Model for Lateral Swelling Pressure in Unsaturated Expansive Soils

Masood Abdollahi, S.M.ASCE[1]; and Farshid Vahedifard, F.ASCE[2]

Abstract: Unsaturated expansive soils annually cause major economic losses and structural damages to foundations and earthen structures, primarily due to exhibiting large amounts of swelling and shrinkage when subjected to variations in water content. If an expansive soil is laterally constrained, lateral swelling pressure can exacerbate the unfavorable characteristics of expansive soil by increasing the lateral earth pressure applied to geotechnical structures. This study presents a model to determine lateral swelling pressure considering transient flow in deformable (swelling) unsaturated soils. The transient profiles of water content and suction versus depth during the wetting process are determined considering a coupled hydromechanical behavior. The interplay between the soil deformation, water content, and hydraulic conductivity is accounted for by linking the soil swelling-shrinkage characteristic curve (SSSCC), the soil water retention curve (SWRC), and the hydraulic conductivity function (HCF). An analytical model for quantifying lateral swelling pressure is then developed using a suction stress-based effective stress, the SWRC, and extended Hook's law for unsaturated soils. The presented model is built upon the conceptual model that lateral swelling pressure is controlled by infiltration-induced changes in suction stress of a laterally-constrained unsaturated soil. The model offers a generalized framework to determine lateral swelling pressure of expansive soils under different degrees of saturation while accounting for the effect of deformation on water content and suction profiles. The model is validated against results attained from three sets of experimental tests available in the literature. Through the three case studies examined, the proposed model exhibited a reasonable accuracy over a wide range of degrees of saturation from 50% to near fully saturated conditions. The findings of this study can contribute toward more accurate evaluations regarding the performance of geotechnical structures in unsaturated expansive soils. **DOI: 10.1061/(ASCE)GT.1943-5606.0002605.** *© 2021 American Society of Civil Engineers.*

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Introduction

The tendency of expansive soils to swell and shrink when subjected to wetting and drying cycles can lead to major issues and severe damages to several geotechnical engineering applications, including shallow and deep foundations, pavements, retaining and basement walls, and slope stability. The annual cost of damages caused by expansive soils in the United States has been reported to be over \$13 billion (Puppala and Cerato 2009). Expansive soils, which mainly include clayey and silty soils with considerable smectite clay minerals, are frequently encountered in several regions around the world and pose a major threat to the integrity of infrastructure systems and foundations. Due to the presence of smectite clay minerals with high water retaining capabilities (e.g., montmorillonite), expansive soils experience large amounts of swelling and shrinkage when subjected to wetting and drying processes. Expansive soils can deform in both lateral and vertical directions. If an expansive soil is laterally constrained, lateral swelling pressure will develop within the soil mass, corresponding to changes in the moisture content. Lateral swelling pressure can exacerbate the unfavorable characteristics of expansive soils by increasing lateral earth pressure applied to geotechnical structures, such as earth retaining structures and basement walls (Chen 1975; Nelson and Miller 1992; Ng et al. 2003; Ozer et al. 2012; Mohamed et al. 2014).

Earth retaining structures are primarily designed and analyzed by accounting for lateral earth pressure induced by overburden and surcharge (e.g., Vahedifard et al. 2015; Abdollahi et al. 2021). However, in expansive soils, lateral swelling pressure will be added to the other components of lateral earth pressure (Abdollahi and Vahedifard 2020). Such an increase in lateral earth pressure due to lateral swelling pressure reduces the safety margin of the earth retaining structure against, for example, sliding and overturning failure modes. Further, it is reported that lateral swelling pressure can adversely affect the integrity of foundations, basement walls, tunnels, and sewers (Liu and Vanapalli 2017). These observations have stimulated several studies involving experimental and numerical efforts to investigate the mechanism of water flow in expansive soils upon infiltration (Kim et al. 1992, 1999; Vu and Fredlund 2003, 2004, 2006) and the corresponding lateral swelling pressures (e.g., Komornik and Zeitlen 1965; Richards and Kurzeme 1973; Chen 1975; Puppala and Cerato 2009; Nelson et al. 2015) over the last few decades.

The main objective of this study is to evaluate lateral swelling pressure in expansive (deformable) soils subjected to transient flow conditions. For this purpose, a coupled hydromechanical approach is used to model the flow of water in deformable swelling soils and determine the changes in water content with time and depth. The coupled flow model considers the interplay between the soil deformation (i.e., changes in void ratio), water content,

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and hydraulic conductivity by linking the soil swelling-shrinkage characteristic curve (SSSCC), soil water retention curve (SWRC), and the hydraulic conductivity function (HCF). The outputs of the flow model are then used to determine suction and suction stress profiles directly. Finally, the proposed lateral swelling pressure model is built upon the conceptual model that lateral swelling pressure is controlled by infiltration-induced changes in suction stress of a laterally-constrained unsaturated soil. The model is validated against those attained from three sets of experimental tests available in the literature.

Background

Two main characteristics of unsaturated expansive soils (i.e., low permeability and high tendency to volume change) necessitate employing rigorous methods in the evaluation of water flow in these soils. Some early attempts were made to apply the classic Richards (1931) equation, originally developed to model water flow in nondeforming soils, to deforming expansive soils (e.g., Smiles and Rosenthal 1968; Raats and Klute 1969). However, it is well recognized that hydraulic properties of soil (e.g., water content and hydraulic conductivity) continuously change as the soil deforms (Kim et al. 1999; Nuth and Laloui 2008a; Su et al. 2020). Jaramillo (1989) and Jaramillo et al. (1990a, b) investigated the water flow in expansive soils under wetting conditions through a set of laboratory tests. Based on their testing measurements, Jaramillo (1989) proposed a model for hydraulic conductivity of deformable swelling soils through an Eulerian and Lagrangian framework. Kim et al. (1992, 1999) and Garnier et al. (1997) later extended the Richards (1931) equation by incorporating variations in the void ratio and proposed coupled hydromechanical models to capture changes in the water content of deformable expansive soils.

Several studies have introduced modifications to the traditional odometer and triaxial test setups to measure the swelling pressure in both vertical and lateral directions (e.g., Sapaz 2004; Xie et al. 2007; Abbas et al. 2015; Monroy et al. 2015). Large-scale tests and in situ investigations have also been employed to determine the lateral earth pressure while considering lateral swelling pressure effects (Katti et al. 1983; Brackley and Sanders 1992; Mohamed et al. 2014). Despite several experimental testing efforts, there are only a few studies in the literature presenting predictive models to determine lateral swelling pressure. Such predictive models are useful to calculate lateral swelling pressure and then incorporate it into the analysis and design of geotechnical structures adjacent to expansive soils. Ikizler et al. (2014) proposed a model using the artificial neural network and adaptive neuro-fuzzy inference systems to predict the transmitted vertical and lateral swelling pressure in expansive soils. Liu and Vanapalli (2017) proposed an analytical solution to calculate lateral swelling pressure using the vertical swelling pressure, which they referred to as the minimum vertical stress required to prevent the vertical swelling during soaking. Liu and Vanapalli (2019) modified the latter work to quantify lateral swelling pressure when the backfill is in an unsaturated state after infiltration. However, the Liu and Vanapalli (2019) model relies on the fully saturated condition as a reference point to calculate the changes in lateral swelling pressure between two unsaturated states.

The existing methods for the determination of lateral swelling pressure mainly require several empirical and semiempirical parameters (e.g., Jiang and Qin 1991; Hong 2008; Liu and Vanapalli 2019), which must be determined through relatively complex experiments, such as triaxial and swelling tests, or estimated based on engineering experience and judgment. Further, the majority of these models (Skempton 1961; Sudhindra and Moza 1987; Nelson et al. 2015; Liu and Vanapalli 2017) can only determine lateral swelling pressure when the soil becomes fully saturated, which is not commonly seen in unsaturated backfill soils in real-world conditions. Expansive soils experience the highest lateral swelling pressure when they are fully saturated. However, in field conditions, unsaturated expansive clays may take a long time to become fully saturated, or they may never reach a fully saturated condition. Such expansive soils undergo lateral swelling pressure during wetting while still under unsaturated conditions, which can affect their service state behavior. Further, no previous attempt has been made to analytically determine the lateral swelling pressure while accounting for the transient flow of water in deformable, unsaturated, expansive soils.

Conceptual Model and Formulation

Over the last few decades, several studies (e.g., Bolzon et al. 1996; Loret and Khalili 2002; Khalili et al. 2004; Nuth and Laloui 2008b; Sheng et al. 2003, 2004; Vahedifard et al. 2018, 2019) have demonstrated the validity of the effective stress concept to capture both shear strength and volume change behavior of unsaturated soils. Specific to the problem studied in the current paper (i.e., swelling), the effective stress principle is shown to offer a sound basis to describe and model the swelling and shrinkage behavior of expansive soils (Mašín and Khalili 2015; Nowamooz et al. 2016; Zhang and Lu 2020). In this study, we present an analytical solution for lateral swelling pressure by employing a suction stress-based representation of effective stress (Lu and Likos 2004, 2006). The validity of the suction stress concept to deal with volume change has been demonstrated through laboratory tests employing drying cake and vapor condensation techniques over a wide range of suction (Lu and Kaya 2012; Lu and Dong 2017; Zhang and Lu 2020; Dong and Lu 2021).

