldsymbol{\beta}} := \arg \min_{\alpha \in \mathbb{R}^n, \beta \in \mathbb{R}^{n \times n}} \sum_{t=1}^{T} \left\| \alpha - \boldsymbol{\beta} p^{(t)} - d^{(t)} \right\|_2^2 \tag{21}$$

$$\text{s.t. } \beta_{ii} > 0, \forall i = 1, \cdots, n \tag{22}$$

$$\beta_{ij} \leq 0, \forall i, j = 1, \cdots, n \text{ and } i \neq j \tag{23}$$

$$\beta_{ii} + \sum_{j \neq i} \beta_{ij} > 0, \forall i = 1, \cdots, n \tag{24}$$

Garage capacity does not affects demand structure learned from the regression. Suppose the demand vector is adjusted from $d^{(t)}$ to $\boldsymbol{\kappa} d^{(t)}$ for all t, where $\boldsymbol{\kappa} \in \mathbb{R}^{n \times n}$ is a diagonal matrix numerically scaling up the demands. Regression outcome changes proportionally. Parameters $\hat{\alpha}, \hat{\beta}$, and $\hat{\boldsymbol{\Sigma}}$ become $\boldsymbol{\kappa}\hat{\alpha}, \boldsymbol{\kappa}\hat{\beta}$, and $\boldsymbol{\kappa}\hat{\boldsymbol{\Sigma}}\boldsymbol{\kappa}^\top$, respectively. Then, followed from Proposition 2, the information values $V = (V_1, V_2, \cdots, V_n)^\top$ scale up to $\boldsymbol{\kappa} V$. That is, the numerical value of V_i is multiplied by the garage $i's$ capacity if we adjust for the garage's capacity.

We compute $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\Sigma}}$ from the regression model. We report their diagonal values in Figure 4 (since the full regression result is high-dimensional). The mean square errors of the high-dimensional regression is given by the diagonal elements of $\hat{\boldsymbol{\Sigma}}$. The average mean square error is 38.32 (occupancy%-squared).

A comparison between the information value for each garage under two information structures is given in Figure 5. The dark-colored bars represent the information value when garages keep their private information, while the light-colored bars represent the additional benefit from information sharing. As shown in the figure, all 28 garages benefit

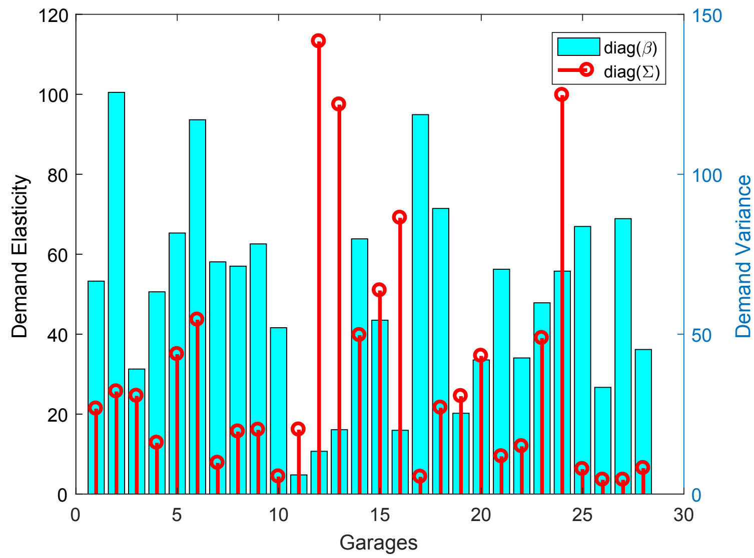

Figure 4: Learned parameters: self-elasticity (bars) and variance of private signals (stems)

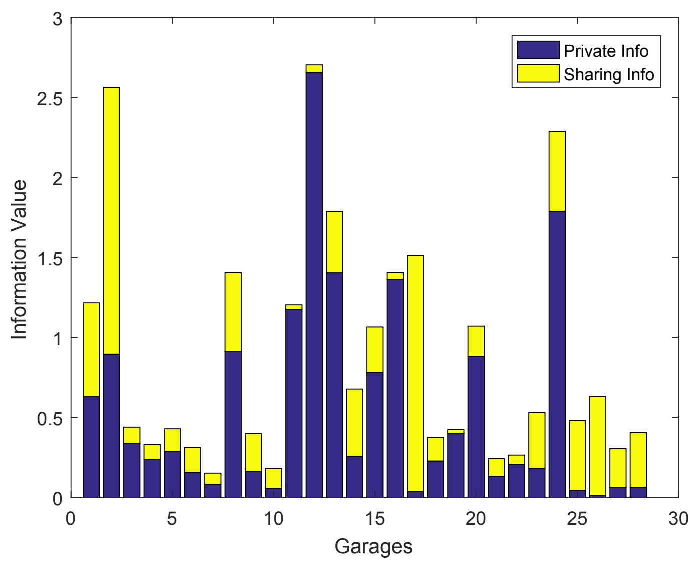

Figure 5: Information value: private (dark-colored) vs. sharing (light-colored)

from information sharing, but the additional gain in information value varies depending on the demand structure. Garages with higher self-elasticities and lower demand uncertainties (e.g., garage No.1,2,17,25,26,27,28) tend to enjoy larger percentages of increase in information value upon complete information sharing. For these garages, their own demand uncertainty is predictable but they are more sensitive to the price competition. Typically, these garages represent the parking lots located in busy downtown areas. Their inherent demand pattern is predictable for a fixed time window, but drivers strategically choose the parking lots with lower rates in the vicinity. Information sharing helps these garages better predict their competitors' rates and price their own garages accordingly. Conversely, the garages with low self-elasticity and higher demand uncertainty (e.g., garage No.11,12,13,16,24) benefit less from information sharing. Even though the extra profit from information sharing is less, there is still a positive gain for doing so.

6 Conclusion

In this paper, we study the effect of information sharing among competing parking garages. For the two-garage scenario, it is proved that they are always better off sharing signals. For the general model with n garages, we quantify the value of information by defining it as the portion of the expected payoff that is directly influenced by the information structure. By examining two specific graph structures, we find that information sharing not only improves the garages' overall payoff but also reduces the aggregate cost for customers. This implies that information sharing has the potential to alleviate road congestion. Furthermore, when the interactions among garages are more significant, information sharing exhibits greater power. Intuitively, information sharing help to navigate customers toward the right garages. However, our analysis also suggests that it helps garages further exploit customer welfare.

The numerical study on SFpark data shows that garages with higher self-elasticities and lower demand uncertainties benefit most from information sharing. These garages are likely to be the parking lots in busy downtown areas, whose demand is predictable but price-sensitive. For other garages, the benefit from information sharing is less significant but still positive. Numerical results suggest that information sharing tends to be attractive to garages and congestion improving in reality. The results provide evidence for Smarking to convince garage owners and city municipalities to participate in building such a centralized information sharing platform in the form of dashboards and mobile apps.

In the online supplement, we generalize the pricing competition model in various directions (e.g., involving garage capacity, noisy demand signal, garage coalition, and endogenous information structure). Most extensions confirm our findings from the base model. Future work is required to holistically understand the optimality of information sharing. Besides, incorporating imperfect (noisy and biased) forecasts of private demand will extend our work to better reflect reality.

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Supplemental Online Materials to "Competitive Spatial Pricing for Urban Parking Systems: Network Structures and Asymmetric Information" by Yuguang Wu, Qiao-chu He, and Xin Wang

A Extensions

A.1 Capacitated garages with multiple periods

Suppose that we model garage capacity explicitly and consider a planning horizon of multiple periods, for instance, the entire morning rush hour until the garages are filled up. We index time periods by $t = 1, 2, ..., T$. The baseline demand $\alpha^{[t]}$ as well as the random shock $\theta_1^{[t]}$ are both changing inter-temporally. The random shocks $\{\theta_i^{[t]}\}$'s are independent and identically distributed, with identical variance σ^2, and correlation $\rho^{[t]} = \rho, \forall t$. The underlying economy is stationary such that $\beta^{[t]} = \beta$ and $\hat{\beta}^{[t]} = \hat{\beta}, \forall t$. Garages are symmetric with the same capacity W. In this case, the garages' equilibrium is characterized by the following best-response functions:

$$\max_{p_1^{[t]}, t=1,2,...,T} \pi_1 = \sum_{t=1}^{T} d_1^{[t]} \left(p_1^{[t]} - c \right), \tag{25}$$

$$\max_{p_2^{[t]}, t=1,2,...,T} \pi_2 = \sum_{t=1}^{T} d_2^{[t]} \left(p_2^{[t]} - c \right), \tag{26}$$

$$\sum_{t=1}^{T} d_1^{[t]} \leq W_1, \sum_{t=1}^{T} d_2^{[t]} \leq W_2, \tag{27}$$

wherein

$$\begin{aligned} d_1^{[t]} &= \alpha^{[t]} - \hat{\beta} p_1^{[t]} + \beta p_2^{[t]} + \theta_1^{[t]}, \\ d_2^{[t]} &= \alpha^{[t]} + \beta p_1^{[t]} - \hat{\beta} p_2^{[t]} + \theta_2^{[t]}. \end{aligned} \tag{28}$$

We summarize the results for this extension in the following proposition.

Proposition 5. *The following results for the symmetric capacitated model over multiple periods hold:*

1. *When* $c > \frac{2\hat{\beta}-\beta}{\hat{\beta}(\beta-\hat{\beta})} \cdot \frac{W}{T} - \frac{\hat{\beta}}{\hat{\beta}(\beta-\hat{\beta})} \cdot \frac{\sum_{t=1}^{T} \alpha^{[t]}}{T}$, *the capacity constraints are not binding, the parking rates in equilibrium remain the same as in the static incapacitated model:*

$$p_i^{[t]} = \frac{\alpha^{[t]} + \hat{\beta} c}{2\hat{\beta} - \beta} + \frac{\theta_i^{[t]}}{2\hat{\beta} - \beta\rho}, \tag{29}$$

$\forall t = 1, 2, ..., T$ *and* $i = 1, 2$.

