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JOINT OPTIMIZATION OF SPECTRALLY CO-EXISTING MULTI-CARRIER RADAR AND COMMUNICATION SYSTEMS IN CLUTTERED ENVIRONMENTS

Fangzhou Wang⋆ *Hongbin Li*⋆ *Braham Himed*†

⋆ Stevens Institute of Technology, Hoboken, NJ 07030, USA
†AFRL/RYMD, Dayton, OH 45433, USA

ABSTRACT

We consider the joint optimization of multi-carrier radar and communication systems with shared spectrum. The systems operate in a cluttered environment, where the radar and communication receivers observe not only cross-interference but also multipath and/or clutter signals, which may arise from the system's own transmission. We propose a non-alternating approach to jointly optimize the radar and communication transmission power allocated to each sub-carrier. Numerical results demonstrate the proposed joint designs offer significant performance gain over the conventional sub-carrier allocation-based approach.

***Index Terms*—** radar and communication coexistence, multi-carrier signal, power allocation, cluttered environment

1. INTRODUCTION

In the past decade, the quest for ever increasing transmission rates in terrestrial communications has produced a tremendous demand for additional bandwidth from the wireless sector. This has prompted the need for the coexistence between radar and communication systems using shared spectrum (e.g. [1–5]). Spectral coexistence, if improperly implemented, can cause significant interference and performance degradation for both systems [6–8]. The problem can be addressed by a joint design approach that simultaneously adjusts the design parameters of both systems to optimize relevant performance metrics, e.g., the throughput for the communication system and the signal-to-interference-plus-noise ratio (SINR) for the radar [9–11].

Multi-carrier waveforms are not only widely used in communication systems, but they have become increasingly popular in radar as well, due to several advantages such as easy implementation, frequency and waveform diversity [12, 13]. At any time instant, since the desired sub-carriers can be digitally selected at the transmitter, narrowband interference mitigation can be achieved by simply turning off the affected sub-carriers. In [14], orthogonal frequency division multiplexing (OFDM) waveforms with pulse-to-pulse agility have been investigated for Doppler processing from the radar point of view. The integrated sidelobe level (ISL) was used as an optimization metric to develop a sparse spectrum allocation algorithm for an OFDM-type radar [15]. Multi-carrier waveforms were also employed by radar and communication systems in [16] to tackle coexistence applications.

In this paper, we consider spectrum sharing between a multi-carrier radar and a communication system operating in a cluttered environment, where the communication or radar receiver observes not only the cross-interference from its counterpart but also multipath and clutter signals, which arise from the system's own transmission. Specifically, we propose to jointly optimize the radar and communication transmission power allocated to each sub-carrier by maximizing the radar output SINR while maintaining a minimum communication throughput constraint, along with a total transmission power constraint and sub-channel peak power constraints for each system. To solve the optimization problem, we reformulate it by combining the radar and communication power variables into a single stacked variable. This allows us to bypass a conventional alternating optimization procedure, which is computationally intensive. The resulting problem is then solved by using a quadratic transform method along with a sequential convex programming (SCP) technique. Simulation results validate the effectiveness of the proposed joint design over a sub-carrier allocation-based method.

2. SIGNAL MODEL

Consider a radar system that coexists with a communication system in a cluttered environment, where both systems share a frequency band of bandwidth B Hz and employ multi-carrier waveforms with N sub-carriers. The sub-carrier spacing is $\Delta f = B/N$. Under the considered set-up, the communication or radar receiver (RX) receives not only the direct useful signal, but the direct cross-interference and reflections from the environment as well.

Let $\mathbf{p}_{\rm c} = [p_{{\rm c},1}, \cdots, p_{{\rm c},N}]^T$ denote the communication powers allocated to the N sub-carriers, which are to be designed. Then, the transmitted communication signal can be represented as $x_{\rm c}(t) = q_{\rm c}(t)\sum_{n=1}^{N} d_n\sqrt{p_{{\rm c},n}}e^{j2\pi(f_{\rm c}+n\Delta f)t} \triangleq \sum_{n=1}^{N} x_{{\rm c},n}(t)$, where $q_{\rm c}(t)$ is the communication waveform, $f_{\rm c}$ the carrier frequency, and d_n the symbol carried by the

n-th sub-carrier. Without loss of generality, we assume $E\{|d_n|^2\} = 1$.

