

Economics of Strategic Network Infrastructure Sharing: A Backup Reservation Approach

Jing Hou, Li Sun, Tao Shu, Yong Xiao, and Marwan Krunz

Abstract—In transitioning to 5G, the high infrastructure cost, the need for fast rollout of new services, and the frequent technology/system upgrades triggered wireless operators to consider adopting the cost-effective network infrastructure sharing (NIS), even among competitors, to gain technology and market access. NIS is a bargaining mechanism whose terms and conditions must be carefully determined based on mutual benefits in a market with uncertainties. In this work, we propose a strategic NIS framework for contractual backup reservation between a small/local network operator with limited resources and uncertain demands, and a more resourceful operator with excessive capacity. The backup reservation agreement requires the local operator (say, operator A) to reserve a certain amount of resources (e.g., spectrum) for future sharing from the resource-owning operator (say, operator B). In return, operator B guarantees availability of its reserved resources to meet the need of operator A. We characterize the bargaining between the operators in terms of the optimal reservation prices and quantities with and without consideration of their competitions in market share, respectively. The conditions under which competing operators have incentive to cooperate are explored. The impact of competition intensity and redundant capacity on performance under backup reservation are also investigated. Our study shows that NIS through backup reservation improves both resource utilization and profits of operators, with the potential to support higher target service levels for end users. We also find that, under certain conditions, operator B may still have the incentive to share its resources even at the risk of impinging on its own users.

Index Terms—Network infrastructure sharing, backup reservation, competition, game theory

I. INTRODUCTION

Network infrastructure sharing (NIS) is commonly believed to be a cost-effective solution for mobile network operators (MNOs) to quickly roll out new services, increase coverage, deploy new technologies, and improve resource utilization in a dynamic and uncertain network environment. NIS has been commonly referred to as the sharing of network infrastructure, resources, and functions among operators, such as supporting infrastructure including site locations, power supply, shelters, antenna masts, as well as spectrum and processing capacities of base stations [1], etc. The potential benefits of NIS have

been well recognized by both MNOs and major standards developing organizations (SDOs) such as 3GPP and ITU-T, and its standardization and deployment are underway. For example, 3GPP Release 13 specifies several RAN (radio access network) sharing architectures to expedite new service rollout and reduce system upgrade cost [2]. MNOs throughout the world, such as AT&T, T-Mobile, and Verizon, have participated in network sharing. The FCC has adopted new rules for millimeter wave spectrum among multiple MNOs [3]. The trend of NIS is accelerating in the era of 5G due to its strong potential for cost saving [4], [5], e.g., China Telecom and China Unicom have recently reached an agreement in deploying a 5G network by sharing their network infrastructure [6].

Despite the obvious motivation to deploy NIS, the benefits of NIS have been hindered by various strategic and operational factors. The operators need to consider the following challenges that will affect the economic incentives to implement NIS. Firstly, conventional models of resource sharing, especially spectrum sharing in buy-in situations, are typically based on auction mechanisms. The organization and execution of auctions, however, are complicated, time-consuming, and may incur high overhead due to the amount of exchanged information and coordination required between the auctioneer and bidders. As a result, auction is often considered impractical, if not infeasible, for resource sharing on a short timescale, which could have been more desirable and useful sharing mechanisms for operators dealing with highly dynamic and uncertain user demand. To reduce sharing overhead and ensure rapid reaction to network dynamics, novel sharing models need to be explored.

Furthermore, due to the profit-driven nature of MNOs, the optimal amount of resources that should be shared among operators is not only an engineering decision problem depending on QoS requirements of end users, but also an economic one that targets at offering mutual benefits among all participating operators. Such a problem is difficult to solve beforehand due to the uncertainties and dynamics of users' demand and resource availability. In particular, a priori over-investment in sharing would result in resource wasting and reduced profit, while under-investment cannot guarantee a satisfactory QoS especially in high traffic demand. As such, instead of pursuing a conventional performance-oriented stochastic resource optimization framework, as has been well-investigated in the literature [7], [8], a novel network-economic sharing model taking into account traffic and network uncertainties from both engineering and economic perspectives becomes indispensable.

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Jing Hou, Li Sun and Tao Shu are with the Department of Computer Science and Software Engineering, Auburn University, Auburn, AL 36849 USA (e-mail: {jzh0141, lzs0070, tshu}@auburn.edu)

Yong Xiao is with the School of Electronic Information and Communications at the Huazhong University of Science and Technology, Wuhan, China 430074 (e-mail: yongxiao@hust.edu.cn)

Marwan Krunz is with the Department of Electrical and Computer Engineering, the University of Arizona, Tucson, AZ 85721 USA (e-mail: krunz@email.arizona.edu)

So far, the issue of modeling and optimizing competition among operators in an NIS framework has not been well addressed. Most existing sharing models implicitly assume that operators serve independent markets (i.e., user populations). In reality, however, competition for resources and users among MNOs is complex and how to analyze and incentivize the resource sharing is still an open problem [3]. When competition exists, it is not yet clear whether the resource owner always has the incentive to share its resources with the competitors or not, due to the risk of damaging its own market share and profit. Furthermore, even when sharing takes place, how the profit, if any, should be split among the competing operators is also a key question that is yet to be answered.

In an attempt to address the above challenges, in this paper we propose a novel contract-based backup reservation model for NIS in an uncertain and competitive market. In our model, the MNO with overloaded traffic demand (for example, a virtual network operator, VNO) can request another MNO (i.e., owner) with excessive resources to reserve a certain amount of resources for future sharing. For the resources that have been reserved in advance, the VNO can use as much as it wants to meet the need of its users, while for the capacity that has not been reserved, the owner offers no guarantee of sharing. Depending on the actual demand, the VNO may not use all the reserved capacity [9] (hence, the term “backup”). In this case, the owner MNO can use the leftover capacity if needed. Such a resource sharing strategy improves the flexibility in handling uncertain demand, resulting in improved capacity and resource utilization efficiency [10], [11], with reduced operational overhead. We study such an NIS scheme when adopted by one VNO with uncertain demand and one owner with resource surplus. One potential application of our model corresponds to the scenario where a VNO with relatively limited resource supply, wishes to access the resources and infrastructure of a major MNO by signing a backup reservation contract. To reduce the complexity in designing service level agreement (SLA) and implementation, pairwise contract between two operators, has already been commonly adopted in the wireless industry. For example, the Ultra Mobile (an VNO) signed a sharing contract with T-Mobile [12], and the VNO AirVoice Wireless has made an agreement with AT&T [13]. In fact, contracts between two entities in supply chains have been adopted in a wide range of industries, while those involving three or more agents are not so popular because of the difficulty of designing the multi-operator contract and/or implementing it. The contract for a multi-echelon supply chain or between multiple operators with more complex business relationships and bargaining power is beyond the scope of this paper. We will discuss issues when extending the proposed solution into more general scenarios in Section IX.

Our main contributions are briefly summarized as follows:

First, to provide more flexibility in NIS and adaptation to demand/supply uncertainty, we introduce the concept of backup reservation into the sharing game and propose a novel contract mechanism to support efficient and profitable NIS. The equilibrium contract parameters related to the operators’ sharing decisions under demand uncertainties are derived.

Second, we tackle the issue of incentivizing multi-operator cooperation for NIS in competitive market environments. The conditions under which operators will benefit from such a resource sharing scheme are examined, which include the competition intensity between operators, the service price and level, and demand variation faced by different operators. Our work is among the first to compare NIS decisions under independent and competitive market scenarios.

Third, we propose a comprehensive solution to optimize the operator’s strategic long-term market planning related to service positioning together with the operational planning related to the NIS decisions for maximum profits. Though most researchers study purely operational decisions related to NIS scheme, the operator’s long-term marketing strategy and its resource sharing decisions influence each other. To study the impact of such an NIS scheme, we propose a three-stage Stackelberg game approach to model the operators’ strategic and operational decisions and incorporate the outcome of bargaining in backup reservation NIS into the marketing planning process. This is different from existing works that view the network infrastructure market separately from the wireless service market. To the best of our knowledge, this paper is the first to quantitatively analyze how the sharing scheme in resource market affects the operator’s marketing strategy and how competition in service markets affects the resource sharing decisions.

Finally, we show that under a backup reservation contract, if the VNO’s service price is higher than that of the owner’s, the owner is willing to take a risk of losing its own users to share the resources. This is in contrast to the finding in [14] in which the authors state that setting aside resources (e.g., spectrum) exclusively for secondary users will likely impinge on the current users of the primary system and may not be in the interest of its business model. Our numerical results also reveal that NIS through backup reservation leads to both increased resource utilization and improved target service level for VNO users. These benefits for the whole system can be further improved in a competitive scenario.

Our findings provide insights into both the engineering and the economic aspects of NIS. The framework proposes a guideline for operators to determine their NIS partners and contracts that lead to an efficient and profitable cooperation in a volatile and competitive market. Our work also provides insights on optimizing the market strategies when NIS is implemented. Note that to make our description more concrete, our presentation is based on the case of NIS. However, our model can be directly extended into more general context related to multi-MNO resource sharing.

The remainder of this paper is organized as follows. We describe the system model and formulate the problem in Section II. Sharing decisions in independent and competitive markets are presented in Sections III and IV, respectively. Then the impacts of competition are discussed in Section V. Section VI further presents the numerical results. Section VII extends the results by examining the impacts of demand uncertainty and different decision sequences. We review related work in Section VIII and draw conclusions in Section IX. All proofs are provided in the supplementary materials.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Overview

Our model applies to a network that consists of multiple cells/areas. We consider a VNO that has no or limited resources and will have to rely on renting resources from other operators to provide wireless services to end users in n service areas, $n \geq 1$. Throughout, we add the index i to quantities associated with service area i , $i = 1, \dots, n$. In a given service area, the anticipated number of users as well as the average traffic demand is uncertain. For a given target service level, if the VNO's available resources cannot meet the demand of all the users, the unsatisfied users will be ignored or will migrate to the VNO's competitors, resulting smaller market share and reduced profit. The VNO can however obtain more of the resources by negotiating for a backup reservation contract with a wireless service provider that has sufficient resources (the resource-owning operator, ROO). In the contract, the VNO reserves a certain amount of resources capacity from the ROO at a reservation price, and then uses the resources to serve its users when necessary, as illustrated in Fig. 1. As such, the ROO needs to determine the reservation price to maximize its own expected profit and the VNO encounters a decision about whether it would be worthwhile to make a reservation or not and how much to reserve.

We consider a marketing-planning period of T selling cycles.¹ The model's timing proceeds as follows. At the beginning of the period, the VNO selects its target market by determining its target service level and the corresponding service price. The marketing strategy stays fixed in the whole period. Then in each selling cycle $t = 1, \dots, T$, before the demand is known, the ROO sets the unit resource reservation price, and the VNO determines its reservation quantity as a response. After the demand at the VNO is realized, the VNO uses the reserved capacity to meet its own demand if needed. If the reserved part of the resources is not used by the VNO, the ROO can still use it to serve its own traffic demand. However, if the VNO needs the reserved resources, higher priority should be given to its users. That is, the VNO is guaranteed the share of the reserved capacity at the ROO. Finally, demands are satisfied, or the potential users are lost due to the lack of resources for supporting the target service level. It is notable that our model allows for dynamics taking places at different timescales: the market positioning or pricing process is performed at a relatively large time period (called the market-planning period); while the resource reservation and sharing processes are performed more frequently at a relatively small time period (called the selling cycle). The reservation decisions are made based on the demand distribution in each selling cycle. Although it is more desirable to consider how past reservation decisions may affect future user behaviors and the long-term profits, in this work we assume that the decision of $R_{i,t}$ is optimized to maximize the expected profit in each selling cycle t . In other words, we assume that a decision

TABLE I
LIST OF NOTATION.

Notation	Explanation
$M_i^{(V)}$	VNO's resource capacity
$M_i^{(R)}$	ROO's resource capacity
S_i	ROO's idle capacity
$l^{(V)}$	Avg. # of users sharing one unit resource at the VNO
$l^{(R)}$	Avg. # of users sharing one unit resource at the ROO
$p_c^{(V)}$	VNO's service price per user
$p_c^{(R)}$	ROO's service price per user
$p_r^{(V)}$	VNO's service price per unit resource
$p_r^{(R)}$	ROO's service price per unit resource
$x_i^{(c)}$	Potential number of users at the VNO
x_i	Potential resource demand at the VNO, with mean u_i
R_i	Reserved capacity level (a decision variable for the VNO)
w_{ri}	Unit reservation price (a decision variable for the ROO)
w	Unit capacity usage fee paid by the VNO
θ	competition intensity between operators
$\Pi_1^{(V)}$	VNO's expected profit without backup reservation
$\Pi_1^{(R)}$	ROO's expected profit without backup reservation
$\Pi_2^{(V)}$	VNO's expected profit with backup reservation
$\Pi_2^{(R)}$	ROO's expected profit with backup reservation
$\Pi_{CM}^{(V)}$	VNO's expected profit under coordination mechanism
$\Pi_{CM}^{(R)}$	ROO's expected profit under coordination mechanism
$\Delta\Pi$	Total improved profit as the benefit of backup reservation
β	Two operators' target service level difference, $\beta = l^{(V)}/l^{(R)}$
P	Average received power
n_0	Noise power
h	Steepness of the users' satisfactory curve

made in the current selling cycle will not impact the demand of future cycles. This treatment allows us to focus on the impacts of competition and makes the problem tractable. The more challenging scenario of time-dependent decision making with the consideration of long-term consequences of reservation decisions will be studied in a future work.