2. *Otherwise, the parking rates in equilibrium are given by*

$$\lim_{T\to\infty} p_i^{[t]} \to \frac{\alpha^{[t]}}{2\hat{\beta}-\beta} + \frac{W/T}{\left(\beta-\hat{\beta}\right)} - \frac{\hat{\beta}}{\left(2\hat{\beta}-\beta\right)\left(\beta-\hat{\beta}\right)} \cdot \frac{\sum_{t=1}^{T}\alpha^{[t]}}{T} + \frac{\theta_i^{[t]}}{2\hat{\beta}-\beta\rho}, \quad (30)$$

almost surely, $\forall t = 1, 2, ..., T$ *and* $i = 1, 2$.

3. *When* $W_1 \neq W_2$*, an one-period snapshot is equivalent to an incapacitated static model with heterogeneous cost* c_1 *and* c_2*, wherein* c_1 *and* $c_2 \geq c$.

4. *Compared with the static incapacitated version, while the baseline rates suffer from a constant downward distortion, the response to a private signal remains the same, and thus, the incentives for information sharing remains the same.*

5. *When* $\frac{\beta}{\hat{\beta}} \in (0, 1]$*, the parking rates as well as the aggregate payoff over* T *periods (*$\lim_{T\to\infty} \mathbb{E}\pi$*) decrease in total capacity* W*, but increase in average market potentials* $\frac{\sum_{t=1}^{T}\alpha^{[t]}}{T}$*. (The converse is true when* $\frac{\beta}{\hat{\beta}} \in (1, 2)$*.)*

From this proposition, as a robustness check, we are assured that the results from a single-period snap-shot extend naturally toward multiple-periods. The proof is via a dual approach wherein we use Lagrangian multipliers to calculate the shadow price of limited capacity. When $c > \frac{2\hat{\beta}-\beta}{\hat{\beta}\left(\beta-\hat{\beta}\right)} \cdot \frac{W}{T} - \frac{\hat{\beta}}{\hat{\beta}\left(\beta-\hat{\beta}\right)} \cdot \frac{\sum_{t=1}^{T}\alpha^{[t]}}{T}$, the cost of selling one unit capacity is less costly than its shadow price, and thus capacity constraint is not binding. In the same vein, when $W_1 \neq W_2$, an one-period snapshot is equivalent to an incapacitated static model with heterogeneous cost c_1 and c_2, wherein c_1 and $c_2 \geq c$. The additional costs $c_1 - c$ and $c_2 - c$ capture the shadow price for limited capacity.

In fact, if we extend the restriction of elasticities to $\beta < 2\hat{\beta}$ (allow the cross-price elasticity to be greater than price elasticity), the capacity W (or average market potentials $\frac{\sum_{t=1}^{T}\alpha^{[t]}}{T}$) poses opposite effect toward parking rates/payoffs when $\beta > \hat{\beta}$. In fact, let $\kappa = W/T - \frac{\beta}{\left(2\beta-\hat{\beta}\right)} \cdot \frac{\sum_{t=1}^{T}\alpha^{[t]}}{T}$ (which is increasing in total capacity W and decreasing in average baseline rates $\frac{\sum_{t=1}^{T}\alpha^{[t]}}{T}$) capture the extra capacity which satisfies per-period price-sensitive demand. When $\beta > \hat{\beta}$, i.e., price elasticity is more dominant than cross-price elasticity, the inverse price-demand relationship within a garage dominates. Since an higher extra capacity κ satisfies higher price-sensitive demand, this implies a lower parking rate. Therefore, the parking rates (and consequently the aggregate payoff) decrease in the extra capacity κ, and thus increases in W and decreases in $\frac{\sum_{t=1}^{T}\alpha^{[t]}}{T}$. Conversely, when $\beta > \hat{\beta}$, i.e., the strategic complement between two garages dominates, and thus an higher capacity κ implies higher parking rates from the other garage.

A.2 Noisy demand forecasting

In the basic model, we assume that private signals are received via a noiseless information channel. We find this to be a harmless assumption by examining real-time parking demand data, as private demand forecasts tend to be fairly accurate using historical data (with error rates around 5%). Nevertheless, we generalize the basic model in this section to incorporate noisy demand forecast. Suppose that there are two symmetric garages, $\beta_{11} = \beta_{22} = \hat{\beta}$, $\beta_{12} = \beta_{12} = -\beta$, and $c_1 = c_2 = c$:

$$\begin{aligned} d_1 &= \alpha - \hat{\beta} p_1 + \beta p_2 + \theta_1, \\ d_2 &= \alpha + \beta p_1 - \hat{\beta} p_2 + \theta_2, \end{aligned} \tag{31}$$

We further assume that $(\theta_1, \theta_2)^\top$ are drawn from symmetric bivariate normal distribution $\mathcal{N}\left(0, \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right)$. As a standard stability constraint, we require $0 \leq \beta < \hat{\beta}$. In addition, the garages cannot accurately forecast their demand. Instead, each garage observe a noisy signal $x_i = \theta_i + \epsilon_i$, where $\epsilon_i \sim N(0, \gamma^2)$. We assume observation channels are independent, i.e., $\epsilon_1 \perp \epsilon_2$.

Proposition 6. *The following statements are true for the symmetric uncapacitated model with two garages and noisy demand forecasting:*

1. *Baseline parking rate structure remains the same as that with perfect demand forecasting.*

2. *In addition, all comparative statics in Proposition 1 hold, i.e.,* $\frac{\partial B}{\partial \rho} > 0$, $\frac{\partial B}{\partial \sigma} > 0$, $\frac{\partial \mathbb{E}\pi}{\partial \hat{\beta}} < 0$, $\frac{\partial \mathbb{E}\pi}{\partial \beta} > 0$ *and* $\frac{\partial \mathbb{E}\pi}{\partial \rho} > 0$.

3. *Garages respond more aggressively toward private signals when they are more accurate, and the payoff increases, i.e.,* $\frac{\partial B}{\partial \gamma} < 0$ *and* $\frac{\partial \mathbb{E}\pi}{\partial \gamma} < 0$.

The first two statements confirm our structural results in the basic model with perfect demand forecasts, which serve as robustness check with general demand forecasting accuracy. Not surprisingly, garages respond more aggressively toward private signals when they are more accurate, and the payoff increases since the value of information increases. However, information sharing is not always desirable in general, and information sharing being unprofitable is more likely to happen when forecasting noise increases. Note that information sharing is always favorable if the demand correlation is positive, since the knowledge sharing helps garages reduce demand uncertainty and further take advantage of their monopoly power. Fixing forecasting accuracy and price elasticities, there is a negative region of demand correlation such that information sharing is not preferred. Usually, garages tend to positively respond to the other's signal when information is shared, i.e., $B_{12} > 0$, since the competitor's demand surge potentially increases her demand through cross-price elasticity. However, under some environment where demand signals are negatively correlated, they form an equilibrium where garages strongly negatively respond to the shared signal. In the long run, this causes both garages to price lower than the theoretical optimum. This underutilization of the demand on average reduces the expected payoff for both garages.

A.3 Garage coalition model

It is worth noting that multiple garages in an urban area may be controlled by a single entity and thus not independent in the pricing game. In this subsection, we show that our insights on information sharing do not heavily rely on the assumption of independent garages. The model in Section 3.4 can be generalized to a pricing competition among garage coalitions. (A garage coalition is a parking firm that owns several garages in the city.) We derive the pricing equilibrium of the coalition competition.

Consider $\mathcal{K}$ as a set of independent garage coalitions controlling all garages $N = \{1, 2, \cdots, n\}$. Each coalition $K \in \mathcal{K}$ corresponds to a subset of N, and $\mathcal{K}$ is a partition of N. Every coalition has access to a certain set of demand information, decides the prices of her garages, and maximizes her total expected payoff. Let $N(K)$ be the information index set of coalition K. That is, information θ_j can be used for the pricing of garage $i \in K$ if and only if $j \in N(K)$. Given information $\theta_{N(K)}$, every coalition K decides the price vector p_K to maximize her own expected total payoff

$$\mathbb{E}_\theta \left[\Pi(K) \middle| \theta_{N(K)} \right] = \mathbb{E}_\theta \left[\sum_{i \in K} \pi_i \middle| \theta_{N(K)} \right] = \mathbb{E}_\theta \left[\sum_{i \in K} d_i p_i \middle| \theta_{N(K)} \right]. \tag{32}$$

Proposition 7. *Suppose coalition K observes signals $\theta_{N(K)}$. Then, the equilibrium pricing strategy is given by $p(\theta) = A + \boldsymbol{B}\theta$, where coefficients $A \in \mathbb{R}^n$ and $\boldsymbol{B} \in \mathbb{R}^{n \times n}$ are determined by*

$$A = \boldsymbol{Q}^{-1} \left(\alpha + (\boldsymbol{Q} - \beta) c \right), \tag{33}$$

$$\begin{cases} Q_{K\star} \boldsymbol{B} \Sigma_{\star N(K)} - \Sigma_{KN(K)} = \boldsymbol{0}, \forall K \in \mathcal{K}, \\ B_{Kj} = 0, \forall j \notin N(K), \end{cases} \tag{34}$$

and

$$\boldsymbol{Q} := \beta + diag\left[\left(\beta_{KK}^\top \right)_{K \in \mathcal{K}} \right]. \tag{35}$$

The equilibrium payoff is given by

$$\mathbb{E}_\theta \left[\Pi(K) \right] = \langle A_K - c_K, \beta_{KK} (A_K - c_K) \rangle + \left\langle B_{K\star} \boldsymbol{\Sigma} B_{K\star}^\top, \beta_{KK} \right\rangle, \forall K \in \mathcal{K}. \tag{36}$$

where $\langle \cdot, \cdot \rangle$ is the Frobenius inner product of two vectors/matrices of the same dimension(s).

