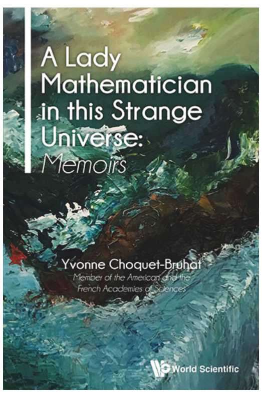
BOOK REVIEW

A Lady Mathematician in this Strange Universe: Memoirs

Reviewed by Lydia Bieri



These are the memoirs of Yvonne Choquet-Bruhat, a celebrated mathematician and physicist, who has made groundbreaking contributions to general relativity. Not only has she pioneered many aspects of mathematical general relativity (GR), but she has also been a role model for women and has opened up new avenues for women in science and mathematics. In addition to her results on Einstein's equations, Choquet-Bruhat has worked on general partial differential equations, geometry, and various aspects of physics. Choquet-Bruhat became the first woman to be elected to the French Academy of Sciences in 1979. She received the *Grand-croix de l'ordre national de la Légion d'honneur* (Grand Cross of the National Order of the Legion of Honor, the highest recognition in France) in 2015, and she is also an elected member of the American Academy of Arts and Sciences. In this book, Choquet-Bruhat writes in an upfront and natural way about her life and work. Family and friends have always been important to her, and she makes them the main figures in her journey through mathematics and physics—through history and also through spacetime. The book cover reads "...At once reflective, enlightening and bittersweet...." I agree and would add "fascinating" as well.

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Communicated by Notices *Book Review Editor Stephan Ramon Garcia.*

DOI: https://dx.doi.org/10.1090/noti2055

Figure 1. Yvonne Choquet-Bruhat during a workshop in Oberwolfach, 2006.

The book begins with a prologue, where the author recounts an amusing scene from her childhood in which she was reflecting on questions about the consciousness of the self versus the surrounding world. Choquet-Bruhat ties this memory to philosophical questions about whether mathematics is a product of the human mind or exists independently from us. However, she immediately bypasses this question (for which there does not exist a scientific answer) by suggesting that mathematics and the sciences are the best means to understand the world. She then explains her perspective on how human beings construct reality, revealing her deep understanding of the dynamical aspects of research as well as her passion for mathematics and science. She writes: "...I wished to understand something of this strange universe in which we live, and what we human beings do in it...." The means to do so are mathematics, physics, and science in general.

In the first chapters, we learn about her ancestors and family. Yvonne Choquet-Bruhat was born as the second of three children on December 29, 1923 in Lille, a city in the very north of France. Her father was Georges Bruhat, and her mother was Berthe Hubert. When Yvonne was two years old, the family moved with their daughters Jeanne and Yvonne to Paris. Her brother François came into the world a few years later. Georges Bruhat worked as a physics professor at the University of Lille, and later at the Sorbonne in Paris. He became vice-director of the École Normale Supérieure (ENS) in the mid-1930s. Georges Bruhat was a loving father and family man. He was organized and structured his days in a way that maximized both the time spent with his family and the time spent at work. Despite all his duties, he always found time to sit down with his children to study, do homework, or discuss science. Berthe Hubert taught literature and philosophy at the high school level and higher (Première Supérieure), first in Chambéry, then in Poitiers, Lille, and Paris. She is characterized as a woman who did not hide her thoughts and had a deep understanding of human problems. Both Yvonne's parents cared well for their children and supported them in their endeavors.

Georges Bruhat and Berthe Hubert provided their children with an intellectual family environment, which was augmented by nannies, as well as relatives and friends of the parents who would come to stay with them. They took family vacations in the Alps and at the Mediterranean, and spent time at their house in Dammartin (near Paris). This comfortable lifestyle was quite different from how their ancestors lived. Some of her father's ancestors, including her great-grandfather, worked breaking stones for roads. To escape this hard life, the next generation had military careers or became teachers. Her mother's family members were mainly farmers for several generations. Yvonne's maternal grandfather became a teacher who was able to send both of his children to universities.