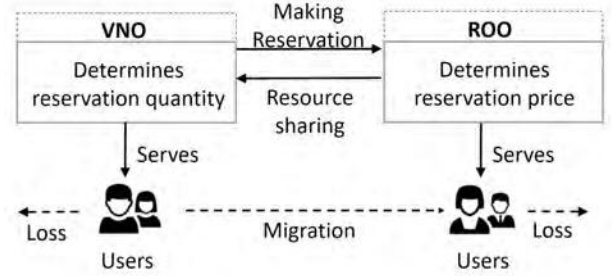


Fig. 1. Backup reservation scheme between VNO and ROO.

B. Operators' Resource Capacity

Considering the heterogeneous demand in different areas, our model assumes that the operators' capacities differ across different areas. Therefore, the optimal reservation decisions are made differently in specific to each servicing area, while the target service level is made for the whole network as a marketing strategy. We are assuming each service area is sufficiently large (for example, a service area covers a whole city), so that an arbitrary location in area A is sufficiently far away from an arbitrary location in area B . Under such a setting, the assumption of independent service areas is reasonable in general, because network infrastructures and resources only have a local scope. For example, the occupied/idle status of a given frequency channel in two geographically distant areas

¹The duration of one marketing-planning period is decided by the VNO's long-term plan for the market strategy, e.g., one year. A selling cycle is the minimum time scale, e.g., one month per cycle. The user demand remains unchanged in one selling cycle, but may change across cycles.

are likely to be independent from each other, and therefore the idle capacity in one area may not be idle in a different service area. There are some special/corner cases in which service areas are not independent. For instance, two adjacent areas may have correlated traffic, if most of the traffic originating from one area terminates in the other. We do not consider such corner cases in this work and leave it for future research.

In a given service area, the VNO's own network resource capacity is denoted by $M_i^{(V)}$, $i = 1, \dots, n$. For the ROO, we consider a wireless service provider that operates in the service areas and that has developed a group of loyal customers or comparatively stable expected demand. This ROO has sufficient resource capacity to meet its own constant demand. Denote its total resource capacity as $M_i^{(R)}$ and the redundant capacity as $S_{i,t}$. The ROO typically allocate more resources in network planning than what are needed by the actual traffic load for two reasons. Firstly, over-provisioning is a commonly used strategy to ensure acceptable network performance against traffic variations caused by equipment failures or transient traffic surges [15]. In addition, as part of the service provider's expansion and growth plan, over-provisioning is a conservative approach to meet the needs of future growing demand in the long-term.

In the backup reservation contract, the VNO and its users have a higher priority in using the reserved capacity over the ROO's own users. If the ROO overly commits its capacity to the reservation (i.e., a reservation quantity larger than $S_{i,t}$) with not enough capacity left (i.e., less than $M_i^{(R)} - S_{i,t}$ units) for its own users, then the ROO may experience capacity starvation and partial user loss. This requires the two operators to carefully decide their reservation contract parameters to ensure mutual benefits in resource sharing. As in [16], the notion of infrastructure is quite general. It can be composed of resources such as links, servers, and buffers. Its quantity might be the bandwidth of a communication link or the cycles available in a computational grid.

C. Target Service Level

In the VNO's long-term market planning, it needs to determine a target service level that is related to its quality of service. A higher target service level indicates a higher service quality, and vice versa. Note that the concept of service level is general. It is not affected by any particular medium access control (MAC) protocol, but instead captures the basic behavior of all MAC mechanisms. This target service level is announced by the VNO to the market as an average service level, and is not necessarily equivalent to the instantaneous level experienced by every user at any given time. So it is reasonable to expect that when there is a traffic demand burst from users, the instant service level may deteriorate.

Intuitively, the service level or QoS enjoyed by a user deteriorates with the number of users accessing the same resource. Therefore, we represent the VNO's target service level as the average number of its own users, $l^{(V)}$, that will consume one unit of resource capacity. For example, the VNO may decide that an average traffic demand of five users can be satisfied by one unit of network resource ($l^{(V)} = 5$). The

users' satisfaction level is closely related to $l^{(V)}$. The smaller the value $l^{(V)}$, the higher service level VNO offers, and the higher satisfaction level users would have.

Let $U(l^{(V)})$ be a measure of the users' average satisfaction level. We assume $\frac{dU}{dl^{(V)}} < 0$, $\frac{d^2U}{dl^{(V)2}} < 0$. For example, if one band of spectrum is shared among $l^{(V)}$ users, the average data rate is $\ln(1 + \frac{P}{n_0 + P(l^{(V)} - 1)})$, where P is the average received power and n_0 is the noise level. The satisfaction level U can be defined as an increasing function of the difference between the achievable data rate and the users' data rate requirement. In line with [17], [18], we use the sigmoid function, which has been widely used to approximate the user's satisfaction with respect to service qualities [19]. Specifically,

$$U(l^{(V)}) = \frac{1}{1 + e^{-h[\ln(1 + \frac{P}{n_0 + P(l^{(V)} - 1)}) - \ln(1 + \frac{P}{n_0 + P(l_{\max}^{(V)} - 1)})]}} \quad (1)$$

where $l_{\max}^{(V)}$ is the maximum value of $l^{(V)}$, which specifies the lowest service level users can accept; and h is the steepness of the satisfactory curve [17]. We suppose $l^{(R)}$, a constant, is the average number of users sharing one unit of resource at the ROO, which has already been strategically determined².

D. Service Pricing

For each user, the VNO charges price $p_c^{(V)}$ as the service fee. This is equivalent to an average price per resource $p_r^{(V)} = l^{(V)} p_c^{(V)}$. Our model allows the VNO to adjust its price $p_c^{(V)}$ but it also needs to change the corresponding service level at the same time. That is, in maximizing the VNO's expected profit, the price $p_c^{(V)}$ is supposed to match its service level. The reason is that: it is the market that determines that the service price should reflect the quality of the service. That means an operator cannot arbitrarily increase the price without improving the service. Otherwise, the operator would be at the risk of losing its potential users. Therefore, the service price can be set as the users' willingness to pay for the service, and normally increases with the service level. It is reasonable to assume that $p_c^{(V)} = \alpha U(l^{(V)})$ where α is the equivalent service value per degree of users' satisfaction. There is a tradeoff for the VNO's decisions: a higher price indicates a higher target service level and thus more resource consumption, which implies a larger user loss rate with limited resource capacity, but it also means a higher marginal revenue. The ROO's service price is $p_r^{(R)}$ per unit resource, and on average the traffic of $l^{(R)}$ users consume one unit of resource capacity.

E. Demand Uncertainty and Lost Demand

On the VNO's side, the number of its users during a selling cycle t is denoted by $x_{i,t}^{(c)}$. We assume $x_{i,t}^{(c)}$ is a random variable, and its distribution can be obtained through either an extensive market investigation based on publicly available data and customer surveys or data mining on sale history.

²Our model allows the possibility of changing the ROO's target service level by taking into account a longer marketing period than that of the VNO.

Under a service level $l^{(V)}$, the total resource demand can therefore be written as $x_{i,t} = x_{i,t}^{(c)}/l^{(V)}$ units per cycle. Let $F_{i,t}(\cdot)$ and $f_{i,t}(\cdot)$ be the cumulative distribution function and probability density function of $x_{i,t}$, respectively, with $\mu_{i,t}$ be its expected value. The distribution depends on both the distribution of $x_{i,t}^{(c)}$ and the service level. For instance, if $x_{i,t}^{(c)}$ follows a uniform distribution $U(a_0, b_0)$, then $x_{i,t}$ follows $U(a_0/l^{(V)}, b_0/l^{(V)})$. To accommodate the difference in the demand distributions, we do not assume any specific function for the demand distribution and it can differ across different selling cycles. That is, the analysis in this paper applies to any general functions of $F_{i,t}(\cdot)$ and $f_{i,t}(\cdot)$.

If the VNO's limited resources cannot serve all incoming users at the target service level, then some users will be lost to a competitor, possibly the ROO itself. In our model the user capacity is defined as the maximum number of users whose QoS can be satisfied statistically in a selling cycle (i.e., a soft QoS guarantee bound), rather than the maximum instantaneous number of users that can be served at the same time. In practice, the realization of this user capacity is related to the equilibrium of the dynamic process of users (or subscribers) coming and leaving. In this dynamic process, new users keep coming and they are all accepted by the operator. However, as the amount of demand exceeds the operator's capacity, the actual service level cannot meet the committed service guarantee (i.e., the prescribed target service level that corresponds to the service price). So existing users would start to look for another service provider and leave the current operator. Many wireless operators offer various satisfaction guarantee policies so that users have great flexibility in choosing the service providers. For example, both Sprint and T-mobile provide a free 30-day trial of their wireless service; the subscribers of Verizon may terminate service for any reason within 14 days of activation; and operators like AT&T provide no-contract based service to users. Under these policies, unsatisfied users have the freedom to stop using the current service and to switch to other service providers. As in many marketing literature (see, e.g., [20]), instead of focusing on the dynamics of the demand throughout the selling cycle, we are more interested in the equilibrium outcome when the demand matches the supply at the target service level. That is, the final satisfied demand is limited by both the capacity with respect to the target service level and the potential demand.

We define lost demand as the corresponding demand of users that cannot be satisfied at the claimed service level. The loss rate can be expressed as $1 - \frac{\text{number of satisfied users}}{\text{number of arriving users}}$. Similar to [21], we consider a long-term average demand that is independent of short-term wireless characteristics. That is, a burst of users' demand in a selling cycle may cause a larger user loss rate; while a burst of data traffic demand among the users would not incur any demand loss.

Since the reservation decisions are repeated for every selling cycle, to simplify our presentation, but without loss of generality, we consider only one selling cycle (i.e. $T = 1$) and ignore the time-associated subscript (t) in the following model. An extension to a general case of multi-cycle model is straightforward, where the sum of the VNO's expected profits

across all selling cycles is maximized for the whole market-planning period. As for the ROO who has built a stable user base, the number of potential users is assumed to be constant and can be calculated as $(M_i^{(R)} - S_i)l^{(R)}$. The notation used throughout the paper is listed in Table I³.

F. Backup Reservation Scheme and Problem Formulation

As a condition of the backup reservation contract, the VNO pays a fixed monetary value $w_{ri}R_i$, regardless of the actual amount of resources used in the cycle. R_i is the reserved capacity level chosen by the VNO. If the VNO's current resource capacity can meet the actual resource demand, no backup resource from the ROO is needed; otherwise, the VNO should use the reserved resources at the ROO. Note that the ROO may or may not require the VNO to reserve the redundant capacity S_i before using it, depending on the market situations as will be specified later.

The interactions between two operators are formulated in a three-stage hierarchical order of decision making. Stage 1: the VNO makes the decision of its target service level $l^{(V)}$ as a high-level market positioning strategy. Stage 2: knowing the VNO's target market, the ROO sets the unit reservation price w_{ri} at each selling cycle by considering their possible competition in that market segment. ROO announces w_{ri} to the VNO. In stage 3, the VNO determines its reservation quantity R_i and pays the ROO a monetary amount $w_{ri}R_i$. After the demand at the VNO in each selling cycle is realized, the VNO determines the quantity of reserved resources to be used in the cycle and pays the capacity usage fee to the ROO. The unit capacity usage fee is denoted by w , $w < \min\{p_r^{(R)}, p_r^{(V)}\}$. In order to concentrate our analysis on the resource reservation problem, we consider a fixed resource sharing model, that is, w is exogenously determined by the resource market. The decoupling of the reservation price from the sharing price (usage fee), without altering the existing process of NIS, would allow us to better focus on the benefits of the backup reservation scheme.

We assume both the demand and the capacity can be estimated by the operators through an analysis of the supply market and the users market. In the case that this information is considered private and sensitive, one possible way to implement the proposed mechanism is to delegate the computation of the proposed mechanism to a trusted third party, who is a central controller in charge of the trading process and supports decision-making for the operators based on their inputs, but does not disclose the information submitted by one operator to the other operator. In such a way the proposed mechanism can be performed without disclosing the private/sensitive information to the other operator. Such a trusted third-party solution has been widely used in the literature, e.g., an auctioneer in online spectrum auctions. The problem under asymmetric information is normally solved through contract design with different sets of prices and reservation quantities. By satisfying both individual rationality and

³To be general, we assume the quantities throughout the paper take normalized values with the metric units chosen depending on the application. Therefore, all the quantities are unit-less.

incentive compatibility constraints in moral hazard models, the operators would truthfully reveal their private demand information, as studied in [16]. In order to concentrate on the benefits of reservation-based NIS, we will focus on the contract design under symmetric information, which is widely assumed as in the literature (see, for example, [22], [21], [14]).

In the rest of this paper, we aim to answer all of the following questions:

- 1) How would the reservation-based sharing scheme affect the VNO's determination of its target service level and the two players' expected profits?
- 2) Can both operators benefit from the backup reservation? When does the VNO prefer not to make any reservation even at the risk of losing some users? When does the ROO reject the reservation request?
- 3) How do the redundant capacity at the ROO and the service prices at the two operators affect the reservation price and the reservation quantity?
- 4) If the service markets are competitive, do the operators still benefit from backup reservation? How does the market competition intensity affect both operators' decisions and expected profits?