This proposition extends the pricing equilibrium (Proposition 2) to the case where garages are owned by competing coalitions. Here, Equations (33) & (34) are the extensions of Equations 8 & 9. The definition of $\boldsymbol{Q}$ has slightly changed. In (35), diag $\left[\left(\beta_{KK}^{\top}\right)_{K\in\mathcal{K}}\right]$ is an $n\times n$ block diagonal matrix whose main-diagonal blocks are square matrices $\beta_{KK}^{\top}, \forall K \in \mathcal{K}$. In particular, if every coalition K contains only one garage, this proposition reduces to Proposition 2. In Equation (36), $\left\langle B_{K\star}\boldsymbol{\Sigma}B_{K\star}^{\top}, \beta_{KK}\right\rangle$ corresponds to the information value for coalition K in the coalition model.

The connection between the original model and the coalition model lies in the dimensionality of the pricing decision. The coalition model is high-dimensional: Instead of deciding a single price p_i, each entity decides a set of prices p_K to maximize the total payoff $\sum_{i\in K}\pi_i$. The base model is a special case of a single dimension pricing. Therefore, we start from the base model without coalition formation to capture the main insight related to information sharing, which is robust shown in this extension. Furthermore, we will show in the next example additional insights derived from this coalition version.

Consider a symmetric duopoly setting where each garage coalition owns m garages in the city. The total $2m$ garages are assumed to have a symmetric influence on each other. Specifically, $\beta_{ii} = 1$, $\sigma_{ii} = 1$, and $\sigma_{ij} = \rho$, $\beta_{ij} = -\beta, \forall i \neq j$. (Note that Assumption 1 requires $\frac{1}{2m-1} \leq \rho \leq 1$ and $0 \leq \beta \leq \frac{1}{2m-1}\hat{\beta}$.) Then, under the coalition setting, we can show properties similar to the ones in Proposition 1. In Figure 6, we extend the 4th observation in Proposition 1 to the coalition competition. As the demand signals become more positively correlated, the information value (also the expected payoff since the baseline payoff is independent of ρ) increases. Figure 7 illustrate the additional value gain if information sharing is adopted. This result echoes Proposition 1 in that information sharing always generates positive value even if coalition formation is allowed. It also confirms that information sharing is more beneficial when the cross-elasticity β is higher. The value gain from sharing is non-monotone in the signal correlation. Because information is most useful when demand signals are weakly correlated. In the extreme case when demand signals are perfectly correlated, information sharing has 0 marginal value since competitor's information is already contained in the knowledge of demand correlation.

A.4 Optimal information assignment

Proposition 2 presents the exact equilibrium solution for an arbitrary observation matrix $\boldsymbol{M}$. A natural question one would be curious about is: among a set of possible information structures, which one of them would result in an equilibrium that maximizes the expected payoff of a particular garage (or their total expected payoff). The result for two-garage model in Section 3.3 indicates that information sharing is beneficial to both garages. However, this does not always hold in general.

From the perspective an information service provider, a natural question to ask is who should know what and how much they should know. The insights from answering

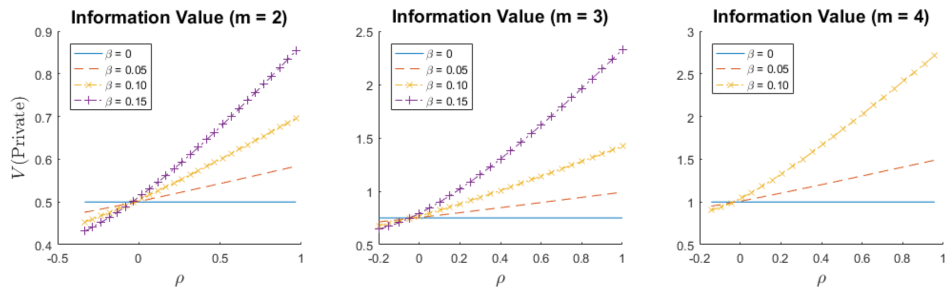


Figure 6: Information value under private information structure increases in the demand correlation ρ

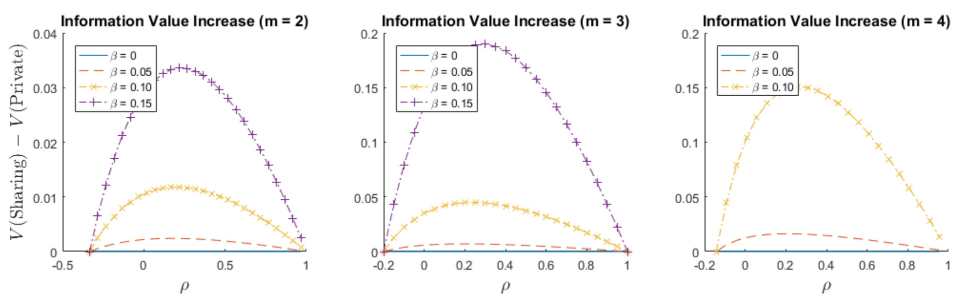


Figure 7: Information value increased from sharing

such questions facilitate the design of information systems. In general, to identify an information structure that maximizes the total information value, we need to solve the following optimization problem

$$\max_{B,M} \left\{ B_{i\star} \Sigma B_{i\star}^{\top} : \boldsymbol{B} \text{ and } \boldsymbol{M} \text{ satisfy (9)} \right\}. \tag{37}$$

This is a mixed integer program with quadratic objective function (convex-maximization). Or generally, we can get rid of binary decision variables M and rewrite it as an Quadratically Constrained Quadratic Program (QCQP)

$$\max_{\boldsymbol{B} \in \mathbb{R}^{n \times n}} \left\{ B_{i\star} \Sigma B_{i\star}^{\top} : (\boldsymbol{QB\Sigma} - \boldsymbol{\Sigma}) * \boldsymbol{B} = \boldsymbol{O} \right\}. \tag{38}$$

Solving the above programs awaits future computational studies, which in itself, is of great interests. In this research, we focus on the strategic aspects of this operational challenge. In Section A.5, we consider a certain type of symmetric information structure — group information sharing. Assume we have one (or multiple) information exchange platform. Every garage can choose to be a member of the platform and share her private signal within the group. In section A.5, we will see that such platform benefits their members and appeals to garages not in the group.

A.5 Information exchange platform

We return to the general model where all parameters can be asymmetric. In this subsection, we discuss the motivation for garages to form an information sharing group. Proposition 8 states that if all garages are using the information exchange platform, then no one has an incentive to quit from the group.

Proposition 8. *All agents joining the group is a Nash equilibrium.*

The equilibrium in Proposition 8 refers to a Nash equilibrium of the group entering strategy. Proposition 8 applies to arbitrary demand correlations as well as cross-elasticities, which is more general than existing literature (Raith, 1996) in this regard, to our best knowledge. It states that given every garage inside the platform, no one can achieve higher profit by unilaterally withdrawing from it. For a two-garage system, Proposition 8 apparently shows that sharing their signal is in the interest of both garages. For a system with more than 2 garages, Proposition 8 ensures that everyone in the group is an equilibrium. The study of the uniqueness and global optimality of such an equilibrium remains for future research.

B Proofs.

In this appendix, we provide detailed proofs of the main results. We use * in the proofs to denote component-wise multiplication of two matrices of identical dimensions.

Proof of Propositions 1 and 2.

Proof. Propositions 1 is a special case of Proposition 2. Here we prove Proposition 2.

Garage i maximizes expected payoff by taking first-order condition:

$$\frac{\partial \pi_i}{\partial p_i} = \left(\alpha_i - \sum_j \beta_{ij} p_j + \theta_i \right) + \beta_{ii} \left(p_i - c_i \right). \tag{39}$$

Garage i observes information θ_{N_i}. Then, her conditional expectation of the entire θ vector is given by

$$\mathbb{E} \left[\theta \left| \theta_{N_i} \right. \right] = \Sigma_{\star N_i} \Sigma_{N_i N_i}^{-1} \theta_{N_i}. \tag{40}$$

Therefore, her anticipation of the pricing vector p is

$$\mathbb{E} \left[p \left| \theta_{N_i} \right. \right] = E \left[A + B\theta \left| \theta_{N_i} \right. \right] = A + B \Sigma_{\star N_i} \Sigma_{N_i N_i}^{-1} \theta_{N_i}. \tag{41}$$

Setting the expected first-order condition to 0, namely $\mathbb{E} \left[\frac{\partial \pi_i}{\partial p_i} \left| \theta_{N_i} \right. \right] = 0$, we obtain

$$\left(\alpha_i - \beta_{i\star} \left(A + B \Sigma_{\star N_i} \Sigma_{N_i N_i}^{-1} \theta_{N_i} \right) + \Sigma_{i N_i} \Sigma_{N_i N_i}^{-1} \theta_{N_i} \right) - \beta_{ii} \left[A + B \Sigma_{\star N_i} \Sigma_{N_i N_i}^{-1} \theta_{N_i} - c \right]_i = 0. \tag{42}$$

Matching the coefficient of θ_{N_i}, we get

$$\left(-Q_{i\star} B \Sigma_{\star N_i} + \Sigma_{i N_i} \right) \Sigma_{N_i N_i}^{-1} \theta_{N_i} - Q_{i\star} A + \alpha_i - [\beta * I]_{i\star} c = 0, \tag{43}$$

where $*$ represents entry-wise multiplication, and $Q = \beta + \beta * I$ is the matrix defined in the proposition. This is a linear equation with respect to θ_{N_i}. And it holds for any θ_{N_i}. Thus, we can get the decision A, B by solving

$$\begin{cases} -Q_{i\star} A + \alpha_i + [\beta * I]_{i\star} c = 0 \\ -Q_{i\star} B \Sigma_{\star N_i} + \Sigma_{i N_i} = 0 \end{cases}, \forall i \in N. \tag{44}$$

Rewriting the first equation above in vector form, we have

$$A = Q^{-1} \left(\alpha + [\beta * I] \, c \right). \tag{45}$$

Together with the 0 entry constraint, we obtain the system of linear equations which determines our B matrix

$$\begin{cases} Q_{i\star} B \Sigma_{\star N_i} - \Sigma_{i N_i} = 0, \forall i \in N \\ B_{ij} = 0, \forall M_{ij} = 0 \end{cases}. \tag{46}$$

Note that, excluding those $B_{ij} = 0$ entries, we have $\sum_{i,j} M_{ij}$ unknowns and $\sum_{i} |N_i| = \sum_{i} M_{ij}$ equations. Thus, there exists at least one solution to this system. In general, the solution is unique.