The same carrier frequency f_c, inter-carrier spacing Δf, and a radar waveform $q_r(t)$ are used by the multi-carrier radar system. Then, the transmitted radar signal can be written as $x_r(t) = q_r(t)\sum_{n=1}^{N}\sqrt{p_{r,n}}e^{j2\pi(f_c+n\Delta f)t} \triangleq \sum_{n=1}^{N} x_{r,n}(t)$, where $\mathbf{p}_r = [p_{r,1}, \cdots, p_{r,N}]^T$ denote the radar powers, which are to be determined.

Under the considered setup, the signal received at the communication RX on the n-th sub-carrier is given by

$$y_{c,n}(t) = \sum_{k=1}^{K_{cc}} \alpha'_{cc,n,k} x_{c,n}(t-\tau_{cc,k}) + \sum_{k=1}^{K_{rc}} \beta'_{rc,n,k} x_{r,n}(t-\widetilde{\tau}_{rc,k}) + w'_{c,n}(t), \quad (1)$$

where $\alpha'_{cc,n,k}$ is the channel coefficient of the k-th communication path with propagation delay $\tau_{cc,k}$, K_{cc} denotes the total number of communication paths, $\beta'_{rc,n,k}$ is the channel coefficient from the radar transmitter (TX) to the communication RX due to the k-th clutter scatterer with propagation delay $\widetilde{\tau}_{rc,k}$, K_{rc} denotes the total number of clutter scatterers, and $w'_{c,n}(t)$ is the additive channel noise.

We assume that the propagation delay spread $\Delta\tau$, i.e., the difference between the smallest delay and largest delay, from the communication/radar TXs to the communication/radar RX is small with respect to (w.r.t.) the pulse duration T. The assumption is usually satisfied in a multi-carrier system since each sub-carrier is a narrowband system with a bandwidth $\Delta f \ll 1/\Delta\tau$ [17, Section 12.1]. In other words, this assumption implies $|\tau_{cc,k} - \tau_{cc,1}| \ll T$, for $k > 1$, and $|\widetilde{\tau}_{rc,k} - \tau_{cc,1}| \ll T, \forall k$, in (1).

After down-conversion, $y_{c,n}(t)$ passes through a matched filter (MF) matched to the line of sight (LOS) communication waveform $q_c(t-\tau_{cc,1})$ and is sampled at the symbol rate, which yields [18, Section 5.1]

$$y_{c,n} = \alpha_{cc,n} d_n \sqrt{p_{c,n}} + \beta_{rc,n}\sqrt{p_{r,n}} + w_{c,n}, \quad (2)$$

where $\alpha_{cc,n} = \int_T \sum_{k=1}^{K_{cc}} \alpha'_{cc,n,k} q_c(t-\tau_{cc,k}) q_c^*(t-\tau_{cc,1}) dt$, which integrates $\alpha'_{cc,n,k}$ and the auto-correlation of the communication waveform, $\beta_{rc,n}$ is similarly defined, which lumps $\beta'_{rc,n,k}$ and the cross-correlation between the radar and communication waveforms, and $w_{c,n}$ is a zero-mean white additive noise with variance σ_c^2.

