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Experimental and Analytical Delay-Adaptive Control of a 7-DOF Robot Manipulator

Alexander Bertino[1], Peiman Naseradinmousavi[2], and Miroslav Krstic[3]

***Abstract*— We study the analytical and experimental trajectory-tracking control of a 7-DOF robot manipulator with an unknown long actuator delay. In order to compensate for this unknown delay, we formulate a delay-adaptive prediction-based control strategy in order to simultaneously estimate the unknown delay while driving the robot manipulator towards the desired trajectory. To the best of the authors' knowledge, this paper is the first to present a delay-adaptive approach for a nonlinear system with multiple inputs. Through Lyapunov analysis, we first obtain a local asymptotic stability result of the proposed controller. Then, through the results of both our simulation and experiment, we demonstrate that the proposed controller is capable of tracking the desired trajectory with desirable performance despite a large initial delay mismatch, which would cause non-adaptive prediction based controllers to become unstable.**

I. INTRODUCTION

In this paper, we address the experimental and analytical control of a 7-DOF robot manipulator subjected to an unknown constant input delay. Such delays are frequently observed in the control of remote manipulators [1]–[5], where a long, slowly time-varying (often assumed to be constant) communication delay is likely present. In order to account for a known delay, a variety of predictor based approaches have been developed for linear systems [6]–[11], nonlinear systems [12]–[15], as well as systems with a time-varying delay [16]–[18]. In recent papers [19]–[23], adaptive control strategies were developed in order to estimate an unknown delay while simultaneously compensating for this delay with a predictor based approach. In the paper [23] by Bresch-Pietri *et al.*, this strategy was extended to nonlinear dynamics subjected to a constant input delay. In this effort, we extend the local technique developed by Bresch-Pietri *et al.* [23] for an unmeasured distributed input, in order to handle the case of trajectory tracking with multiple actuators. This extension preserves the local delay-adaptive stability result of the original method. Furthermore, through the experimental verification of this delay-adaptive control strategy on Baxter, a 7-DOF redundant robot manipulator, we demonstrate desirable controller performance even in the presence of a significant delay mismatch of 0.9 seconds (0s initial prediction, 0.9s actual delay). Thus, the delay-adaptive control strategy is both theoretically sound and effective in practice, significantly improving the tracking performance of the predictor based approach when the delay is unknown. This is the main contribution of this paper.

The organization of this paper is as follows. In Section II, we present a brief overview of the dynamics of Baxter's right manipulator. In Section III, we formulate the delay adaptation task in mathematical terms, as well as state several assumptions on the system dynamics, feedback law, and desired trajectories that are utilized in the Lyapunov analysis of the delay-adaptive method. In Section IV, we present the delay-adaptation approach, and demonstrate the local delay-adaptive stability of the method through a Lyapunov analysis utilizing the $\mathcal{L}_1$ norm. In Section V, we present the simulation and experimental results of the proposed method implemented on Baxter's right manipulator, accounting for a large delay mismatch of 0.9 seconds. Finally, in Section VI, we present the case that the proposed delay-adaptive method has the potential to significantly increase the transient performance of a robot manipulator subjected to an unknown delay, through the compensation of an initial delay mismatch.

Notations: In the following, we use the common definitions of class $\mathcal{K}$ and $\mathcal{K}_\infty$ given in [24]. $|\cdot|$ and $|\cdot|_1$ refers to the Euclidean and $\mathcal{L}_1$ norms respectively, the matrix norm is defined accordingly, for $M \in \mathcal{M}_\ell(\mathbb{R})(\ell \in \mathbb{N}^*)$, as $|M| = \underset{|x|\leq 1}{sup}|Mx|$ and the spatial $\mathcal{L}_1$ and $\mathcal{L}_2$ norms are defined as follows:

$$\|u(t)\|_1 = \int_0^1 |u(x,t)|_1 dx$$
$$\|u(t)\|_2 = \sqrt{\int_0^1 u(x,t)^T u(x,t) dx}$$

For $(a,b) \in \mathbb{R}^2$ such that $a < b$, we define the standard projector operator on the interval $[a,b]$ as a function of two scalar arguments f (denoting the parameter being updated) and g (denoting the nominal update law) in the following manner:

$$Proj_{[a,b]}(f,g) = g\begin{cases} 0 & \text{if } f = a \text{ and } g < 0 \\ 0 & \text{if } f = b \text{ and } g > 0 \\ 1 & \text{otherwise} \end{cases}$$

II. MATHEMATICAL MODELING

The redundant manipulator, which is being studied here, has 7-DOF as shown in Fig. 1. The Euler-Lagrange formulation leads to a set of 7 coupled nonlinear second-order ordinary differential equations:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

where, $q, \dot{q}, \ddot{q} \in \mathbb{R}^7$ are angles, angular velocities and angular accelerations of joints, respectively, and $\tau \in \mathbb{R}^7$ indicates the vector of joints' driving torques. Also, $M(q) \in \mathbb{R}^{7\times 7}$ is a symmetric mass-inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{7\times 7}$ is a matrix of Coriolis coefficients, and $G(q) \in \mathbb{R}^7$ is a vector of gravitational loading.