The expected payoff of garage i conditioned on her observation θ_{N_i} is given by

$$\mathbb{E}\left[\pi_i \,|\theta_{N_i}\right] = E\left[\left(\alpha_i - \beta_{i\star} p + \theta_i\right)\left(p_i - c_i\right)|\theta_{N_i}\right] = \left(\alpha_i - \beta_{i\star} E\left[p\,|\theta_{N_i}\right] + \theta_i\right)\left(p_i - c_i\right), \quad (47)$$

which is a convex quadratic function of p_i. Thus, substitute $p = A + B\theta$ which satisfies the first-order condition, we obtain the expected payoff under equilibrium

$$\mathbb{E}\left[\pi_i \,|\theta_{N_i}\right] = \beta_{ii}(p_i - c_i)^2 = \beta_{ii}(A_i + B_{iN_i}\theta_{N_i} - c_i)^2. \quad (48)$$

The expected payoff before the observation of θ_{N_i} is

$$\mathbb{E}\left[\pi_i\right] = \mathbb{E}_{\theta_{N_i}}\left[\beta_{ii}(A_i + B_{iN_i}\theta_{N_i} - c_i)^2\right] = \beta_{ii}\left((A_i - c_i)^2 + B_{iN_i}\Sigma_{N_iN_i}B_{iN_i}^{\top}\right). \quad (49)$$

□

Proof of Lemma 2.

Proof. Rewrite equation (3) in a simpler format,

$$x_{ij} = \frac{l}{2} + \frac{\beta_0}{\lambda}\left(\left(v_i - v_j\right) - \left(p_i - p_j\right)\right). \quad (50)$$

The expectation of v is exogenously given. Thus, $\Delta\mathbb{E}\left[v\right] = 0$. Since $\mathbb{E}\left[\theta\right] = 0$, and followed from Proposition 2,

$$\mathbb{E}\left[p\right] = \mathbb{E}\left[A + \boldsymbol{B}\theta\right] = A.$$

A is independent of the information matrix M as stated in (8). Therefore, $\Delta\mathbb{E}\left[p\right] = 0$, and then $\Delta\mathbb{E}\left[x_{ij}\right] = 0$.

Note that

$$\begin{aligned} C_{ij} &= \mathbb{E}\left[\int_0^{x_{ij}} \left(c_t x + c_w \lambda x^2\right)\lambda dx + \int_0^{l - x_{ij}} \left(c_t x + c_w \lambda x^2\right)\lambda dx\right] \\ &= \lambda\mathbb{E}\left[\frac{1}{2}c_t x_{ij}^2 + \frac{1}{3}c_w\lambda x_{ij}^3 + \frac{1}{2}c_t\left(l - x_{ij}\right)^2 + \frac{1}{3}c_w\lambda\left(l - x_{ij}\right)^3\right] \\ &= \lambda\left(c_t + c_w\lambda l\right)\mathbb{E}\left[x_{ij}^2\right] + \text{constant term}. \end{aligned} \quad (51)$$

Thus, $\Delta C_{ij} = \lambda (c_t + c_w \lambda l) \cdot \Delta \{\text{var}[x_{ij}]\} = \frac{\lambda^2}{2\beta_0}\Delta\{\text{var}[x_{ij}]\}$. For the aggregate cost summed over all links,

$$\begin{aligned}\sum_{ij\in E}\text{var}[x_{ij}] &= \left(\frac{\beta_0}{\lambda}\right)^2 \sum_{ij\in E}\text{var}[(v_i - p_i) - (v_j - p_j)] \\ &= \left(\frac{\beta_0}{\lambda}\right)^2 \sum_{ij\in E}\left(\text{var}[v_i - p_i] + \text{var}[v_j - p_j] - 2\text{cov}(v_i - p_i, v_j - p_j)\right) \\ &= \left(\frac{\beta_0}{\lambda}\right)^2 \langle L, \text{var}[v-p]\rangle .\end{aligned} \tag{52}$$

Hence, $\Delta\left\{\sum_{ij\in E} C_{ij}\right\} = \frac{\beta_0}{2}\langle L, \text{var}[v-p]\rangle$.

Similarly,

$$\begin{aligned}U_{ij} + C_{ij} &= \mathbb{E}\left[\int_0^{x_{ij}} \lambda dx + \int_0^{l-x_{ij}} (v_j - p_j)\lambda dx\right] \\ &= \lambda \mathbb{E}\left[((v_i - p_i) - (v_j - p_j)) x_{ij}\right] + \text{constant term} \\ &= \frac{\lambda^2}{\beta_0}\mathbb{E}\left[x_{ij}^2\right] + \text{constant term} \\ &= 2C_{ij} + \text{constant term.}\end{aligned} \tag{53}$$

Thus, $\Delta U_{ij} = \Delta C_{ij}$. ∎

Proof of Proposition 3.

Proof. Both Proposition 3 and 4 are derived from solving (9) for $M = I_n$ and $M = ee^\top$. Then, (11) to solve the information value and (16) to obtain aggregate cost. We omit the algebra for solving B and present the solution directly.

In the private information scenario, $B_{ii} = \frac{3}{16}\beta_0^{-1}, \forall i$. Then,

$$v_i = 2\beta_0 B_{ii}^2 \Sigma_{ii} = \frac{27}{64}\beta_0. \tag{54}$$

$$\langle L, \text{var}[v-p]\rangle = \frac{29}{64}n \cdot \frac{\beta_0}{2}. \tag{55}$$

For the circular model, the B solution is symmetric, i.e., B_{ij} only depends on the distance between i and j but not i or j. Thus, we simply give the solution $B_{1\star}$.

$$B_{1\star} = \begin{cases} \frac{\beta_0^{-1}}{4y_{n/2} - 2y_{n/2-1}}\left[y_{n/2}, y_{n/2-1}, \cdots, y_1, y_0, y_1, \cdots, y_{n/2-1}\right], & \text{if } n \text{ is even,} \\ \frac{\beta_0^{-1}}{4y_{n/2} - 2y_{n/2-1}}\left[y_{n/2}, y_{n/2-1}, \cdots, y_{1/2}, y_{1/2}, \cdots, y_{n/2-1}\right], & \text{if } n \text{ is odd.}\end{cases} \tag{56}$$

Here y_k is a constant defined in Table 2.

Then, by manipulating the hyperbolic functions, the two cases merge to a single analytical format in terms of information value and aggregate cost.

$$v_i = \frac{2}{3(y_n - 1)} \left(\frac{1}{3}(4y_{n-1} + y_n) + n - 3 \right) \beta_0 \to \left(2 - \frac{8}{9}\sqrt{3} \right) \beta_0 \approx 0.4604\beta_0 \text{ as } n \to \infty. \quad (57)$$

$$\langle L, \text{varr} \rangle = \frac{2n}{3(y_n - 1)} \left(\frac{\bar{y}_{n-1} - \bar{y}_n}{\bar{y}_1} + y_n - n \right) \cdot \frac{\beta_0}{2\lambda} \to \frac{2n}{3\sqrt{3}} \cdot \frac{\beta_0}{2} \text{ as } n \to \infty. \quad (58)$$

□

Proof of Proposition 4.

Proof. For private information case, let index 1 denote the center garage. Then,

$$\begin{aligned} B_{11} &= \tfrac{2}{7m-1}\beta_0^{-1}, \\ B_{jj} &= \tfrac{3m-1}{7m-1}\beta_0^{-1}, \forall j \neq 1. \end{aligned} \quad (59)$$

The individual information values are,

$$\begin{aligned} v_1 &= \left(\tfrac{2m}{7m-1}\right)^2 (m+1)\beta_0, \\ v_j &= 2\left(\tfrac{3m-1}{7m-1}\right)^2 \beta_0, \forall j \neq 1. \end{aligned} \quad (60)$$

The aggregate information value is

$$v_1 + mv_j = \frac{2m\left(11m^2 - 4m + 1\right)}{(7m-1)^2}\beta_0 \to \frac{22}{49}m\beta_0 \text{ as } m \to \infty. \quad (61)$$

The aggregate cost is

$$\langle L, \text{var}[v - p] \rangle = \frac{4m^2(5m-3)}{(7m-1)^2} \cdot \frac{\beta_0}{2} \to \frac{20}{49}m \cdot \frac{\beta_0}{2} \text{ as } m \to \infty.$$

For the complete information case, we also list the intermediate and final solutions.

$$B = \left(\frac{1}{3m} \begin{bmatrix} 2 & e^\top \\ e & ee^\top/2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \right) \beta_0^{-1}. \quad (62)$$

$$\begin{aligned} v_1 &= \tfrac{1}{9}(m+1)\beta_0, \\ v_j &= \tfrac{13m-5}{36m}\beta_0, \forall i \neq j. \end{aligned} \quad (63)$$

$$v_1 + mv_j = \frac{17m - 1}{36}\beta_0 \to \frac{17}{36}m\beta_0 \text{ as } m \to \infty. \quad (64)$$

$$\langle L, \text{var}[v - p] \rangle = \frac{13m - 5}{36} \cdot \frac{\beta_0}{2}. \quad (65)$$

□

Proof of Proposition 5.