Suction stress is part of effective stress merely due to soil-water interaction (Zhang and Lu 2020). In this study, we conceptualize that lateral swelling pressure is primarily controlled by infiltration-induced changes in suction stress of a laterally constrained unsaturated soil. Setting the lateral deformation to zero allows one to capture the most critical condition (i.e., maximum lateral swelling pressure) as zero, or minima lateral swelling pressure will develop if the soil is totally free to move laterally. Suction stress lumps and upscales all interparticle forces, including physicochemical forces, surface tension forces, and forces arising from negative pore-water pressure (Lu and Likos 2006; Zhang and Lu 2020). Implementing suction stress into the proposed model allows one to independently account for the effect of matric suction and effective degree of saturation on lateral swelling pressure. Heave deformation, vertical, and lateral swelling pressures in expansive soils are shown to directly correspond to infiltration-induced changes in suction between initial and final conditions (Cui et al. 2002; Hong 2008; Tu and Vanapalli 2016). However, several studies (e.g., Khalili et al. 2004; Zhou et al. 2012; Lu and Kaya 2014) have shown that the hardening/softening behavior of unsaturated soils is governed not only by matric suction but also by a degree of saturation. Matric suction can be measured or determined along the depth by knowing the initial condition and flow rate with respect to time. Expansive soils are shown to behave differently than nonswelling soils in transporting water (Garnier et al. 1997). Thus, when investigating the flow of water and the subsequent change in the matric suction in expansive soils, it is important to consider the effect of the changing volume and void ratio on the hydraulic properties of the soil.

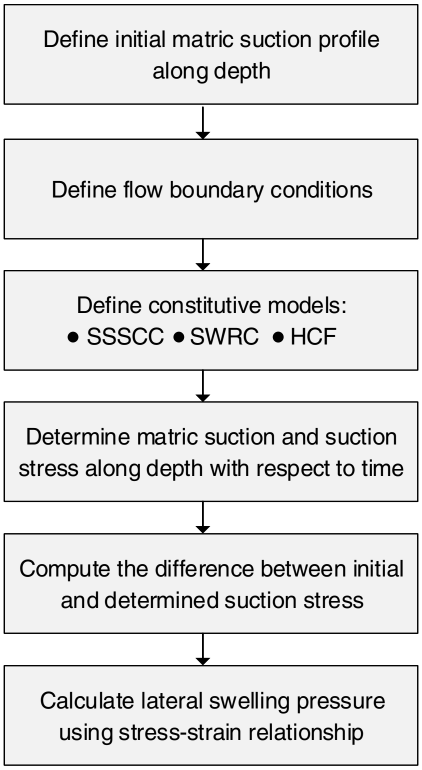

Fig. 1. Flowchart of the proposed model for determining lateral swelling pressure in expansive soils during infiltration.

In other words, a coupled hydromechanical approach is required to describe the flow of water in deformable swelling soils.

Based on the aforementioned conceptual model, we developed an analytical model to determine lateral swelling pressure by integrating the following components:

- A coupled hydromechanical model is employed to determine the changes in water content in expansive soils subject to infiltration under varying degrees of saturation. The flow model considers the interplay between the soil deformation (changes in void ratio) and water content, and the hydraulic conductivity by linking SSSCC, SWRC, and HCF. Further, the model is developed considering the fact that displacement boundary conditions will influence the flow behavior. Because the maximum lateral swelling pressure is mobilized when the soil is restricted in the horizontal direction, a one-dimensional fluid flow is considered in which the soil is restrained horizontally and only allowed to deform vertically.
- The suction and suction stress profiles are directly determined using the output of the flow model. The suction stress-based representation of the effective stress is used to characterize the effective stress at different matric suctions and degrees of saturation.
- Hooke's law extended to unsaturated conditions is used to establish a stress-strain relationship for a laterally constrained unsaturated soil element.
- Finally, the stress-strain relationship is used along with the suction stress profiles obtained in the previous steps to develop an analytical model for the determination of lateral swelling pressure under different flux boundary conditions and degrees of saturation. The proposed lateral swelling pressure model is built upon the conceptual model that lateral swelling pressure is dominated by infiltration-induced changes in suction stress of a laterally-constrained unsaturated soil.

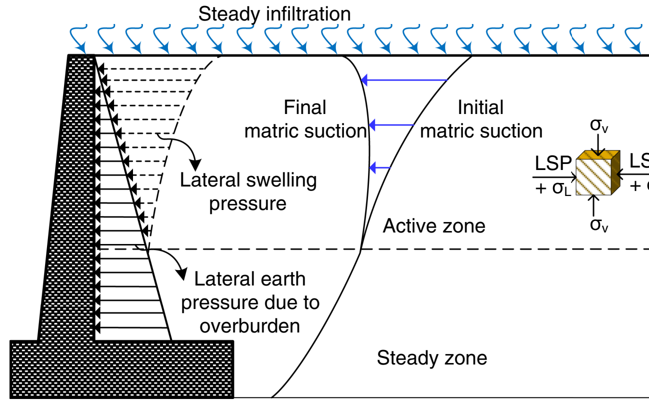

Fig. 2. Components of lateral earth pressure due to overburden and swelling in expansive soils under transient unsaturated flow.

The aforementioned process is illustrated in Fig. 1. Each step will be further explained in detail throughout the paper. Fig. 2 schematically shows different components of lateral earth pressure, including lateral swelling pressure, in an unsaturated expansive backfill soil subject to infiltration. The following sections provide detailed derivation and formulations for each of these components.

Suction Profiles versus Depth in Expansive Soils under Transient Flow

Richards' equation (Richards 1931) is commonly used to describe transient water flow in unsaturated soils. The one-dimensional form of this equation for expansive soils can be written (Kim et al. 1999)

$$\frac{\partial \vartheta}{\partial t} = \frac{\partial}{\partial z}\left[k\frac{\partial \emptyset}{\partial z}\right] \quad (1)$$

where ϑ = moisture ratio; t = time; $\emptyset$ = total water head; z = vertical coordinate and positive upward; and k = hydraulic conductivity. The moisture ratio is defined as V_w/V_s, where V_w is the volume of water, and V_s is the volume of solids, or it can be calculated as $\omega \times G_s$, where ω is the gravimetric water content, and G_s is the specific gravity of solid particles. The moisture ratio is extensively used as a primary variable for studying expansive soils (Kim et al. 1999; Lu and Dong 2017; Chen and Lu 2018) because it provides a convenient measure of water content while incorporating changes in the void ratio (as needed in expansive soils).

Eq. (1) describes the movement of water in the Eulerian coordinate frame in which the water flows in a porous media with respect to a fixed location in space. However, using the Eulerian approach to define water flow in expansive soils may lead to inaccurate results because the soil continuously changes its volume when it absorbs water. In this case, it is important to account for changes in the soil's hydraulic properties with respect to its deformation (Sposito and Giráldez 1976; Garnier et al. 1997). To address this issue, one can approach the problem through the Lagrangian perspective. Following the Lagrangian framework, Kim et al. (1999) proposed a modified version of Richards' equation as follows

$$\frac{\partial \vartheta}{\partial t} = \frac{\partial}{\partial m}\left[k^*\frac{\partial \emptyset}{\partial m}\right] \quad (2)$$

where e = void ratio; and m = material coordinate whose derivative is defined

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$$dm = \frac{dz}{1+e} \quad (3)$$

and $k^* = k/(1+e)$ is the hydraulic conductivity in the deformed soil and can be derived using either the Eulerian method (Kim et al. 1999), with respect to moving particles, or the Lagrangian method using material diffusivity, D_m (Philip 1969)

$$k^* = k\left[1 - (\theta/\theta_s)(D_s/D_w)\left(\frac{\partial\theta_s}{\partial\theta}\right)\right] \quad (4)$$

$$k^* = D_m\left(1 - \frac{\theta}{\theta_s}\frac{d\theta_s}{d\theta}\right)\left(\theta_s^{-2}\frac{\partial\theta}{\partial h}\right) \quad (5)$$

where θ = volumetric water content; θ_s = volumetric solid content; D_s and D_w = apparent capillary diffusivity of pore water and solid phase; D_m = material diffusivity; and h = matric suction head defined

$$h = \frac{(u_a - u_w)}{\rho_w g} \quad (6)$$

where u_a = air pressure; u_w = pore-water pressure; the term $(u_a - u_w)$ = matric suction; ρ_w = density of water; and g = gravitational acceleration.

The total head in Eq. (2), Ø, can be written as a summation of the capillary and gravitational components. According to Philip (1969), for shrinking and swelling soils, an additional component, namely, overburden head (Ω), needs to be included in the total head

$$\emptyset = h + z + \Omega \quad (7)$$

where Ω = required movement of the particles against gravity and any other external surcharge and can be determined as follows

$$\Omega = \left(\frac{de}{d\vartheta}\right)p(z) \quad (8a)$$

$$p(z) = p(0) - \int \gamma dz \quad (8b)$$

$$\gamma = \frac{\vartheta\rho_w + \rho_S}{1+e} \quad (8c)$$

where $p(z)$ = total vertical stress at a depth of z; $p(0)$ = external load on the soil surface; γ = apparent wet specific density of soil; and ρ_s = specific density of particles.