Next, consider the radar received signal. Assume a target is moving with a radial velocity v at range R from the radar. The round-trip delay between the radar and target is $\tau_{rr} = 2R/c$, where c is the speed of light. Then, the received signal at the radar RX on the n-th sub-carrier can be written as [12]

$$y_{r,n}(t) = \bar{\alpha}\alpha'_{rr,n} x_{r,n}\big(\varepsilon(t-\tau_{rr})\big) + \sum_{k=1}^{K_{rr}} \beta'_{rr,n,k} x_{r,n}(t-\widetilde{\tau}_{rr,k}) + \sum_{k=1}^{K_{cr}} \beta'_{cr,n,k} x_{c,n}(t-\widetilde{\tau}_{cr,k}) + w'_{r,n}(t), \quad (3)$$

where $\bar{\alpha}$ is the radar cross-section (RCS), $\alpha'_{rr,n}$ is a complex coefficient of the target path, $\varepsilon = 1 + \frac{2v}{c}$ is a scaling factor for the target Doppler shift, $\beta'_{rr,n,k}$ denotes the complex scattering coefficient of the k-th out of K_{rr} clutter scatterers due to radar illumination with propagation delay $\widetilde{\tau}_{rr,k}$, $\beta'_{cr,n,k}$ and $\widetilde{\tau}_{cr,k}$ are the scattering coefficient and, respectively, propagation delay associated with the k-th out of K_{cr} clutter scatterers due to the communication illumination, and $w'_{r,n}(t)$ is the additive channel noise.

The radar signal $y_{r,n}(t)$ is down-converted, Doppler compensated, filtered by a MF matched to the radar waveform $q_r(t-\tau_{rr})$, and sampled at the pulse rate. The MF output can be written as [19, Section 4.2]:

$$y_{r,n} = \alpha_{rr,n}\sqrt{p_{r,n}} + \beta_{rr,n}\sqrt{p_{r,n}} + \beta_{cr,n} d_n \sqrt{p_{c,n}} + w_{r,n}, \quad (4)$$

where, similar to (2), $\alpha_{rr,n}$ is a parameter that integrates $\bar{\alpha}$, $\alpha'_{rr,n}$, and the auto-correlation of the radar waveform, $\beta_{rr,n}$ lumps $\beta'_{rr,n,k}$ and the auto-correlation of radar waveforms, $\beta_{cr,n}$ integrates $\beta'_{cr,n,k}$ and the cross correlation of radar and communication waveforms, and $w_{r,n}$ is the output noise with zero mean and variance σ_r^2.

In this paper, the problem of interest is to jointly design the power allocation vectors $\mathbf{p}_r$ and $\mathbf{p}_c$ based on the radar-communication coexistence model of (2) and (4).

3. PROPOSED JOINT DESIGN

In this section, we first formulate the problem by jointly designing parameters of both systems to tackle the cross-interference induced by the coexistence. Then, a non-alternating approach is proposed to solve the joint design problem.

3.1. Problem Formulation

The figure of merit for the communication system is the achievable channel throughput, which is given by $C(\mathbf{p}_r, \mathbf{p}_c) = \sum_{n=1}^{N}\log_2\left(1 + \frac{\gamma_{cc,n}p_{c,n}}{\eta_{rc,n}p_{r,n}+1}\right)$, where $\gamma_{cc,n} = \mathbb{E}\{|\alpha_{cc,n}|^2\}/\sigma_c^2$, $\eta_{rc,n} = \mathbb{E}\{|\alpha_{rc,n}|^2\}/\sigma_c^2$, and $\mathbb{E}\{\cdot\}$ is the statistical expectation operator. For radar, the figure of merit is SINR, which is given by $\text{SINR}(\mathbf{p}_r, \mathbf{p}_c) = \sum_{n=1}^{N}\frac{\gamma_{rr,n}p_{r,n}}{\eta_{rr,n}p_{r,n}+\eta_{cr,n}p_{c,n}+1}$, where $\gamma_{rr,n} = \mathbb{E}\{|\alpha_{rr,n}|^2\}/\sigma_r^2$, $\eta_{rr,n} = \mathbb{E}\{|\beta_{rr,n}|^2\}/\sigma_r^2$, and $\eta_{cr,n} = \mathbb{E}\{|\beta_{cr,n}|^2\}/\sigma_r^2$. The joint power allocation problem is formulated as maximizing the radar SINR under throughput and power constraints:

$$\max_{\mathbf{p}_r,\mathbf{p}_c} \ \text{SINR}(\mathbf{p}_r, \mathbf{p}_c), \quad (5a)$$

