

Network Structure and its Impact on Commodity Markets[†]

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Connections across commodity markets create the potential for risk to propagate and for failures to cascade as successive market agents fail. The structure of these networks is, however, often hidden and not directly observable. This article describes methods to uncover this hidden structure and the implications that these hidden connections may have for predicting risk propagation and cascading failures. The results are described in the context of electricity, gasoline, and financial markets. They illustrate the potential of this methodology to help address energy and commodity policy issues and their environmental implications.

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1. Introduction

Many different types of commodities flow from sources to destinations across complex networks representing a variety of modes for delivery. In many cases, the structure of the network is not observable, for example, in the case of US electricity markets, due to the Critical Infrastructure Information Act of 2002 (United States Congress, 2002). In other cases, as is the case with gasoline, the variety of modes (e.g., pipeline, tanker, rail, and truck) and their flexibility make identification difficult. Even when current network structure is identifiable, additional information on unobservable transaction costs may be necessary to consider counterfactual analyses. This study describes approaches to use observable market information on transactions and prices to uncover hidden network structure and perceived transaction costs.

These investigations can be broadly seen as a form of structural modeling to identify critical parameters in optimization and equilibrium settings. Optimization methods can be directly applied in such settings (e.g., Su and Judd 2012). The optimization literature includes many approaches to identify

parameters, particularly for linear programs (e.g., Zhang and Liu (1996)). Mostly those parameters are objective or constraint right-hand side parameters, but they may also include left-hand side constraint coefficients that represent network structure (see Birge et al. 2017).

This study will describe three examples of identifying network structure from this inverse optimization approach and some of the implications from this identification for policy considerations. The first example involves electric power networks and follows the development in Birge et al. (2017), which uses an example in the Midcontinent Independent System Operator (MISO) network. The second example, which is described in more detail in Birge et al. (2020), includes the gasoline supply chain and focuses on the Southeastern United States portion of this supply network. The third example, which has more detail in Birge (2021), describes a network of financial holdings and particularly relates the discovery of network decisions and the influence of regulatory actions to the potential for cascading events. Overall, the examples illustrate the potential of this methodology to uncover the network structure and motivations of market participants that can in turn inform policy analyses, such as those focused on environmental concerns.

The paper is organized as follows. Section 2 presents some basic material on inverse optimization and the discovery of network structure. Section 3–5 then describe the three commodity examples of, respectively, electricity, gasoline, and financial assets. Section 6 then presents conclusions.

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2. Inverse Optimization Basics

In this section, we consider a basic model that can represent the choices of a firm or the equilibrium for the interactions of multiple agents assuming that a characterization of equilibrium (through a set of optimality conditions, e.g., under the conditions in Rosen (1965), or for a single objective in the case of potential games as in Monderer and Shapley (1996)). For ease of exposition, we consider here a single convex objective, $c(x)$, for minimization and a set of linear constraints, $Ax = b, x \geq 0$. The basic optimization model is then:

$$\min c(x) \quad (1)$$

$$\text{s. t. } Ax = b, \quad (2)$$

$$x \geq 0. \quad (3)$$

The Karush–Kuhn–Tucker optimality conditions for optimal primal solution x^* and optimal dual solution π^* are then:

$$Ax^* = b, x^* \geq 0, \quad (4)$$

$$\nabla c(x^*) - \pi^* A \geq 0, \quad (5)$$

$$(\nabla c(x^*) - \pi^* A)x^* = 0, \quad (6)$$

where $\nabla c(x^*)$ corresponds to the gradient of the objective function c at x^* , (4) corresponds to primal feasibility, (5) corresponds to dual feasibility, and (6) corresponds to complementarity.

The general approach for inverse optimization is to use observations of x^* and π^* to infer information about the objective $c(x)$ and the constraint coefficients A and b . In some cases, the observations may be for different instances, for example, constraint coefficients $b^t, t = 1, \dots, T$. If it is known that A (which may correspond to the network structure) is unchanged, then identification of the elements of A may be possible using observations of x^{*t} and π^{*t} for $t = 1, \dots, T$. In other cases, we may be able to identify coefficients of particular forms of $c(x)$ or to find ranges of these values. The next sections illustrate these identification processes in different contexts.