III. SHARING DECISIONS WITH INDEPENDENT MARKETS

As a benchmark, we first consider the scenario in which two operators serve two completely independent markets. That is, the two operators provide totally different services that are irreplaceable to each other but are using the same kind of resources. A good example is the wireless TV broadcast service and the wireless data communication service over the TV white space. In this case the two operators are targeting different user populations, i.e., there is no competition between the VNO and ROO. To analyze the decisions of backup reservation, we consider two cases: i) the VNO does not request any reservation, and the amount of shared resources from the ROO does not exceed S_i , and ii) in case the required amount of resources from the VNO exceeds S_i , it needs to reserve in advance the extra part; and when the resource demand is larger than $M_i^{(V)}$, the total amount of $R_i + S_i$ resources can be shared if needed.

A. Utility Functions

1) *Without Backup Reservation:* In the case of no backup reservation, VNO users that cannot be served with $M_i^{(V)} + S_i$ amount of resources are simply ignored. With an average number of $l^{(V)} \int_{M_i^{(V)}+S_i}^{+\infty} (x_i - M_i^{(V)} - S_i) f_i(x_i) dx_i$ users lost due to resource scarcity, the VNO's expected profit can be written as

$$\begin{aligned} \Pi_1^{(V)} = & \sum_{i=1}^n [-w \int_{M_i^{(V)}}^{M_i^{(V)}+S_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i \\ & - w S_i \int_{M_i^{(V)}+S_i}^{+\infty} f_i(x_i) dx_i + \mu_i p_r^{(V)} \\ & - p_r^{(V)} \int_{M_i^{(V)}+S_i}^{+\infty} (x_i - M_i^{(V)} - S_i) f_i(x_i) dx_i] \end{aligned} \quad (2)$$

where the first two terms inside the summation represent the expected capacity usage fee when the VNO's own resources cannot satisfy the actual demand and the last two terms represent the revenue from the users. The VNO's decision is represented as $l_1^{(V)*} = \arg \max_{l_1^{(V)} \leq l_{\max}^{(V)}} \Pi_1^{(V)}$.

Similarly, the ROO's expected profit is

$$\begin{aligned} \Pi_1^{(R)} = & \sum_{i=1}^n [(M_i^{(R)} - S_i) p_r^{(R)} + w S_i \int_{M_i^{(V)}+S_i}^{+\infty} f_i(x_i) dx_i \\ & + w \int_{M_i^{(V)}}^{M_i^{(V)}+S_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i]. \end{aligned} \quad (3)$$

2) *With Backup Reservation:* We now consider the case in which the VNO reserves R_i units of resources in advance and the ROO accepts the reservation request. For the VNO, the actual number of lost users in one selling cycle can be written as $l^{(V)}(x_i - M_i^{(V)} - S_i - R_i)^+$, where $(\cdot)^+ = \max(\cdot, 0)$, and the actual amount of used resources shared from the ROO is $\min\{(x_i - M_i^{(V)})^+, S_i + R_i\}$. The VNO's expected profit is

$$\begin{aligned} \Pi_2^{(V)} = & \sum_{i=1}^n [-w_{ri} R_i + \mu_i p_r^{(V)} \\ & - p_r^{(V)} \int_{M_i^{(V)}+S_i+R_i}^{+\infty} (x_i - M_i^{(V)} - S_i - R_i) f_i(x_i) dx_i \\ & - w(S_i + R_i) \int_{M_i^{(V)}+S_i+R_i}^{+\infty} f_i(x_i) dx_i \\ & - w \int_{M_i^{(V)}}^{M_i^{(V)}+S_i+R_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i] \end{aligned} \quad (4)$$

where $w_{ri} R_i$ is the reservation cost irrespective of the usage amount, and the last two terms indicate the expected capacity usage fee under high demand. In the following analysis, we do not consider the trivial case when the highest possible demand is smaller than $M_i^{(V)} + S_i$ and no backup reservation is needed.

The ROO's expected profit $\Pi_2^{(R)}$ consists of three parts: the reservation cost paid by the VNO, the revenue earned through sharing resources with the VNO, and the revenue of offering services to its own users. Note that more shared resource with the VNO would also indicate a larger user loss rate, and thus less revenue from users. Accordingly,

$$\begin{aligned} \Pi_2^{(R)} = & \sum_{i=1}^n [w_{ri} R_i + w \int_{M_i^{(V)}}^{M_i^{(V)}+S_i+R_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i \\ & + w(S_i + R_i) \int_{M_i^{(V)}+S_i+R_i}^{+\infty} f_i(x_i) dx_i \\ & + p_r^{(R)} \int_{M_i^{(V)}+S_i}^{M_i^{(V)}+S_i+R_i} (R_i - (x_i - M_i^{(V)} - S_i)) f_i(x_i) dx_i \\ & + R_i p_r^{(R)} \int_0^{M_i^{(V)}+S_i} f_i(x_i) dx_i + p_r^{(R)} (M_i^{(R)} - S_i - R_i)]. \end{aligned} \quad (5)$$

B. Stackelberg Game and Equilibrium Solution Analysis

It is straightforward to see that the amount of shared resources to be used by the VNO in a selling cycle is $\min\{(x_i - M_i^{(V)})^+, S_i + R_i\}$. To make optimal decisions on the reservation quantity and price, as well as the VNO's target service level, a three-stage Stackelberg game is employed to describe the bargaining process. In the following analysis, we adopt backward induction to derive the Stackelberg equilibrium solution. First, in stage 3: for a given target service level and reservation price, we derive the VNO's optimal reservation quantity; second, with the prediction of the VNO's response, the ROO's optimal reservation price is analyzed in stage 2; finally, in stage 1, the VNO determines the target service level. These three stages are now described in detail.

Stage 3 (VNO sets the reservation quantity): Given the target service level and the reservation price, by maximizing the VNO's expected profit in (4), we have:

Lemma 1. In independent markets, for a given reservation price w_{ri} and the target service level, the optimal reservation quantity must satisfy:

$$R_i^* = \max\{\min\{F_i^{-1}(1 - \frac{w_{ri}}{p_r^{(V)} - w}) - S_i - M_i^{(V)}, M_i^{(R)} - S_i\}, 0\}. \quad (6)$$

Note that the VNO does not need to reserve any of the ROO's redundant capacity S_i . In fact, the maximum reserved quantity cannot exceed $M_i^{(R)} - S_i$. For $R_i > 0$, the unit reservation price should be low enough so that $w_{ri} < (p_r^{(V)} - w) \int_{M_i^{(V)} + S_i}^{+\infty} f_i(x_i) dx_i$. The explanation is intuitive: only if the reservation price is lower than the expected revenue (i.e., price minus usage fee), will the VNO make a reservation.

Stage 2 (ROO sets the reservation price): The ROO needs to decide the optimal reservation price that maximizes its expected profit. If it does not expect to share more resources than S_i , it will set $w_{ri} = (p_r^{(V)} - w) \int_{M_i^{(V)} + S_i}^{+\infty} f_i(x_i) dx_i$; otherwise, an optimal value of w_{ri} will be given with the consideration of the VNO's response in (6).

Proposition 1. In independent markets, if the optimal reservation quantity is smaller than $M_i^{(R)} - S_i$, then the optimal unit reservation price w_{ri} should satisfy:

$$R_i(w_{ri})(p_r^{(V)} - w)^2 f_i(M_i^{(V)} + S_i + R_i(w_{ri})) = w_{ri}(p_r^{(V)} - p_r^{(R)}) \quad (7)$$

where $R_i(w_{ri}) = F_i^{-1}(1 - \frac{w_{ri}}{p_r^{(V)} - w}) - M_i^{(V)} - S_i$.

From (7), for the reservation quantity to be positive, we must have $p_r^{(V)} > p_r^{(R)}$. That is, if the VNO sets a service price per unit resource lower than that of the ROO, a high reservation price would be set by the ROO so that the VNO would opt for no reservation. This way, the resources at the VNO would be sold at the higher price, $p_r^{(R)}$. On the other hand, if the ROO sets a price higher than $p_r^{(R)}$, the ROO would set a low reservation price to encourage backup reservation. Therefore, the shared resources are always used by the operator with a higher price. In other words, the ROO is always better off under backup reservation as long as $p_r^{(V)} > p_r^{(R)}$.

Stage 1 (VNO sets the target service level): When the predictions of R and w_r are made, the VNO needs to determine

the optimal value of $l^{(V)}$ or $p_r^{(V)}$, denoted by

$$l_2^{(V)*} = \arg \max \Pi_2^{(V)} \quad s.t. \quad l_2^{(V)} \leq l_{\max}^{(V)}. \quad (8)$$

Because $p_r^{(V)} = l^{(V)} p_c^{(V)}(l^{(V)}) = l^{(V)} \alpha U(l^{(V)})$, $\frac{dp_r^{(V)}}{dl^{(V)}} = p_c^{(V)}(l^{(V)}) + l^{(V)} \frac{dp_c^{(V)}(l^{(V)})}{dl^{(V)}}$, $\frac{d^2 p_r^{(V)}}{dl^{(V)2}} = l^{(V)} \frac{d^2 p_c^{(V)}}{dl^{(V)2}} + 2 \frac{dp_c^{(V)}}{dl^{(V)}} < 0$, we can find a range of $[l_A^{(V)}, l_B^{(V)}]$ within which $p_r^{(V)} \geq p_r^{(R)}$ can be satisfied, where $l_A^{(V)}$ or $l_B^{(V)}$ satisfies $p_r^{(V)} = p_r^{(R)}$. If $l_A^{(V)} = l_B^{(V)}$, then no backup reservation is made.

It is intuitive to see that with the backup reservation scheme, more resources are available to the VNO, which encourages the VNO to offer a higher level of service. Due to the complexity of solving problem (8), it is hard to provide a closed-form expression of $l^{(V)*}$. A heuristic algorithm for calculating the optimal value of $l^{(V)}$ is as follows:

Step 1) Calculate the thresholds of making backup reservation $p_r^{(V)} \geq p_r^{(R)}$, where $l^{(V)} \in [l_A^{(V)}, l_B^{(V)}]$;

Step 2) For $l^{(V)} \notin [l_A^{(V)}, l_B^{(V)}]$, find $l_{21}^{(V)*} = \arg \max \Pi_1^{(V)}$; if $l_{21}^{(V)*} > l_{\max}^{(V)}$, then $l_{21}^{(V)*} = l_{\max}^{(V)}$;

Step 3) For $l^{(V)} \in [l_A^{(V)}, l_B^{(V)}]$, first calculate the value of R_i and w_{ri} using (6) and (7), then combine them with (4) and find $l_{22}^{(V)*} = \arg \max \Pi_2^{(V)}$; if $l_{22}^{(V)*} > l_{\max}^{(V)}$, then $l_{22}^{(V)*} = l_{\max}^{(V)}$;

Step 4) Compare $\Pi_1^{(V)}|_{l^{(V)}=l_{21}^{(V)*}}$ and $\Pi_2^{(V)}|_{l^{(V)}=l_{22}^{(V)*}}$; select the one with a larger value for the final expected profit, and the corresponding $l^{(V)}$ as the optimal solution.

The basic idea is to first find the service level interval that $R_i^* > 0$. Within this interval, the optimal reservation decisions in (6) and (7) are incorporated into the VNO's expected profit function (4). Outside the interval, the VNO's expected profit function is written as (2). Then we find the optimal target service level within or outside of the interval that maximizes the VNO's expected profit respectively. Thus we can find the optimal target service level with the maximum expected profit. We will rely on numerical analysis in Section VI to further investigate the equilibrium outcome of the target service level.

IV. SHARING DECISIONS BETWEEN COMPETITIVE OPERATORS

In this section, we are considering that the two operators are indirect competitors, whose services are different but substitutable (such as mobile and fixed broadband services). In this situation, we suppose the users always prefer to subscribe to their first-choice service provider, and a user chooses an alternative service only when his or her primary service provider is "out of stock". That is, the VNO's potential users would switch to the ROO only when their demand cannot be satisfied by the VNO [23]. Let θ be the fraction of unsatisfied users that switch to the ROO, where $0 < \theta \leq 1$, which measures the competition intensity between operators [24], with $\theta = 1$ indicating the highest competition intensity⁴. Note

⁴The user's switching rate θ can be estimated by studying user switching behavior and the market share dynamics. For example, factor analysis and logistic regression can be used to investigate the impacts of various factors such as the switching cost and users' evaluation of the service quality. User utility model that takes the influence factors into account can be built to help determine the value of θ . In particular, for the case of two competing operators, a hotelling model can be used to capture the users' preference and further determine the users switching rate.

that in our model the competition level θ is a statistical index that measures user's VNO-to-ROO switching rate over the time period of one selling cycle. It is an input argument that is used in our model to support the service providers' NIS decision making. Its value is fixed in one selling cycle, but can change from cycle to cycle. In our model, θ largely depends on the ROO's service level: A higher target service level at the ROO side (under the same price) is expected to attract more users from its competitor, i.e., a larger θ value. However, since in our model we assume that the ROO's target service level has already been strategically determined, the value of θ is given exogenously. This value could change with the ROO's target service level in a different decision cycle.