$$\text{s.t.} \ \sum_{n=1}^{N} p_{r,n} \le P_r, \ \sum_{n=1}^{N} p_{c,n} \le P_c, \quad (5b)$$

$$0 \le p_{r,n} \le \xi_r, \ 0 \le p_{c,n} \le \xi_c, \ \forall\, n, \quad (5c)$$

$$C(\mathbf{p}_r, \mathbf{p}_c) \ge \kappa, \quad (5d)$$

where (5b) represents the total power constraint for each system, (5c) denotes sub-channel peak power constraints, and (5d) is a communication throughput constraint. Note that the proposed scheme assumes cooperation between the radar and communication systems by sharing certain channel related information, e.g., $\gamma_{\text{cc},n}$, $\eta_{\text{rc},n}$, etc.

3.2. Non-Alternating Approach

The joint design problem (5) is highly non-convex w.r.t. the design variables because both the objective function and the constraint (5d) are non-convex. The above problem may be solved by employing an alternating optimization procedure [20, 21], which iteratively solves (5) w.r.t. $\mathbf{p}_{\text{r}}$ while keeping $\mathbf{p}_{\text{c}}$ fixed, and vice versa, until convergence is reached. However, this alternating method is computationally intensive and does not guarantee convergence. This is particularly the case for the considered cluttered environment, where the clutter term in the SINR depends on the power allocation variable $\mathbf{p}_{\text{r}}$, which makes the optimization problem significantly more challenging even with fixed $\mathbf{p}_{\text{c}}$. To address these challenges, we consider a different approach that is described next.

Specifically, let us define $\boldsymbol{\eta}_{\text{c},n} = [\eta_{\text{rc},n},\ 0]^T$, $\boldsymbol{\eta}_{\text{r},n} = [\eta_{\text{rr},n},\ \eta_{\text{cr},n}]^T$, $\boldsymbol{\gamma}_{\text{r},n} = [\gamma_{\text{rr},n},\ 0]^T$, $\boldsymbol{\gamma}_{\text{c},n} = [0,\ \gamma_{\text{cc},n}]^T$, and $\mathbf{P} = [\mathbf{p}_{\text{r}}^T;\ \mathbf{p}_{\text{c}}^T]$ is a $2 \times N$ matrix. Then, (5) can be written as

$$\max_{\mathbf{P}} \ \sum\nolimits_{n=1}^{N} \frac{\boldsymbol{\gamma}_{\text{r},n}^T \mathbf{P}\mathbf{s}_n}{\boldsymbol{\eta}_{\text{r},n}^T \mathbf{P}\mathbf{s}_n + 1}, \quad \text{s.t. (5b), (5c),} \tag{6a}$$

$$\sum\nolimits_{n=1}^{N} \log_2 \Big(1 + \boldsymbol{\gamma}_{\text{c},n}^T \mathbf{P}\mathbf{s}_n / \big(\boldsymbol{\eta}_{\text{c},n}^T \mathbf{P}\mathbf{s}_n + 1\big)\Big) \geq \kappa, \tag{6b}$$

where $\mathbf{s}_n$ is an $N \times 1$ selection vector, e.g., $\mathbf{s}_n(i)$ is 1 for $i = n$ and 0 otherwise. Note that (6) is a fractional programming (FP) with the cost function being a sum of multiple ratios.

The multiple-ratio FP problem (6) is non-convex since the objective function is a sum of ratios, which is non-convex, and the throughput constraint (6b) imposes a non-convex feasible set. To solve (6), we can reformulate the objective function and employ an inner iteration based on convex relaxation for the throughput constraint. First, for the objective function, a quadratic transform can be used [22]. This approach introduces a set of slack variables $\boldsymbol{\lambda} = [\lambda_1, \cdots, \lambda_N]^T$ to deal with the non-convexity. Specifically, problem (6) is equivalent to

$$\max_{\mathbf{P}, \boldsymbol{\lambda}} \ F(\boldsymbol{\lambda}, \mathbf{P}), \quad \text{s.t. (5b), (5c), (6b),} \tag{7}$$