3. Electric Power Networks

This section follows the development in Birge et al. (2017), which considers identification of the MISO transmission network. In this case, MISO explicitly solves a model of the form in Equations (1)–(3), which is known as the economic dispatch problem, for each time interval (typically 5 minutes for the real-time or spot markets and 1 hour for a forward market known as the day-ahead market). The objective $c(x)$ is also released in the form of a piecewise linear bid (within

a feasible interval) or offer from each market participant. MISO also releases information on x^{*t} in the form of generation and consumption at different locations and π^{*t} in terms of *locational marginal prices* (LMPs) at generation and consumption nodes and congestion prices on binding constraints.

To specify the relationships in Equations (1)–(3), we suppose generation and consumption bids are described by a supply function c_j defined over a feasible interval $[\ell_j, u_j]$, where $c_j(x_j)$ is the announced total cost to participant j of producing x_j MWh. To allow for both generation and consumption, we will now allow x_j to become negative (i.e., $\ell_j < 0$). We also allow for multiple participants $j \in z(i)$ at each location i and let $b_i = \sum_{j \in z(i)} x_j$. We then re-write the constraints in terms of the net generation at each location and arrive at the following formulation:

$$\min_{x, b} \sum_{j \in Q} c_j(x_j) \quad (7)$$

$$\text{s.t. } b_i = \sum_{j \in z(i)} x_j \quad \forall i \in \mathcal{L}, \quad (8)$$

$$\sum_{i \in \mathcal{L}} (1 - \delta_i) b_i = 0, \quad (9)$$

$$Db \leq d, \quad (10)$$

$$x \geq \ell, \quad (11)$$

$$x \leq u. \quad (12)$$

We assume a set x^{*t} of observations of the generation and production quantities and (π^{*t}, ρ^{*t}) for observations of dual variables associated with constraints (9) and (10) at instances $t = 1, \dots, T$. Since the pricing results are identifiable relative to a given reference location r , we suppose that location r is fixed. To accommodate transmission losses δ_i in Equation (9), we suppose a relative productivity level, $q_i = \frac{1-\delta_i}{1-\delta_r}$, for each location i relative to r . We can then replace (9) by

$$\sum_{i \in \mathcal{L}} q_i b_i = 0. \quad (13)$$

and obtain the following result (Theorem 1 in Birge et al. 2017) which establishes a condition for the identifiability of the network structure parameters q , D and d .

THEOREM 1. *For an arbitrarily selected reference location r and a set of T samples of market outcomes $\{(x^{*t}, \pi^{*t}, \rho^{*t})_{t=1, \dots, T}\}$, consider equations:*

$$\sum_{k \in \mathcal{K}} D_{ki} \rho_k^{*t} = q_i \pi_r^{*t} - \pi_i^{*t}, \quad \forall i \in \mathcal{L}, t = 1, \dots, T, \quad (14)$$

$$\sum_{p \in z(i)} x_p^{*t} = b_i^{*t}, \quad \forall i \in \mathcal{L}, t = 1, \dots, T, \quad (15)$$

$$\sum_{i \in \mathcal{L}} q_i b_i^{t*} = 0; \quad (16)$$

then, D and q are identified if the above system of equations has rank $(|\mathcal{K}| + 1)|\mathcal{L}|$. The remaining unknown parameters of the primal program, d , may be recovered from $D_k b^{t*} = d_k$.

The results in Birge et al. (2017) use data from MISO to predict LMP (π) and allocation decisions (x) using the structure found from (14). On this basis, across a range of cases from 2010 and 2011, the results indicated a 92% correlation between the predictions and actual LMPs. These values can then be used to determine market reactions to external sources of supply and demand, changes in the transmission configuration, and other relevant issues for policy makers.

4. Gasoline Supply Network

In this section, we consider the identification of critical elements of the gasoline supply network using the model presented in Birge et al. (2020). In this case, we divide the variables in x into x_d , $d \in D$ to represent demand nodes and x_s , $s \in S$ to represent supply nodes. we then consider $-c_d(x_d)$ to represent the utility of consumption at d and $c_s(x_s)$ to represent the cost of supply at s . In addition to x_d and x_s , we also consider a set of flow decisions x_{ij} for all edges $ij \in E$ that connect nodes i and j in the supply network. We assume these are subject to a constant marginal cost c_{ij} and constraints $0 \leq x_{ij} \leq u_{ij}$ (where reverse flow from j to i would be represented as x_{ji} and could have the same or potentially different constraints).