Darcy's law can be used to determine discharge (q) for the deforming soil

$$q = -k^*\nabla\emptyset \quad (9)$$

Combining a one-dimensional vertical form of Eq. (9) with Ø defined by Eq. (7), Kim et al. (1999) introduced the general water flow equation in deformable swelling soils under transient infiltration as follows

$$\frac{\partial\vartheta}{\partial t} = \frac{\partial}{\partial m}\left[\frac{k}{1+e}\left(\frac{dh}{dm} + SF1 + SF2\frac{\partial\vartheta}{\partial m}\right)\right] \quad (10a)$$

$$SF1 = (1+e) - \left(\vartheta + \frac{\rho_s}{\rho_w}\right)\frac{de}{d\vartheta} \quad (10b)$$

$$SF2 = \frac{d^2e}{d\vartheta^2}\int_0^m\left(\vartheta + \frac{\rho_s}{\rho_w}\right)dm \quad (10c)$$

where $SF1$ = components of gravitational $(1+e)$ and the overburden head $[\vartheta + (\rho_s/\rho_w)](de/d\vartheta)$; and $SF2$ = component of the overburden head arising from the second-order derivatives of the SSSC. The term $SF2$ is zero during the normal shrinkage and swelling phase (Kim et al. 1992). Thus, Eq. (10a) can be reduced

$$\frac{\partial\vartheta}{\partial t} = \frac{\partial}{\partial m}\left[\frac{k}{1+e}\left(\frac{dh}{dm} + SF1\right)\right] \quad (11)$$

Solving Eq. (11) requires its transformation into a simpler form with a single dependent variable. Using the chain rule of calculus, the left-hand side of Eq. (11) can be written

$$\frac{\partial\vartheta}{\partial t} = C(\vartheta)\frac{\partial h}{\partial t} \quad (12)$$

where $C(\vartheta)$ = differential soil water capacity. Substituting Eq. (12) into Eq. (11) yields

$$C(\vartheta)\frac{\partial h}{\partial t} = \frac{\partial}{\partial m}\left[\frac{k}{1+e}\left(\frac{dh}{dm} + SF1\right)\right] \quad (13)$$

Eq. (13) can be solved using the finite-difference method by discretizing h with respect to t and m. In this study, the implicit discretization scheme with an explicit linearization of C, k^*, and $SF1$ is adopted. Solving the equation requires the knowledge of the initial condition and two boundary conditions. Neumann and Dirichlet conditions are assigned to the top and bottom boundaries, respectively. In this study, the cell-centered finite-difference method is used to solve Eq. (13) numerically.

To solve Eq. (13), one needs to know the constitutive equations governing the hydraulic properties of unsaturated expansive soil. In the current study, van Genuchten's (1980) SWRC model is adopted to define the relationship between the matric suction and moisture ratio

$$S_e = \frac{\vartheta - \vartheta_r}{\vartheta_s - \vartheta_r} = [1 + (\alpha(u_a - u_w))^n]^{((1/n)-1)} \quad (14)$$

where S_e = effective saturation; ϑ_r (m^3/m^3) and ϑ_s (m^3/m^3) = residual and saturated moisture ratios, respectively; and α (1/kPa) and n = fitting parameters. While the drying path of the SWRC is commonly measured in laboratory tests, the wetting path of the SWRC needs to be used when studying lateral swelling pressure of expansive soils. As demonstrated by several studies, the wetting SWRC can be obtained from the drying SWRC by adjusting the fitting parameters of the SWRC model (e.g., Likos et al. 2014). While any SWRC model can be implemented in the proposed analytical framework, the van Genuchten (1980) model has been employed in this study because of its simplicity (requiring only two fitting parameters) and the extensive use in engineering applications and practice. It is recognized that the van Genuchten (1980) model does not explicitly distinguish adsorption and capillary water retention mechanisms, which may result in unrealistic suction values when the soil is approaching dry conditions (Nitao and Bear 1996). However, lateral swelling pressure generally starts to develop when the infiltration increases the soil saturation from the in situ degree of saturation to fully saturated or near-saturated conditions. Thus, using the SWRC models that explicitly consider adsorption is not critical in most cases when dealing with lateral swelling pressure problems.

Using the HCF proposed by Brooks and Corey (1964), hydraulic conductivity can be expressed as a function of the effective degree of saturation

$$k(\vartheta) = k_s(S_e)^\beta \quad (15)$$

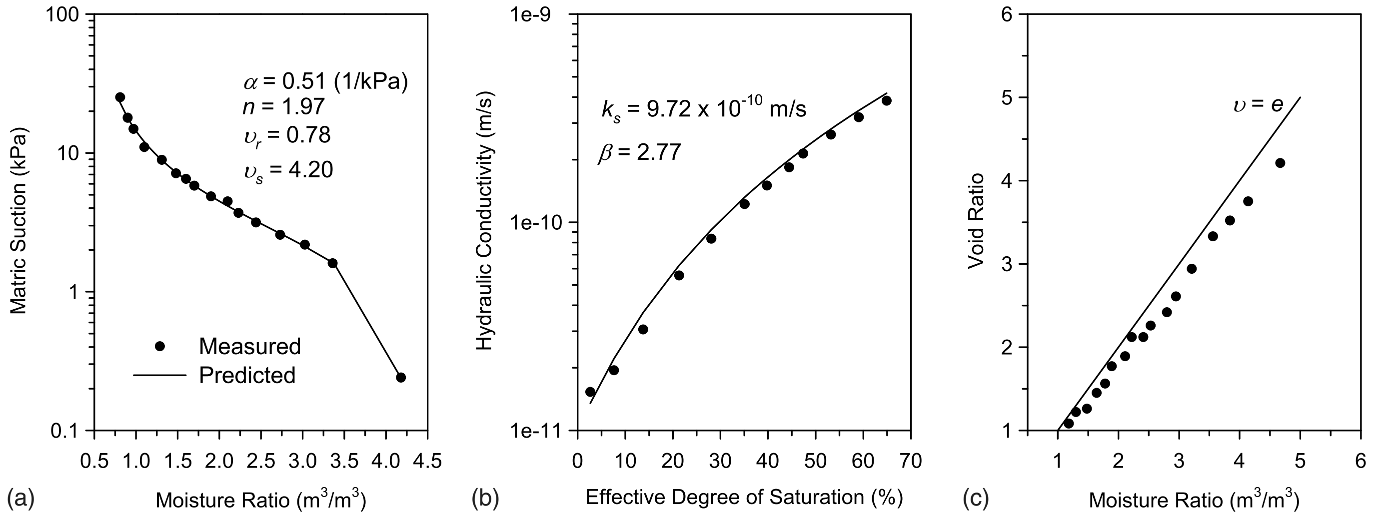

Fig. 3. Constitutive relationships for loam-bentonite sample: (a) SWRC; (b) HCF; and (c) SSSCC. (Measured data from Garnier et al. 1997.)

where k_s = saturated hydraulic conductivity; and β = fitting parameter.

The SSSCC model proposed by Peng and Horn (2005) is used in this study. They used the void ratio and moisture ratio to identify the soil volume change during the absorption or desorption of water. Peng and Horn (2005) defined four regimes to capture the SSSCC. In the direction of the increasing moisture ratio, these regions are zero, residual, proportional, and structural deformation regimes. In the structural and residual deformation ranges, the soil volume change is smaller than the change in the moisture ratio. In the zero-deformation range, the soil volume does not change as it absorbs or loses water. Finally, in the proportional regime, the soil volume change is almost equal to the change in water content, and thus, the air volume remains constant. Peng and Horn (2005) proposed a general model to capture different zones of the SSSCC as follows

$$e = \begin{cases} e_r & \vartheta = 0 \\ e_r + \dfrac{e_s - e_r}{\left[1 + \left(\frac{a\vartheta}{e_s - \vartheta}\right)^{-b}\right]^c} & 0 < \vartheta < \vartheta_s \\ e_s & \vartheta = \vartheta_s \end{cases} \tag{16}$$

where e_r = residual void ratio; e_s = saturated void ratio; and a, b, and c = fitting parameters.

To examine the performance of the proposed formulations, the results are compared against laboratory-measured data reported by Jaramillo (1989) and Jaramillo et al. (1990b), who investigated the change in water content and volume of a mixture of loam and bentonite subjected to vertical infiltration. In their test set up, a 3-cm high sample was placed in a cylinder with a diameter and height equal to 6 cm. A mobile piston, connected to a porous plate, was placed on top of the sample. This configuration allowed the sample to swell freely in the vertical direction. A constant hydraulic head was applied to the soil surface through the porous stone, and the change in the sample water content was observed periodically.

Fig. 3 shows the changes in the matric suction (SWRC), hydraulic conductivity (HCF), and void ratio (SSSCC) of the soil subjected to wetting. In this study, the values are fitted to the laboratory measured data from the same soil reported by Garnier et al. (1997) to find the fitting parameters of the SWRC [Fig. 3(a)], HCF [Fig. 3(b)], and SSSCC [Fig. 3(c)] in the proposed formulation. Fig. 4 depicts a comparison between the measured data reported by Jaramillo (1989) versus the calculated values using the proposed model in the study and the Kim et al. (1999) numerical solution. As seen, the proposed model can capture the overall behavior of soil and return slightly better results than the Kim et al. (1999) solution.