where $F(\boldsymbol{\lambda}, \mathbf{P}) = \sum_{n=1}^{N} \Big(2\lambda_n \sqrt{\boldsymbol{\gamma}_{\text{r},n}^T \mathbf{P}\mathbf{s}_n} - \lambda_n^2 \big(\boldsymbol{\eta}_{\text{r},n}^T \mathbf{P}\mathbf{s}_n + 1\big)\Big)$. Let $\lambda_n^{(\ell-1)}$ and $\tilde{\mathbf{P}}^{(\ell-1)}$ denote the solutions obtained from the $(\ell-1)$-th iteration. Then, $\lambda_n^{(\ell)}$ can be updated by solving:

$$\max_{\boldsymbol{\lambda}} \ F(\boldsymbol{\lambda}, \tilde{\mathbf{P}}^{(\ell-1)}), \tag{8}$$

which has a closed-form solution:

$$\lambda_n^{(\ell)} = \sqrt{\boldsymbol{\gamma}_{\text{r},n}^T \tilde{\mathbf{P}}^{(\ell-1)} \mathbf{s}_n} / \big(\boldsymbol{\eta}_{\text{r},n}^T \tilde{\mathbf{P}}^{(\ell-1)} \mathbf{s}_n + 1\big). \tag{9}$$

In turn, $\tilde{\mathbf{P}}^{(\ell)}$ can be obtained by solving

$$\max_{\mathbf{P}} \ F(\boldsymbol{\lambda}^{(\ell)}, \mathbf{P}), \quad \text{s.t. (5b), (5c), (6b),} \tag{10}$$

Algorithm 1 Proposed non-alternating approach

Input: $\gamma_{\text{rr},n}$, $\gamma_{\text{cc},n}$, $\eta_{\text{rc},n}$, $\eta_{\text{cr},n}$, $\eta_{\text{rr},n}$, P_{r}, P_{c}, ξ_{r}, ξ_{c}, κ, ϵ.
Output: Radar and communication powers $\mathbf{P}$.
Initialization: Initialize $\tilde{\mathbf{P}}^{(0)}$ and set iteration index $\ell = 0$.
repeat
1. Set $\ell = \ell + 1$ and solve problem (9) to obtain $\lambda_n^{(\ell)}$.
2. Initialization: $\ell_{\text{s}} = 0$ and $\hat{\mathbf{P}}^{(\ell_{\text{s}})} = \tilde{\mathbf{P}}^{(\ell-1)}$.
3. **repeat**
   (a) Set $\ell_{\text{s}} = \ell_{\text{s}} + 1$ and solve problem (13) with fixed $\hat{\mathbf{P}}^{(\ell_{\text{s}}-1)}$ and $\lambda_n^{(\ell)}$ to obtain $\hat{\mathbf{P}}^{(\ell_{\text{s}})}$.
4. **until** convergence.
5. Update $\tilde{\mathbf{P}}^{(\ell)} = \hat{\mathbf{P}}^{(\ell_{\text{s}})}$.

until convergence.
return $\mathbf{P} = \tilde{\mathbf{P}}^{(\ell)}$.

Note that the above problem is non-convex since (6b) imposes a non-convex set. We can use a SCP process to relax constraint (6b) by converting it into a convex set along with an inner iteration to solve (10). Specifically, (6b) can be relaxed into a linear form as

$$\sum\nolimits_{n=1}^{N} \log_2 \big(\boldsymbol{\gamma}_{\text{c},n}^T \mathbf{P}\mathbf{s}_n + \boldsymbol{\eta}_{\text{c},n}^T \mathbf{P}\mathbf{s}_n + 1\big) - G(\mathbf{P}, \hat{\mathbf{P}}^{(\ell_{\text{s}}-1)}) \geq \kappa, \tag{11}$$

where $\hat{\mathbf{P}}^{(\ell_{\text{s}}-1)}$ is the power vector from the $(\ell_{\text{s}}-1)$-th inner SCP iteration and