We assume that the ROO has a redundant capacity S_i after satisfying its own users. This redundant capacity can be used later to satisfy the new demand coming from the VNO's market. If the redundant capacity is shared with the VNO, then it is possible that the ROO would lose some of the new users. Therefore, the ROO becomes conservative in sharing its redundant resources with the VNO. In this case, we consider the situation when the ROO requires the VNO to make a backup reservation first before sharing any amount of the resources, and examine the conditions under which a backup reservation benefits both operators.

A. Utility Functions

1) *Without Backup Reservation:* If no backup reservation is made at the ROO, the VNO only serves its users with its own resources. The VNO's decision is $l_1^{(V)*} = \arg \max_{l_1^{(V)} \leq l_{\max}^{(V)}} \Pi_1^{(V)}$, where

$$\Pi_1^{(V)} = p_r^{(V)} \sum_{i=1}^n \left[\int_0^{M_i^{(V)}} x f_i(x) dx + M_i^{(V)} \int_{M_i^{(V)}}^{+\infty} f_i(x) dx \right]. \quad (9)$$

The ROO's expected profit is

$$\begin{aligned} \Pi_1^{(R)} = & \sum_{i=1}^n [(M_i^{(R)} - S_i) p_r^{(R)} + p_r^{(R)} S_i \int_{M_i^{(V)} + \frac{S_i}{\theta\beta}}^{+\infty} f_i(x) dx \\ & + \theta\beta p_r^{(R)} \int_{M_i^{(V)}}^{M_i^{(V)} + \frac{S_i}{\theta\beta}} (x - M_i^{(V)}) f_i(x) dx] \end{aligned} \quad (10)$$

where $\beta = l^{(V)}/l^{(R)}$ indicates the difference between the two operators' service levels. The first term inside the summation in (10) is the revenue from satisfying the ROO's own users, and the following two are from satisfying new users switching from the VNO's market.

2) *With Backup Reservation:* For the VNO, the actual number of unsatisfied users in one selling cycle is $l^{(V)}(x_i - M_i^{(V)} - R_i)^+$, and the actual amount of resources shared with the ROO is $\min\{(x_i - M_i^{(V)})^+, R_i\}$. Then the VNO's expected

profit can be written as

$$\begin{aligned} \Pi_2^{(V)} = & \sum_{i=1}^n [p_r^{(V)} \int_0^{M_i^{(V)} + R_i} x f_i(x) dx \\ & + p_r^{(V)} \int_{M_i^{(V)} + R_i}^{+\infty} (M_i^{(V)} + R_i) f_i(x) dx - w_r R_i \\ & - w R_i \int_{M_i^{(V)} + R_i}^{+\infty} f_i(x) dx \\ & - w \int_{M_i^{(V)}}^{M_i^{(V)} + R_i} (x - M_i^{(V)}) f_i(x) dx]. \end{aligned} \quad (11)$$

As for the ROO, two cases need to be considered:

Case 1: $R_i \leq S_i$. All of ROO's own users can be satisfied, and the ROO may still have capacity to satisfy new demands coming from the other market. The ROO's expected profit in each service area i is

$$\begin{aligned} \Pi_{2i}^{(R)} = & (M_i^{(R)} - S_i) p_r^{(R)} + w_r R_i + w R_i \int_{M_i^{(V)} + R_i}^{+\infty} f_i(x) dx \\ & + w \int_{M_i^{(V)}}^{M_i^{(V)} + R_i} (x - M_i^{(V)}) f_i(x) dx \\ & + \theta\beta p_r^{(R)} \int_{M_i^{(V)} + R_i}^{M_i^{(V)} + R_i + \frac{S_i - R_i}{\theta\beta}} (x - M_i^{(V)} - R_i) f_i(x) dx \\ & + p_r^{(R)} \int_{M_i^{(V)} + R_i + \frac{S_i - R_i}{\theta\beta}}^{+\infty} (S_i - R_i) f_i(x) dx. \end{aligned} \quad (12)$$

The first term is the revenue obtained from the ROO's own users. The second term is the reservation price paid by the VNO. The third item is the capacity usage fee paid by the VNO when the actual demand at the VNO x_i is larger than $M_i^{(V)} + R_i$ and all the reservation quantity R_i is shared. The fourth item is the capacity usage fee paid by the VNO when $x_i \leq M_i^{(V)} + R_i$ and only an amount of $x_i - M_i^{(V)}$ resources is shared. The last two items are the revenue from satisfying the new demand, with an average number of $\theta l^{(V)} \int_{M_i^{(V)} + R_i}^{M_i^{(V)} + R_i + \frac{S_i - R_i}{\theta\beta}} (x - M_i^{(V)} - R_i) f_i(x) dx + l^{(R)} \int_{M_i^{(V)} + R_i + \frac{S_i - R_i}{\theta\beta}}^{+\infty} (S_i - R_i) f_i(x) dx$ new users.

Case 2: $R_i > S_i$. In this case, some of the ROO's own users may not be satisfied when VNO uses more resources than S_i , and clearly the ROO has no redundant capacity to satisfy any new demand. Thus, the ROO's expected profit in a given service area is

$$\begin{aligned} \Pi_{2i}^{(R)} = & (M_i^{(R)} - R_i) p_r^{(R)} + w_r R_i + w R_i \int_{M_i^{(V)} + R_i}^{+\infty} f_i(x) dx \\ & + w \int_{M_i^{(V)}}^{M_i^{(V)} + R_i} (x - M_i^{(V)}) f_i(x) dx \\ & + p_r^{(R)} \int_0^{M_i^{(V)} + S_i} (R_i - S_i) f_i(x) dx \\ & + p_r^{(R)} \int_{M_i^{(V)} + S_i}^{M_i^{(V)} + R_i} (R_i - x + M_i^{(V)}) f_i(x) dx. \end{aligned} \quad (13)$$

The first term is the revenue from the ROO's own users using the resources that are not reserved by the VNO. The second term is the reservation price paid by the VNO. The third item is the capacity usage fee paid by the VNO when the actual demand at the VNO x_i is larger than $M_i^{(V)} + R_i$ and all the reservation quantity R_i is shared. The fourth item is the capacity usage fee paid by the VNO when $x_i \leq M_i^{(V)} + R_i$ and only an amount of $x_i - M_i^{(V)}$ resources is shared. The last two items are the revenue from satisfying the ROO's own users using the reserved resources that are not shared with the VNO. The ROO's total expected profit is $\Pi_2^{(R)} = \sum_{i=1}^n \Pi_{2i}^{(R)}$.

B. Stackelberg Game and Equilibrium Analysis

Similar to the procedures of decision-making under the independent scenario, in the following models, we first derive the Stackelberg equilibrium solution in stages 3 and 2, and then analyze the VNO's optimal service level in stage 1.

Stage 3 (VNO sets the reservation quantity): By maximizing the VNO's expected function of (11), the optimal reservation quantity is given by the following lemma:

Lemma 2. In a competitive market, given the VNO's target service level and the reservation price w_{ri} , the optimal reservation quantity is given by:

$$R_i^* = \max\{\min\{F_i^{-1}(1 - \frac{w_{ri}}{p_r^{(V)} - w}) - M_i^{(V)}, M_i^{(R)}\}, 0\}. \quad (14)$$

Lemma 2 shows that the VNO would cooperate with the ROO through backup reservation as long as the reservation price is low enough such that $w_{ri} < (p_r^{(V)} - w) \int_{M_i^{(V)}}^{+\infty} f_i(x_i) dx_i$. We observe that in a competitive market the ROO's potential redundant capacity S_i does not affect the VNO's reservation quantity. This result is intuitive because the ROO requires the VNO to make a reservation first before sharing any amount of resource, including the ROO's redundant capacity. However, the market competition does not only affect the reservation quantity, but also the reservation price. Therefore, we need to examine the change in the reservation price to evaluate the overall impact of competition.

Stage 2 (ROO sets the reservation price): By substituting R_i in (12), and (13) with (14), and by setting $\frac{d\Pi_{2i}^{(R)}}{dR_i} = 0$, the following result can be obtained.

Proposition 2. In a competitive market, the optimal value of unit reservation price w_{ri}^* takes one of five possible values: $(1 - F_i(M_i^{(V)}))(p_r^{(V)} - w)$, w_{ri1} , $(1 - F_i(S_i + M_i^{(V)}))(p_r^{(V)} - w)$, w_{ri2} , and $(1 - F_i(M_i^{(R)} + M_i^{(V)}))(p_r^{(V)} - w)$. The VNO makes no reservation when $w_{ri} \geq (1 - F_i(M_i^{(V)}))(p_r^{(V)} - w)$, and makes maximum reservation when $w_{ri} \leq (1 - F_i(M_i^{(R)} + M_i^{(V)}))(p_r^{(V)} - w)$, where w_{ri1} satisfies

$$\begin{aligned} & R_i(w_{ri})(p_r^{(V)} - w)f_i(M_i^{(V)} + R_i(w_{ri})) \\ &= w_{ri}(p_r^{(V)} - p_r^{(R)})/(p_r^{(V)} - w) \\ &+ p_r^{(R)}(1 - \theta\beta) \int_{M_i^{(V)} + R_i(w_{ri})}^{M_i^{(V)} + R_i(w_{ri}) + \frac{S_i - R_i(w_{ri})}{\theta\beta}} f_i(x_i) dx_i \end{aligned} \quad (15)$$

w_{ri2} satisfies

$$w_{ri}(p_r^{(V)} - p_r^{(R)}) = R_i(w_{ri})(p_r^{(V)} - w)^2 f_i(M_i^{(V)} + R_i(w_{ri})) \quad (16)$$

$$\text{and } R_i(w_{ri}) = F_i^{-1}(1 - \frac{w_{ri}}{p_r^{(V)} - w}) - M_i^{(V)}.$$

The following results can be derived concerning the ROO's decision when the VNO's response is considered.

Corollary 1. In a competitive market, if $p_r^{(V)} < p_r^{(R)}$ and $\theta\beta > 1$, then the ROO will set a high reservation price that no reservation is made by the VNO.

From Corollary 1, if the VNO's service price per unit resource is lower than that of the ROO's, and their competition intensity is high enough, then the ROO benefits more from the competition than from cooperation, and it is not beneficial to cooperate with the VNO. This results can be explained by the fact that, the ROO has no incentive to share its resources with its competitor when it could make better use of the resources by selling at a higher price, and it could lose the opportunity of obtaining a considerable number of new users otherwise.

Stage 1 (VNO sets the target service level): Now we analyze the VNO's optimal value of $l^{(V)}$ or $p_r^{(V)}$ as in (8), but with a different profit function (11). The algorithm for calculating the optimal target service level is similar to that in Section III.B. We rely on numerical analysis to further compare the results between independent and competitive markets.

V. IMPACTS OF MARKET COMPETITION

In this section, we investigate the following three questions: (1) How are the reservation decisions and the maximum amount of sharing resources under an independent market scenario different from those under a competitive market scenario? (2) How are the conditions of backup reservation affected when there is competition between operators? (3) Under the competitive scenario, how do the reservation decisions depend on the competition intensity?

From the optimal reservation quantities shown in Lemmas 1 and 2, we have: (1) if $w_{ri} > (p_r^{(V)} - w) \int_{M_i^{(V)} + S_i}^{+\infty} f_i(x_i) dx_i$, then $R_i^* > 0$ under the independent scenario, and $R_i^* > S_i$ under the competitive scenario; and (2) if $(p_r^{(V)} - w) \int_{M_i^{(V)} + S_i}^{+\infty} f_i(x_i) dx_i < w_{ri} < (p_r^{(V)} - w) \int_{M_i^{(V)}}^{+\infty} f_i(x_i) dx_i$, then $R_i^* = 0$ under the independent scenario, and $R_i^* > 0$ under the competitive scenario. By comparing (6) with (14), we get the following results.

Corollary 2. Under the same reservation price and the VNO's target service level, the need of the VNO to make a reservation under the competitive scenario is higher than that under the independent scenario; and the reservation quantity in competitive markets is S_i units larger than that in independent markets, resulting in the same amount of shared resources available under both scenarios.

It indicates that given a reservation price, the relationship between the operators' markets does not impact the maximum amount of sharing resources, but affects the reservation quantity. The result is based on the implicit assumption that, under the independent scenario, the VNO does not need to make any reservation before sharing the ROO's redundant capacity. To examine the impacts of market competition on the reservation price, we assume the demand follows a uniform distribution, and the following results can be derived.

Corollary 3. Given the target service level and uniformly distributed demand at the VNO, the optimal reservation price

under the competitive scenario is not smaller than that under the independent scenario, and the maximum amount of resources to be shared under the competitive scenario is not larger than that under the independent scenario.

Corollary 3 shows that the relationship between the two operators does impact the final reservation decisions. Intuitively, the ROO would like to obtain more users from the VNO's market. Therefore, the ROO has less incentive to share resources with the VNO, and it raises the reservation price to lower the amount of shared resources when the two operators compete in the wireless service markets. As to the condition under which a backup reservation is made, the following corollary shows some interesting results.