Our verified coupled nonlinear dynamic model of the robot [14], [15], [25]–[31] is used as the basis of the delay-adaptive approach. Note that the inertia matrix $M(q)$ is symmetric, positive definite, and consequently invertible. This property is used in the subsequent development. The multi-input nonlinear system (1) can be written as 14th-order system of ODEs with the following general state-space

[1]A. Bertino is with the Department of Mechanical Engineering, San Diego State University, San Diego, CA 92115 USA, as well as the Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla CA 92093 USA (e-mail: abertino6245@sdsu.edu)

[2]P. Naseradinmousavi is with the Department of Mechanical Engineering, San Diego State University, San Diego, CA 92115 USA (e-mail: pnaseradinmousavi@sdsu.edu)

[3]M. Krstic is with the Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla CA 92093 USA (e-mail: krstic@ucsd.edu)

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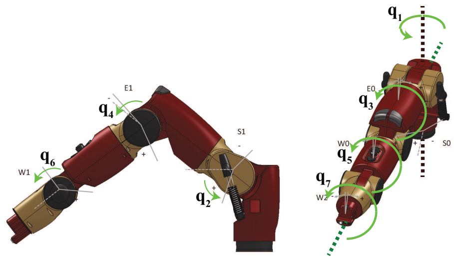

(a) (b)

Fig. 1. The joints' configuration: (a) sagittal view; (b) top view

form:

$$\dot{X} = f_0(X(t), U(t)) \tag{2}$$

where $X(t) = [q_1(t), \cdots, q_7(t), \dot{q}_1(t), \cdots, \dot{q}_7(t)]^T \in \mathbb{R}^{14}$ is the 14-dimensional vector of states, and $U(t) = \tau(t) \in \mathbb{R}^7$ represents the input torques to the system (2).

In order to track a desired trajectory, we reformulate (2) in terms of the error dynamics. We introduce the state error vector $E(t) = [e, \dot{e}]^T \in \mathbb{R}^{14}$, where $e = q_{des} - q \in \mathbb{R}^7$ is the positional error of the robot manipulator, and $q_{des} \in \mathbb{R}^7$ are the reference joint trajectories to track. Utilizing this state error vector, we reexpress (2):

$$\dot{E} = f(E(t), U(t)) \tag{3}$$

Furthermore, we make the following assumption regarding the reference joint trajectories:

Assumption 1. *The desired joint trajectories are designed such that $q_{des}(t) \in \mathbb{R}^7$ is a class $\mathcal{C}^\infty$ function and is bounded for all $t \geq 0$.*

III. Problem Statement

Consider the following nonlinear plant:

$$\dot{E} = f(E(t), U(t - D)) \tag{4}$$

in which D is an unknown delay introduced to the error dynamic model of Baxter's right manipulator (3), belonging to the interval $[\underline{D}, \overline{D}]$, with $\underline{D} \geq 0$. The objective of the delay-adaptive approach is to stabilize the error dynamics with input delay (4), despite the length of the delay being initially unknown. In order to assist the Lyapunov stability analysis in the next section, the following assumptions are made regarding the nonlinear plant (4), the control feedback law, as well as the desired joint trajectories.

Assumption 2. *The nonlinear plant (3) is strongly forward complete.*

Assumption 3. *There exists a feedback law $U(t) = \kappa(E(t))$ such that the nominal delay-free plant (3) is globally exponentially stable and such that κ is a class $\mathcal{C}^2$ function, i.e. there exist $\lambda > 0$ and a class $\mathcal{C}^\infty$ radially unbounded positive definite function V such that for $E \in \mathbb{R}^{14}$*

$$\frac{dV}{dE}(E) f(E, \kappa(E)) \leq -\lambda V(E)$$
$$|E|^2 \leq V(E) \leq c_1 |E|^2$$
$$\left|\frac{dV}{dE}(E)\right| \leq c_2 |E|^2$$

Assumption 4. *The contribution of the desired joint trajectories $q_{des}(t)$ to the dynamics of the nonlinear plant subject to an unknown delay (4), as well as the feedback control law $\kappa(E(t))$, vary slowly as a function of time. That is, $\forall n \geq 0$:*

$$\frac{\partial f}{\partial q_{des}^{(n)}} q_{des}^{(n+1)} \approx 0, \quad \frac{\partial \kappa}{\partial q_{des}^{(n)}} q_{des}^{(n+1)} \approx 0$$

Assumption 2 assures that (4) does not escape in finite time. This assumption is necessary to ensure that the system does not escape before the input $U(t - D)$ reaches the system. Assumption 3 is a stronger than necessary condition used in order to prove the local stability of the delay-adaptive approach. Finally, Assumption 4 ensures that the desired joint trajectories do not change too quickly, in a manner that noticeably alters the error dynamics of the system.