The network yields a set of paths $\mathcal{P}_{(i,j)}$ that link i to j with a cost p_{ij}^q equal to the sum of the transportation costs along a path $q \in \mathcal{P}_{(i,j)}$ from i to j . Between any pair of nodes i and j , we can let $\mathcal{P}_{(i,j)}^*$ be the set of minimum-cost paths with cost p_{ij}^* . Finally, for a specific demand node $d \in D$, we let the set $\mathcal{S}_{(d)} \subseteq S$ denote the set of supply nodes with a directed path to d .

If we assume that this market is competitive, resulting in a welfare-maximizing allocation, then we have the following version of (1)–(3):

$$\begin{aligned} \min_x & - \sum_{d \in D} c_d(x_d) + \sum_{(i,j) \in E} c_{ij} x_{ij} + \sum_{s \in S} c_s(x_s) \\ \text{s.t.} & -x_d + \sum_{id \in E} x_{id} - \sum_{dj \in E} x_{dj} = 0, \quad \forall d \in D, \\ & x_s + \sum_{is \in E} x_{is} - \sum_{sj \in E} x_{sj} = 0, \quad \forall s \in S, \\ & 0 \leq x_{ij} \leq u_{ij}, \quad \forall (i,j) \in E, \\ & x_d \geq 0, \quad \forall d \in D, \\ & x_s \geq 0, \quad \forall s \in S. \end{aligned} \quad (17)$$

Equilibrium prices π associated with the equality constraints in (17), $\rho \geq 0$ associated with non-negativity of x_d and x_s , and $\sigma = \sigma^+ - \sigma^-$ ($\sigma^+ \geq 0$ and $\sigma^- \geq 0$) associated with the bounds on x_{ij} can be found from the optimality conditions as follows:

$$\pi_d = -c'_d(x_d) + \rho_d, \quad \forall d \in D, \quad (18a)$$

$$\pi_s = c'_k(x_k) - \rho_s, \quad \forall s \in S, \quad (18b)$$

$$\pi_j - \pi_i = c_{ij} - \sigma_{ij}^+ + \sigma_{ij}^-, \quad \forall (i,j) \in E, \quad (18c)$$

$$\sigma_{ij}^+ x_{ij} = 0, \quad \forall (i,j) \in E, \quad (18d)$$

$$\sigma_{ij}^- (u_{ij} - x_{ij}) = 0, \quad \forall (i,j) \in E, \quad (18e)$$

$$\rho_d x_d = 0, \quad \forall d \in D, \quad (18f)$$

$$\rho_s x_s = 0, \quad \forall s \in S, \quad (18g)$$

where c'_d and c'_k correspond to the first derivatives of c_d and c_k , respectively.

If we sum equation (18c) over a path q from node n_1 to n_2 that traverses links in a set E_q , this results in

$$\begin{aligned} \pi_{n_2} - \pi_{n_1} &= \sum_{(i,j) \in E_q} c_{ij} - \sum_{(i,j) \in E_q} \sigma_{ij}^+ + \sum_{(i,j) \in E_q} \sigma_{ij}^- \\ &= p_{n_1 n_2}^q - \sum_{(i,j) \in E_q} \sigma_{ij}^+ + \nu_{n_1 n_2}^q, \end{aligned} \quad (19)$$

where $p_{n_1 n_2}^q = \sum_{(i,j) \in E_q} c_{ij}$ represents the transportation cost from n_1 to n_2 along path q and $\nu_{n_1 n_2}^q = \sum_{(i,j) \in E_q} \sigma_{ij}^-$ denotes the sum of shadow prices along path q . From the optimality conditions, we then have:

$$\pi_{n_2} - \pi_{n_1} \leq p_{n_1 n_2}^q + \nu_{n_1 n_2}^q, \quad \forall q \in P(n_1, n_2), \quad (20)$$

$$\pi_{n_2} - \pi_{n_1} = p_{n_1 n_2}^q + \nu_{n_1 n_2}^q, \quad \forall q \in P^+(n_1, n_2), \quad (21)$$

where $P^+(n_1, n_2)$ is the set of paths from n_1 to n_2 for which there exists positive flow in the optimal market allocation.

Equations (20) and (21) imply that prices between any two locations differ by at most the transportation costs between those two locations if none of the links used for transportation between those two locations are at capacity (i.e., so that $\nu_{n_1 n_2}^q = 0$ for any path q with flow). The result is then that we can describe prices π_s^t at any location s and time t as a function of current market conditions (which we denote as η^t), the transportation cost for serving a location in an efficient allocation (given as ρ_s), the capacity of that transportation service (represented as an interval $[-\alpha(s), \alpha(s)]$), and surcharges (denoted w_s^t) that arise from congestion. These conditions are formalized in the following proposition (Proposition 3 in Birge et al. (2020)).