Yvonne Choquet-Bruhat went through the rigorous French school system. One has to remember that at the time boys and girls were schooled separately. Teachers had a lot of influence. Already as a girl, Yvonne cultivated an independent mind and made friends for life. For a while, life was good. The author describes pleasures and problems throughout those years and names a subchapter "The Happy Years." This happiness would end in 1939 with the beginning of World War II and the German attack on France on May 10, 1940. Paris fell in June of that year, and France signed an armistice that left Paris in the occupied zone and the southern part of France under the Vichy Regime.

Georges and Berthe tried to protect their children as much as possible during this dark period, sending them to stay with their uncle in Poitiers in 1939. Just before the Germans invaded Paris, Georges Bruhat departed for Bordeaux with his physics laboratory. Yvonne's mother and grandmother then joined the children in Poitiers and they all headed for Bordeaux as well. It was a journey with interruptions, where at one point Yvonne found herself separated from the others, but finally the family was reunited in Bordeaux. Later, Georges Bruhat decided to go back to Paris and use his position of leadership at ENS to protect not only his laboratory from the Germans, but more importantly, to protect his students from the Nazis. He ended up helping many Jews and members of the Resistance travel to unoccupied France, where he found refuge for them. From there they could leave for Britain, where many of them joined the Allies.

On June 6, 1944 (D-Day), the Allied forces landed in Normandy. Paris was liberated in August 1944. It took almost another year until the war in Europe ended with Germany's surrender on May 8, 1945. Worldwide, the war came to an end on September 2, 1945. However, 1944 brought a radical and brutal change to the Bruhats' lives. The shock for the Bruhat family came in the form of the Gestapo a few days before the liberation of Paris. Upon refusing to help find a student who was a member of the Resistance, Georges Bruhat was deported to a concentration camp in Germany. He was transferred to the camp in Sachsenhausen, where he died at the beginning of 1945. The family tried to get news of him and hoped that after the liberation he would come back. They learned of the horrible loss in spring of that year. Needless to say, things were difficult for them. Especially Berthe Hubert had enormous trouble coping with the situation.

Yvonne felt close to her father, who was able to give her the kind of warmth that she often missed from her mother. The deportation and tragic death of her father was a traumatic experience. The author discloses that even after all these years it was still painful to write about it.

In spite of the tragic loss of Georges, life went on for the Bruhat family. Yvonne studied at the École Normale Supérieure de Jeunes Filles (the partner institution of the ENS for women) and graduated in 1946. After that, she became an assistant at the École Normale Supérieure, and then in 1949 she became a research assistant at the Centre National de la Recherche Scientifique (CNRS), the French National Center for Scientific Research. André Lichnerowicz, a professor at the Sorbonne in Paris, invited Yvonne to become his doctoral student after he saw her brilliant performance on the exam for future high school teachers. She writes: "Lichnerowicz asked me what I would like to do in the future. I answered, 'I would like to do a thesis on theoretical physics but I don't know if I am able to do it.' He said: 'You will do one if you do as I say. I work on General Relativity and I am interested in philosophy.' I said, 'It suits me very well.'" Thus, she began working in GR with André Lichnerowicz, earning her doctoral degree in 1951.

In her memoir, Choquet-Bruhat writes about the difficult situation faced by women in mathematics and science at the time. They could aim to obtain a teaching position. However, it was basically impossible for a woman to obtain a research position. She had always been fascinated by the interplay of mathematics and physics, but she had contemplated becoming a medical doctor for a while. It was not clear for a long time what career Yvonne would choose. The fortunate encounter with Lichnerowicz marked the beginning of her career in mathematics and physics.

In addition to Lichnerowicz, the person who proved to be most influential in Choquet-Bruhat's career was Jean Leray, whom she met in 1947, when she was a student in one of his courses. "He was a pillar in my life, a master and friend who supported me for more than fifty years." Leray suggested that she should work on a problem that later became the main part of her thesis. Encouraged by Jean Leray, Choquet-Bruhat solved in her dissertation the Cauchy problem for Einstein's equations in the nonanalytic case. The main content of her thesis, which was published [1] in 1952, became one of the most important results in the history of general relativity. We will get back to this in more detail later in this review.