To analyze the closed-loop stability despite delay uncertainties, we use the systematic Lyapunov tools introduced in [12] and first reformulate plant (4) in the form:

$$\begin{cases} \dot{E} & = f(E(t), u(0, t)) \\ D u_t(x, t) & = u_x(x, t) \\ u(1, t) & = U(t) \end{cases} \tag{5}$$

by introducing the following distributed input:

$$u(x, t) = U(t + D(x - 1)), \quad x \in [0, 1] \tag{6}$$

Thus, the input delay is now represented as a coupling with a transport PDE driven by an input with unknown convection speed $1/D$.

IV. Delay-Adaptive Controller Formulation

Due to the fact that the distributed input is unmeasured, we introduce an estimate of the distributed input:

$$\hat{u}(x, t) = U(t + \hat{D}(t)(x - 1)), \quad x \in [0, 1] \tag{7}$$

where $\hat{D}(t)$ is the current estimate of the input delay. In order to stabilize (5), we must first predict the state of the system (5) once the delayed input reaches the system. In order to achieve this, we introduce a distributed predictor estimate:

$$\hat{p}(x, t) = E(t + \hat{D}(t)x) = E(t) + \hat{D}(t) \int_0^x f(\hat{p}(y, t), \hat{u}(y, t)) dy \tag{8}$$

If the input delay was known, the control law $U(t) = \kappa(E(t+D))$ could be used to stabilize the system, exactly compensating for the delay present in the system. Therefore, by the certainty equivalence principle, we choose the control law as:

$$U(t) = \kappa(E(t + \hat{D}(t))) = \kappa(\hat{p}(1, t)) \tag{9}$$

In order to derive an adaptation update law for the estimated delay, we utilize the following instantaneous cost function, initially proposed in [20] for a linear plant:

$$\varphi : (t, \hat{D}) \in \Big[t_0, \infty\Big[\rightarrow \Big|X_P(t, \hat{D}) - X(t)\Big| \tag{10}$$

where $X_P(t, \hat{D})$ is a ϕ seconds ahead predictor of the system state, starting from the state $X(t - \phi)$ and assuming the input delay is $\hat{D}(t)$. By taking the gradient of the cost function with respect to the estimated delay, one can obtain:

$$\tau_D(t) = -\Big(X_P(t, \hat{D}) - X(t)\Big)^T \frac{\partial X_P}{\partial \hat{D}}(t, \hat{D}) \tag{11}$$

where

$$X_P(t, \hat{D}) = X(t - \phi) + \int_{t-\phi}^{t} f_0\Big(X_P(s), U(s - \hat{D})\Big) ds \tag{12}$$

$$\frac{\partial X_P}{\partial \hat{D}}(t,\hat{D}) = -\int_{t-\phi}^{t} \frac{\partial f_0}{\partial U}\Big(X_P(s), U(s-\hat{D})\Big)\dot{U}(s-\hat{D})ds \tag{13}$$

Utilizing this computed gradient τ_D, the rate of change of the delay estimate can be computed as:

$$\dot{\hat{D}}(t) = \gamma Proj_{[\underline{D},\overline{D}]}\Big\{\hat{D}(t), \tau_D(t)\Big\} \tag{14}$$

where $\gamma > 0$ is the adaptation rate of the delay estimate, and the projection operator is utilized in order to ensure that the delay estimate remains in the interval $[\underline{D}, \overline{D}]$. By utilizing a steepest descent argument [32], one can obtain the following properties of (11), provided that the initial delay estimate is close enough to the true value of the delay:

Proposition 1. *There exist positive parameters $H > 0$ and $\tilde{D}^* > 0$ such that, if $\tilde{D}(0) < \tilde{D}^*$:*

$$\tilde{D}(t)\tau_D(t) \geq 0, \quad |\tau_D| \leq H, \quad |\dot{\tau}_D| \leq H \tag{15}$$

where $\tilde{D}(t) = D - \hat{D}(t)$ is the current estimation error of the delay.

Utilizing the delay estimate update law (14), as well as the control law (9), we are now ready to present the stability theorem for the delay-adaptive controller operating on a robot manipulator with unknown input delay.

Theorem 1. *Consider the closed-loop system consisting of the error dynamics of the robot manipulator (3), control law (9), delay estimate update law (14), and desired joint trajectories $q_{des}(t)$ satisfying Assumptions 1-4. Define the functional:*

$$\Gamma(t) = \big|E(t)\big| + \int_{t-max\{D,\hat{D}(t)\}}^{t} \big|U(s)-\kappa(0)\big|ds + \int_{t-max\{D,\hat{D}(t)\}}^{t} \Big|\dot{U}(s)\Big|ds + \int_{t-\hat{D}(t)}^{t} \Big|\ddot{U}(s)\Big|ds + \tilde{D}(t)^2$$

Then, there exist $\gamma^* > 0$ (potentially dependent on initial conditions), $\rho > 0$ and a class $\mathcal{K}_\infty$ function α^* such that, if $\Gamma(0) \leq \rho$ and $\gamma < \gamma^*$, then

$$\forall t \geq 0, \quad \Gamma(t) \leq \alpha^*(\Gamma(0))$$
$$E(t) \underset{t\to\infty}{\rightarrow} 0$$

A. Lyapunov Analysis

In this section, the proof of Theorem 1 bears strong resemblance to the proof of Theorem 3 in [23], since the delay-adaptive control approach in our paper is an extension of their approach. Furthermore, the Lemmas 1 and 2 in this section are mostly equivalent in representation to the Lemmas 4 and 5 in [23]. Thus, in the interest of brevity, the proofs of these lemmas are not provided in this paper.