Corollary 4. Under the independent scenario, the ROO agrees to cooperate with the VNO through backup reservation as long as $p_r^{(V)} > p_r^{(R)}$; while under the competitive scenario, the condition that the backup-reservation scheme benefits the ROO depends on various factors such as the competition intensity, the demand, the service prices and the resource capacities. Specifically, with a uniformly distributed number of users $U(a_i, b_i)$ at the VNO, if the difference between their service prices is large enough that

$$(p_r^{(V)} - p_r^{(R)}) > S_i p_r^{(R)} (1 - \frac{1}{\theta\beta}) / (\frac{b_i}{l^{(V)}} - M_i^{(V)}) \quad (17)$$

or the competition intensity is low enough that

$$\theta < \frac{S_i p_r^{(R)} / \beta}{S_i p_r^{(R)} - (p_r^{(V)} - p_r^{(R)})(b_i / l^{(V)} - M_i^{(V)})} \quad (18)$$

then the ROO is willing to cooperate with the VNO through backup reservation.

Corollaries 4 points out that the condition of backup reservation-based cooperation under the independent scenario is totally determined by the operators' service price difference, while the conditions under the competitive scenario are comprehensively affected by multiple factors.

Intuitively, under the competitive scenario, if the ROO would not get a substantial number of new users from the VNO's market, cooperation might bring more profits to the ROO than gaining additional market shares from its competitor would do. Corollary 4 shows that only when the user's switching rate is low enough that (18) is satisfied, the ROO gains less from the market competition than from the cooperation. This implies the threshold of the competition intensity below which the ROO is willing to cooperate with the VNO.

In contrast to the the results under the independent scenario, when $\theta\beta > 1$, even if $p_r^{(V)} > p_r^{(R)}$, the ROO would not cooperate with the VNO, unless their price difference is larger than the threshold shown in (17). On the other hand, the two operators can still benefit from the backup reservation when $p_r^{(V)} < p_r^{(R)}$, as long as (18) is satisfied.

The condition of cooperation is not straightforward due to the requirements on both the price difference and the competition intensity. Moreover, as the competition intensity θ increases, the VNO's price p_r^V needs to be higher to facilitate the cooperation. Intuitively, with more intense competition between the operators, it is expected that more users switch to the ROO when the VNO is "out of stock", and therefore

the ROO benefits more from the competition than from the cooperation. Corollary 5 shows that the ROO would enhance the reservation price to decrease cooperation as a response to a larger competition intensity.

Corollary 5. With a uniformly distributed number of users $U(a_i, b_i)$ at the VNO, the reservation quantity decreases with the competition intensity, and the reservation price increases with the competition intensity.

VI. NUMERICAL RESULTS

In this section, we numerically illustrate the equilibrium of the Stackelberg game, and evaluate the system performance at equilibrium to provide more insight. First, we are interested in comparing our scheme with counterpart schemes that are used in practice. The main goal is to shed light on the benefits of our proposed backup-reservation scheme in NIS. Then we evaluate the impacts of various parameters on the benefits of NIS through backup reservation. In all of the examples, we present the results in both the independent and competitive scenarios to show the impacts of market competition.

We consider an example of spectrum sharing between two operators. Channels are assumed to be orthogonal, thus relegating the problem of channel interference. We will focus on the basic model with only one service market of uncertain user demand that follows a uniform distribution $U(a_0, b_0)$, and so we omit the subscript i here. The base parameter set is provided as: $M^{(V)} = 200$, $M^{(R)} = 600$, $S = 100$, $w = 300$, $p_r^{(R)} = 400$, $P = 100$, $\alpha = 80$, $h = 120$, $n_0 = 50$, $l_{\max}^{(V)} = 10$, $\theta = 1$, $a_0 = 1500$, $b_0 = 4500$, where the VNO's service price has a nonlinear relationship with its service level as in Section II. Since we are only interested in the optimal outcomes, we ignore the subscript (*) in the rest of this section.

A. Comparisons with Other Reservation Schemes

Firstly, we compare our proposed reservation scheme with the following two reservation schemes: a mean demand (MD) satisfaction scheme and a linear price-based scheme. The former one indicates that the reservation quantity is made so that the mean demand can always be satisfied, i.e. $R + M^{(V)} + S \geq \mu$ under the independent scenario, or $R + M^{(V)} \geq \mu$ under the competitive scenario, regardless of the reservation price. This scheme may be adopted by the operator for its simplicity. In the linear price-based scheme, the reservation quantity is linearly dependent on the reservation price as in $R = D - M^{(V)} - \alpha w_r$, where D is the highest possible resource demand (note in independent markets with uniform demand, $D = \frac{b_0}{l^{(V)}} - S$). The ROO would set the reservation price lower than $p_r^{(V)} - w$. The comparison results for the VNO are shown in Fig. 2 with different values of the reservation price under an arbitrary value set of $l^{(V)} = 8.0$, $a_0 = 1000$, and $b_0 = 5000$, which illustrates clearly the dominance of our reservation scheme over other two schemes with respect to the VNO's expected profits, in both independent and competitive markets. It is worth mentioning that, the ROO's expected profit may be larger when the VNO chooses the mean demand satisfaction or the linear price-based scheme. Because in these two strategies, the VNO might reserve more than its optimal reservation quantity.

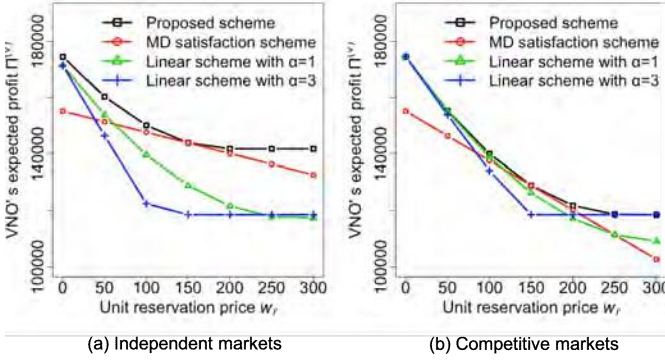


Fig. 2. VNO's expected profit under three reservation schemes vs. w_r .

B. Sensitivity Analysis

1) *Impact of Demand Variance*: Let the ROO's improved profit from the backup reservation scheme $\Delta\Pi^{(R)} = \Pi_2^{(R)} - \Pi_1^{(R)}$ be the value of backup reservation for the ROO. The results under three levels of demand uncertainty (high ($U(1000, 5000)$), moderate ($U(1500, 4500)$) and low ($U(2000, 4000)$)) are: 8981, 7771, 6654 in independent markets and 15313, 14729, 13935 in competitive markets, respectively. It is revealed that the ROO is more willing to cooperate under higher demand risks. In addition, we can also observe that the ROO benefits more from the reservation under the competitive scenario than under the independent scenario.

2) *Impact of Redundant Resource Capacity*: To see how the ROO's potential redundant capacity affects the outcome performance, we change the value of S from 100 to 300, with the constant value of $M^{(R)} = 600$ and $\theta = 0.8$. Table II reports the equilibrium results, from which three important observations can be made.

First of all, under the independent scenario, both the reservation price and quantity decrease with the ROO's redundant capacity S . This is because there is less need for the VNO to make reservations with more redundant resources available at the ROO. While under the competitive scenario, the reservation quantity increases with S due to the ROO's stronger incentive to share its redundant capacity and thus the reducing reservation price. Besides, despite the different values of $l^{(V)}$ in two scenarios, compared to the independent scenario, the reservation price is higher and the maximum amount of resources to be shared is smaller under the competitive scenario, which confirms the results in Corollary 3.

Secondly, denote the total improved profit $\Delta\Pi = \Pi_2^{(R)} + \Pi_2^{(V)} - \Pi_1^{(R)} - \Pi_1^{(V)}$ as the benefit of backup reservation for the two operators, the value of which is always higher under the competitive scenario. $\Delta\Pi$ decreases with S under the independent scenario since less reservation is made, while it increases under the competitive scenario. This is because, larger redundant capacity indicates less need to compromise the ROO's own user demand when the VNO shares the spectrum, thereby increasing the benefit of the backup reservation. Hence, the implementation of a backup reservation scheme is most effective in improving profits under the competitive scenario or the independent scenario with small redundant capacity. Finally, if we use $\Delta l^{(V)} = l_1^{(V)} - l_2^{(V)}$ to denote the

improvement in the target service level when implementing backup reservation scheme, it can be seen in our numerical examples that the users typically benefit more from the reservation-based NIS when the operators compete.

TABLE II
IMPACT OF REDUNDANT CAPACITY ON THE RESERVATION DECISIONS AND THE VALUE OF BACKUP RESERVATION.

S	Independent markets				Competitive markets			
	w_r	R	$\Delta\Pi$	$\Delta l^{(V)}$	w_r	R	$\Delta\Pi$	$\Delta l^{(V)}$
100	129	104	12287	0.051	178	144	24680	0.069
200	80	66	4967	0.033	170	152	29466	0.088
300	34	28	1031	0.017	158	169	36519	0.123

3) *Impact of Market Competition*: NIS is supposed to benefit the collaborators through cost sharing and larger market share. However, previous studies have assumed the independence between the markets of the operators. Our theoretical analysis in Section V has shown us that the conditions of using the backup reservation scheme are different when the operators compete. In the present section, we further explore the impacts of competition intensity on the reservation decisions. Fig. 3 shows that the impact of the competition intensity θ is affected by the potential redundant capacity S . When S is small, the impact of competition intensity θ is negligible, because the VNO reserves all of its redundant capacity and no new users are expected to switch to the ROO; but when S is large, as the competition intensity increases, the ROO would increase the reservation price to inhibit the reservation and to keep more resources for new users.

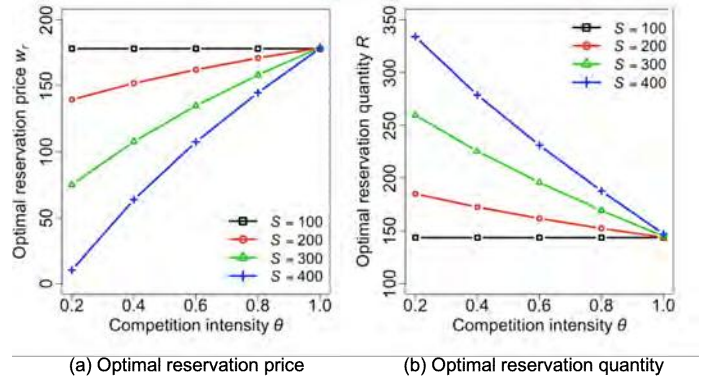


Fig. 3. Impact of market competition on the reservation decisions.

4) *Impact of the ROO's Market Strategy*: Next we study the impacts of the ROO's market strategy, indicated by the price and the target service level, on the equilibrium results. As shown in Fig. 4, by increasing $p_r^{(R)}$ from 300 to 650, the total improved profit for the two operators decreases in either independent or competitive markets. This is because, it is more profitable to use the limited resources to satisfy the ROO's users than to satisfy the VNO's when $p_r^{(R)} - p_r^{(V)}$ is large enough. This observation actually adheres to the result in Proposition 1.

Under the independent scenario, the optimal reservation decisions are independent on the ROO's target service level. However, Fig. 5 reveals that under the competitive scenario, when S is large, the optimal reservation quantity increases with $l^{(R)}$. This result complies with our result in Proposition 2: When S is small enough, $R > S$ and (16) is satisfied,

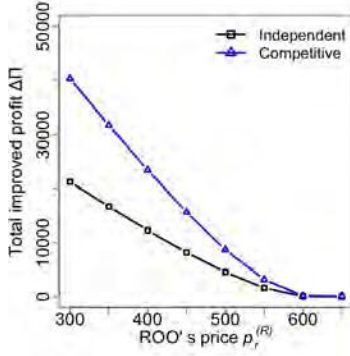


Fig. 4. Impact of the ROO's price $p_r^{(R)}$ on the total improved profit.

the impact of $l^{(R)}$ is negligible since no spectrum is left at the ROO to serve new users. When S is large, the revenue from satisfying the new users decreases with $l^{(R)}$. Therefore, the ROO would lower the reservation price to induce larger reservation quantity for more revenue obtained from NIS.

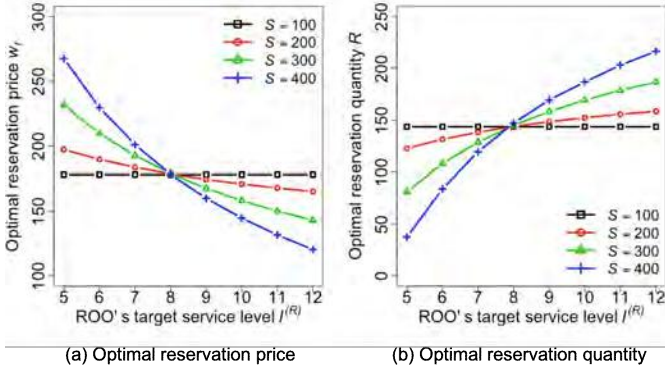


Fig. 5. Impact of $l^{(R)}$ on the optimal reservation decisions.

Moreover, we can use $E\{1 - \frac{\text{unused spectrum}}{\text{total spectrum}}\}$ to represent the spectrum efficiency. We plot the resource efficiency as a function of R (shown below as Fig. 6(a)). In addition, numerical results have been obtained to study how the resource efficiency changes with other parameters such as S and θ . These results are plotted in Fig. 6(b). From these figures, it can be observed that resource efficiency is increased by implementing the backup reservation scheme. The percentage of increase in resource efficiency increases with S and decreases with θ under the competitive scenario. This observation can be explained as follows: as a larger proportion of the VNO's unsatisfied users would turn to the ROO, there is less space to increase the resource efficiency through the backup reservation scheme. With more redundant capacity, the total resource efficiency is expected to increase with a larger reservation quantity. This observation is also consistent with our results in Fig. 3(b) regarding the impact of R .