In 1947, Choquet-Bruhat married Léonce Fourès, who was also a mathematician, and together they moved into the small apartment where her mother lived. It was a difficult time for Yvonne. Her mother was suffering from the loss of her husband and had become angry at the world, which affected everyone around her. Also, Yvonne was struggling with health problems. She felt very tired, lost weight, and was exhausted by her teaching job, yet her doctor could not find an explanation. Years later an X-ray revealed that she in fact had tuberculosis, whose effects persisted for several years. A happy event occurred for Choquet-Bruhat in 1950, when her daughter Michelle was born. As for the couple, however, the book explains how they diverged over the years, and eventually divorced in 1960.

In 1961, Yvonne married Gustave Choquet, whom she had met through Léonce. Gustave, too, was a mathematician and a climber, as was Léonce. Yvonne writes that one of the things that she admired in Gustave was his affectionate way of caring for his three children from a previous marriage. Yvonne and Gustave had two children together, a son Daniel, born in 1962, and a daughter Geneviève, born in 1966. The couple was determined to create a warm and loving environment for all of their children. They were a happy family, enjoying traveling together as well as going camping in California after they had spent a term in Berkeley. The photo in Figure 2 stems from their time in California.

Figure 2. Yvonne Choquet-Bruhat and Gustave Choquet in Berkeley, 1974.

Choquet-Bruhat's doctoral thesis [1] represented a breakthrough in the Cauchy problem (initial value problem) for the Einstein equations. These equations lie at the heart of GR, describing the physical laws of our universe by its geometric properties. The universe itself, or the part of it that we wish to study, is described as a spacetime manifold with a metric whose curvature encodes the properties of the gravitational field. Thereby, time, space, and matter or energy are unified into this manifold, where the latter two give it the curvature structure. Indeed, there is no "absolute" entity or frame as in Newtonian mechanics in which objects move. Instead everything is part of the spacetime itself contributing to its geometric structure.

A first glimpse at the Einstein equations may not give the impression that they are so challenging. They couple Ricci $R_{\mu\nu}$ and scalar R curvature to a stress-energy tensor $T_{\mu\nu}$ that includes other physical fields (such as electromagnetic fields or a fluid), $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ with indices running from 1 to 4. Extra equations are coupled to this system for the matter-energy fields on the right-hand side. We aim to find solutions to these equations for physical situations. Thus, under assumptions given by the latter, the solution spacetime is a four-dimensional, oriented, differentiable manifold M with a Lorentzian metric tensor, g. (The trivial example would be Minkowski spacetime of special relativity with the flat diagonal metric $(-1,1,1,1)$.) To describe cosmological phenomena, these equations take an extra term involving a cosmological constant multiplied by the metric. If no other sources are present, the above system reduces to the Einstein vacuum equations $R_{\mu\nu} = 0$. Note that solutions of the latter may contain black holes, domains of spacetime with highly concentrated curvature such that nothing can escape their boundaries.

There are immediate consequences of the Einstein equations on various levels, including the bending of light by gravitation. This led to Arthur Eddington's successful expedition in 1919, where he confirmed that the sun's gravitational field deflects light as predicted by GR. This was a milestone. Several results were derived in the pioneering years. Many exact solutions were found and studied for their physical properties, though there was still much more that one would like to know. Those solutions, for example, cannot address dynamical properties of the gravitational field. In order to understand dynamics and radiation, we have to investigate large classes of spacetimes.