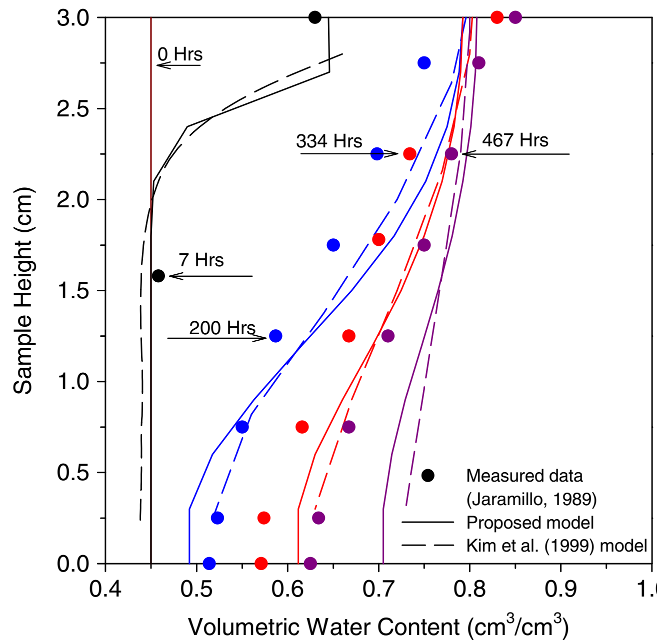

Fig. 4. Comparison between measured and predicted volumetric water content along the soil sample height upon wetting at different times.

Effective Stress in Unsaturated Soils

By extending Bishop's (1959) effective stress expression, the suction stress-based effective stress for unsaturated soils can be defined (Lu and Likos 2004, 2006)

$$\sigma' = \sigma - u_a - \sigma^s \tag{17}$$

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where σ and σ' = total and effective stress, respectively; and σ^s = suction stress. The pore-air pressure is assumed to be equal to the atmospheric pressure and set to zero in this study ($u_a = 0$). Lu et al. (2010) defined σ^s for unsaturated soils

$$\sigma^s = -S_e(u_a - u_w) \tag{18}$$

Extended Hooke's Law for Unsaturated Soils

We use the extended Hooke's law to establish an effective stress-strain relationship for an unsaturated soil element that is laterally constrained. Hooke's law is a linear stress-strain constitutive equation, which is commonly used to establish a relationship between vertical and horizontal stress components under at-rest conditions (no lateral movement). Lateral swelling pressure in unsaturated soils highly depends on the lateral deformation. The maximum lateral swelling pressure (i.e., the most critical condition) develops when the expansive soil is laterally constrained during wetting. If the soil is free to move in the lateral direction, no or minimal lateral swelling pressure will develop. For this reason, the lateral deformation is commonly set to zero in similar derivations to obtain the formulation of lateral swelling pressure at the most critical condition. Such conditions can be reasonably represented using the at-rest state. It is noted that the soil behavior, in general, is highly nonlinear and plastic beyond a small range of strains. However, considering the nonlinear elastoplastic behavior in the derivations leads to complex formulations, which warrants employing advanced numerical methods.

Several researchers investigated the volume change of unsaturated soils, assuming an elastic behavior for a considerable range of loading conditions (e.g., Fredlund and Morgenstern 1976, 1977; Lloret et al. 1987). The linear elastic assumption has been extensively employed in several constitutive models for describing the behavior of swelling soils (e.g., Abed 2008; Adem and Vanapalli 2016; Qi and Vanapalli 2018). Two distinct levels are recognized for expansive soils: (1) the microstructural level, which is responsible for the swelling and shrinkage of soil, and (2) the macrostructural level, which is responsible for major structural rearrangement (Gens and Alonso 1992). Gens and Alonso (1992) showed that the coupling between microstructure and macrostructure levels results in the accumulation of elastoplastic strains at the macrostructure level. However, the microstructure level is saturated, and its volumetric strain remains elastic and reversible. The application of drying and wetting cycles results in swelling and shrinkage at the microstructure level. It is shown that an equilibrium state is reached after a few cycles of wetting and drying, after which the changes in the volumetric strain become completely elastic and reversible (Dif and Bluemel 1991; Tripathy et al. 2002; Alonso et al. 2005; Nowamooz and Masrouri 2008; Estabragh et al. 2015; Al-Dakheeli and Bulut 2019). Tripathy et al. (2002) performed a set of laboratory tests and showed that the equilibrium state could be reached as early as at the fourth cycle. At the equilibrium state, the macrostructure level reaches a stable condition, and the volumetric strain of soil is fully governed by the microstructure level. Al-Dakheeli and Bulut (2019) used this concept and proposed a simple relation between the slope of the SWRC and the elastic volumetric strain of expansive soils. Furthermore, several studies (e.g., Vu and Fredlund 2004, 2006; Zhang 2004) were able to successfully capture the heave volumetric deformation of expansive soils through the consolidation theory and the assumption of elastic constitutive behavior for expansive soils.

Pioneering works by Terzaghi (1925, 1926, 1931) concluded that swelling of expansive soils is mainly an elastic deformation resulting from the soil's tendency to adsorb water. Several modeling efforts (e.g., Adem and Vanapalli 2013; Liu and Vanapalli 2017, 2019) have built upon Terzaghi's work and used an elastic model to predict the swelling pressures of expansive soils. Using the elastic assumption not only simplifies the problem, allowing for the development of an analytical solution, but also leads to a more conservative solution for the calculation of lateral swelling pressure. This assumption is supported through results of laboratory testing as well (e.g., Rawat et al. 2019).

Hooke's law can be extended to unsaturated conditions by incorporating the suction stress-based effective stress representation as follows (Lu and Likos 2004; Shahrokhabadi et al. 2019)

$$\varepsilon_x = \frac{\sigma_x - u_a}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z - 2u_a) - \frac{1-2\mu}{E}\sigma^s \tag{19}$$

$$\varepsilon_y = \frac{\sigma_y - u_a}{E} - \frac{\mu}{E}(\sigma_x + \sigma_z - 2u_a) - \frac{1-2\mu}{E}\sigma^s \tag{20}$$

$$\varepsilon_z = \frac{\sigma_z - u_a}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y - 2u_a) - \frac{1-2\mu}{E}\sigma^s \tag{21}$$

where ε_x, ε_y, and ε_z = elastic strains in the x, y, and z directions, respectively; σ_x, σ_y, and σ_z = elastic total stresses in the horizontal and vertical directions; and E and μ = Young's modulus and Poisson's ratio.

For a homogenous unsaturated soil layer in a half-space domain, the following two assumptions can be used (Lu and Likos 2004):

- The horizontal stresses are equal ($\sigma_x = \sigma_y = \sigma_h$).
- The horizontal strains are negligible ($\varepsilon_x = \varepsilon_y = \varepsilon_h = 0$).

Imposing these two assumptions reduces Eqs. (19)–(21) to (Lu and Likos 2004; Shahrokhabadi et al. 2019)

$$\varepsilon_v = \frac{\sigma_v - u_a}{E} - \frac{2\mu}{E}(\sigma_h - u_a) - \frac{1-2\mu}{E}\sigma^s \tag{22}$$

$$\varepsilon_h = \frac{\sigma_h - u_a}{E} - \frac{\mu}{E}(\sigma_v + \sigma_h - 2u_a) - \frac{1-2\mu}{E}\sigma^s \tag{23}$$

Closed-Form Model for Lateral Swelling Pressure

To determine the lateral swelling pressure, a representative soil element is considered behind the retaining structure. Lateral earth pressure (LEP) exerted on the soil element during infiltration is comprised of two different horizontal pressure components as follows:

- LSP: lateral pressure due to infiltration-induced changes in suction stress ($\Delta\sigma^s = \sigma^s_{\text{final}} - \sigma^s_{\text{initial}}$ and $\sigma^s_{\text{initial}} > \sigma^s_{\text{final}}$).
- σ_L: lateral pressure caused by overburden and surcharge (σ_v).

It is assumed that the backfill soil can move freely in the vertical direction. Thus, the vertical stress is only due to the overburden and/or surcharge, and the swelling has no effect on it. Using Eq. (23), the horizontal stress-strain relation in the soil element can be written as follows:

- For the soil element subjected to LSP only

$$\varepsilon_h = 0 = \frac{LSP}{E_{\text{ave}}} - \frac{\mu}{E_{\text{ave}}}LSP - \frac{1-2\mu}{E_{\text{ave}}}(\sigma^s_{\text{final}} - \sigma^s_{\text{initial}}) \tag{24}$$

- For the soil element subjected to overburden/surcharge pressure only

$$\varepsilon_h = 0 = \frac{-\sigma_L}{E} + \frac{\mu}{E}(\sigma_v + \sigma_L) \tag{25}$$

where E_{ave} = average of various values of unsaturated elastic modulus between initial and final state. Solving Eqs. (24) and (25), two different components of lateral earth pressure can be obtained as follows

$$LSP = \left(\frac{1-2\mu}{1-\mu}\right)(\sigma^S_{\text{final}} - \sigma^s_{\text{initial}}) \tag{26a}$$

$$\sigma_L = \left(\frac{\mu}{1-\mu}\right)\sigma_v \tag{26b}$$

The total lateral earth pressure is the summation of the lateral pressures due to swelling and overburden and/or surcharge as follows

$$LEP = LSP + \sigma_L = \left(\frac{1-2\mu}{1-\mu}\right)(\sigma^S_{\text{final}} - \sigma^s_{\text{initial}}) + \frac{\mu}{1-\mu}\sigma_v \tag{27}$$

We introduce an upper bound for lateral swelling pressure such that the calculated lateral earth pressure (including lateral swelling pressure and the component due to overburden and/or surcharge) cannot exceed the passive earth pressure. When an expansive soil element is constrained behind a retaining wall, the horizontal stress during infiltration can exceed the vertical stress, a condition that can be seen when the soil is in a passive state. As the infiltration continues and the soil moves toward the saturation condition, the difference between the initial and final suctions increases, which results in higher values of lateral swelling pressure. Consequently, lateral earth pressure (i.e., confinement pressure) becomes greater and greater until Mohr's circle intersects the shear failure envelope and the soil element fails. Thus, lateral earth pressure acting on a retaining structure cannot be greater than passive earth pressure. It should be added that as the soil's water content increases during infiltration, suction decreases, and the soil's shear strength possesses lower values. Therefore, passive earth pressure in the final state must be considered to find the upper bound limit for lateral earth pressure values. Passive earth pressure in unsaturated soils can be calculated (Lu and Likos 2004)

$$\text{PEP} = (\sigma_v - u_a)K_p + 2c'\sqrt{K_p} + \sigma^s(K_p - 1) \tag{28}$$

where PEP = passive earth pressure; K_p = coefficient of Rankine's passive earth pressure; and c' = effective cohesion of soil.