Note that the benefit of backup reservation scheme is to significantly improve profits, and our results in Fig. 7 show that this benefit increases with S and decreases with θ under the competitive scenario. The reason is that, with more redundant capacity or a smaller value of competition intensity, the ROO faces a lower risk of hurting its own users or of losing new users through NIS, which indicates that their cooperation

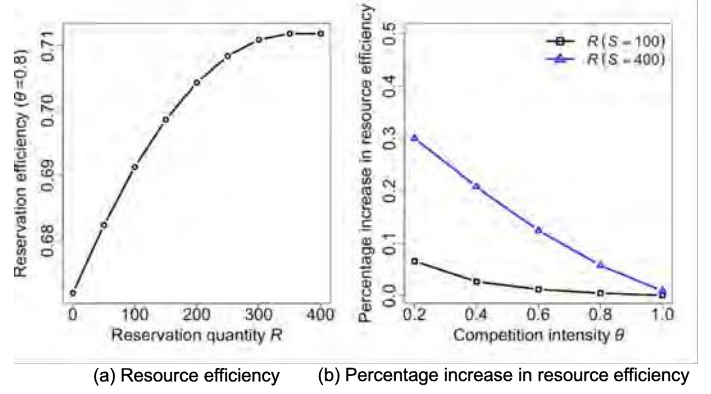


Fig. 6. (a) Resource efficiency vs. R ; and (b) Percentage increase in resource efficiency vs. θ .

becomes more profitable for the ROO. Therefore, a lower price is set to encourage a larger reservation quantity, such that both parties benefit more from the backup reservation scheme.

Moreover, from Fig. 3, Fig. 6(b), and Fig. 7, it can be observed that under low to mild surplus capacity (e.g., $S = 100 \sim 200$), the estimation accuracy of θ only has minor impact on the decision making, as the optimal reservation price, reservation quantity, resource efficiency gain, and total profit gain do not change significantly for small perturbations of the θ value (i.e., all these decision-vs.- θ curves have small/flat slopes). Therefore it is clear that the proposed model is robust against estimation errors in θ .

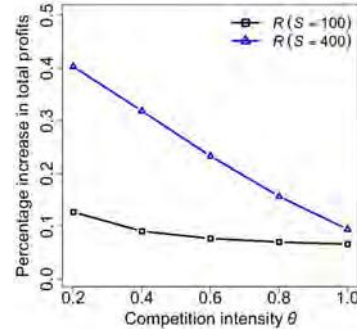


Fig. 7. Percentage increase in total profits under competitive scenario.

VII. ALTERNATIVE MODELS AND ASSUMPTIONS

In this section, we discuss two alternative modeling assumptions and some limitations of our analysis. In particular, we examine (1) the impacts of the ROO's demand uncertainty, and (2) a different timing of moves in the Stackelberg game when the ROO makes the reservation price first.

A. Uncertain ROO's demand

In our base model, the ROO's demand is assumed to be fixed. In this section, we study the more general case in which the ROO's demand is uncertain. We assume that the true value of its demand is unknown, but it is publicly known to be either high, and the users consume all the ROO's resource capacity, or low, which results in a redundant capacity of S_i . The probability of a high market demand is denoted by q_i .

The three-stage decision process is similar to that depicted in Section II. The analysis of stage 1 is the same as subsection III.B. We now analyze the optimal backup reservation contract parameters for stages 2 and 3 in both independent and competitive scenarios. It is worth to note that, under the independent scenario, due to the demand uncertainty, the ROO does not always have redundant resources to be shared with the VNO. In this case, naturally we consider the situation when the VNO is required to make a reservation before sharing any amount of resource. In Appendix H, we provide the service providers' expected profit functions for each scenario. Regarding the optimal unit reservation price, we can derive a similar result following the same steps of Proposition 2, but with a different value for w_{ri1}^* , given in the following Proposition.

Proposition 3. With uncertain demand at the ROO, the optimal unit reservation price w_{ri1}^* that leads to a reservation quantity $R \in (0, S)$ satisfies the following equations:

1) In a independent market,

$$R_i(w_{ri1})(p_r^{(V)} - w)f_i(M_i^{(V)} + R_i(w_{ri1})) = w_{ri1} - ((1 - q_i)p_r^{(R)} - w) \int_{M_i^{(V)} + R_i(w_{ri1})}^{+\infty} f_i(x_i)dx_i. \quad (19)$$

2) In a competitive market,

$$R_i(w_{ri1})(p_r^{(V)} - w)f_i(M_i^{(V)} + R_i(w_{ri1})) = w_{ri1}(p_r^{(V)} - p_r^{(R)})/(p_r^{(V)} - w) + qp_r^{(R)}(1 - \theta\beta) \int_{M_i^{(V)} + R_i(w_{ri1})}^{M_i^{(V)} + R_i(w_{ri1}) + \frac{S_i - R_i(w_{ri1})}{\theta\beta}} f_i(x_i)dx_i. \quad (20)$$

By comparing the results under the independent and competitive scenarios, we observe that, under the same reservation price and the VNO's target service level, the relationship between the operators' markets does not impact the reservation quantity. The reason is that, both in independent and competitive scenarios, the ROO needs to be conservative in sharing resources since its own demand is uncertain. These results is different with the one stated in Corollary 2 in the case of fixed demand. Besides, under the independent scenario, different from the fixed demand case, the two operators may still benefit from the backup reservation scheme even if $p_r^{(V)} < p_r^{(R)}$ due to the demand uncertainty. In a competitive market, if $p_r^{(V)} < p_r^{(R)}$ and $\theta\beta > 1$, then the ROO will set a high reservation price that no reservation is made by the VNO. This is consistent with what we observed when the ROO's demand is fixed.

In the following numerical studies, we are interested in investigating the impacts of the demand uncertainty on the reservation decisions and the system performance. If we use the relative variation, i.e., the variance divided by the absolute value of the mean, to denote the demand variance, then a larger value of q_i indicates a larger demand uncertainty. Using the same problem setting for only one service market and input parameters values as in Section VI, we obtain the optimal reservation decisions with different values of q at $S = 300$ and $\theta = 1$. Fig. 8(a) illustrates that, as q increases,

the ROO encourages a larger backup reservation quantity by setting a lower price under the independent scenario. That is, more risk from the demand side makes it more attractive to implement the backup reservation scheme. However, under the competitive scenario, the impacts of the demand uncertainty is negligible because the ROO would get new users from the VNO's market and the demand risk is mitigated. Fig. 8(b) shows that the total improved profit increases with the demand uncertainty, more apparently in an independent market.

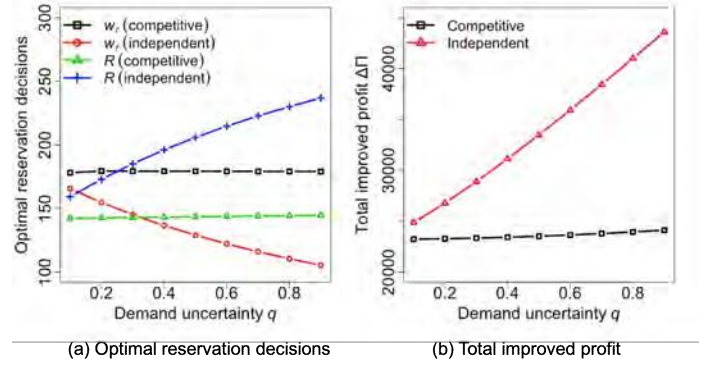


Fig. 8. Impact of ROO's demand uncertainty.

B. ROO moves first

Our base model assumes that the ROO provides the backup reservation scheme after the VNO sets its target service level. This decision sequence is suitable for the situation when the VNO's market planning is strategic (i.e., performed at a relatively large time period) and the target service level needs to be made before the operational reservation decisions. An alternative decision sequence is allowing the ROO to be the first mover by declaring the reservation price, and then the VNO determines its target service level and the reservation quantity. This situation occurs when the VNO is a new entrant operator and the ROO would like to take the potential first-mover advantage. In this subsection, We discuss the robustness of our analytical results with respect to the above alternative decision sequence and examine whether there is a first mover advantage for the ROO in our Stackelberg game model.

In the alternative decision sequence, it is difficult to solve the optimal decisions in closed-form, but they can be solved easily using numerical methods. Table III shows that, consistent with what we observed in our base model, the backup reservation scheme benefits the two operators and the users especially when the ROO has small redundant capacity or the market is competitive. Besides, the impacts of the redundant capacity S or the competition intensity θ on the final reservation decisions when the ROO moves first are similar to those in our base model, as illustrated by Fig. 9(a).

Fig. 9(b) plots the increased profit of the ROO ($\Delta\Pi^{(R)} = \Pi^{(R)}(\text{ROO moves first}) - \Pi^{(R)}(\text{VNO moves first})$) by letting it making decision first. It shows that the ROO does not always enjoy the first-mover advantage in our reservation model. Whether or not the ROO benefits from making the decision first is determined by the values of its redundant capacity and the competition intensity. Specifically, if the ROO has large

TABLE III
IMPACT OF REDUNDANT CAPACITY WHEN THE ROO MOVES FIRST.

S	Independent markets				Competitive markets ($\theta = 0.8$)			
	w_r	R	$\Delta\Pi$	$\Delta I^{(V)}$	w_r	R	$\Delta\Pi$	$\Delta I^{(V)}$
100	126	110	12801	0.09	175	148	25353	0.113
200	75	74	5701	0.08	166	159	30643	0.123
300	23	45	1847	0.06	151	178	37779	0.139

redundant capacity and the competition is intense enough, it is beneficial for the ROO to delay its reservation price decision after the VNO has decided its target service level. This is due to the fact that the ROO can adjust optimally its reservation price with more information of the VNO's demand.

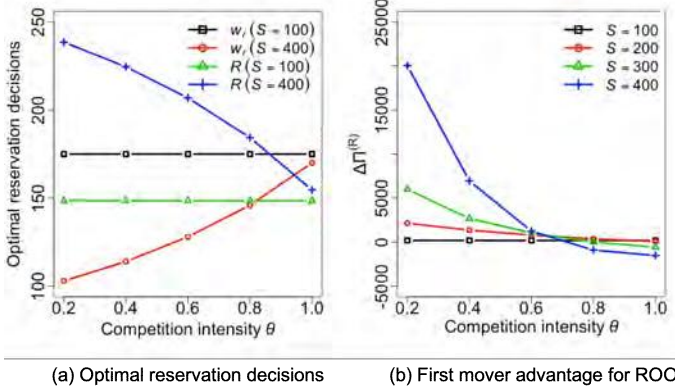


Fig. 9. Results when ROO moves first.

VIII. RELATED WORK

In recent years, NIS has emerged as an important research area with interest from both academic and industrial. Existing literature on sharing models have identified a wide range of factors that determine how network infrastructure are allocated and affect the effectiveness of sharing strategy. Current studies typically focus on examining them in one of the two categories: service market profile and infrastructure market profile. The studies in the former include but are not limited to the service demand pattern, market shares, regulations on the market concentration, interference and traffic cost [25], [26], [27]. For instance, [28] argues that the market shares of the operators should be taken into consideration to guarantee coalition stability, but most models assume fixed market shares or user numbers [28], [29], [30]. [31] shows that the existence of tight competition regulations on the market concentration could reduce the benefits of jointly updating and managing the operators' networks, while how the market competition among service providers affects their network resource sharing decisions has not been investigated. The studies in the later refer to the relationships among the operators which cannot be avoided when multiple operators are considered in resource sharing or trading market. The existing literature normally considers the price competition/cooperation among multiple resource suppliers [32], [33], [34], [30]. Our study tries to investigate both the competition in services and the cooperation in resource sharing. Moreover, how the factors in both markets, including the uncertain demand risks, the long-term backup contract design, and the potential redundant

resource capacity, affect the sharing decision configurations and economic performances is explored in our research.

In determining how the resources or benefits should be allocated among the operators in NIS, one stream of research, mostly in the area of spectrum sharing for unlicensed bands, looks for the sharing rules that lead to fairness and efficiency [35], [36], [37], [38]. Another stream of research conducts a cooperative game approach [28], [39]. However, many challenges still remain in real world NIS, which include preserving a liable, mutually beneficial relationship for the operators, reserving an appropriate backup capacity and ensuring supply in cases of uncertain resource demand. To combat these challenges, we generalize and develop the well-structured partnership between partners to ensure supply availability in NIS mechanism under demand uncertainty. Different from many papers in the literature, our interest is in the interactions between the operators participating in both the resource sharing cooperation and the service competition. Through backup contract design and parameter optimization, the operators' effort and willingness to collaborate should increase.

The long-term contract design between operators in NIS is still relatively under-explored. There has been some work that relies on different types of contracts to incentivize spectrum sharing, like revenue sharing [40], and insurance contracts [26]. However, the contract parameters are assumed to be exogenous, which determine how the economic benefits are allocated between the partners. Our work tries to optimize the contract parameters for a realistic profit allocation mechanism. [30] obtains the optimal service prices to maximize the cross-carrier VNO's profit, but how the resource allocation can be implemented through pricing for maximum ISPs' profit are not fully studied. Of particular relevance is the work of [21] on the spectrum reservation contract. Their contract is assigned between two different types of players: one third-party broker and one unlicensed white space device; while our model focuses on the co-opetition relationships between two operators providing similar services to the users. An altogether different approach to the spectrum sharing/trading problem is the one taken by the mechanism design literature. There, the primary users offer a direct mechanism that allocates its resources as a function of secondary users' reports of their private information, such as transmission efficiency [22], and preference for a given spectrum quality [41]. The contract parameters are optimized to minimize the impact of the private information for the resource providers. Our paper contributes to this literature by applying the theory of backup contract design to the problem of network infrastructure sharing in an uncertain and competitive environment. Our contract-based approach provides a richer form of characterizing relationships among operators than the previous auction-based approach, and enables us to evaluate the feasibility and effectiveness of cooperation among competing operators.