To gain more insight into these equations and to realize the impact of Choquet-Bruhat's PhD result, let us travel back in time to 1915, when Einstein derived them [9], [10], and try to imagine our reaction: If this geometric equation describes our world, then how do we make use of it? We would like to understand the dynamics of the gravitational field, know the evolution of systems like a galaxy or stars, and more generally solve the initial value problem. However, the Einstein equation in its brief form cannot be used right away to do so. It took many years, as well as many mathematical steps by a number of contributors, until in 1952 Choquet-Bruhat [1] set up the Cauchy problem for the Einstein equations in a general form and obtained a local result. The main result from Choquet-Bruhat's dissertation [1], published in 1952, proved a local existence and uniqueness theorem for the Einstein equations and has since been essential for further investigations of the Cauchy problem. Her proof introduced a useful coordinate system, called wave coordinates, in which Einstein's vacuum equations appear clearly as a hyperbolic system of nonlinear (quasilinear) partial differential equations.

Many people had worked on the problem that Yvonne eventually solved in her thesis; G. Darmois studied the analytic case. We recall that real analytic functions are infinitely differentiable and they are determined by their values in an open set. Therefore, they cannot detect any propagation phenomena or causality. One needs to generalize these results in order to describe physics. Darmois noticed the existence of constraints that initial data have to satisfy in the analytic spacelike Cauchy problem, also pointing out that these constraints propagate. A. Lichnerowicz extended Darmois' work. T. de Donder and C. Lanczos introduced wave coordinates that Darmois later used. D. Hilbert, H. Weyl, and many more people contributed. The big leap forward was to go beyond the analytic case because of the reasons above. Leray suggested that Yvonne should study J. Schauder's results to extend those ideas to GR. Choquet-Bruhat ended up studying the paper by Sergey Sobolev where he had introduced a method to obtain nonanalytic solutions for linear PDEs similar to the linearized Einstein equations in wave coordinates. Choquet-Bruhat took these ideas further to solve nonlinear PDEs in her thesis.

Not only is this the beginning of answering the crucial questions raised above, but it is also the first proof establishing that for the nonlinear Einstein equations gravitational waves propagate at finite speed and that causality holds. Rough ideas about causality had been mentioned earlier by Hermann Weyl, but it had not been mathematically established for the Einstein equations. In 1916, Einstein looked at the linearized equations and found wave solutions. However, he himself knew that linearization can introduce artifacts that are not present in the general case. We owe the first full gravitational wave result to Choquet-Bruhat. These waves were detected for the first time in 2015 by the Laser Interferometer Gravitational-Wave Observatory (LIGO) team, marking the beginning of a new era, where they will be the messengers from parts of the universe that telescopes cannot see.

The $3 + 1$ splitting of the four-dimensional spacetime manifold into time and three-dimensional spacelike Riemannian hypersurfaces that results from Choquet-Bruhat's proof is called the "$3 + 1$ formalism." It was introduced in the analytic case by G. Darmois, generalized by A. Lichnerowicz, further generalized by Choquet-Bruhat (published in 1948), and fully resolved by Choquet-Bruhat shortly thereafter. In fact, she also showed that this holds for $n + 1$ dimensions. Note that the Einstein equations do not have any such a priori structure, but the $3 + 1$ (respectively, $n + 1$) splitting relies on the choice of a time coordinate.

This formalism has had huge impacts in many directions. In the 1950s and 1960s it served as the foundation of Hamiltonian formulations of GR by P. Dirac, R. Arnowitt, S. Deser, and C. Misner. This is often called the ADM mechanism (referring to Arnowitt, Deser, and Misner), ignoring the pioneering works of Choquet-Bruhat. The Einstein equations decouple into a set of constraint equations that initial data have to satisfy, and which propagate, and a set of evolution equations for this data. In the 1970s, J. York developed a general method, called the conformal method, to solve these constraint equations. It was introduced by Ch. Racine (a student of Darmois) and partially extended by Lichnerowicz. Later, the Bowen–York data (which York developed with J. Bowen) became very popular. A lot of work in GR has been devoted to the constraint equations. Again, the names for these methods did not include the first contributors. In this book, Choquet-Bruhat expresses her regrets about that, yet at the same time she does not brood and instead stands above issues of this sort. Indeed, it is a feature of her character that occurs in different moments throughout her professional as well as personal life, in which she deals with difficulties through constructive solutions rather than by holding grudges.