Illustrative Example

This section presents an example to illustrate the application of the proposed model. A 6-m high basement wall with an expansive backfill (Denver bentonite) is considered. It is assumed that the soil is subjected to an infiltration rate equal to its saturated hydraulic conductivity $(q/k_s = 1)$. Table 1 shows the hydrological and mechanical properties of the soil used in this example. Fig. 5 depicts the SWRC [Fig. 5(a)], HCF [Fig. 5(b)], and SSSCC [Fig. 5(c)] of the soil using the constitutive models used in the proposed formulations. The measured data points shown in Fig. 5 are adopted from Chen and Lu (2018). It is assumed that the initial degree of saturation, S_e, is equal to 0.52 and constant behind the wall.

The changes in the moisture ratio and the corresponding lateral swelling pressure are shown in Fig. 6. As seen, during infiltration, the shallow depths first become saturated and develop the highest values of lateral swelling pressure. As the infiltration continues, the soil layer moves toward saturation. As a result, the water content increases, and suction stress decreases gradually. As the difference between the two states becomes larger, higher lateral swelling pressures develop behind the wall. Furthermore, Fig. 6 shows that it takes a long time before the soil layer reaches the saturated state (in this case, more than 5 months). Thus, it is crucial to have a model that can predict the lateral swelling pressure under varying

Table 1. Hydrological and mechanical of properties of Denver-Bentonite used in the example problem

SWRC					HCF		SSSCC				
α (1/kPa)	n	m	ϑ_r	ϑ_s	k_s (m/s)	β	e_r	e_s	a	b	c
0.43	1.17	0.15	0	2.24	5.44×10^{-7}	8.50	0.77	2.25	427.0	0.75	84.30

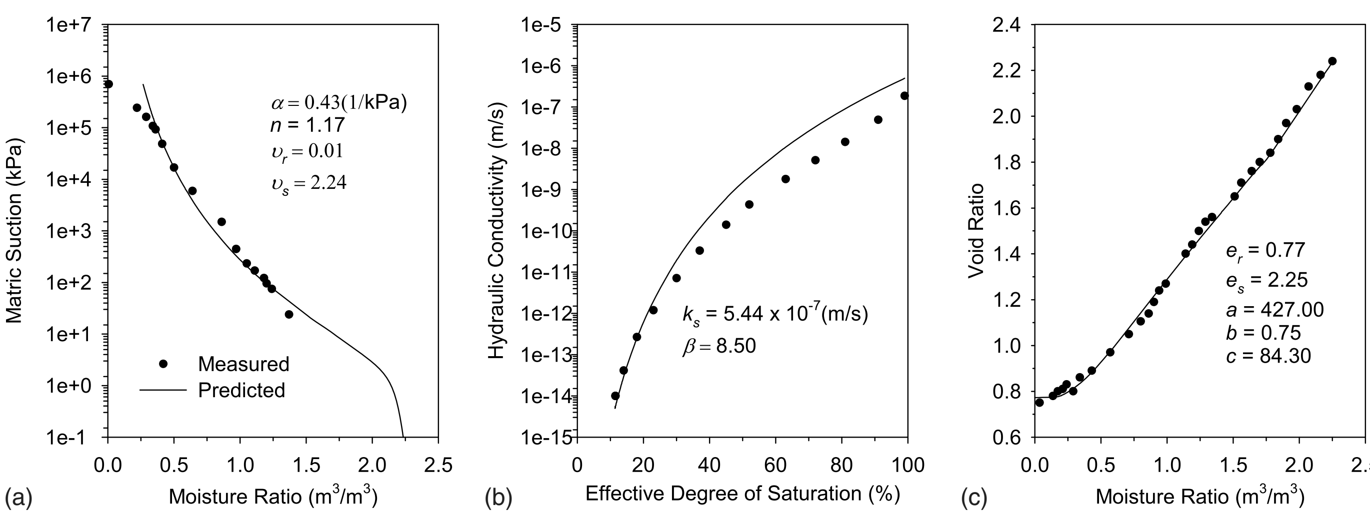

Fig. 5. Constitutive relationships of Denver bentonite: (a) SWRC; (b) HCF; and (c) SSSCC. (Measured data from Chen and Lu 2018.)

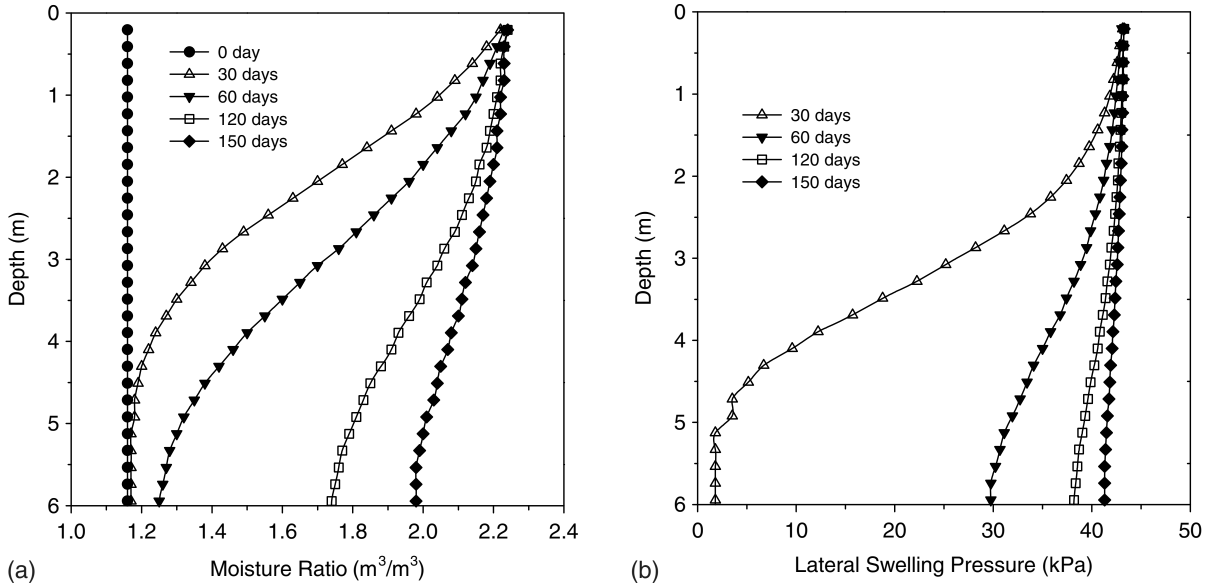

Fig. 6. Depth profiles of (a) moisture ratio; and (b) lateral swelling pressure at different times upon infiltration.

degrees of saturation when the moisture ratio increases due to infiltration, but the soil is not yet in the saturated condition.

Validation

In this section, results from the proposed model are validated against three sets of experimental tests available in the literature. The first case study is reported by Mohamed et al. (2014), who describe the field measurements of lateral swelling pressure behind a retaining structure during infiltration. The second case study is reported by Richards and Kurzeme (1973), who present two-year measurements of lateral swelling pressure behind a reinforced concrete basement wall with expansive clay as the backfill. Finally, the third case study is reported by Rawat et al. (2019), who represents a 1-year measurement of lateral swelling pressures in a soil column of Calcigel bentonite-sand mixture subjected to hydration from one end.