Proof. We begin by writing a general dual form, using Lagrangian multipliers λ_1 and $\lambda_2 > 0$ to relax the capacity constraints:

$$\begin{aligned}\max_{p_1^{[t]},t=1,2,...,T} L_1 &= \sum_{t=1}^{T}\left(\alpha^{[t]} - \hat{\beta}^{[t]}p_1^{[t]} + \beta^{[t]}p_2^{[t]} + \theta_1^{[t]}\right)\left(p_1^{[t]} - c\right) \\ &\quad -\lambda_1\left[W_1 - \sum_{t=1}^{T}\left(\alpha^{[t]} - \hat{\beta}^{[t]}p_1^{[t]} + \beta^{[t]}p_2^{[t]} + \theta_1^{[t]}\right)\right], \end{aligned} \tag{66}$$

$$\begin{aligned}\max_{p_2^{[t]},t=1,2,...,T} L_2 &= \sum_{t=1}^{T}\left(\alpha^{[t]} - \hat{\beta}^{[t]}p_2^{[t]} + \beta^{[t]}p_1^{[t]} + \theta_2^{[t]}\right)\left(p_2^{[t]} - c\right) \\ &\quad -\lambda_2\left[W_2 - \sum_{t=1}^{T}\left(\alpha^{[t]} - \hat{\beta}^{[t]}p_2^{[t]} + \beta^{[t]}p_1^{[t]} + \theta_2^{[t]}\right)\right]. \end{aligned} \tag{67}$$

We can decompose these problems by $t = 1, 2, ..., T$, and each sub-problem can by solved by

$$\begin{aligned} p_1^{[t]} &= \frac{\alpha^{[t]} + \hat{\beta}^{[t]}\left(c + \lambda_1\right)}{2\hat{\beta}^{[t]} - \beta^{[t]}} + \frac{\theta_1^{[t]}}{2\hat{\beta}^{[t]} - \beta^{[t]}\rho^{[t]}}, \\ p_2^{[t]} &= \frac{\alpha^{[t]} + \hat{\beta}^{[t]}\left(c + \lambda_2\right)}{2\hat{\beta}^{[t]} - \beta^{[t]}} + \frac{\theta_2^{[t]}}{2\hat{\beta}^{[t]} - \beta^{[t]}\rho^{[t]}}. \end{aligned} \tag{68}$$

Plug in prices

$$\begin{aligned} \sum_{t=1}^{T} \alpha^{[t]} - \hat{\beta}^{[t]}\left[\frac{\alpha^{[t]} + \hat{\beta}^{[t]}\left(c + \lambda_1\right)}{2\hat{\beta}^{[t]} - \beta^{[t]}} + \frac{\theta_1^{[t]}}{2\hat{\beta}^{[t]} - \beta^{[t]}\rho^{[t]}}\right] & \\ +\beta^{[t]}\left[\frac{\alpha^{[t]} + \hat{\beta}^{[t]}\left(c + \lambda_2\right)}{2\hat{\beta}^{[t]} - \beta^{[t]}} + \frac{\theta_2^{[t]}}{2\hat{\beta}^{[t]} - \beta^{[t]}\rho^{[t]}}\right] + \theta_1^{[t]} &= W_1, \end{aligned} \tag{69}$$

$$\sum_{t=1}^{T} \alpha^{[t]} - \hat{\beta}^{[t]}\left[\frac{\alpha^{[t]} + \hat{\beta}^{[t]}\left(c + \lambda_2\right)}{2\hat{\beta}^{[t]} - \beta^{[t]}} + \frac{\theta_2^{[t]}}{2\hat{\beta}^{[t]} - \beta^{[t]}\rho^{[t]}}\right] \tag{70}$$

$$+\beta^{[t]}\left[\frac{\alpha^{[t]} + \hat{\beta}^{[t]}\left(c + \lambda_1\right)}{2\hat{\beta}^{[t]} - \beta^{[t]}} + \frac{\theta_1^{[t]}}{2\hat{\beta}^{[t]} - \beta^{[t]}\rho^{[t]}}\right] + \theta_2^{[t]} = W_2, \tag{71}$$

under stationary conditions: $\hat{\beta}^{[t]} = \hat{\beta}$, $\beta^{[t]} = \beta$, $\rho^{[t]} = \rho$, we have

$$\begin{aligned}
\frac{\hat{\beta}}{2\hat{\beta}-\beta}\sum_{t=1}^{T}\alpha^{[t]}+\frac{T\hat{\beta}\beta}{2\hat{\beta}-\beta}(c+\lambda_2)-\frac{T\hat{\beta}^2}{2\hat{\beta}-\beta}(c+\lambda_1)+\frac{\hat{\beta}+\beta(1-\rho)}{2\hat{\beta}-\beta\rho}\sum_{t=1}^{T}\theta_1^{[t]} &= W_1,\\
\frac{\hat{\beta}}{2\hat{\beta}-\beta}\sum_{t=1}^{T}\alpha^{[t]}+\frac{T\hat{\beta}\beta}{2\hat{\beta}-\beta}(c+\lambda_1)-\frac{T\hat{\beta}^2}{2\hat{\beta}-\beta}(c+\lambda_2)+\frac{\hat{\beta}+\beta(1-\rho)}{2\hat{\beta}-\beta\rho}\sum_{t=1}^{T}\theta_2^{[t]} &= W_2 \quad (72)
\end{aligned}$$

When $W_1 = W_2 = W$, and $\lim_{T\to\infty}\frac{\sum_{t=1}^{T}\theta_i^{[t]}}{T} \to E\theta_i^{[t]}$ almost surely, due to Strong Law of Large Numbers, we have

$$\frac{\hat{\beta}}{2\hat{\beta}-\beta}\sum_{t=1}^{T}\alpha^{[t]}+\frac{T\hat{\beta}\left(\beta-\hat{\beta}\right)}{2\hat{\beta}-\beta}(c+\lambda)+\frac{\hat{\beta}+\beta(1-\rho)}{2\hat{\beta}-\beta\rho}\sum_{t=1}^{T}\theta_i^{[t]}=W,$$

$$\lambda=\frac{2\hat{\beta}-\beta}{\hat{\beta}\left(\beta-\hat{\beta}\right)}\cdot\frac{W}{T}-\frac{\left(2\hat{\beta}-\beta\right)\left[\hat{\beta}+\beta(1-\rho)\right]}{\hat{\beta}\left(\beta-\hat{\beta}\right)\left[2\hat{\beta}-\beta\rho\right]}\cdot E\theta_i^{[t]}-\frac{\hat{\beta}}{\hat{\beta}\left(\beta-\hat{\beta}\right)}\cdot\frac{\sum_{t=1}^{T}\alpha^{[t]}}{T}-c, \quad (73)$$

whenever this is non-negative. Plug this in the pricing strategies, and $E\theta_i^{[t]}=0$, we obtain

$$\begin{aligned}
p_1^{[t]} &= \frac{\alpha^{[t]}}{2\hat{\beta}-\beta}+\frac{W/T}{\left(\beta-\hat{\beta}\right)}-\frac{\hat{\beta}}{\left(2\hat{\beta}-\beta\right)\left(\beta-\hat{\beta}\right)}\cdot\frac{\sum_{t=1}^{T}\alpha^{[t]}}{T}+\frac{\theta_1^{[t]}}{2\hat{\beta}-\beta\rho},\\
p_1^{[t]} &= \frac{\alpha^{[t]}}{2\hat{\beta}-\beta}+\frac{W/T}{\left(\beta-\hat{\beta}\right)}-\frac{\hat{\beta}}{\left(2\hat{\beta}-\beta\right)\left(\beta-\hat{\beta}\right)}\cdot\frac{\sum_{t=1}^{T}\alpha^{[t]}}{T}+\frac{\theta_2^{[t]}}{2\hat{\beta}-\beta\rho}. \quad (74)
\end{aligned}$$

When $W_1 \neq W_2$, a similar procedure returns $\lambda_1 \neq \lambda_2$, a one-period snapshot is equivalent to an incapacitated static model with heterogeneous cost $c_1 = c+\lambda_1$, and $c_2 = c+\lambda_2$. For finite T,

$$p_i^{[t]}=\frac{\alpha^{[t]}+\hat{\beta}\left(c+\lambda_i\right)}{2\hat{\beta}-\beta}+\frac{\theta_i^{[t]}}{2\hat{\beta}-\beta\rho}, \quad (75)$$

wherein λ_i will be function of both $\theta_1^{[t]}$ and $\theta_2^{[t]}$, which is inconsistent, and thus we need $\mathbb{E}\left(p_2^{[t]}|\theta_1^{[t]}\right)$ to solve garage 1's maximization problem. This problem is fundamentally more complicated and awaits future research.

Alternatively, when

$$\frac{2\hat{\beta}-\beta}{\hat{\beta}\left(\beta-\hat{\beta}\right)}\cdot\frac{W}{T}-\frac{\left(2\hat{\beta}-\beta\right)\left[\hat{\beta}+\beta(1-\rho)\right]}{\hat{\beta}\left(\beta-\hat{\beta}\right)\left[2\hat{\beta}-\beta\rho\right]}\cdot E\theta_i^{[t]}-\frac{\hat{\beta}}{\hat{\beta}\left(\beta-\hat{\beta}\right)}\cdot\frac{\sum_{t=1}^{T}\alpha^{[t]}}{T}-c<0, \quad (76)$$

we have non-binding capacity constraint ($\lambda = 0$). This case is trivial, with

$$p_i^{[t]} = \frac{\alpha^{[t]} + \hat{\beta} c}{2\hat{\beta} - \beta} + \frac{\theta_i^{[t]}}{2\hat{\beta} - \beta\rho}. \tag{77}$$

Compared with the static incapacitated version:

$$\lim_{T\to\infty} p_i^{[t]} - p_i = \frac{W/T}{\left(\beta - \hat{\beta}\right)} - \frac{\hat{\beta}}{\left(2\hat{\beta} - \beta\right)\left(\beta - \hat{\beta}\right)} \cdot \frac{\sum_{t=1}^{T} \alpha^{[t]}}{T} - \frac{\hat{\beta} c}{2\hat{\beta} - \beta} < 0,$$

which means the response to a private signal remains the same, while the baseline rates suffer from a constant downward distortion. The aggregate payoff is

$$\lim_{T\to\infty} \mathbb{E}\pi = \sum_{t=1}^{T} \hat{\beta} \left(\left[\frac{\alpha^{[t]}}{2\hat{\beta} - \beta} - \left(c - \left[\frac{W/T}{\left(\beta - \hat{\beta}\right)} - \frac{\hat{\beta}}{\left(2\hat{\beta} - \beta\right)\left(\beta - \hat{\beta}\right)} \cdot \frac{\sum_{t=1}^{T} \alpha^{[t]}}{T} \right] \right) \right]^2 + \frac{\hat{\beta}\sigma^2}{(2\hat{\beta} - \beta\rho)^2} \right),$$

which is decreasing in W and increasing in $\frac{\sum_{t=1}^{T} \alpha^{[t]}}{T}$ when $\hat{\beta} > \beta$, the converse is true when $\frac{\beta}{2} < \hat{\beta} \leq \beta$. □

Proof of Proposition 6.