Later on, Choquet-Bruhat collaborated on these topics with York and other relativists including J. Isenberg, D. Pollack, P. Chruściel, and J. M. Martín-García [4], [7]; see [2] for more details. Choquet-Bruhat's work became a crucial base for numerical relativity. H. Friedrich suggested using generalized harmonic coordinates for numerical simulations of the two-body problem (such as two black holes spiraling in, merging, and sending out gravitational waves). This was implemented by Frans Pretorius in 2005 to produce the first successful numerical code to model gravitational radiation from binary mergers. That same year and ever since, many groups have been successful using different approaches.

In 1969, Choquet-Bruhat and Robert Geroch [6] proved the global existence of a unique globally hyperbolic maximal development for every given initial data set for the Einstein equations, constituting another breakthrough. This meant that the maximal development of any Cauchy data is unique up to a diffeomorphism that fixes the initial hypersurface. From here, the following crucial questions could be attacked: What happens for initial data of a physical system? Will these evolve under the Einstein equations without singularities for all time or will black holes form? The former was proven to be true by D. Christodoulou and S. Klainerman in 1993 for asymptotically flat, suitably small (and nontrivial) initial data, namely, that there exists a unique, causally geodesically complete and globally hyperbolic solution to the Einstein equations that itself is globally asymptotically flat. The latter was shown by Christodoulou in 2008, namely, that highly concentrated gravitational waves form a closed trapped surface and eventually a black hole. These areas of research have become very popular, attracting many mathematicians to work on them currently.

Choquet-Bruhat's first journey to the United States in 1951 was an important experience. Initiated by J. Leray, Yvonne and Léonce were offered positions at the Institute for Advanced Study (IAS) in Princeton. Together with their daughter Michelle, they embarked for New York. Choquet-Bruhat writes: "Life in Princeton was, for me, a happy time." What a relief it was after the hardship of the war not to worry about material life. She appreciated the American vitality and the engaging academic environment in Princeton. We learn that she made many friends and especially liked the warm human relations. Among her friends we find I. Segal (who also became a collaborator), G. Mackey, and J. Tits. The latter coauthored an influential work with Yvonne's brother François, called the Bruhat–Tits theory. While in Princeton, Choquet-Bruhat witnessed early steps in computing after having met J. von Neumann, who had designed the computer called the "IAS machine." A delightful part of the book explains how Choquet-Bruhat met Einstein at IAS. During their first conversation Einstein asked Choquet-Bruhat to explain her work to him. "…I summarized, in French, my thesis on Einstein's blackboard. Einstein listened with interest, and congratulated me for the results which gave rigorous proofs of properties he had expected from the gravitational field. When I left, Einstein told me I would be welcome in his office any time I felt like knocking on his door.…"

After their stay at IAS, the couple accepted positions in mathematics at the University of Marseille. In 1958, Choquet-Bruhat became a professor at the University in Reims, where she stayed for only about a year. Then she was given the chair of celestial mechanics at the Sorbonne in Paris.

In addition to describing Choquet-Bruhat's life and work, the book provides insights into the political and social developments in the late 1960s at the Sorbonne and in Paris from her point of view as a professor. She describes meetings with students, sympathizing with them on issues like the need for reforms in the educational system, but also chaotic discussions that would not lead anywhere. We also see how the universities in Paris were restructured, in particular how Paris 6 and 7 were established. Choquet-Bruhat moved to Jussieu (Paris 6), which later was called Pierre et Marie Curie, from which she retired in 1990. From 2003 on, Yvonne used office space at the Institut des Hautes Études Scientifiques (IHES) that its director, J. P. Bourguignon, had reserved for her. She has enjoyed working at IHES and interacting with its members and visitors, in particular with her friend T. Damour, who holds one of the very few professorships.

In 1957, the famous Chapel Hill Congress, organized by Bryce and Cécile DeWitt, hosted the main specialists in GR at the time, including, of course, Choquet-Bruhat. It was in Chapel Hill, North Carolina that the idea of creating the General Relativity and Gravitation Society was born, though it only took shape fourteen years later. Since then the society has promoted research in this area via many channels, including a major conference that is held triennially and two journals that have become highly influential.