PROPOSITION 1. *The set of equilibrium prices for nodes $s \in S$ over a market with a fixed network structure and link costs can be expressed as*

$$\pi_s^t = \eta^t + \rho_s + \epsilon_s^t + w_s^t, \forall s \in S, t \in T, \quad (22)$$

where $\epsilon_s^t \in [-\alpha_s, \alpha_s]$ and $w_s^t \geq 0$ for some values η^t which depends on the market conditions at time t , ρ_s and α_s which depend on the network structure.

The implications of this proposition are that prices remain in a *neutral band* relative to one another (given by α_s) except in cases of network congestion in which the surcharges w_s^t may separate a given location's price from the neutral band. Birge et al. (2020) provides a procedure to estimate these quantities using the following optimization model:

$$\underset{\eta^t, \rho_s, \epsilon_s^t, w_s^t, \alpha_s \in S}{\text{minimize}} \sum \alpha_s \quad (23a)$$

$$\text{s.t. } \lambda_s^t = \eta^t + \rho_s + \epsilon_s^t + w_s^t, \forall s \in S, t \in T, \quad (23b)$$

$$|\epsilon_s^t| \leq \alpha_s, \forall s \in S, t \in T, \quad (23c)$$

$$w_s^t \in W_s^t, \forall s \in S, t \in T, \quad (23d)$$

where W_s^t is assumed to be $\{0\}$ for most periods (i.e., assumes that transportation capacity is mostly non-binding) and then can be unbounded in others. An additional set of constraints represented with integer variables then provides an overall mixed integer linear optimization model that identifies surcharges assuming a given fraction of time that surcharges occur.

The results in Birge et al. (2020) describe the application of this model to identify surcharge periods within the Southeastern United States. The results show that generally prices between any pair of locations remain within the neutral band but that certain events, such as the Colonial Pipeline disruption in 2016 and hurricanes during 2017, resulted in temporary surcharges for those locations that were most affected by the reductions in transportation capacity caused by these events. The results are of interest particularly for identifying locations for which the extent of alternative (non-pipeline) resources, such as ship, rail, and truck terminals for gasoline and the overall capacity of these resources, might not be as readily identified as with pipeline flows. For example, in the Colonial Pipeline disruption in 2016, while Nashville and Atlanta had significant and lasting surcharges according to the model, northern North Carolina and Virginia locations had little impact, a potential indication of alternative resources (e.g., the Plantation Pipeline and combinations of ship, rail, and truck transportation) that can serve these locations. In the case of the 2017 hurricanes, the impact of Hurricane Harvey, which made landfall in Texas, was most

severely felt with significant surcharges along the Colonial Pipeline between central Mississippi and central North Carolina, indicating their relative dependence on this resource compared to locations closer to ports and alternative resources. For Hurricane Irma, which made landfall in Florida that year, while Florida cities experienced surcharges, they were relatively minor and short-lived compared to the effects of Hurricane Harvey, suggesting that the resources (mostly tanker ship terminals) in this area are quite substantial and resilient to such events.

5. Global Financial Network

The network of global financial institutions creates a complex form of commodity network (with convertible currency as the commodity). In examining these networks, a key question is how much do institutions from each country hold of assets in other countries and how might the investments from these countries respond to changes in regulations that govern them and how might those responses impact the overall stability and resilience of the global financial network. Acharya (2009) and others note that regulatory measures should consider the institutions' endogenous response to requirements, but these reactions may be correlated and lead to a cascade of reactions. Birge (2021) provides a model for determining how institutions in different countries may react to regulatory changes by inferring their relative risk preferences and effective transaction costs for foreign investment, using their current cross-country investment level, building on the network model of Elliott et al. (2014) and using inverse optimization to identify institutional preferences and perceived costs for their investment. To identify the perceived risk preferences and costs for transacting in other locations, the approach follows the inverse optimization models given above.

Formally, the model assumes n institutions, $i = 1, \dots, n$, with cross-holdings c_{ij} (forming the matrix C) that represent the fraction of institution j owned by i (where $c_{ij} \geq 0$, $c_{ii} = 0$, and $\sum_{i=1}^n c_{ij} \leq 1$). The institutions also have holdings outside the network given as \hat{C} , a diagonal matrix such that $\hat{c}_{ii} = 1 - \sum_{j=1}^n \hat{c}_{ji}$ and $\hat{c}_{ij} = 0$ for $j \neq i$, m primitive assets with prices p_k , $k = 1, \dots, m$, that are held by k according to a matrix D with elements d_{ik} for the fraction of asset k owned by i .