In order to perform the Lyapunov stability analysis, a backstepping transformation is first used in order to reformulate (5).

Lemma 1. *The backstepping transformation of the distributed input estimate (7)*

$$\hat{w}(x,t) = \hat{u}(x,t) - \kappa(\hat{p}(x,t)) \tag{16}$$

in which the distributed predictor estimate is defined in (8), together with the control law (9), transforms plant (5) into:

$$E(t) = f(E(t), \kappa(E(t)) + \hat{w}(0,t) + \tilde{u}(0,t)) \tag{17}$$

$$\begin{cases} \hat{D}(t)\hat{w}_t &= \hat{w}_x + \dot{\hat{D}}(t)q_1(x,t) - q_2(x,t)f_{\tilde{u}}(t) \\ \hat{w}(1,t) &= 0 \end{cases} \tag{18}$$

$$\begin{cases} D\tilde{u}_t &= \tilde{u}_x - \tilde{D}(t)p_1(x,t) - \dot{\hat{D}}(t)p_2(x,t) \\ \tilde{u}(1,t) &= 0 \end{cases} \tag{19}$$

in which

$$\tilde{u}(x,t) = u(x,t) - \hat{u}(x,t) \tag{20}$$

is the distributed input estimation error. The expressions for the terms p_1, p_2, q_1, q_2, and $f_{\tilde{u}}$ can be found in [33].

Additionally, the governing equations for several spatial derivatives of the distributed input estimation error (20) and the backstepping transformation (16) are required in order to proceed with the Lyapunov analysis.

Lemma 2. *The spatial derivatives of the distributed input estimation error (20) and of the backstepping transformation (16) satisfy*

$$\begin{cases} D\tilde{u}_{xt} &= \tilde{u}_{xx} - \tilde{D}(t)p_3(x,t) - \dot{\hat{D}}(t)p_4(x,t) \\ \tilde{u}_x(1,t) &= \tilde{D}(t)p_1(1,t) \end{cases} \tag{21}$$

$$\begin{cases} \hat{D}(t)\hat{w}_{xt} &= \hat{w}_{xx} + \dot{\hat{D}}(t)q_3(x,t) - q_4(x,t)f_{\tilde{u}}(t) \\ \hat{w}_x(1,t) &= -\dot{\hat{D}}(t)q_1(1,t) + q_2(1,t)f_{\tilde{u}}(t) \end{cases} \tag{22}$$

$$\begin{cases} \hat{D}(t)\hat{w}_{xxt} &= \hat{w}_{xxx} + \dot{\hat{D}}(t)q_5(x,t) - q_6(x,t)f_{\tilde{u}}(t) \\ \hat{w}_{xx}(1,t) &= -\dot{\hat{D}}(t)q_3(1,t) + q_4(1,t)f_{\tilde{u}}(t) + \hat{D}(t)q_7(t) \end{cases} \tag{23}$$

in which $p_3 = p_{1,x}$, $p_4 = p_{2,x}$, $q_3 = q_{1,x}$, $q_4 = q_{2,x}$, $q_5 = q_{3,x}$, $q_6 = q_{4,x}$, and $q_7 = \hat{w}_{x,t}(1,t)$. The expressions for these functions can be found in [33].

For the purpose of the Lyapunov analysis, we consider the following Lyapunov-Krasovskii functional candidate:

$$\begin{aligned} W(t) &= V_0(E) + b_0 D\int_0^1 (1+x)\big|\tilde{u}(x,t)\big|_1 dx \\ &+ b_1 D\int_0^1 (1+x)\big|\tilde{u}_x(x,t)\big|_1 dx + b_2\hat{D}(t)\int_0^1 (1+x)\big|\hat{w}(x,t)\big|_1 dx \\ &+ b_3\hat{D}(t)\int_0^1 (1+x)\big|\hat{w}_x(x,t)\big|_1 dx \\ &+ b_4\hat{D}(t)\int_0^1 (1+x)\big|\hat{w}_{xx}(x,t)\big|_1 dx + \tilde{D}(t)^2 \end{aligned} \tag{24}$$

in which $V_0 = \sqrt{V}$, which was previously defined in Assumption 3. Utilizing the properties of Assumption 3, along with the state space model of the robot manipulator (3), the following inequality can be obtained:

$$\dot{V}_0 \leq= \frac{\lambda}{2}|E| + \frac{c_2}{2}\Big|M(q)^{-1}\Big|\big|\tilde{u}(0,t) + \hat{w}(0,t)\big|_1 \tag{25}$$