Case Study 1

Mohamed et al. (2014) examined the lateral earth pressure, considering the effect of lateral swelling pressure against a retaining wall in a city project in Assiut, Egypt. The backfill behind the retaining wall was compacted in 0.25-m thick layers to achieve the bulk unit weight of 14.3 kN/m^3. Fig. 7(a) shows the configuration of the field test conducted by Mohamed et al. (2014). To investigate the effect of lateral swelling pressure on lateral earth pressure, they increased the water content of soil by gradually adding water whenever the soil surface started drying. They measured lateral swelling pressure behind the retaining structure at time intervals of 24, 48, 72, and 96 h. Their initial measurements show that the backfill was

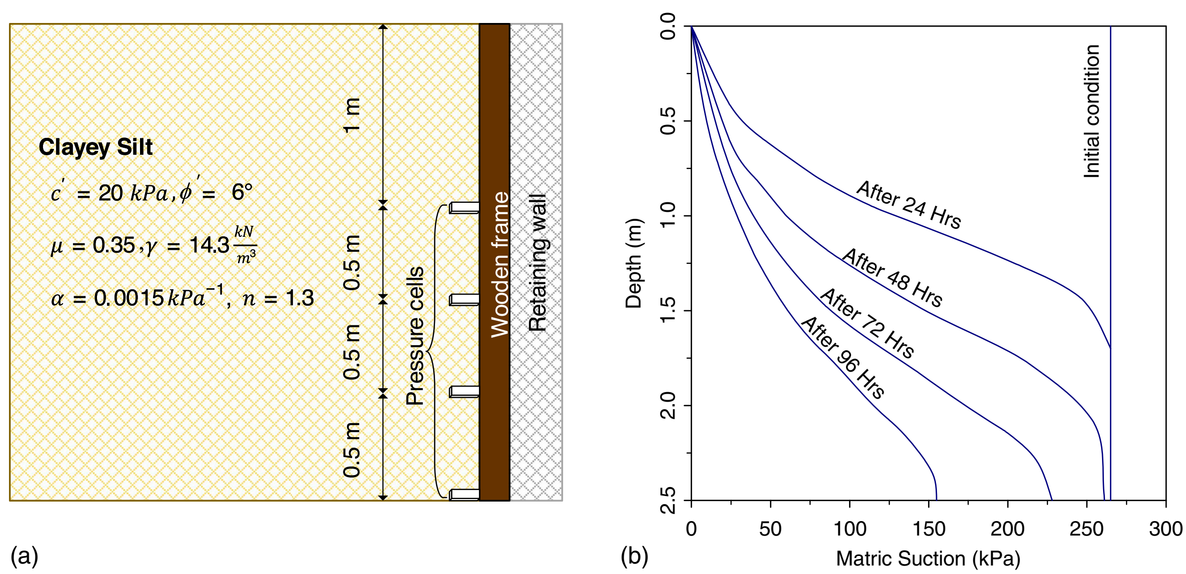

Fig. 7. (a) Overall configuration of a field test conducted by Mohamed et al. (2014); and (b) estimated suction profile along depth ring infiltration (data from Liu and Vanapalli 2019).

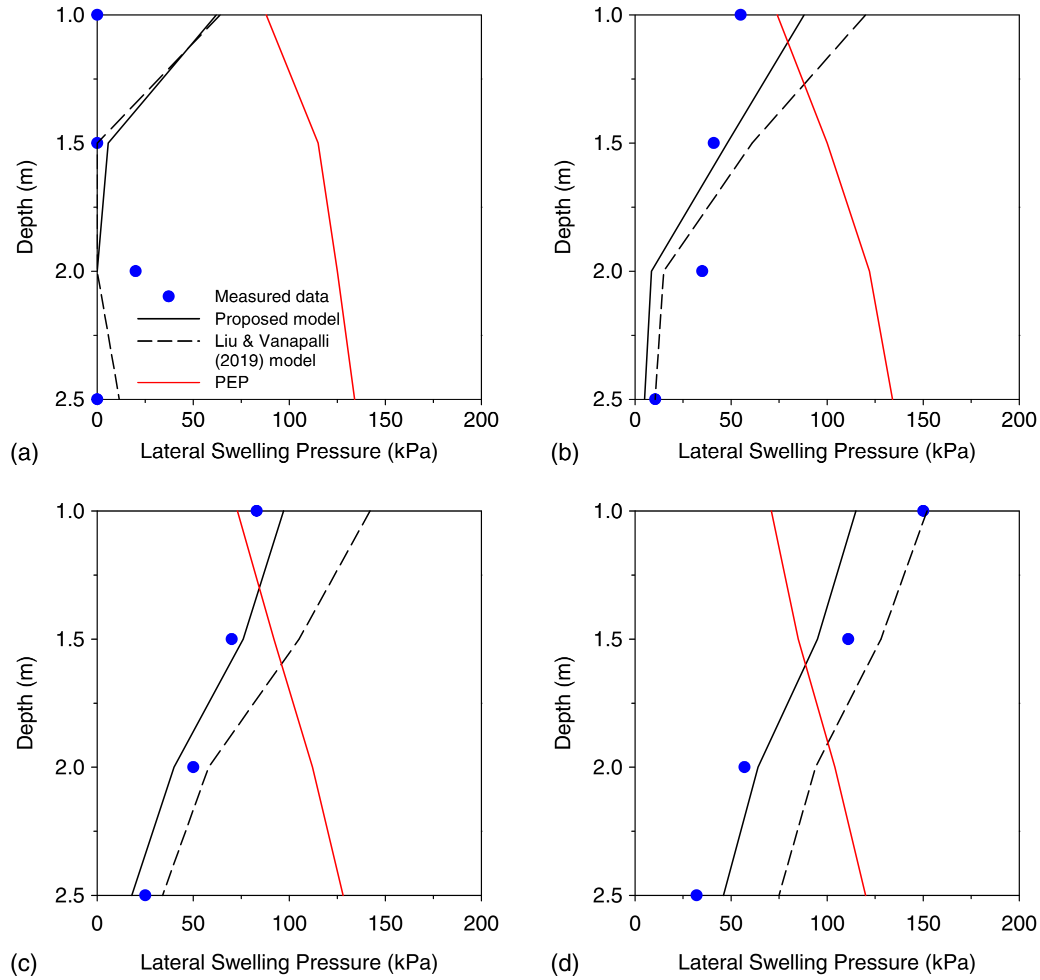

Fig. 8. Measured and predicted lateral swelling pressure along depth for Case study 1 after (a) 24 h; (b) 48 h; (c) 72 h; and (d) 96 h. (Measured data from Mohamed et al. 2014.)

first in the active state, and the lateral earth pressure had negative values. However, when the water was added, the soil expanded, and the lateral earth pressure increased.

The SWRC parameters and an in-depth suction profile during the wetting process were not provided by Mohamed et al. (2014). However, Liu and Vanapalli (2019) estimated the SWRC using the grain size distribution and Atterberg limits of the soil reported by Mohamed et al. (2014). Liu and Vanapalli (2019) assumed the value of the saturated hydraulic conductivity of soil to be 10^{-7} m/s and used a finite-element program to estimate the changes in the suction profile versus the depth over time [Fig. 7(b)]. The SWRC data and estimated suction profiles versus depth reported by Liu and Vanapalli (2019) are used in the current study to determine lateral swelling pressure using the proposed model. Because the variation of suction along the depth with respect to time was provided, there is no need to solve the flow equation for this case.

Fig. 8 compares the measured lateral swelling pressure values with those predicated using the Liu and Vanapalli (2019) model, as well as the model proposed in this study. Fig. 9 shows a comparison between the measured lateral swelling pressures and predicted values using the proposed model versus the degrees of saturation. In general, the predicted values from the proposed model show a good agreement with the measured data. The differences between the predicted versus measured results may be partly attributed to the

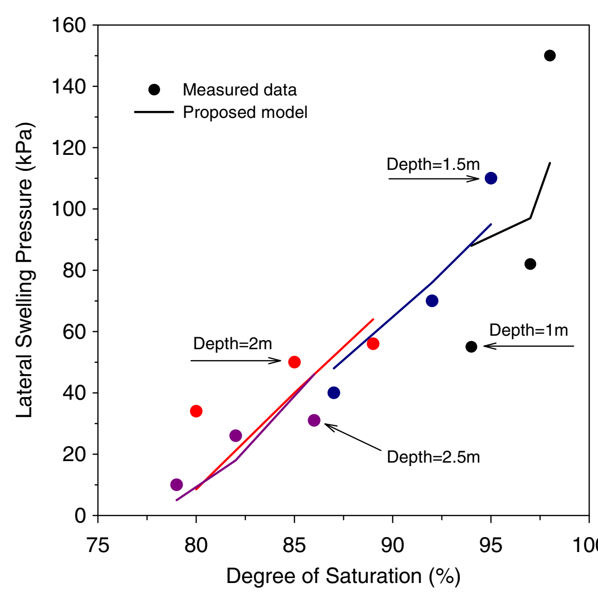

Fig. 9. Measured and predicted lateral swelling pressure versus degree of saturation for Case study 1. (Measured data from Mohamed et al. 2014.)

assumptions made regarding the SWRC parameters and suction profiles due to the lack of measured data. As shown in Fig. 8, the proposed model provides comparable and, in most cases, more accurate predictions than the Liu and Vanapalli (2019) model for this case study. Further, the proposed model offers a robust, simpler, and more practical alternative for the following reasons:

- The proposed model is the first attempt in the literature to determine the lateral swelling pressure under transient flow in swelling (deformable) soils by incorporating surface flux boundary conditions while accounting for the interplay between the soil deformation, water content, and hydraulic conductivity. The proposed model needs no additional fitting parameters than those needed for the SWRC. The model can determine the lateral swelling pressure of unsaturated deformable soils at different times and degrees of saturation.
- The proposed model does not need E and P_s, whereas each of these two parameters is needed at two different states (at states a and b for a total of four parameters) in the Liu and Vanapalli (2019) model. As noted, E is a suction-dependent variable. Further, the vertical swelling pressure (P_s) should be obtained from a free swell-load backtest. The P_s parameter can also estimate using a semiempirical model proposed by Tu and Vanapalli (2016). However, such a semiempirical estimation may impose more uncertainty and possibly error to lateral swelling pressure predictions.
- While offering a rigorous theoretical model, the Liu and Vanapalli (2019) model still relies on the fully saturated condition as a reference point to calculate the changes in lateral swelling pressure between two unsaturated states, whereas the proposed model can directly determine lateral swelling pressure between two unsaturated states without the need to reference the fully saturated state.