Part of the work has been presented at the IEEE SECON 2019 conference, Boston, MA. Compared to the conference version [42], this paper provides more model and numerical analysis with more insightful results, including the analysis of the model with demand uncertainty at the ROO's side, the results with a different time sequence and a thorough

investigation of the impacts of competition intensity.

IX. DISCUSSIONS AND CONCLUSIONS

In this paper, we propose an optimal NIS schemes based on the equilibrium of the backup reservation game between two MNOs. The decision-making conditions are identified for MNOs to adopt and implement the backup reservation contract under both independent and competitive scenarios. The interactive decisions of the two operators, namely the reservation price and the reservation quantity, are derived under uncertain demand risks. The strategic backup reservation based infrastructure sharing framework is shown to have the potential to improve both the capacity utilization of the resource-owning operator and the virtual operator's service level to the users. The benefits of such a scheme are determined by various factors such as the demand uncertainty, the potential redundant capacity, and the competitive intensity. These findings lay the foundation for a clear guideline for the operators to choose their partners and determine their action plans when NIS is desired in volatile and competitive markets. For future research, it would be worthwhile to relax the assumption of the availability of complete information between MNOs.

Furthermore, extending this study to three or more operators will be of interest to assess the generality of the conjectures. In the case of multiple VNOs competing for the resources of a single ROO, two possible strategies may be considered: (1) Iteration of partner selection and reservation: The ROO chooses the most beneficial VNO j to cooperate through backup reservation, according to its own redundant resources S , the VNOs' demand patterns, and service prices; and then VNO determines the reservation quantity R_j , after which the ROO updates the redundant resource quantity to $(S - R_j)^+$, and chooses the next VNO to cooperate. The above process is repeated until no benefit improvement can be earned from the NIS for the ROO. This method is easier to implement, but still leaves space for further resource utilization improvement. (2) Capacity allocation and flexible contract: The ROO would design a contract with each VNO that specifies the reservation price, the bounds of reservation quantity, as well as penalties for deviating from these agreed-upon quantities. This flexible contractual setting is supposed to further enhance the resource utilization but more sophisticated. In the case of multiple ROOs, the optimal reservation decisions would change according to the equilibrium in the ROOs' price competition game. We leave these as potential topics for future research.

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opinions, findings, conclusions, or recommendations expressed in this paper are those of the author(s) and do not necessarily reflect the views of NSF.

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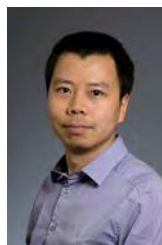
Jing Hou is currently a Ph.D. student in the Department of Computer Science and Software Engineering at Auburn University. She received the B.S. degree in Computing Science from Nanjing Tech University, Nanjing, China in 2004 and the Ph.D. degree in Systems Engineering from Southeast University, Nanjing, China in 2011, respectively. Her research interest includes wireless communications and network economics.



Li Sun is currently a Ph.D. student in the Department of Computer Science and Software Engineering at Auburn University. He received the M.S. and Ph.D. degrees in Systems Engineering from Southeast University, China in 2008 and 2013, respectively. He used to be a visiting scholar in RWTH Aachen University, Germany from 2009 to 2011. His research interest includes machine learning, optimization, and their applications in wireless networking systems.



Tao Shu is currently an associate professor in the Department of Computer Science and Software Engineering at Auburn University. He received his Ph.D. in Electrical and Computer Engineering from The University of Arizona in 2010. He received the B.S. and M.S. degrees in Electronic Engineering from the South China University of Technology, Guangzhou, China in 1996 and 1999, respectively, and the Ph.D. degree in Communication and Information Systems from Tsinghua University, Beijing, China in 2003. His research aims at addressing security and performance issues in wireless networking systems, with strong emphasis on system architecture, protocol design, and performance modeling and optimization.



Yong Xiao (S'09-M'13-SM'15) is a professor in the School of Electronic Information and Communications at the Huazhong University of Science and Technology (HUST), Wuhan, China. He is also the associate group leader in the network intelligence group of IMT-2030 (6G promoting group). His research interests include machine learning, game theory, and their applications in cloud/fog/mobile edge computing, green communication systems, wireless networks, and Internet-of-Things (IoT).



Marwan Krunz is the Kenneth VonBehren Endowed Professor in ECE and a professor of computer science at the University of Arizona. He directs the Broadband Wireless Access and Applications Center (BWAC), a multi-university NSF/industry center that focuses on next-generation wireless technologies and applications. He also holds a courtesy appointment as a professor at the University Technology Sydney. He previously served as the site director for the Connection One center. Dr. Krunz's research emphasis is on resource management, network protocols, and security for wireless systems. He has published more than 300 journal articles and peer-reviewed conference papers, and is an inventor on 12 patents. His latest h-index is 60. He is an IEEE Fellow, an Arizona Engineering Faculty Fellow, and an IEEE Communications Society Distinguished Lecturer (2013–2015). He received the NSF CAREER award. He recently served as the Editor-in-Chief for the IEEE Transactions on Mobile Computing. He also served as editor for numerous IEEE journals. He was TPC chair INFOCOM04, SECON05, WoWMoM06, and Hot Interconnects 9. He was the general vice-chair for WiOpt 2016 and general co-chair for WiSec12. Dr. Krunz is also an entrepreneur, serving as chief technologist/scientist for two startup companies that focus on 5G and beyond systems and signal intelligence.

APPENDIX A

This appendix contains the Proof for Lemma 1.

Proof: Because $\frac{d\Pi_2^{(V)}}{dR_i} = -w_{ri} - (w - p_r^{(V)}) \int_{M_i^{(V)}+S_i+R_i}^{+\infty} f_i(x_i)dx_i$, and $\frac{d^2\Pi_2^{(V)}}{dR_i^2} = (w - p_r^{(V)})f_i(M_i^{(V)}+S_i+R_i) < 0$, the optimal reservation quantity satisfies $\frac{d\Pi_2^{(V)}}{dR_i} = 0$, i.e., $\int_{M_i^{(V)}+S_i+R_i}^{+\infty} f_i(x_i)dx_i = \frac{w_{ri}}{p_r^{(V)}-w}$. ■

APPENDIX B

This appendix contains the Proof for Proposition 1.

Proof: If $0 < R_i < M_i^{(R)} - S_i$, substitute R_i in $\Pi_2^{(R)}$ with (6), we can re-write the ROO's profit as a function of R_i . From $\frac{d\Pi_2^{(R)}}{dR_i} = p_r^{(V)} - p_r^{(R)} - (p_r^{(V)} - p_r^{(R)}) \int_0^{M_i^{(V)}+S_i+R_i} f_i(x_i)dx_i + R_i(w - p_r^{(V)})f_i(M_i^{(V)}+S_i+R_i) = 0$, we can get the results shown in Proposition 1. ■

APPENDIX C

This appendix contains the Proof for Lemma 2.

Proof: Because $\frac{d\Pi_2^{(V)}}{dR_i} = -w_{ri} - (w - p_r^{(V)}) \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i)dx_i$, $\frac{d^2\Pi_2^{(V)}}{dR_i^2} = (w - p_r^{(V)})f_i(M_i^{(V)}+R_i) < 0$, the optimal reservation quantity satisfies $\int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i)dx_i = \frac{w_{ri}}{p_r^{(V)}-w}$, from which we can derive the result in Lemma 2. ■

APPENDIX D

This appendix contains the Proof for Corollary 1.

Proof: If $p_r^{(V)} < p_r^{(R)}$ and $\theta\beta > 1$, the value of R_i that satisfies (15) or (16) is negative. Because $\frac{d\Pi_{2i}^{(R)}}{dR_i}|_{R_i=S_i} = (p_r^{(V)} - p_r^{(R)}) \int_{M_i^{(V)}+S_i}^{+\infty} f_i(x_i)dx_i - (p_r^{(V)} - w)S_i f_i(M_i^{(V)}+S_i) < 0$, the optimal value of R_i is 0 for the ROO. ■

APPENDIX E

This appendix contains the Proof for Corollary 3.

Proof: Under the independent scenario, we can derive that the optimal reservation quantity with a uniformly distributed number of users $U(a_i, b_i)$ is $(b_i/l^{(V)} - S_i - M_i^{(V)}) \frac{p_r^{(R)} - p_r^{(V)}}{p_r^{(R)} - 2p_r^{(V)} + w}$. That is, the maximum amount of resources to be shared between the two operators is $(b_i/l^{(V)} - S_i - M_i^{(V)}) \frac{p_r^{(R)} - p_r^{(V)}}{p_r^{(R)} - 2p_r^{(V)} + w} + S_i$.

Under the competitive scenario, since $\frac{d^2\Pi_{2i}^{(R)}}{dR_i^2} = \frac{(2 - \frac{1}{\theta\beta})p_r^{(R)} - 2p_r^{(V)} + w}{b_i/l^{(V)} - a_i/l^{(V)}}$, $\frac{d^2\Pi_{2i}^{(R)}}{dR_i^2} = (p_r^{(R)} - 2p_r^{(V)} + w) \frac{1}{b_i/l^{(V)} - a_i/l^{(V)}}$, $\Pi_{2i}^{(R)}$ is either convex or concave functions of R_i .

We denote $A = \frac{d\Pi_{2i}^{(R)}}{dR_i}|_{R_i=0} = (p_r^{(V)} - p_r^{(R)}) \int_{M_i^{(V)}+\frac{S_i}{\theta\beta}}^{b_i/l^{(V)}} f_i(x_i)dx_i + (p_r^{(V)} - \theta\beta p_r^{(R)}) \int_{M_i^{(V)}}^{M_i^{(V)}+\frac{S_i}{\theta\beta}} f_i(x_i)dx_i$, $B = \frac{d\Pi_{2i}^{(R)}}{dR_i}|_{R_i=S_i} = (p_r^{(V)} - p_r^{(R)}) \int_{M_i^{(V)}+S_i}^{b_i/l^{(V)}} f_i(x_i)dx_i - (p_r^{(V)} - w)S_i f_i(M_i^{(V)}+S_i)$, and

$C = \frac{d\Pi_{2i}^{(R)}}{dR_i}|_{R_i=M_i^{(R)}} = (p_r^{(V)} - p_r^{(R)}) \int_{M_i^{(V)}+M_i^{(R)}}^{b_i/l^{(V)}} f_i(x_i)dx_i - (p_r^{(V)} - w)M_i^{(R)} f_i(M_i^{(V)}+M_i^{(R)})$. If the user number at the VNO follows uniform distribution $U(a_i, b_i)$, we have

$$A = (p_r^{(V)} - p_r^{(R)})(b_i/l^{(V)} - M_i^{(V)}) - S_i p_r^{(R)}(1 - \frac{1}{\theta\beta}) \quad (A.1)$$

$$B = (p_r^{(V)} - p_r^{(R)})(b_i/l^{(V)} - M_i^{(V)}) - S_i(2p_r^{(V)} - p_r^{(R)} - w) \quad (A.2)$$

and

$$C = B + (S_i - M_i^{(R)})(2p_r^{(V)} - p_r^{(R)} - w). \quad (A.3)$$

At equilibrium, where $p_r^{(V)} = l^{(V)}p_c^{(V)}$,

(i) if $A < 0$, $B < 0$, and $C < 0$, then the VNO does not make any reservation;

(ii) if $A > 0$, $B < 0$, and $C < 0$, then

$$w_{ri}^* = (p_r^{(V)} - w) \frac{(b_i - M_i^{(V)}l^{(V)})(p_r^{(V)} - w - p_r^{(R)} + \frac{p_r^{(R)}}{\theta\beta}) + (S_i - \frac{S_i}{\theta\beta})p_r^{(R)}}{(2p_r^{(V)} - w - 2p_r^{(R)} + \frac{p_r^{(R)}}{\theta\beta})(b_i - a_i)} \quad (A.4)$$

$$R_i^* = \frac{(b_i/l^{(V)} - M_i^{(V)})p_r^{(V)} - (S_i + b_i/l^{(V)} - M_i^{(V)} - \frac{S_i}{\theta\beta})p_r^{(R)}}{2p_r^{(V)} - w - 2p_r^{(R)} + \frac{p_r^{(R)}}{\theta\beta}}; \quad (A.5)$$

(iii) if $A > 0$, $B > 0$, and $C < 0$, then

$$w_{ri}^* = \frac{(l^{(V)}M_i^{(V)} - b_i)(p_r^{(V)} - w)^2}{(p_r^{(R)} - 2p_r^{(V)} + w)(b_i - a_i)} \quad (A.6)$$

$$R_i^* = (b_i/l^{(V)} - M_i^{(V)}) \frac{p_r^{(R)} - p_r^{(V)}}{p_r^{(R)} - 2p_r^{(V)} + w}; \quad (A.7)$$

(iv) if $A > 0$, $B > 0$, and $C > 0$, then the ROO makes the maximum reservation $R_i^* = M_i^{(R)}$ and

$$w_{ri}^* = \frac{l^{(V)}(M_i^{(R)} + M_i^{(V)} - b_i/l^{(V)})(w - p_r^{(V)})}{b_i - a_i}; \quad (A.8)$$

(v) if $A < 0$, and $C > 0$, then the ROO either makes maximum reservation or no reservation with

$$w_{ri}^* = (p_r^{(V)} - w) \frac{b_i - l^{(V)}M_i^{(V)}}{b_i - a_i}; \quad (A.9)$$

(vi) if $A < 0$, $B > 0$, $C < 0$, then either R_i^* satisfies (A.7) or $R_i^* = 0$;

(vii) if $A > 0$, $B < 0$, $C > 0$, then either R_i^* satisfies (A.5) or $R_i^* = M_i^{(R)}$.