By taking the derivative of (24) and utilizing integration by parts, the following inequality can be obtained:

$$\begin{aligned} \dot{W}(t) \leq{}& \frac{\lambda}{2}|E| + \frac{c_2}{2}\Big|M(q)^{-1}\Big|\big|\tilde{u}(0,t)_1 + \hat{w}(0,t)\big| - b_0\big\|\tilde{u}(t)\big\|_1 \\ &- b_0\big|\tilde{u}(0,t)\big|_1 + b_0\Big|\tilde{D}(t)\Big|\int_0^1 (1+x)\big|p_1(x,t)\big|_1 dx \\ &+ b_0\Big|\dot{\hat{D}}(t)\Big|\int_0^1 (1+x)\big|p_2(x,t)\big|_1 dx - b_1\big\|\tilde{u}_x(t)\big\|_2^2 \\ &+ b_1\big|\tilde{u}_x(1,t)\big|_1 - b_1\big|\tilde{u}_x(0,t)\big|_1 + b_1\Big|\tilde{D}(t)\Big|\int_0^1 (1+x)\big|p_3(x,t)\big|_1 dx \end{aligned}$$

$$
\begin{aligned}
&+ b_1 \left|\dot{\hat{D}}(t)\right| \int_0^1 (1+x)\left|p_4(x,t)\right|_1 dx - b_2\left\|\hat{w}(t)\right\|_1 - b_2\left|\hat{w}(0,t)\right|_1 \\
&+ b_2 \left|\dot{\hat{D}}(t)\right| \int_0^1 (1+x)\left|q_1(x,t)\right|_1 dx \\
&+ b_2 \int_0^1 (1+x)\left|q_2(x,t) f_{\tilde{u}}(t)\right|_1 dx - b_3\left\|\hat{w}_x(t)\right\|_2^2 \\
&- b_3 \hat{w}_x(0,t)^T \hat{w}_x(0,t) + b_3\left|\hat{w}_x(1,t)\right|_1 + b_3\left|\dot{\hat{D}}(t)\right| \int_0^1 (1+x)\left|q_3(x,t)\right|_1 dx \\
&+ b_3 \int_0^1 (1+x)\left|q_4(x,t) f_{\tilde{u}}(t)\right|_1 dx + b_4\left|\hat{w}_{xx}(1,t)\right|_1 \\
&- b_4\left|\hat{w}_{xx}(0,t)\right|_1 - b_4\left\|\hat{w}_{xx}(t)\right\|_1 \\
&+ b_4\left|\dot{\hat{D}}(t)\right| \int_0^1 (1+x)\left|q_5(x,t)\right|_1 dx + b_4 \int_0^1 (1+x)\left|q_6(x,t) f_{\tilde{u}}(t)\right|_1 dx \\
&+ b_2 \dot{\hat{D}}(t) \int_0^1 (1+x)\left|\hat{w}(x,t)\right|_1 dx + b_3 \dot{\hat{D}}(t) \int_0^1 (1+x)\left|\hat{w}_x(x,t)\right|_1 dx \\
&+ b_4 \dot{\hat{D}}(t) \int_0^1 (1+x)\left|\hat{w}_{xx}(x,t)\right|_1 dx - 2\dot{\hat{D}}(t)\tilde{D}(t) \qquad (26)
\end{aligned}
$$

In order to bound the positive terms in the previous expression, we define the following alternative functional:

$$
\begin{aligned}
W_0(t) = &\left|E(t)\right| + \left\|\tilde{u}(t)\right\|_1 + \left\|\tilde{u}_x(t)\right\|_1 + \left\|\hat{w}(t)\right\|_1 \\
&+ \left\|\hat{w}_x(t)\right\|_1 + \left\|\hat{w}_{xx}(t)\right\|_1 \qquad (27)
\end{aligned}
$$

Then, by applying Lemmas 6-8 in [23], and introducing $\eta = \min\{\lambda/2, b_0, b_1, b_2, b_3, b_4\}$, one can bound (26) in terms of class $\mathcal{K}_\infty$ functions $\alpha_{4:21}(W_0(t))$ of (27):

$$
\begin{aligned}
\dot{W}(t) \leq &-\Bigg(\eta W_0(t) - \left|\dot{\hat{D}}(t)\right|\Big(b_0\alpha_4(W_0(t)) + b_1\alpha_7(W_0(t)) \\
&+ b_2\alpha_8(W_0(t)) + b_3\alpha_{12}(W_0(t)) + b_4\alpha_{14}(W_0(t)) \\
&+ 2(2b_3 + b_4)W_0(t) + b_3\alpha_{10}(W_0(t))\Big) \\
&- b_4\left(\left|\dot{\hat{D}}(t)\right| + \dot{\hat{D}}(t)^2 + \left|\dot{\hat{D}}(t)\right|^3\right)\alpha_{16}(W_0(t)) \\
&- b_4\left|\ddot{D}(t)\right|\alpha_{19}(W_0(t)) - \left|\tilde{D}(t)\right|\Big(b_0\alpha_3(W_0(t)) \\
&+ b_1\alpha_6(W_0(t)) + b_1\alpha_{11}(W_0(t)) + b_1\alpha_5(W_0(t)) \\
&+ b_1\left|\dot{\hat{D}}(t)\right|\alpha_{10}(W_0(t)) + b_4\alpha_{20}(W_0(t))\Big)\Bigg) \\
&- \left|\tilde{u}_x(0,t)\right|_1\Big(b_1 - b_4\alpha_{17}(W_0(t))\Big) - \left|\tilde{u}(0,t)\right|_1\Big(b_0 \\
&- b_2\alpha_9(W_0(t)) - b_3\alpha_{11}(W_0(t)) - b_3\alpha_{13}(W_0(t)) - b_4\alpha_{15}(W_0(t)) \\
&- b_4\alpha_{18}(W_0(t)) - \alpha_{21}(W_0(t))\Big) - \left|\hat{w}(0,t)\right|_1\Big(b_2 - \alpha_{21}(W_0(t))\Big) \\
&- \left|\hat{w}_x(0,t)\right|_1\Big(b_3 - b_4\alpha_{17}(W_0(t))\Big) - \left|\hat{w}_{xx}(0,t)\right|_1 b_4\left(1 - \left|\dot{\hat{D}}(t)\right|\right) \\
&- 2\dot{\hat{D}}(t)\tilde{D}(t) \qquad (28)
\end{aligned}
$$