Case Study 2

Richards and Kurzeme (1973) investigated lateral swelling pressure exerted on the concrete basement structure of the Gauger Street Mail Exchange, which was founded in Hindmarsh soil in Adelaide, South Australia. Hindmarsh is an expansive clayey soil known for many structural problems in the area. Lateral swelling pressures and the soil suction behind the basement wall were monitored by Richards and Kurzeme (1973) over two years, using pressure cells and psychrometers, respectively. To measure the matric suction, several boreholes were excavated behind the retaining wall, and psychrometers were placed at depths of 2, 4, 6, and 7.5 m. Fig. 10 shows the soil profile and positions of the psychrometers and pressure cells behind the retaining wall. Matric suction values obtained at the site are directly used to estimate the lateral swelling pressure without the need to solve the flow equation. The properties of Hindmarsh clay were not reported by Richards and Kurzeme (1973). Thus, the properties of Hindmarsh clay reported and estimated by other researchers are used in the current study (e.g., Cox 1970; Jaksa 1995; Liu and Vanapalli 2019). The van Genuchten (1980) SWRC model is used to fit the in situ measurement database provided by Jaksa (1995). The soil properties are summarized in Table 2.

The readings by the pressure cells Nos. 302 and 303 at the depths of 6.6 and 7.4 m are used to validate the proposed model. Richards and Kurzeme (1973) measured lateral swelling pressure and suction changes behind the retaining wall between August 1971 and September 1973. The reported data are shown in Fig. 11. Observed changes in the suction profiles versus depth with time can be attributed to seasonal changes, fluctuations in the water table, and weather events (e.g., rainfalls) during this time period.

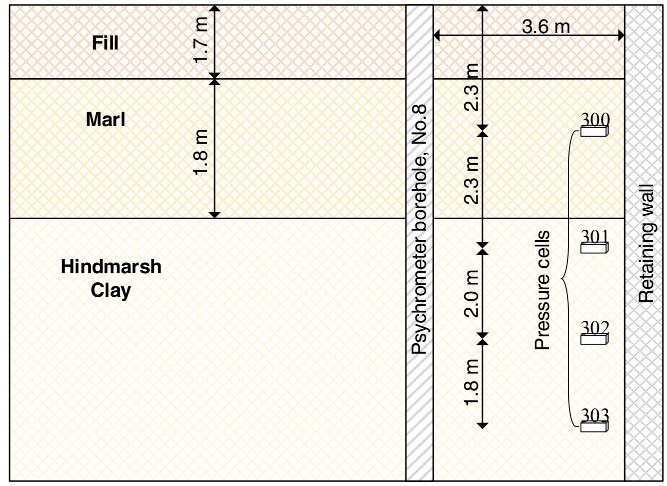

Fig. 10. Retaining wall and in situ measurement sensors for Case study 2. (Adapted from Richards and Kurzeme 1973.)

Table 2. Properties of Hindmarsh clay used for Case study 2

Property (unit)	Value(s)
Liquid limit, LI (%)	16–30[a]
Plastic index, PI	40–70[a]
Specific gravity, G_s	2.77
Unit dry weight, γ (kN/m^3)	13.8–18.2
α (1/kPa)[b]	0.0012
n[b]	1.58

[a]Data from Cox (1970).
[b]Based on data by Jaksa (1995).

Fig. 12 provides a comparison between the measured and predicted lateral swelling pressures versus time [Fig. 12(a)] and degree of saturation [Fig. 12(b)]. In general, a good agreement can be seen between the predicted lateral swelling pressure and the in situ data recoded using the cell pressures Nos. 302 and 303. As no information on the shear strength parameters was available, the variations in the PEP are not presented in this example. As demonstrated by Fig. 12(a), the proposed model yields comparable and, in most cases, more accurate predictions than the Liu and Vanapalli (2019) model for this case study as well. It is noted that when the soil moves toward saturation, the lateral swelling pressure increases. As seen in Fig. 12(b), the proposed model can reasonably capture the measured lateral swelling pressure between degrees of saturation of 50% and 90%.

Case Study 3

Rawat et al. (2019) conducted a water infiltration test on a compacted Calcigel bentonite-sand mixture (50:50) to measure the swelling pressure developed during the transient hydration process. Considering the soil swelling upon hydration as a coupled hydro-mechanical process, Rawat et al. (2019) continuously measured axial stresses at both ends of the soil sample, lateral stress, water content, and relative humidity for almost one year. The tested material is common bentonite in the southern part of Germany with 60%–70% montmorillonite. It has a plasticity index of 135%–137% and a specific gravity of 2.80. The soil sample was compacted with an initial water content of 9% in three layers using uniaxial static compaction under 30 MPa in the vertical direction to achieve the dry

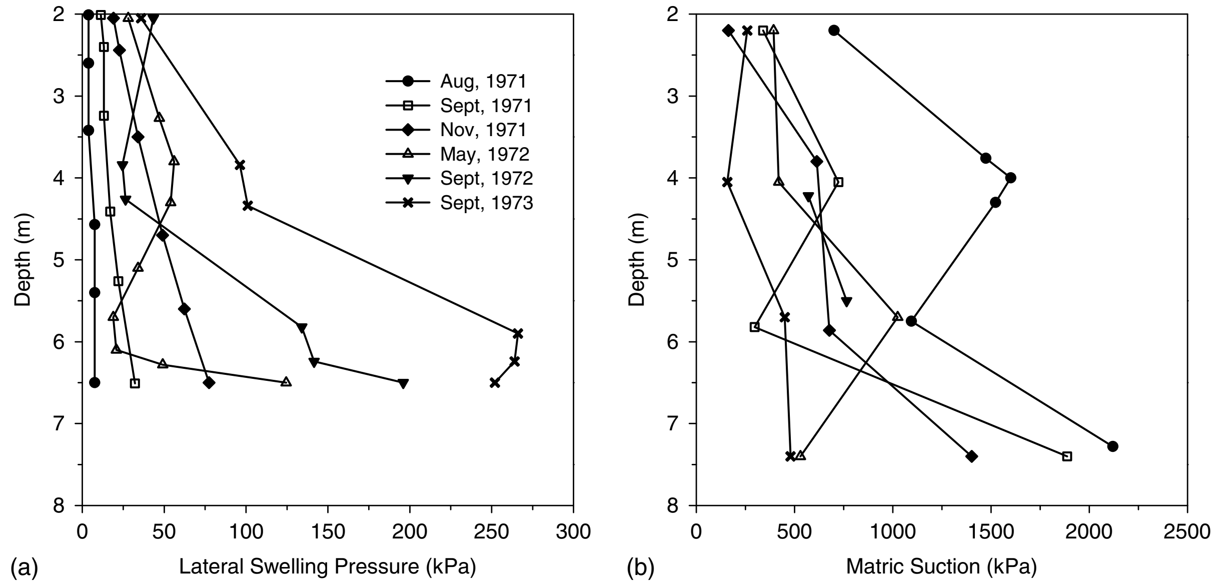

Fig. 11. Variation of (a) lateral swelling pressure; and (b) matric suction along depth in 2 years for Case study 2 (data from Richards and Kurzeme 1973).

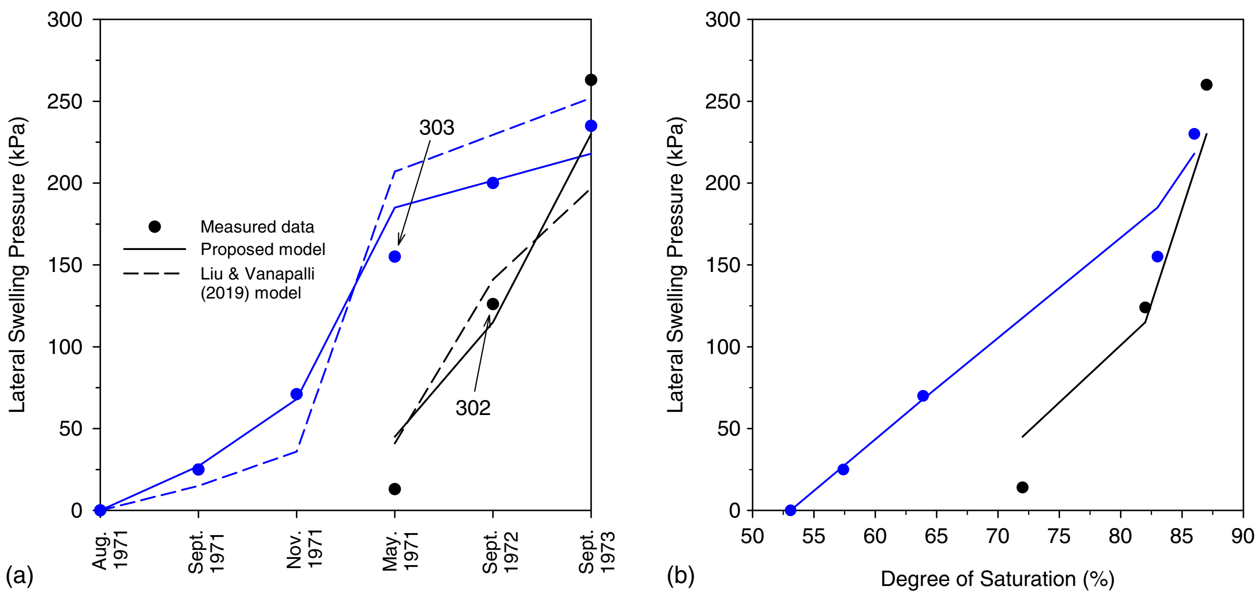

Fig. 12. Comparison between measured versus predicted lateral swelling pressure for Case study 2 at pressure cell 302 (depth = 6.6 m) and pressure cell 303 (depth = 7.4 m) versus (a) time; and (b) degree of saturation. (Measured data from Richards and Kurzeme 1973.)

density of 18.5 kN/m^3 and the initial void ratio of 0.47. The prepared soil sample had a diameter of 150 mm and a height of 300 mm. The initial suction of the soil sample was measured to be 26.9 MPa using the child mirror hydroscope. The sample was then put into a column-type test device (Fig. 13). The soil was hydrated from the bottom end under a 15 kPa hydration pressure. The air outlet at the bottom plug was kept closed, while the air outlet at the top plug was kept open to evacuate the air during the hydration process. The test was performed under constant volume conditions for a period of 349 days. Rawat et al. (2019) measured the soil suction, water content, relative humidity, and lateral swelling pressure at Sections X1, X2, and X3 positioned at 50, 150, and 250 mm heights from the bottom of the sample.