The two possible positive reservation quantities smaller than the maximum reservation quantity under the competitive scenario are those in (A.5) and (A.7). It is easy to prove that both values are smaller than $(b_i/l^{(V)} - S_i - M_i^{(V)}) \frac{p_r^{(R)} - p_r^{(V)}}{p_r^{(R)} - 2p_r^{(V)} + w} + S_i$. If the ROO makes maximum reservation $M_i^{(R)}$ under the competitive scenario, we have $(p_r^{(V)} - p_r^{(R)})(\frac{b_0}{l^{(V)}} - M_i^{(V)}) - M_i^{(R)}(2p_r^{(V)} - p_r^{(R)} - w) > 0$, then $(\frac{b_0}{l^{(V)}} - S_i - M_i^{(V)})(p_r^{(V)} - p_r^{(R)}) > (M_i^{(R)} - S_i)(2p_r^{(V)} - p_r^{(R)} - w)$, which means that the

ROO also makes maximum reservation $R_i^* = M_i^{(R)} - S_i$ under the independent scenario. In this case, the amount of resources that can be shared are the same for the two situations. Since the maximum amount of shared resources under the competitive scenario is smaller or equals to that under the independent scenario, according to Corollary 2, the optimal reservation price is not smaller under the competitive scenario. ■

APPENDIX F

This appendix contains the Proof for Corollary 4.

Proof: Under the independent scenario, the condition of a backup reservation scheme can be derived from (7). Under the competitive scenario, from the proof for Corollary 3, we can find that if $A > 0$, $R_i^* > 0$. By solving $A > 0$, we can therefore deduce the results in Corollary 4. ■

APPENDIX G

This appendix contains the Proof for Corollary 5.

Proof: Firstly, it is easy to prove that either B in (A.2) or C in (A.3) does not change with θ , while A in (A.1) decreases in θ . Therefore, as θ increases, the value of A either stays positive or negative, or changes from positive to negative.

Case 1: when A stays negative, (1) if $B < 0$ and $C < 0$, then the VNO does not make any reservation and the ROO's reservation price stays at $w_{ri}^* = (p_r^{(V)} - w) \frac{b_i - l^{(V)} M_i^{(V)}}{b_i - a_i}$; (2) if $B > 0$ and $C < 0$, then either R_i^* satisfies (A.7) which is not impacted by θ or $R_i^* = 0$; similarly, w_{ri}^* satisfies either (A.6) or (A.9), with the latter value larger than the former one. (3) if $C > 0$, then the ROO either makes maximum reservation or no reservation, and w_{ri}^* satisfies either (A.8) or (A.9), with the latter value larger than the former one. As θ increases, $\Pi_{2i}^{(R)}|_{R_i=0}$ increases but the value of $\Pi_{2i}^{(R)}$ when R_i^* satisfies (A.7) or $\Pi_{2i}^{(R)}|_{R_i=M_i^{(R)}}$ stays unchanged. Therefore R^* is non-increasing with θ , and w_{ri}^* is non-decreasing with θ .

Case 2: when A stays positive, then the optimal reservation quantity takes one of the three values:

$$(b_i/l^{(V)} - M_i^{(V)}) \frac{p_r^{(R)} - p_r^{(V)}}{p_r^{(R)} - 2p_r^{(V)} + w} \text{ and } M_i^{(R)}. \text{ Because the latter two values are independent of } \theta, \text{ we only need to prove that } R_i^*(\theta) = \frac{(b_i/l^{(V)} - M_i^{(V)})p_r^{(V)} - (S_i + b_i/l^{(V)} - M_i^{(V)} - \frac{S_i}{\theta\beta})p_r^{(R)}}{2p_r^{(V)} - w - 2p_r^{(R)} + \frac{p_r^{(R)}}{\theta\beta}},$$

is decreasing in θ . First, we can derive that $\frac{dR_i^*}{d\frac{1}{\theta\beta}} = \frac{-Bp_r^{(R)}}{(2p_r^{(V)} - w - 2p_r^{(R)} + \frac{p_r^{(R)}}{\theta\beta})^2} > 0$, where

$$B = (p_r^{(V)} - p_r^{(R)})(b_i/l^{(V)} - M_i^{(V)}) - S_i(2p_r^{(V)} - p_r^{(R)} - w) < 0. \text{ Because } \frac{d\frac{1}{\theta\beta}}{d\theta} < 0, \text{ we have } \frac{dR_i^*}{d\theta} = \frac{dR_i^*}{d\frac{1}{\theta\beta}} \frac{d\frac{1}{\theta\beta}}{d\theta} < 0. \text{ As for the value of } w_{ri}^*, \text{ because } \frac{dw_{ri}^*}{d\frac{1}{\theta\beta}} = Bl^{(V)}p_r^{(R)} \frac{p_r^{(V)} - w}{(b_i - a_i)(2p_r^{(V)} - w - 2p_r^{(R)} + \frac{p_r^{(R)}}{\theta\beta})^2} < 0 \text{ when } w_{ri}^* \text{ satisfies (A.4), we can derive that } \frac{dw_{ri}^*}{d\theta} > 0.$$

Case 3: when A changes from positive to negative, because R^* either stays unchanged or decreases to zero, we conclude that R^* is non-increasing with θ . Similarly, w_{ri}^* increases with θ . ■

APPENDIX H

This appendix contains the operators' profit analysis for Subsection VII.A.

A. Independent Scenario

1) *Without Backup Reservation:* The VNO's expected profit is written as in (9). The ROO's expected profit in each service area i is

$$\begin{aligned} \Pi_1^{(R)} &= q((M_i^{(R)} - S_i)p_r^{(R)} + wS_i \int_{M_i^{(V)} + S_i}^{+\infty} f_i(x_i)dx_i \\ &\quad + w \int_{M_i^{(V)}}^{M_i^{(V)} + S_i} (x_i - M_i^{(V)})f_i(x_i)dx_i) \\ &\quad + (1 - q)p_r^{(R)}M_i^{(R)}. \end{aligned} \quad (\text{A.10})$$

2) *With Backup Reservation:* The VNO's expected profit is written as in (11). As for the ROO, three cases need to be considered:

Case 1: With low demand, and $R_i \leq S_i$. The ROO's expected profit is

$$\begin{aligned} \Pi_{2Ai}^{(R)} &= w \int_{M_i^{(V)}}^{M_i^{(V)} + R_i} (x_i - M_i^{(V)})f_i(x_i)dx_i \\ &\quad + wR_i \int_{M_i^{(V)} + R_i}^{+\infty} f_i(x_i)dx_i + w_rR_i \\ &\quad + p_r^{(R)}(M_i^{(R)} - S_i). \end{aligned} \quad (\text{A.11})$$

Case 2: With low demand, and $R_i > S_i$. The ROO's expected profit is

$$\begin{aligned} \Pi_{2Bi}^{(R)} &= w \int_{M_i^{(V)}}^{M_i^{(V)} + R_i} (x_i - M_i^{(V)})f_i(x_i)dx_i \\ &\quad + wR_i \int_{M_i^{(V)} + R_i}^{+\infty} f_i(x_i)dx_i + w_rR_i \\ &\quad + p_r^{(R)} \int_0^{M_i^{(V)} + S_i} (M_i^{(R)} - S_i)f_i(x_i)dx_i \\ &\quad + p_r^{(R)} \int_{M_i^{(V)} + S_i}^{M_i^{(V)} + R_i} (M_i^{(V)} + M_i^{(R)} - x_i)f_i(x_i)dx_i \\ &\quad + p_r^{(R)} \int_{M_i^{(V)} + R_i}^{+\infty} (M_i^{(R)} - R_i)f_i(x_i)dx_i. \end{aligned} \quad (\text{A.12})$$

Case 3: With high demand. In the case of no surplus capacity, the ROO's expected profit in each service area i is

$$\begin{aligned}\Pi_{2Ci}^{(R)} = & w \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i \\ & + w R_i \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i) dx_i + w_r R_i \\ & + p_r^{(R)} M_i^{(R)} \int_0^{M_i^{(V)}} f_i(x_i) dx_i \\ & + p_r^{(R)} \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (M_i^{(V)} + M_i^{(R)} - x_i) f_i(x_i) dx_i \\ & + p_r^{(R)} \int_{M_i^{(V)}+R_i}^{+\infty} (M_i^{(R)} - R_i) f_i(x_i) dx_i.\end{aligned}\quad (\text{A.13})$$

By taking the demand uncertainty into consideration, the ROO's expected profit in service area i can be written as

$$\Pi_{2i}^{(R)} = \begin{cases} (1 - q_i) \Pi_{2Ci}^{(R)} + q_i \Pi_{2Ai}^{(R)} & \text{if } R_i \leq S_i, \\ (1 - q_i) \Pi_{2Ci}^{(R)} + q_i \Pi_{2Bi}^{(R)} & \text{if } R_i > S_i. \end{cases} \quad (\text{A.14})$$

The ROO's total expected profit is $\Pi_2^{(R)} = \sum_{i=1}^n \Pi_{2i}^{(R)}$.

B. Competitive Scenario

1) *Without Backup Reservation:* The VNO's expected profit is written as in (9). The ROO's expected profit in each service area i is

$$\begin{aligned}\Pi_1^{(R)} = & q_i [(M_i^{(R)} - S_i) p_r^{(R)} + p_r^{(R)} S_i \int_{M_i^{(V)} + \frac{S_i}{\theta\beta}}^{+\infty} f_i(x_i) dx_i \\ & + \theta\beta p_r^{(R)} \int_{M_i^{(V)}}^{M_i^{(V)} + \frac{S_i}{\theta\beta}} (x_i - M_i^{(V)}) f_i(x_i) dx_i] \\ & + (1 - q_i) p_r^{(R)} M_i^{(R)}.\end{aligned}\quad (\text{A.15})$$

2) *With Backup Reservation:* The VNO's expected profit is written as in (11). As for the ROO, three cases need to be considered:

Case 1: With low demand, and $R_i \leq S_i$. The ROO's expected profit is

$$\begin{aligned}\Pi_{2Ai}^{(R)} = & (M_i^{(R)} - S_i) p_r^{(R)} + w_r R_i + w R_i \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i) dx_i \\ & + w \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i \\ & + \theta\beta p_r^{(R)} \int_{M_i^{(V)}+R_i}^{M_i^{(V)}+R_i + \frac{S_i - R_i}{\theta\beta}} (x_i - M_i^{(V)} - R_i) f_i(x_i) dx_i \\ & + p_r^{(R)} \int_{M_i^{(V)}+R_i + \frac{S_i - R_i}{\theta\beta}}^{+\infty} (S_i - R_i) f_i(x_i) dx_i.\end{aligned}\quad (\text{A.16})$$

Case 2: With low demand, and $R_i > S_i$. The ROO's expected profit is

$$\begin{aligned}\Pi_{2Bi}^{(R)} = & (M_i^{(R)} - R_i) p_r^{(R)} + w_r R_i + w R_i \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i) dx_i \\ & + w \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i \\ & + p_r^{(R)} \int_0^{M_i^{(V)}+S_i} (R_i - S_i) f_i(x_i) dx_i \\ & + p_r^{(R)} \int_{M_i^{(V)}+S_i}^{M_i^{(V)}+R_i} (R_i - x_i + M_i^{(V)}) f_i(x_i) dx_i.\end{aligned}\quad (\text{A.17})$$

Case 3: With high demand. The ROO's expected profit in each service area i is

$$\begin{aligned}\Pi_{2Ci}^{(R)} = & w_r R_i + w R_i \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i) dx_i \\ & + w \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i \\ & + M_i^{(R)} p_r^{(R)} \int_0^{M_i^{(V)}} f_i(x_i) dx_i \\ & + p_r^{(R)} \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (M_i^{(R)} - x_i + M_i^{(V)}) f_i(x_i) dx_i \\ & + p_r^{(R)} (M_i^{(R)} - R_i) \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i) dx_i.\end{aligned}\quad (\text{A.18})$$

By taking the demand uncertainty into consideration, the ROO's expected profit in service area i can be written as

$$\Pi_{2i}^{(R)} = \begin{cases} (1 - q) \Pi_{2Ci}^{(R)} + q \Pi_{2Ai}^{(R)} & \text{if } R_i \leq S_i, \\ (1 - q) \Pi_{2Ci}^{(R)} + q \Pi_{2Bi}^{(R)} & \text{if } R_i > S_i. \end{cases} \quad (\text{A.19})$$