In order to ensure that the last 5 terms of (28) are initially negative, we choose parameters such that:

$$
\begin{aligned}
&b_0 > b_2\alpha_9(W_0(0)) + b_3\alpha_{11}(W_0(0)) + b_3\alpha_{13}(W_0(0)) \\
&\quad + b_4\alpha_{15}(W_0(0)) + b_4\alpha_{18}(W_0(0)) + \alpha_{21}(W_0(0)) \\
&b_1 > b_4\alpha_{17}(W_0(0)), b_2 > \alpha_{21}(W_0(0)), b_3 > b_4\alpha_{17}(W_0(0)) \qquad (29)
\end{aligned}
$$

To further reduce (28), we apply Proposition 1 and introduce the following functions:

$$
\begin{aligned}
\alpha_1^*(W_0(t)) = &H\Big(b_0\alpha_4(W_0(t)) + b_1\alpha_7(W_0(t)) \\
&+ b_2\alpha_8(W_0(t)) + b_3\alpha_{12}(W_0(t)) + b_4\alpha_{14}(W_0(t)) \\
&+ 2(2b_3 + b_4)W_0(t)\Big) + b_3H^2\alpha_{10}(W_0(t)) \\
&+ b_4(H^2 + H^4 + H^6)\alpha_{16}(W_0(t)) + b_4H\alpha_{19}(W_0(t)) \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
\alpha_2^*(W_0(t)) = &b_0\alpha_3(W_0(t)) + b_1\alpha_6(W_0(t)) + b_1\alpha_{11}(W_0(t)) \\
&+ b_1\alpha_5(W_0(t)) + b_1H^2\alpha_{10}(W_0(t)) + b_4\alpha_{20}(W_0(t)) \qquad (31)
\end{aligned}
$$

For $\left|W_0(t)\right| \leq W_0(0)$, (28) reduces to:

$$
\begin{aligned}
\dot{W}(t) \leq &-\Big(\eta W_0(t) - \gamma\alpha_1^*(W_0(t)) - \left|\tilde{D}(t)\right|\alpha_2^*(W_0(t))\Big) \\
&- b_4(1 - \gamma H)\left|\hat{w}_{xx}(0,t)\right|_1 \qquad (32)
\end{aligned}
$$

Using the following inequality, which results from Young's inequality:

$$
\left|\tilde{D}(t)\right| \leq \frac{\epsilon}{2} + \frac{1}{2\epsilon}\tilde{D}(t)^2 \leq \frac{\epsilon}{2} + \frac{1}{2\epsilon}W(t) \qquad (33)
$$

one can obtain the following inequality:

$$
\begin{aligned}
\dot{W}(t) \leq &-\Bigg(\eta W_0(t) - \gamma\alpha_1^*(W_0(t)) - \\
&\left(\frac{\epsilon}{2} + \frac{1}{2\epsilon}W(t)\right)\alpha_2^*(W_0(t))\Bigg) - b_4(1 - \gamma H)\left|\hat{w}_{xx}(0,t)\right|_1 \qquad (34)
\end{aligned}
$$

Thus, by choosing for a given $\nu \in [0,1]$:

$$
\gamma < \gamma^* = \max\left\{1, \frac{1}{H}, \frac{\nu\eta}{\max\limits_{x\in[0,W_0(0)]} \alpha_1^{*\prime}(x)}\right\} \qquad (35)
$$

$$
\epsilon < 2\frac{\nu\eta - \gamma \max\limits_{x\in[0,W_0(0)]} \alpha_1^{*\prime}(x)}{\max\limits_{x\in[0,W_0(0)]} \alpha_2^{*\prime}(x)} \qquad (36)
$$

$$
W(0) \leq 2\epsilon \frac{\nu\eta - \gamma \max\limits_{x\in[0,W_0(0)]} \alpha_1^{*\prime}(x) - \frac{\epsilon}{2}\max\limits_{x\in[0,W_0(0)]} \alpha_2^{*\prime}(x)}{\max\limits_{x\in[0,W_0(0)]} \alpha_2^{*\prime}(x)} \qquad (37)
$$

one can ensure that:

$$
\dot{W}(t) \leq -(1-\nu)\eta W_0(t) \qquad (38)
$$

and consequently the following inequality which concludes the Lyapunov analysis:

$$
W(t) \leq W(0) \qquad (39)
$$

In order to provide a stability result in terms of Γ, Assumption 3 can be used to prove the existence of two class $\mathcal{K}_\infty$ functions α_3^* and α_4^* such that $\alpha_3^*(\Gamma(t)) \leq W(t) \leq \alpha_4^*(\Gamma(t))$. It follows that:

$$
\Gamma(t) \leq \alpha_3^{*-1}(W(0)) \leq \alpha_3^{*-1}(\alpha_4^*(\Gamma(0))) \qquad (40)
$$