With these assumptions, the vector w of values of each institution satisfies:

$$w = Dp + Cw, \quad (24)$$

which is solved (assuming $I - C$ is invertible) as

$$w = (I - C)^{-1} Dp. \quad (25)$$

In Elliott et al. (2014) and Birge (2021), the market value v_i of each organization i held by external investors is given as $v = \hat{C}w$ or

$$v = \hat{C}(I - C)^{-1}Dp, \quad (26)$$

or, for $A = \hat{C}(I - C)^{-1}$, $v = ADp$.

The procedure in Elliott et al. (2014) allows for cascades of defaults. The cascades result if the value v_i of any organization i falls below a critical value \bar{v}_i , so that the value of the primitive assets decreases by a fraction $0 < \beta_i < 1$ resulting in a loss $b_i(v, p) = \beta_i(p)\mathbf{1}_{v_i < \bar{v}_i}$. The equilibrium values including such potential losses then are given by:

$$v = \hat{C}(I - C)^{-1}(Dp - b(p, v)), \quad (27)$$

which, as Elliott et al. (2014) observes, has possibly many solutions, although a simple algorithm which iteratively updates $b(p, v)$ for each subsequent failures can achieve the *best-case* equilibrium in which the fewest number of organizations fail.

In Birge (2021), C is assumed to result from incentive-compatible choices of the institutions, which may change depending on the relative values of the assets and in reaction to requirements, such as capital requirements α . Writing C explicitly in terms of α , p , and v , yields the following updated version of (27):

$$v = \hat{C}(\alpha, p, v)(I - C(\alpha, p, v))^{-1}(Dp - b(p, v)), \quad (28)$$

for which the sequential algorithm of updating values for each subsequent default while simultaneously updating $C(\alpha, p, v)$ can obtain an equilibrium.

To determine how $C(\alpha, p, v)$ may change in response to requirements and values requires a model of the objectives (and constraints) of each institution so that their investment decisions are incentive compatible. This requires identifying how each organization responds to risk and what costs they perceive in placing investments with different organizations. The assumption used in Birge (2021) is that each institution maximizes a mean-variance utility function subject to constraints, such as capital requirements. The formulation includes their risk tolerance, given as a parameter γ_i for institution i , as well as perceived costs τ_{ij} for institution i to invest in j . Normalizing their assets to have unit value, i then determines amounts x_{ij} in each j (with x_i representing the vector (x_{i1}, \dots, x_{in})) with expected returns $r = (r_1, \dots, r_n)$, covariance matrix Σ , equity E_i with rate e_i , non-core liabilities (or short-term debt) S_i with rate n_i , and assuming deposits D_i with rate d_i to obtain the following optimization model:

$$\begin{aligned} & \max_{x_i, E_i, S_i} (r - \tau_i)^T x_i - \frac{\gamma_i}{2} x_i^T \Sigma x_i - d_i D_i - e_i E_i - n_i S_i \\ & \text{s. t. } \mathbf{1}^T x_i = 1; \\ & \quad \mathbf{1}^T x_i - E_i - S_i = D_i; \\ & \quad E_i \geq \alpha_i; \\ & \quad x_i, S_i \geq 0. \end{aligned} \quad (29)$$

If the capital constraint is binding, $E_i = \alpha_i \mathbf{1}^T x_i$, the optimality conditions for (29) can be written with $\mu_i \geq 0$:

$$(r - \tau_i) - (\gamma_i \Sigma + \alpha_i e_i \mathbf{1}^T) x_i + \mu_i = 0, \quad (30)$$

$$\mu_i^T x_i = 0, \quad (31)$$

where x can be scaled to ensure $\mathbf{1}^T x_i = 1$.