Proof. From Proposition 2, we can obtain the equilibrium pricing strategy $p_i = A + Bx_i, \forall i$ =1,2, and $\mathbb{E}\pi_1 = \mathbb{E}\pi_2 = \mathbb{E}\pi = \hat{\beta}\left((A-c)^2 + B^2\sigma^2\right)$. Garage i maximize expected payoff by taking first-order condition:

$$\frac{\partial \mathbb{E}\left(\pi_1 | x_1\right)}{\partial p_1} = \alpha - \hat{\beta} p_1 + \beta \mathbb{E}\left(p_2 | x_1\right) + \mathbb{E}\left(\theta_1 | x_1\right) - \hat{\beta}\left(p_1 - c\right), \tag{78}$$

Plug in $p_i = A + Bx_i$, $\mathbb{E}\left(\theta_1 | x_1\right) = \frac{1/\gamma^2}{1/\sigma^2 + 1/\gamma^2} x_1 = \frac{\sigma^2}{\sigma^2 + \gamma^2} x_1$, and

$$\mathbb{E}\left(x_2 | x_1\right) = \mathbb{E}\left(\theta_2 + \epsilon_2 | x_1\right) = \mathbb{E}\left(\theta_2 | x_1\right) = \mathbb{E}\left[\mathbb{E}\left(\theta_2 | \theta_1, x_1\right) | x_1\right] = \rho \mathbb{E}\left(\theta_1 | x_1\right) = \frac{\rho\sigma^2}{\sigma^2 + \gamma^2}. \tag{79}$$

The first-order condition becomes:

$$\alpha - 2\hat{\beta}\left(A + Bx_1\right) + \beta\left[A + B\frac{\rho\sigma^2}{\sigma^2 + \gamma^2} x_1\right] + \frac{\sigma^2}{\sigma^2 + \gamma^2} x_1 + \hat{\beta} c = 0. \tag{80}$$

Matching coefficients:

$$\alpha - 2\hat{\beta} A + \beta A + \hat{\beta} c = 0,$$

$$-2\hat{\beta}B+\beta B\frac{\rho\sigma^2}{\sigma^2+\gamma^2}+\frac{\sigma^2}{\sigma^2+\gamma^2}=0, \tag{81}$$

we have

$$A=\frac{\alpha+\hat{\beta}c}{2\hat{\beta}-\beta}, B=\frac{\sigma^2}{\left(2\hat{\beta}-\beta\rho\right)\sigma^2+2\hat{\beta}\gamma^2}. \tag{82}$$

To summarize:

$$p_i=\frac{\alpha+\hat{\beta}c}{2\hat{\beta}-\beta}+\frac{\sigma^2}{\left(2\hat{\beta}-\beta\rho\right)\sigma^2+2\hat{\beta}\gamma^2}\cdot x_i, \tag{83}$$

$$\begin{aligned}\mathbb{E}\pi &= \hat{\beta}\left[\frac{\alpha+\hat{\beta}c}{2\hat{\beta}-\beta}-c\right]^2+\left[\frac{\sigma^2}{\left(2\hat{\beta}-\beta\rho\right)\sigma^2+2\hat{\beta}\gamma^2}\right]^2\cdot\hat{\beta}\mathbb{E}x_i^2 && (84)\\ &= \hat{\beta}\left[\frac{\alpha-(\hat{\beta}-\beta)c}{2\hat{\beta}-\beta}\right]^2+\frac{\hat{\beta}\sigma^4(\sigma^2+\gamma^2)}{\left[\left(2\hat{\beta}-\beta\rho\right)\sigma^2+2\hat{\beta}\gamma^2\right]^2}. && (85)\end{aligned}$$

It can be checked that $\frac{\partial B}{\partial\rho}>0$, $\frac{\partial B}{\partial\sigma}>0$, $\frac{\partial B}{\partial\gamma}<0$, $\frac{\partial\mathbb{E}\pi}{\partial\hat{\beta}}<0$, $\frac{\partial\mathbb{E}\pi}{\partial\beta}>0$ and $\frac{\partial\mathbb{E}\pi}{\partial\rho}>0$.

Suppose that garages share information. We can obtain the equilibrium pricing strategy $p_1=A+B_1x_1+B_2x_2$, $p_2=A+B_2x_1+B_1x_2$. We have

$$\begin{aligned}\mathbb{E}\left(\theta_1|x_1,x_2\right) &= \mathbb{E}\left[\mathbb{E}\left(\theta_1|x_1,x_2,\theta_2\right)|x_1,x_2\right]\\ &= \mathbb{E}\left[\mathbb{E}\left(\theta_1|x_1,\theta_2\right)|x_1,x_2\right], && (86)\end{aligned}$$

with marginal distribution being $\theta_1|\theta_2\sim N(\rho\theta_2,\sigma^2\left(1-\rho^2\right))$,

$$\mathbb{E}\left(\theta_1|x_1,\theta_2\right)=\frac{\sigma^2\left(1-\rho^2\right)}{\sigma^2\left(1-\rho^2\right)+\gamma^2}\cdot x_1+\frac{\gamma^2\rho\theta_2}{\sigma^2\left(1-\rho^2\right)+\gamma^2}, \tag{87}$$

$$\begin{aligned}\mathbb{E}\left(\theta_1|x_1,x_2\right) &= \frac{\sigma^2\left(1-\rho^2\right)}{\sigma^2\left(1-\rho^2\right)+\gamma^2}\cdot\mathbb{E}\left(x_1|x_1,x_2\right)+\frac{\gamma^2\rho}{\sigma^2\left(1-\rho^2\right)+\gamma^2}\cdot\mathbb{E}\left(\theta_2|x_1,x_2\right)\\ &= \frac{\sigma^2\left(1-\rho^2\right)}{\sigma^2\left(1-\rho^2\right)+\gamma^2}\cdot x_1+\frac{\gamma^2\rho}{\sigma^2\left(1-\rho^2\right)+\gamma^2}\cdot\frac{\sigma^2}{\sigma^2+\gamma^2}\cdot x_2. && (88)\end{aligned}$$

$$E\left(\theta_1\,|x_1,x_2\,\right)=\frac{\sigma^2}{\left(\sigma^2+\gamma^2\right)^2-\left(\rho\sigma^2\right)^2}\left[\sigma^2\left(1-\rho^2\right)+\gamma^2,\rho\gamma^2\right]\left[\begin{array}{c}x_1\\x_2\end{array}\right].$$

Garage i maximize expected payoff by taking first-order condition:

$$\frac{\partial \mathbb{E}\left(\pi_1|x_1,x_2\right)}{\partial p_1} = \alpha - 2\hat{\beta}(A+B_1x_1+B_2x_2)+\beta\mathbb{E}\left(A+B_2x_1+B_1x_2|x_1,x_2\right)+\mathbb{E}\left(\theta_1|x_1,x_2\right)+\hat{\beta}c. \tag{89}$$

Matching coefficients:

$$\alpha - \left(2\hat{\beta}-\beta\right)A+\hat{\beta}c = 0 \Rightarrow A = \frac{\alpha+\hat{\beta}c}{2\hat{\beta}-\beta}, \tag{90}$$

$$-2\hat{\beta}(B_1x_1+B_2x_2)+\beta\mathbb{E}\left(B_2x_1+B_1x_2|x_1,x_2\right)+\frac{\sigma^2}{\left(\sigma^2+\gamma^2\right)^2-\left(\rho\sigma^2\right)^2}\left[\sigma^2\left(1-\rho^2\right)+\gamma^2,\rho\gamma^2\right]\left[\begin{array}{c}x_1\\x_2\end{array}\right]=0$$

$$-2\hat{\beta}B_1+\beta B_2+\frac{\sigma^2}{\left(\sigma^2+\gamma^2\right)^2-\left(\rho\sigma^2\right)^2}\cdot\left[\sigma^2\left(1-\rho^2\right)+\gamma^2\right]=0, \tag{91}$$

$$-2\hat{\beta}B_2+\beta B_1+\frac{\sigma^2}{\left(\sigma^2+\gamma^2\right)^2-\left(\rho\sigma^2\right)^2}\cdot\rho\gamma^2=0, \tag{92}$$

which gives

$$\begin{aligned} B_1 &= \frac{\gamma^2(2\hat{\beta}+\beta\rho)+2\hat{\beta}(1-\rho^2)\sigma^2}{4\hat{\beta}^2-\beta^2}\frac{\sigma^2}{\left(\sigma^2+\gamma^2\right)^2-\left(\rho\sigma^2\right)^2}, \\ B_2 &= \frac{\gamma^2(2\hat{\beta}\rho+\beta)+\beta(1-\rho^2)\sigma^2}{4\hat{\beta}^2-\beta^2}\frac{\sigma^2}{\left(\sigma^2+\gamma^2\right)^2-\left(\rho\sigma^2\right)^2}. \end{aligned} \tag{93}$$

Since $2\hat{\beta} > \beta$, garages respond positively toward signals.

$$\mathbb{E}\pi = \hat{\beta}\left[\frac{\alpha-(\hat{\beta}-\beta)c}{2\hat{\beta}-\beta}\right]^2+\hat{\beta}\sigma^2\frac{\sigma^2(\sigma^2+\gamma^2)}{\left[\left(2\hat{\beta}-\beta\rho\right)\sigma^2+2\hat{\beta}\gamma^2\right]^2}. \tag{94}$$

$$\mathbb{E}\pi' = \hat{\beta}\left[\frac{\alpha-(\hat{\beta}-\beta)c}{2\hat{\beta}-\beta}\right]^2+\hat{\beta}\sigma^2\left(B_1^2+B_2^2+2\rho B_1B_2\right). \tag{95}$$

Recall that $\mathbb{E}\left(\theta_1|x_1,x_2\right)$ has two parts, one associated with forecasting via x_1, the other associated with forecasting indirectly via x_2, since θ_1 and θ_2 is correlated. We can explicitly observe the corresponding information value in the expression of B_1 and B_2. It can be checked that $\mathbb{E}\pi' - \mathbb{E}\pi > 0$ as $\beta/\hat{\beta} \to 2$, i.e., information sharing is desirable when $\beta/\hat{\beta} \to 2$. More comprehensive characterization can be obtained when $\gamma \to 0$, as in Proposition 1. □

Proof of Proposition 7.