Thus, the condition $\Gamma(t) \leq \alpha^*(\Gamma(0))$ in Theorem 1 is verified.

We now use Barbalat's Lemma to prove the convergence of $E(t)$. By integrating (38) from 0 to ∞, it can be seen that both $\left|E(t)\right|$ and $\left|U(t)\right|$ are integrable. Furthermore, from (39), it can be seen that $\left\|\tilde{u}_x(t)\right\|_1$, $\left\|\hat{w}(t)\right\|_1$ and $\left\|\hat{w}_x(t)\right\|_1$ are uniformly bounded. From the application of Lemma 7 in [23], $u(0,t)$ is also uniformly bounded. Using the properties of $u(0,t)$, and applying them to (5), it can be seen that $\left|E(t)\right|$ is uniformly bounded. Thus, we conclude with Barbalat's Lemma that $E(t) \to 0$ as $t \to \infty$.

V. Simulation and Experimental Results

In order to assess the performance of the delay-adaptive approach, we perform both a simulation using ODE methods on Baxter's dynamic equation (1), as well as an experiment. In both the simulation and the experiment, Baxter must track a trajectory designed for a pick and place task in [30], while having an input delay of $D = 0.9$ seconds. Furthermore, we initialize the estimated delay as $\hat{D}(0) = 0.0$ seconds. This large delay mismatch is intentionally chosen in order to demonstrate the ability of the delay-adaptive approach to achieve stability under conditions that would cause a purely predictor based approach to fail.

The experimental, simulated, and desired joint trajectories for several select joints can be seen in Figure 2. Despite the large initial

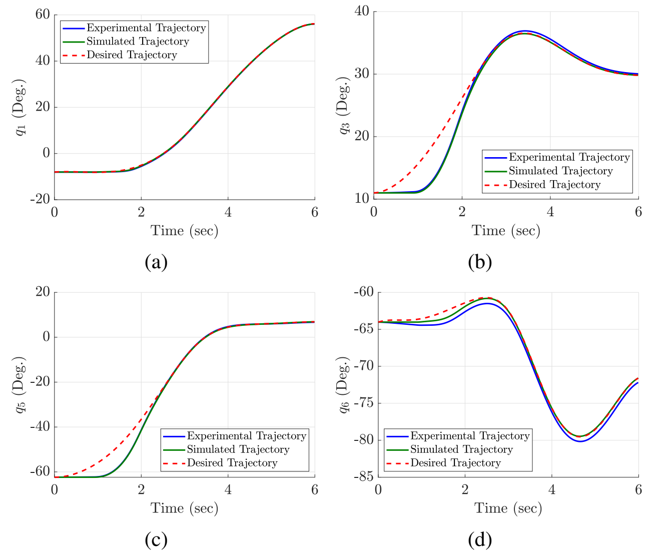

Fig. 2. The experimental (blue line), simulated (green line), and desired (red dashed line) joint trajectories of Baxter: (a) Joint 1, (b) Joint 3, (c) Joint 5, (d) Joint 6

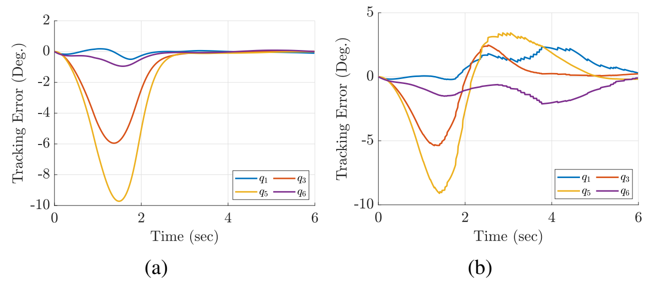

Fig. 3. The simulated (a) and experimental (b) joint tracking errors of select joints of Baxter (Joints 1, 3, 5 and 6).

delay mismatch, the delay-adaptive approach is effective at driving the robot manipulator towards the desired trajectory. After the initial 0.9 seconds of operation, in which the robot manipulator was expectedly stationary due to the input delay, the robot manipulator quickly corrects itself towards the desired trajectory. This behavior can also be observed in Figure 3, as both the simulated and experimental joint tracking errors decrease rapidly after around 1 second of operation. Furthermore, both the experimental and simulated trajectories appear to be smooth, indicating that changes to the estimated delay during adaptation did not cause disturbances in the tracking performance of the manipulator.