As shown in Birge (2021), the optimality conditions, now allow for identification of γ_i and τ_i assuming that Σ_i , x_i , r_i , α_i , and e_i can be observed by

$$\gamma_i = (r_i - \alpha_i e_i) / \Sigma_i x_i, \quad (32)$$

and

$$\tau_{ij} = r_j - \alpha_i e_i - \gamma_i \Sigma_j x_i. \quad (33)$$

These values can then be used for counterfactual analyses, such as changing capital ratio requirements and threshold levels \bar{v} , to iteratively solve for the equilibria in (28). Birge (2021) uses country cross-holding, similar to that in Elliott et al. (2014), for nine countries plus Japan, the United Kingdom, and the United States, using data from Bank of International Settlements (2015). The results in Table 1 show the sequence of defaults as $\bar{v} = \theta v_0$ for a given baseline value v_0 (2008 GDP) is increased from 0.68 to 1.14. The table lists the first set of institutions to default followed by subsequent cascades at a second level and in some cases a third level.

In these initial tests, the institutions were assumed to have their original capital ratios with no increase in capital requirements. A counterfactual exercise, assuming that the institutions maintain their perceived risk tolerance and costs to transact in other locations, can now be done by increasing capital requirements, for example, to α_1 where $\alpha_1(i) = \max(0.06, \alpha_0(i), i = 1, \dots, 9)$, which would entail additional equity holdings in France, Germany, Italy, Japan, and the United Kingdom. Birge (2021) provides the results for this case reproduced in Table 2. The results illustrate the contrasting effect of endogenous investment choices relative to fixed investments in Table 1. Now, instead of Portugal being the second market to suffer losses, Italy, Germany, and Japan fail before Portugal. The reaction of

Table 1 Failure Cascades for Increasing Failure Threshold Values θ with the Level of the Failures for the base Case of Original Capital Ratios α_0 for France (FR), Germany (DE), Greece (GR), Italy (IT), Japan (JP), Portugal (PT), Spain (ES), United Kingdom (GB), and United States (US)

Levels:	Critical Threshold Values θ									
	0.68	0.89	0.91	0.92	0.96	0.99	1.00	1.04	1.14	
First	GR	GR	GR, PT	GR, IT, PT	GR, IT, PT, ES	GR, IT, PT, ES	FR, GR, IT, PT, ES	FR, GR, IT, JP, PT, ES	All except US	
Second	—	PT	IT	ES	FR	FR	JP	DE	US	
Third	—	—	—	—	—	JP	DE	GB	—	

Table 2 Failure Cascades for Increasing Failure Threshold Values θ with the Level of the Failures for the case of Capital Ratios $\alpha_1(i) = \max(\alpha_0(i), 0.06)$, $i = 1, \dots, 9$, for France (FR), Germany (DE), Greece (GR), Italy (IT), Japan (JP), Portugal (PT), Spain (ES), United Kingdom (GB), and United States (US)

Levels:	Critical Threshold Values θ									
	0.65	0.74	0.80	1.00	1.01	1.04	1.08	1.11	1.14	
First	GR	GR	GR, IT	DE, GR, IT	DE, GR, IT	DE, GR, IT, JP, PT	DE, GR, IT, JP, PT, ES	DE, GR, IT, JP, PT, ES	DE, GR, IT, JP, PT, ES	
Second	—	IT	DE	JP	JP	ES	FR	FR	FR, GB	
Third	—	—	—	—	PT	—	—	GB	US	

investors from these countries has been to alter investments (in fact to make them riskier) in reaction to the requirement to hold more capital. The end result is that these institutions are then more exposed to default, an unintended consequence of the tighter capital restrictions.

6. Conclusions

The pricing, production, and consumption of commodities depends strongly on the structure of the network that transports these goods from sources to destinations and of the preferences of consumers and costs for producers and intermediaries. While many components of these systems are directly observable, many others are only revealed through the choices of participants or are hidden from direct observation. Inverse optimization or structural estimation using the conditions for optimality and equilibrium within these markets can be used to uncover unobserved preferences and network structure. This article has described the application of this approach in three situations: an electric power network with unobserved transmission connections, a gasoline network with unobserved transportation capacity, and a financial network with unobserved risk preferences and costs for inter-regional investments. In each case, using optimality conditions from the resulting equilibria lead to identification of critical parameters that can in turn be used to conduct counter-factual analyses as in the financial example that considers the effect of tightening capital requirements.

These examples indicate that a wide range of analyses within commodity networks may be considered

using this framework. One topic of immediate relevance is the evaluation and control of storage resources such as batteries for electric power networks. Making these decisions requires understanding the characteristics of the network, the potential actions of competitors, and the choices of participants, all of which might be uncovered through this identification approach. Many other such examples can benefit from these approaches. With these insights, we can hope to enable more informed decisions from policy makers who will determine the future evolution of commodity networks.

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