$$G(\mathbf{P}, \hat{\mathbf{P}}^{(\ell_{\text{s}}-1)}) \triangleq \log_2(\boldsymbol{\eta}_{\text{c},n}^T \hat{\mathbf{P}}^{(\ell_{\text{s}}-1)} \mathbf{s}_n + 1) + \boldsymbol{\eta}_{\text{c},n}^T (\mathbf{P} - \hat{\mathbf{P}}^{(\ell_{\text{s}}-1)}) \mathbf{s}_n / \big(\ln 2 (\boldsymbol{\eta}_{\text{c},n}^T \hat{\mathbf{P}}^{(\ell_{\text{s}}-1)} \mathbf{s}_n + 1)\big). \tag{12}$$

Thus, during the ℓ_{s}-th inner SCP iteration, the following convex optimization problem is solved to obtain $\hat{\mathbf{P}}^{(\ell_{\text{s}})}$:

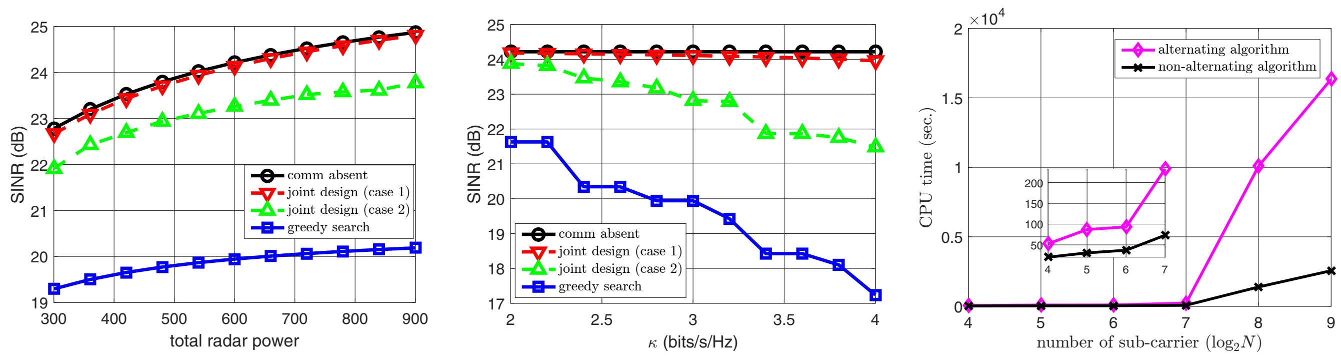
$$\max_{\mathbf{P}} \ F(\boldsymbol{\lambda}^{(\ell)}, \mathbf{P}), \quad \text{s.t. (5b), (5c), (11).} \tag{13}$$

After convergence, $\tilde{\mathbf{P}}^{(\ell)} = \hat{\mathbf{P}}^{(\ell_{\text{s}})}$ is used in (9) to compute λ_n for the next quadratic transform iteration. Our proposed solution to the joint design problem is summarized in **Algorithm 1**.

3.3. Discussions

The computational complexity of the proposed non-alternating algorithm depends on the number of the quadratic transform iterations L as well as the number of the SCP iterations L_{s}. Simulations show that the required number of the inner or outer iteration is relatively small. In addition, the convex problem (13) inside the iteration has a complexity of $\mathcal{O}(N^{3.5})$ when an interior-point method is used [23]. Thus, the overall complexity of the proposed solution is $\mathcal{O}(LL_{\text{s}}N^{3.5})$.

For the design problem (5), its feasibility depends upon whether the maximum achievable throughput (e.g., $C_{\max}$) under the power constraints is no less than the minimum throughput constraint κ, that is, $C_{\max} \geq \kappa$. Clearly, $C_{\max}$ is



(a) (b) (c)

Fig. 1. (a) Radar output SINR versus total radar power when $P_c = 600$ and $\kappa = 2.5$; (b) radar output SINR versus communication throughput constraint when $P_r = P_c = 600$; (c) computer simulation time versus the number of sub-carrier for the conventional alternating algorithm and the proposed non-alternating algorithm;.

achieved when the radar is absent while the communication system uses all sub-carriers to maximize its throughput.