The experimental and simulated joint toque input signals for several select joints can be seen in Figure 4. It is important to note that these torques are significantly lower than the maximum torque output of Baxter's joints, which are 50 Nm for joints 1-4, and 15 Nm for joints 5-7. Thus, the delay-adaptive approach is able to compensate for a large delay mismatch without producing excessive joint torques. In the simulation, slight chattering can be observed in the input joint torque signal. This chattering, which is most prominent during the 1st 2 seconds of the simulation, is caused by large changes in the estimated delay in the beginning of the simulation. This behavior is not of concern however, as the chattering is of a small amplitude, and is mostly eliminated after 2 seconds. In the experiment, a small amount of noise can be observed in the control signal. This is likely due to the indirect measurement of the joint angular velocities through the use of encoders. This

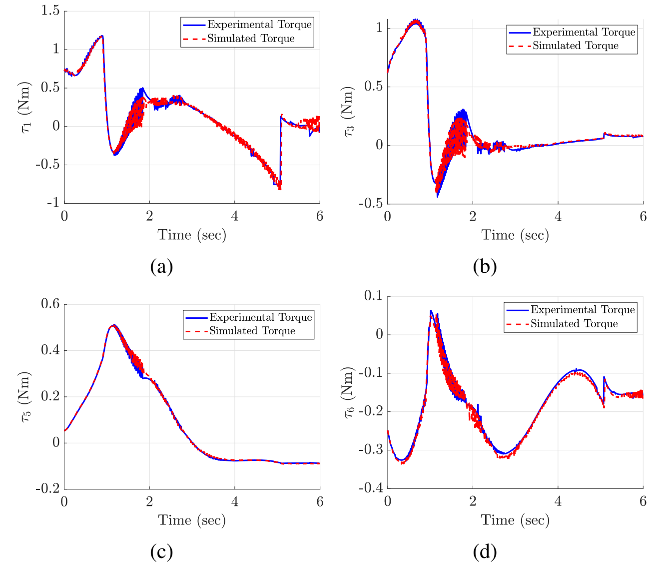

Fig. 4. The experimental (blue line) and simulated (red dashed line) joint torque input signals of Baxter: (a) Joint 1, (b) Joint 3, (c) Joint 5, (d) Joint 6

measurement noise, introduced when taking the derivative of the joint angular positions, is reflected in a small amount of noise being present in the torque signal. This is also not a cause of concern, as the amplitude of the noise is not large enough to noticeably impact tracking performance.

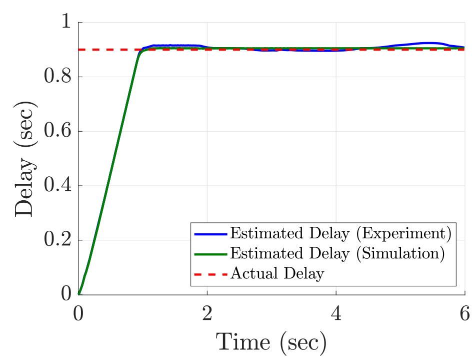

Fig. 5. The adaptation of the estimated delay in experiment (blue line) and simulation (green line), compared to the actual input delay (red dashed line)

The adaptation of the estimated delay in the experiment and simulation can be seen in Figure 5. In both the experiment and simulation, the estimated delay quickly converges to the actual delay. Furthermore, the following important observations can be made regarding the behavior of the adaptation. First of all, the rate of change of the adaptation appears to be constant during the 1st 0.9 seconds of the simulation and experiment, until the estimated delay coincides with the actual delay. Due to the initial state of the robot manipulator being at rest, delays longer than the elapsed time t are indistinguishable from a delay of t seconds. Thus, as a consequence of the properties stated in Proposition 1, the estimated delay is upper bounded by the simulation time in the simulation and experiment. Second of all, we observe that the estimation of the delay in simulation and experiment follow a nearly identical curve. This indicates that the delay-adaptive procedure does not suffer significantly from factors such as measurement noise of joint states which are present in the experiment but not in the simulation. Finally, it can be seen that there is slight overshoot in the estimated delay during the experiment. Although this behavior technically violates Proposition 1, it can reasonably be attributed to discretization of the control law.

VI. Conclusion

In this effort, we investigated the analytical and experimental trajectory-tracking control of a 7-DOF robot manipulator with an unknown long actuator delay. In order to compensate for this unknown delay, we formulated a delay-adaptive prediction-based control strategy in order to simultaneously estimate the unknown delay while driving the robot manipulator towards the desired trajectory. To the best of the authors' knowledge, this paper is the first to present a delay-adaptive approach for a nonlinear system with multiple inputs. Through Lyapunov analysis utilizing the $\mathcal{L}_1$ norm, we obtained a local asymptotic stability result of the proposed controller. Then, we demonstrated through both simulation and experiment that the proposed controller is capable of achieving desirable trajectory tracking performance, even in the case of a large initial delay mismatch. This research represents a large improvement upon the predictor-based approach in the case of an unknown delay, and thus has promising potential for use cases in which the delay is difficult to accurately predict or measure directly.

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