Proof. For conciseness, we present the proof of proposition using a two-coalition formulation. The proof naturally extends to the multiple-coalition cases.

We use a sequence of vector-matrix formulations to prove the proposition. To clarify the notations, $\langle \cdot, \cdot \rangle$ is the Frobenius inner product of two vectors/matrices of the same dimension(s); $z_K = [z]_K$ takes the subvector from vector z based on the index set K. Suppose the two coalitions are denoted by index sets K_1 and K_2

$$\Pi(K_1) = \langle d_{K_1}, p_{K_1} - c_{K_1} \rangle = \langle \alpha_{K_1} - \beta_{K_1\star} p + \theta_{K_1}, p_{K_1} - c_{K_1} \rangle .$$

Define $\boldsymbol{Q} := \boldsymbol{\beta} + \begin{bmatrix} \beta_{K_1K_1}^{\top} & O \\ O & \beta_{K_2K_2}^{\top} \end{bmatrix}$. Utilizing the expression of $\boldsymbol{Q}$ and the linear pricing $p = A + \boldsymbol{B}\theta$, we have Since $\mathbb{E}\left[\theta | \theta_{N(K_1)}\right] = \Sigma_{\star N(K_1)} \Sigma_{N(K_1)N(K_1)}^{-1} \theta_{N(K_1)}$, we have

$$\begin{aligned} \frac{\partial \Pi(K_1)}{\partial p_{K_1}} &= -\beta_{K_1K_1}^{\top}(p_{K_1} - c_{K_1}) + \alpha_{K_1} - \beta_{K_1\star} p + \theta_{K_1} \\ &= \left[-\boldsymbol{Q}p + (\boldsymbol{Q} - \boldsymbol{\beta}) c + \alpha + \theta\right]_{K_1}, \\ &= \left[-\boldsymbol{Q}A + (\boldsymbol{Q} - \boldsymbol{\beta}) c + \alpha + (\boldsymbol{I} - \boldsymbol{Q}\boldsymbol{B}) \theta\right]_{K_1}. \end{aligned}$$

$$\mathbb{E}\left[\left.\frac{\partial \Pi(K_1)}{\partial p_{K_1}}\right| \theta_{N(K_1)}\right] = \left[-\boldsymbol{Q}A + (\boldsymbol{Q} - \boldsymbol{\beta}) c + \alpha + (\boldsymbol{I} - \boldsymbol{Q}\boldsymbol{B}) \Sigma_{\star N(K_1)} \Sigma_{N(K_1)N(K_1)}^{-1} \theta_{N(K_1)}\right]_{K_1}.$$

Under the equilibrium pricing strategy, the R.H.S is 0 for every $\theta_{N(K_1)}$. Therefore, we have

$$\begin{aligned} \left[-\boldsymbol{Q}A + (\boldsymbol{Q} - \boldsymbol{\beta}) c\right]_{K_1} &= 0, \\ \Sigma_{K_1 N(K_1)} - Q_{K_1\star} \boldsymbol{B} \Sigma_{\star N(K_1)} &= 0. \end{aligned}$$

The same relations apply to K_2 as well. Thus, we obtain (33) and (34). (The 0 entries in $\boldsymbol{B}$ are enforced by the information structure.)

K_1's expected payoff given the information $\theta_{N(K_1)}$ is

$$\begin{aligned} \mathbb{E}\left[\Pi(K_1) | \theta_{N(K_1)}\right] &= \mathbb{E}\left[\langle \alpha_{K_1} - \beta_{K_1\star} p + \theta_{K_1}, p_{K_1} - c_{K_1} \rangle | \theta_{N(K_1)}\right] \\ &= \left\langle \mathbb{E}\left[\alpha_{K_1} - \beta_{K_1\star} p + \theta_{K_1} | \theta_{N(K_1)}\right], p_{K_1} - c_{K_1} \right\rangle \\ &= \left\langle \beta_{K_1K_1}^{\top}(p_{K_1} - c_{K_1}), p_{K_1} - c_{K_1} \right\rangle. \end{aligned}$$

The last equality follows from the equilibrium condition

$$\mathbb{E}\left[\left.\frac{\partial \Pi(K_1)}{\partial p_{K_1}}\right| \theta_{N(K_1)}\right] = \mathbb{E}\left[-\beta_{K_1K_1}^{\top}(p_{K_1} - c_{K_1}) + \alpha_{K_1} - \beta_{K_1\star} p + \theta_{K_1} \middle| \theta_{N(K_1)}\right] = 0.$$

Finally, the expected payoff is

$$\begin{aligned}\mathbb{E}_\theta\left[\Pi\left(K_1\right)\right] &= \mathbb{E}_\theta\left[\Pi\left(K_1\right)\middle|\,\theta_{N(K_1)}\right] \\ &= \mathbb{E}_\theta\left[\left\langle \beta_{K_1K_1}^\top\left(p_{K_1}-c_{K_1}\right), p_{K_1}-c_{K_1}\right\rangle \middle|\, \theta_{N(K_1)}\right] \\ &= \left\langle \beta_{K_1K_1}^\top\left(A_{K_1}-c_{K_1}\right), A_{K_1}-c_{K_1}\right\rangle + \mathbb{E}_\theta\left[\left\langle \beta_{K_1K_1}^\top B_{K_1\star}\theta, B_{K_1\star}\theta\right\rangle\right] \\ & \left\langle \beta_{K_1K_1}^\top\left(A_{K_1}-c_{K_1}\right), A_{K_1}-c_{K_1}\right\rangle + \left\langle B_{K_1\star}\boldsymbol{\Sigma}B_{K_1\star}^\top, \beta_{K_1K_1}\right\rangle.\end{aligned}$$

This is equivalent to (36) and we conclude the proof. □ □

Proof of Proposition 8.

Proof. Let $S = N\backslash\{n\}$ be the set group members, and n be the only agent outside the info-sharing group. We need to prove the information value v_n under this structure is less than the value when all agents are in the group.

B_{nn} satisfies

$$\begin{cases} Q_{SS}B_{SS}\Sigma_{SS} + Q_{Sn}B_{nn}\Sigma_{nS} - \Sigma_{SS} = 0 \\ Q_{nS}B_{SS}\Sigma_{Sn} + Q_{nn}B_{nn}\Sigma_{nn} - \Sigma_{nn} = 0 \end{cases}. \tag{96}$$

Eliminate B_{SS}, we get

$$B_{nn} = -\frac{Q_{nS}Q_{SS}^{-1}\Sigma_{Sn} - \Sigma_{nn}}{Q_{nn}\Sigma_{nn} - Q_{nS}Q_{SS}^{-1}Q_{Sn}\Sigma_{nS}\Sigma_{SS}^{-1}\Sigma_{Sn}}. \tag{97}$$

Thus,

$$v_n = \beta_{nn}B_{nn}^2\Sigma_{nn} = \beta_{nn}\left(\frac{Q_{nS}Q_{SS}^{-1}\Sigma_{Sn} - \Sigma_{nn}}{Q_{nn}\Sigma_{nn} - Q_{nS}Q_{SS}^{-1}Q_{Sn}\Sigma_{nS}\Sigma_{SS}^{-1}\Sigma_{Sn}}\right)^2\Sigma_{nn}. \tag{98}$$

If all agents are in the group, utilizing the inverse of block matrix, we have

$$\tilde{B}_{n\star} = \left[Q^{-1}\right]_{n\star} = -\frac{1}{Q_{nn} - Q_{nS}Q_{SS}^{-1}Q_{Sn}}\left[Q_{nS}Q_{SS}^{-1}, -1\right]. \tag{99}$$

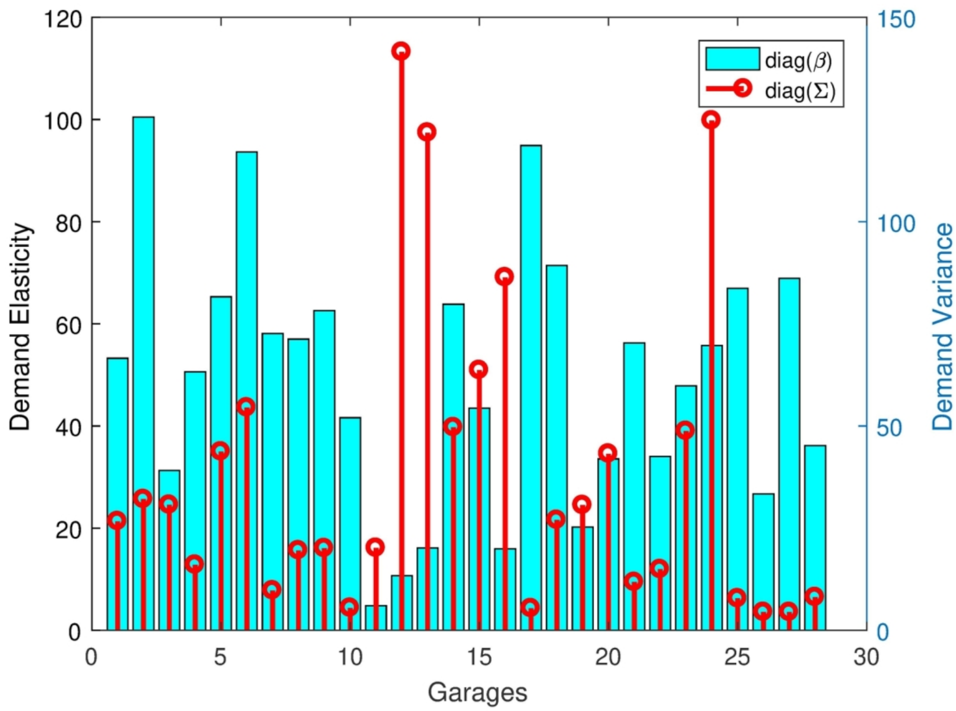
$$\tilde{v}_n = \beta_{nn}\tilde{B}_{n\star}\Sigma\tilde{B}_{n\star}^\top = \frac{Q_{nS}Q_{SS}^{-1}\Sigma_{SS}\left(Q_{nS}Q_{SS}^{-1}\right)^\top - 2Q_{nS}Q_{SS}^{-1}\Sigma_{Sn} + \Sigma_{nn}}{\left(Q_{nn} - Q_{nS}Q_{SS}^{-1}Q_{Sn}\right)^2}\cdot\beta_{nn} \tag{100}$$