Among relativists, the "Battelle Rencontres" meeting between physicists and mathematicians of 1967 in Seattle marked a milestone, organized by J. A. Wheeler and C. DeWitt. Cécile DeWitt was a good friend of Yvonne's. Interdisciplinary collaborations in GR pushed the field forward. Among the participants in 1967 we find R. Bott, S. Helgason, R. Penrose, R. Geroch, S. Hawking, and many other players in these fields.

Choquet-Bruhat has continued to achieve amazing and important results in GR and beyond, in mathematics as well as in physics. It is impossible to do justice to this work in a short review; instead we can only name a few. In solo work and collaborations, she extended her original theorems to the Einstein equations coupled to other fields. Yvonne developed new methods to study nonlinear (hyperbolic) PDEs (including Einstein's equations). Many of these methods provided solid foundations for several theories in physics, such as relativistic hydrodynamics, magnetohydrodynamics, plasmas, nonabelian gauge theories, as well as supergravity. Her results on relativistic fluids have been most influential. Her studies of gravitational waves have also generated major impacts. Choquet-Bruhat investigated high-frequency gravitational waves for the nonlinear Einstein equations using an asymptotic expansion introduced by Leray for linear systems. She constructed gravitational waves where the perturbation and the background fields contribute equally, showing that they propagate like light. Her early methods and ideas have been taken further by her and others to yield important results. With Christodoulou [3] she proved the existence of global solutions of the Yang-Mills, Higgs, and spinor field equations. With V. Moncrief [8] she established nonlinear stability of an expanding universe with S^1 symmetry group. H. Friedrich and Choquet-Bruhat [5] worked on the motion of isolated bodies. She authored and coauthored many books, including the 800-page-volume *General Relativity and the Einstein Equations* [2], as well as her work with C. DeWitt, *Analysis, Manifolds and Physics*, which may be the most popular ones. Her publication list is too long to describe in detail here. See [2] for more references and details on her work.

With overwhelming power Choquet-Bruhat's work has pushed the limits of research in mathematics and physics, thereby achieving the highest standards. She has been decorated with many honors, a few of which I cited in the first paragraph. Traveling is a passion that Yvonne has enjoyed a lot over the years in order to speak at conferences or work at other institutions. She has traveled the world, literally. The descriptions of certain trips under difficult circumstances, interesting encounters, and events make for an extra dimension to this book. For instance, Yvonne describes a journey she took alone in 1975 that brought her first to Teheran, then to Karachi, and from there to several cities in India, eventually heading to Calcutta. She recounts a scene while taking a local bus that left from Agra. On that trip, she was the only tourist and made friends with a couple who invited her to explore the stunning sites together with them. Often, the whole family would voyage together, either professionally or for leisure. She has made friends all over the world.

Choquet-Bruhat is an inspiring researcher, teacher, and person. In many ways she has broken professional as well as social barriers for women and has opened doors for female mathematicians to succeed in academia. Most of the obstacles that she had to overcome do not exist anymore thanks to her persistence and that of other pioneers. She describes her personal relations and professional environment as mostly positive, even though she had to face unfair treatment as well as verbal attacks. Typically, she would counter the latter with a smart remark, keeping a positive attitude.

These memoirs are frank, like its author, and span the enormous spectrum of an exceedingly productive life. A continuing theme is her pleasure in doing mathematics and discussing this work with friends. She shows a lot of understanding as well as respect for the human being in front of her, be it a colleague or a stranger in the street. She is always ready to support a person or a cause. It is impressive how Choquet-Bruhat has overcome all the hardships she has faced along the way. As with almost everything in her life, she would calmly work her way through any obstacles.

She has demonstrated this very same endurance while proving theorems. When she writes about her research, she naturally embeds it into her life with family, friends, and colleagues. In many ways unusual, her memoir touches almost every aspect of human existence.

All in all, Yvonne Choquet-Bruhat is a first-rate, exceptionally creative mathematician and physicist who has led an exciting, active life. In other words, this is the story of a fascinating person.

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