Fig. 14(a) depicts the SWRC of the tested mixture, including the measured data points, as well as the fitted curve using the van Genuchten (1980) model. Using the two-line method (Khalili and Khabbaz 1998) releveled the air entry value (AEV) of the mixture to be about 1.8 MPa, mainly attributed to the large percentage of bentonite (50%). Comparably high AEVs are reported for similar mixtures in the literature (e.g., Agus et al. 2013). Fig. 14(b) shows matric suction versus time developed at Sections X1, X2, and X3. As seen, a sharp suction drop occurred at X1 during the first 60 days of testing.

Rawat et al. (2019) showed that the soil became saturated in areas close to the hydration boundary (Section X1) after 120 days. They found that considering the elastic behavior in these regions could result in an overestimation of the lateral swelling pressure. However, up until the saturation, the swelling soil exhibited an elastic behavior. The distinct behavior of soil at Section X1 was attributed to the rapid rate of change in the water content in this

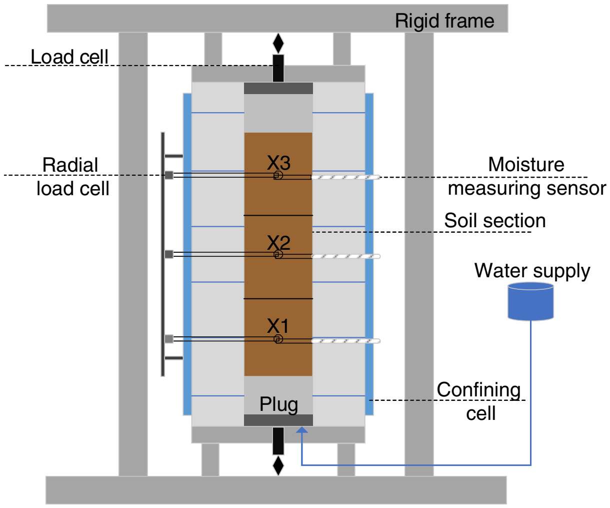

Fig. 13. Schematic illustration of column-type infiltration test setup used in Case study 3. (Adapted from Rawat et al. 2019.)

section upon hydration; whereas, at Sections X2 and X3, the change in the water content had a slower rate, and the lateral swelling pressure in these sections increased almost linearly. Thus, the measured lateral earth pressures only at Sections X2 and X3 are used for the validation purpose in this study. It is noted that the proposed model cannot accurately predict lateral swelling pressure when dealing with soils undergoing large plastic deformations (such as that at X1). Fig. 15 compares the measured lateral swelling pressures reported by Rawat et al. (2019) and those predicted by the proposed model in this study versus time [Fig. 15(a)] and degree of saturation [Fig. 15(b)]. As shown, there is a good agreement ($R^2 = 98\%$) between the measured and predicted lateral swelling pressures.

Conclusions

Lateral swelling pressure in expansive soils during infiltration can significantly lower the factor of safety of a retaining wall, basement wall, or buried geo-structures. In this study, the development

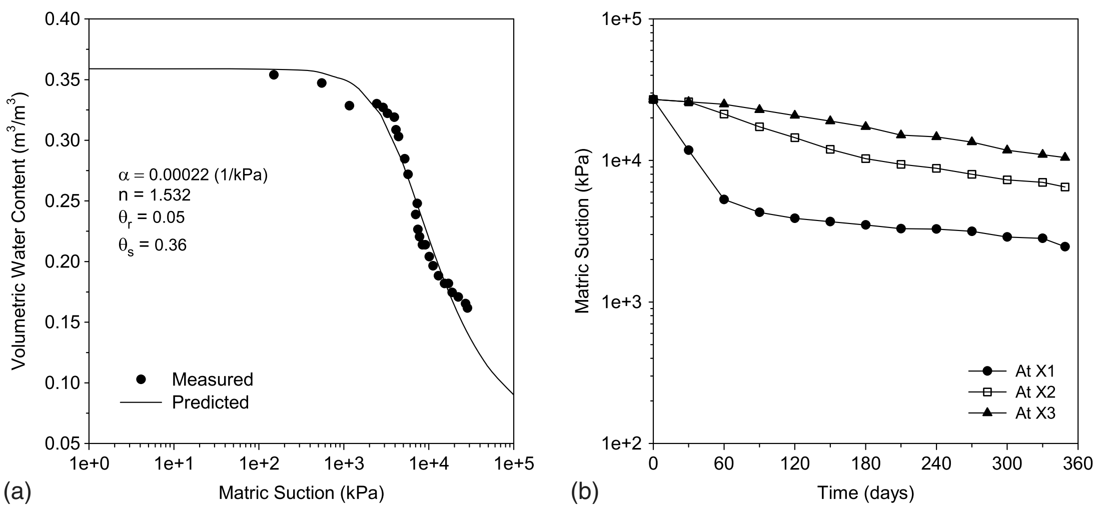

Fig. 14. Measured and predicted data for Case study 3: (a) SWRC of tested soil; and (b) matric suction versus time at Sections X1, X2, and X3. (Measured data from Rawat et al. 2019.)

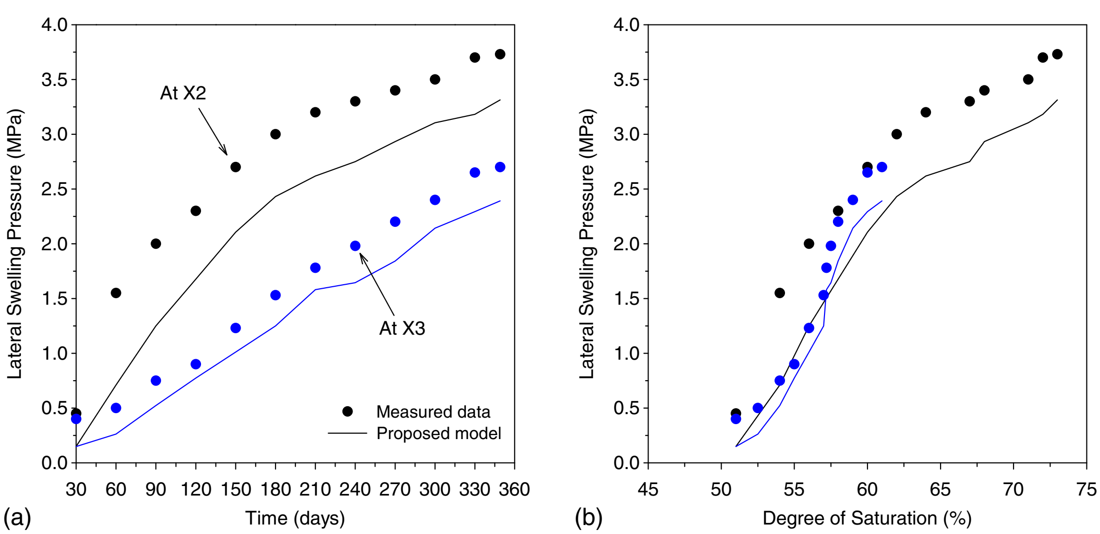

Fig. 15. Comparison between measured versus predicted lateral swelling pressure for Case study 3 at Sections X2 and X3 versus (a) time; and (b) degree of saturation. (Measured data from Rawat et al. 2019.)

of lateral swelling pressure was attributed to infiltration-induced changes in suction stress, which depends on the matric suction and degree of saturation, of a laterally-constrained unsaturated soil. The changes in the matric suction, moisture ratio, and, consequently, suction stress were examined and linked using a coupled hydromechanical approach considering the change of hydraulic properties of deformable soil during infiltration. Using the suction stress-based effective stress representation along with the extended Hook's law, an analytical formulation was developed to derive lateral swelling pressure under different flux boundary conditions and degrees of saturation. The validity of the proposed model was then examined by three sets of independent experimental test results. The model was successful in reasonably predicting the developed lateral swelling pressure for the three discussed case studies in the range of degree of saturation of 50%–90%. This range covers the majority, if not all, of the problems encountered in field conditions. Further studies are suggested to extend the proposed model to cover the overall range of the degrees of saturation.

The model needs few parameters that are related to the physical and measurable properties of soil to predict lateral swelling pressure behind a retaining structure. Both field measurements and flux rates can be used as input parameters to generate the suction profile along the depth and determine the lateral swelling pressure. Unlike the majority of the existing models, the proposed model can predict lateral swelling pressure in expansive soils under varying degrees of saturation. The model offers a reliable and practical tool for the analysis of earthen structures in expansive soils.

Data Availability Statement

All data, models, and code generated or used during the study appear in the published article.

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