Then, we prove $v_n < \tilde{v}_n$.

$$\begin{aligned}\frac{v_n}{\beta_{nn}} &= \left(\frac{Q_{nS}Q_{SS}^{-1}\Sigma_{Sn} - \Sigma_{nn}}{Q_{nn}\Sigma_{nn} - Q_{nS}Q_{SS}^{-1}Q_{Sn}\Sigma_{nS}\Sigma_{SS}^{-1}\Sigma_{Sn}}\right)^2\Sigma_{nn} = \frac{Q_{nS}Q_{SS}^{-1}\frac{\Sigma_{Sn}\Sigma_{nS}}{\Sigma_{nn}}\left(Q_{nS}Q_{SS}^{-1}\right)^\top - 2Q_{nS}Q_{SS}^{-1}\Sigma_{Sn} + \Sigma_{nn}}{\left(Q_{nn} - Q_{nS}Q_{SS}^{-1}Q_{Sn}\frac{\Sigma_{nS}\Sigma_{SS}^{-1}\Sigma_{Sn}}{\Sigma_{nn}}\right)^2} \\ &< \frac{Q_{nS}Q_{SS}^{-1}\frac{\Sigma_{Sn}\Sigma_{nS}}{\Sigma_{nn}}\left(Q_{nS}Q_{SS}^{-1}\right)^\top - 2Q_{nS}Q_{SS}^{-1}\Sigma_{Sn} + \Sigma_{nn}}{\left(Q_{nn} - Q_{nS}Q_{SS}^{-1}Q_{Sn}\right)^2} < \frac{Q_{nS}Q_{SS}^{-1}\Sigma_{SS}\left(Q_{nS}Q_{SS}^{-1}\right)^\top - 2Q_{nS}Q_{SS}^{-1}\Sigma_{Sn} + \Sigma_{nn}}{\left(Q_{nn} - Q_{nS}Q_{SS}^{-1}Q_{Sn}\right)^2} = \frac{\tilde{v}_n}{\beta_{nn}}.\end{aligned} \tag{101}$$

The first inequality follows from $\frac{\Sigma_{nS}\Sigma_{SS}^{-1}\Sigma_{Sn}}{\Sigma_{nn}} < 1$ (since Σ is positive definite) and Q_{nn}, $Q_{nS}Q_{SS}^{-1}Q_{Sn}$, $Q_{nn} - Q_{nS}Q_{SS}^{-1}Q_{Sn} > 0$. The second inequality holds since $\Sigma_{SS} - \frac{\Sigma_{Sn}\Sigma_{nS}}{\Sigma_{nn}}$ is positive definite.

□



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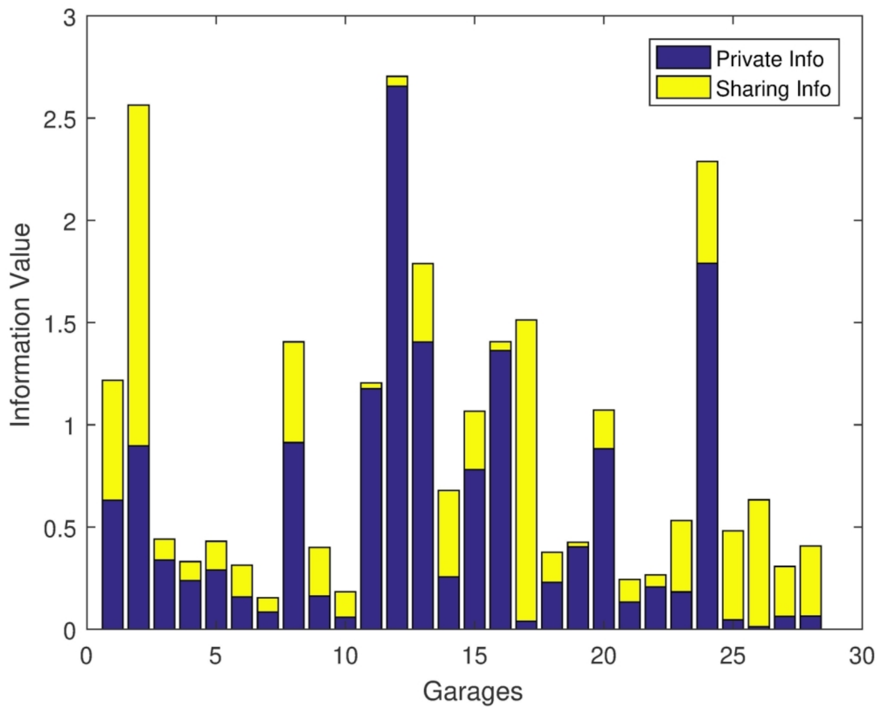
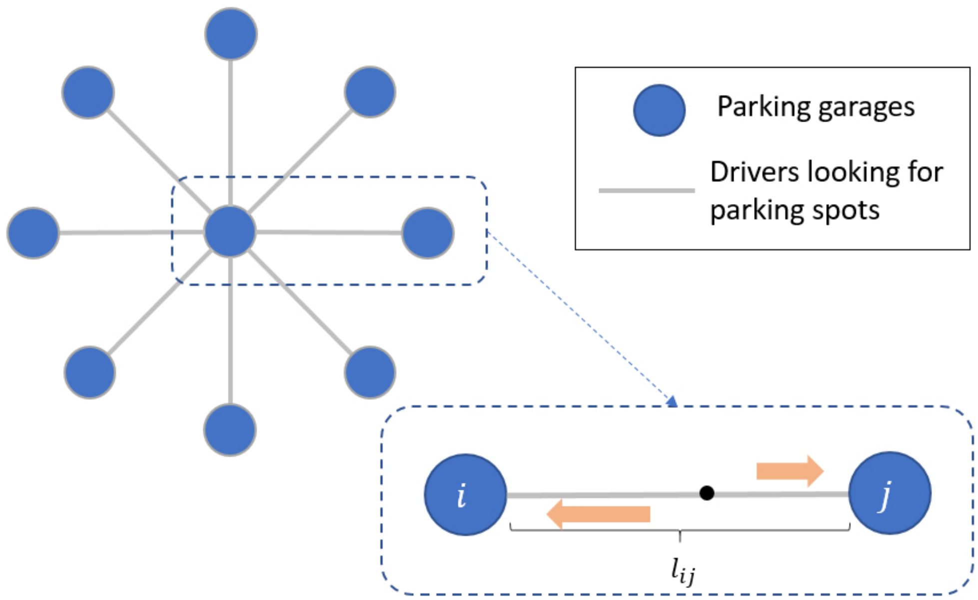
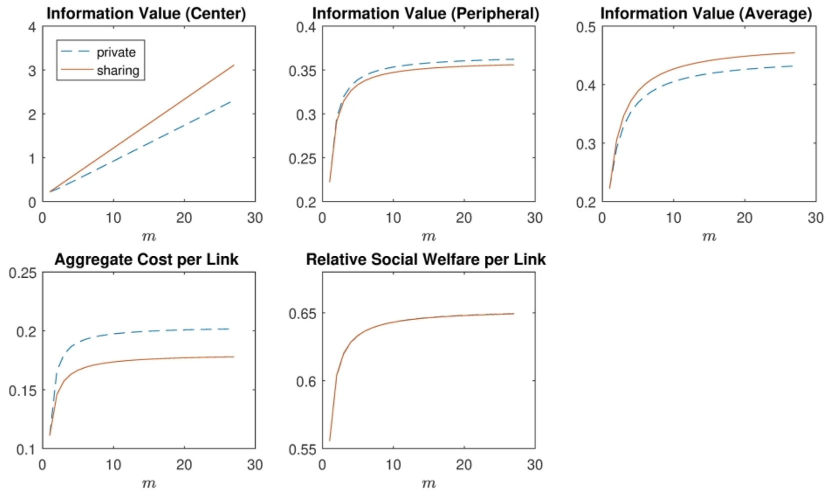


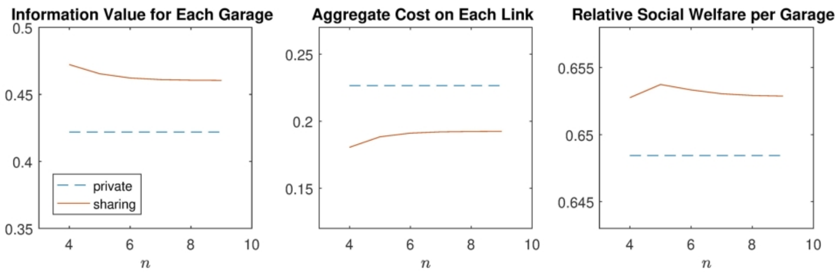
148x111mm (300 x 300 DPI)



582x358mm (38 x 38 DPI)



243x127mm (300 x 300 DPI)



243x66mm (300 x 300 DPI)