Note that the proposed non-alternating solution requires initial values of $\mathbf{p}_c$ and, respectively, $\mathbf{p}_r$. A simple initialization is to consider only the power constraints (5b) and (5c). A better way that also takes into account the throughput constraint (5d) is a *greedy search* method, which is an allocation-based method. Specifically, the communication system uses its best sub-carrier to maintain the throughput constraint while the radar employs the remaining sub-carriers to maximize its SINR.

4. NUMERICAL RESULTS

In this section, we compare the proposed **joint design** with the heuristic **greedy search** method discussed in Section 3.3. In addition, we include the optimum radar output SINR, under the condition when the communication system is absent (denoted as **comm absent**), as an upper bound.

In the simulation, the number of sub-carriers $N = 16$, the convergence tolerance is 0.01, and the noise variance $\sigma_r^2 = \sigma_c^2 = 1$. The sub-carrier channel coefficients $\alpha_{\text{rr},n}$, $\alpha_{\text{cc},n}$, $\beta_{\text{rc},n}$, $\beta_{\text{rr},n}$, and $\beta_{\text{cr},n}$ are generated with Gaussian distribution $\mathcal{CN}(0, \sigma_{\text{rr}}^2)$, $\mathcal{CN}(0, \sigma_{\text{cc}}^2)$, $\mathcal{CN}(0, \sigma_{\text{rc}}^2)$, $\mathcal{CN}(0, \sigma^2)$ and $\mathcal{CN}(0, \sigma_{\text{cr}}^2)$, respectively. The strength of the desired signal for both systems, indicated by σ_{rr}^2 and σ_{cc}^2, are normalized as $\sigma_{\text{rr}}^2 = \sigma_{\text{cc}}^2 = 1$. The clutter strength $\sigma^2 = 0.05$. In the following analysis, we consider two coexistence scenarios characterized by the strength of the cross-interference: *Case 1* (weak cross-interference) with $\sigma_{\text{rc}}^2 = \sigma_{\text{cr}}^2 = 0.01$ and *Case 2* (strong cross-interference) with $\sigma_{\text{rc}}^2 = \sigma_{\text{cr}}^2 = 0.1$. In the simulation, 50 trials of channel realization are utilized to obtain the average performance.

Fig. 1(a) shows the average radar output SINR versus the total radar power. It can be seen that with weak cross-interference (Case 1), the output SINR of the joint design is very close to that of the comm absent scenario since the weak cross-interference creates limited impact from one system to the other. When the cross-interference increases (Case 2), the performance degrades due to the stronger cross impact. In both Case 1 and Case 2, the sub-carrier sharing-based joint design outperforms the greedy search method, which is a sub-carrier allocation-based method. As the total radar transmission power increases, the output SINR of all considered scenarios increases. A similar performance trend can be observed in Fig. 1(b) where the radar output SINR is plotted as a function of the communication throughput constraint κ.

Finally, we consider the computational complexity of the conventional alternating optimization approach, which decomposes the original problem into two sub-problems in $\mathbf{p}_r$ and $\mathbf{p}_c$, and the proposed non-alternating method as discussed in Section 3.2. Fig. 1(c) shows the CPU time measured by Matlab versus the total number of sub-carriers N for Case 1, where $P_r = P_c = 600$ and $\kappa = 1.5$. It can be seen that the complexity of both methods grows as the number of sub-carriers increase. However, the alternating algorithm is seen to take a longer time to converge for all cases considered. In particular, the alternating algorithm is around 8 times slower than the proposed non-alternating method at $N = 512$.

5. CONCLUSIONS

Power allocation based spectrum sharing between multi-carrier radar and communication systems was considered by maximizing the radar output SINR while meeting a communication throughput requirement along with total/peak power constraints. A joint design was proposed to tackle the coexistence problem. Through suitable reformulation, the non-convex joint design was solved by a computationally efficient non-alternating method. Simulation results validated the effectiveness of the proposed spectrum sharing method over the sub-carrier allocation-based greedy search scheme.

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