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Computing a Global Degree of Rate Control for Catalytic Systems

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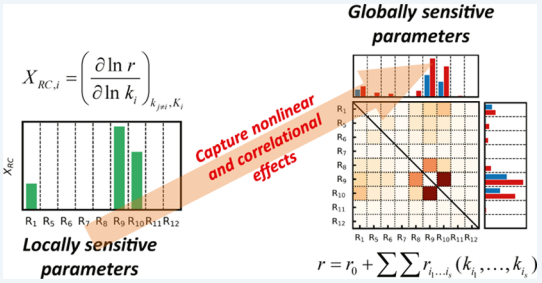
ABSTRACT: In this paper, we discuss the concept and properties of variance-based global sensitivity analysis, as an expansion of local sensitivity metrics (such as the degree of rate control), for modeling and design of catalytic reaction systems. Using an illustrative example and supporting theory, we show that: (i) for small variations in the parameters, global sensitivities are similar to local derivatives; (ii) for larger variations in the parameters (i.e., a larger parameter space), the global sensitivities provide a ranking of importance of parameters and impose a rigorous bound on the errors that arise from fixing one or more parameters to nominal values; and (iii) in general, the global sensitivities can be related to the extrema of local derivatives. We argue that the square root of the total global sensitivity of a parameter, computed by summing the global sensitivity of that parameter acting independently and in combination with others, is a "global" degree of rate control for catalytic systems.



KEYWORDS: *microkinetic modeling, global sensitivity analysis, degree of rate control, uncertainty quantification, data-driven modeling*

INTRODUCTION

Multistep reaction systems are common in catalysis. Numerous approaches have been proposed to elicit microscopic and macroscopic information of such systems; this includes: (i) delineating potential energy surfaces and inferring information using heuristics such as the step with the largest individual barrier being rate-controlling or the difference between the highest state and the initial state being equal to the apparent barrier, etc.; (ii) the energetic span model[1] that, for a single reaction cycle, relates the reaction rate with the free energy difference between the highest and the lowest points of a potential energy surface; (iii) developing rate expressions using the de Donder relations[2,3] or the maximum virtual reaction rate analysis;[4] (iv) developing rate expressions based on presumed abundant intermediates and rate-determining steps,[5,6] and (v) microkinetic modeling.[7−15] While formulating and solving microkinetic models, one often calculates a normalized sensitivity analysis or the kinetic/thermodynamic degree of rate control, by acquiring the local derivatives of the model with respect to its parameters.[16,17]

The kinetic degree of rate control (X_{RC}) of an elementary reaction step, measures the local sensitivity of the rate to perturbations in its transition-state energy (keeping all other free energies fixed). Mathematically

$$X_{\mathrm{RC},i} = \frac{\partial \ln r}{\partial\left(\frac{-G_i^{0,\mathrm{TS}}}{RT}\right)_{G_{i'\neq i}^{0,\mathrm{TS}};G_j^0}} \qquad i, i' \in \{\text{reactions}\};\ j \in \{\text{species}\} \tag{1}$$

where r denotes the net reaction rate r. $G_i^{0,\mathrm{TS}}$, $G_{i'\neq i}^{0,\mathrm{TS}}$, and G_j^0 correspond to the free energy of the transition state of step i, the transition state of step other than i, and species j respectively.

The degree of rate control, introduced by Campbell,[17,18] has proven to be a versatile concept as it allows for calculating the rate-controlling steps and identifying kinetically relevant steps. Over the years, many variants of the original definition have been discussed, including the thermodynamic degree of rate control and the degree of selectivity control.[15,19] Furthermore, recent works have also discussed: (i) relationships among experimental observables such as apparent orders[20] and barriers[21,22] and the degrees of rate control; (ii) the relationship between inferred microscopic and thermodynamic quantities such as surface coverages, apparent entropy of activation, stoichiometric number of a reaction cycle, and the degrees of rate control[20]; (iii) the evaluation of the kinetic isotope effect from these degrees[23]; (iv) the effect of linear scaling and linear free energy relations on the degree of rate control[24,25]; (v) their extension to describing transient systems[26]; and (vi) the utility of X_{RC} values in generating volcano plots quickly.[27]

All of these definitions are based on local derivatives, so they offer insights about the specific catalyst under study. In many cases, such as parameter estimation, uncertainty quantification,

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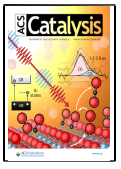

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and catalyst design, a larger parameter space is under consideration. While the degree of rate control provides information on the dominant parameters at one point in this parameter space, it is generally not reflective of the entire space unless the rate function is linear with respect to a few parameters or when the space is not large enough. Global sensitivity measures have been employed while ranking important parameters of a nonlinear model in a given space[28]; they provide a more rigorous framework than the local sensitivity. Such global sensitivity measures could be (i) derivative-based,[29] wherein derivatives are calculated at multiple points and an ensemble mean is taken or (ii) variance-based,[30] wherein the extent to which a parameter contributes to the variation of a function over a space is computed. In both methods, the parameters with high global sensitivity values are considered dominant in the space taken as a whole, and it is prudent to not ignore them while computing the properties or evaluating average characteristics of the space. On the other hand, those parameters that are not globally sensitive can be set to a nominal (reference) value or totally dropped.

Various global sensitivity metrics and methods have been applied in chemistry and catalysis.[31−36] Here, we revisit the definition and properties of variance-based global sensitivities and demonstrate their usefulness in the modeling and design of catalytic systems. On the basis of the properties, we define a global degree of rate control. We begin with a discussion of variance-based global sensitivity analysis.

METHOD

Variance-Based Global Sensitivity Analysis. Consider a continuous real-valued nonlinear multivariate function $f(x)$ where $x = \{x_1, x_2, ..., x_n\}$ are independent and uniformly distributed variables (or parameters) and $x_i \in I = [0, 1]$ such that the parameter space is I^n. If no closed-form expression exists that relates f with x, as is usually the case in many physical systems including the catalytic reaction systems, inferring the influence of each parameter by themselves and in concert with others is nontrivial. Indeed, partial derivatives only offer a local picture; furthermore, locally, parameters can be taken to influence f independently. One way to obtain a global picture of the relationship between x and f comes from the concept of analysis of variance (ANOVA[30]) to decompose the function as

$$f(x) = f_0 + \sum_i f_i(x_i) + \sum_{i<j} f_{ij}(x_i, x_j) + ... + f_{12...n}(x_1, x_2, ..., x_n) \quad (2)$$

or, generally

$$f(x) = f_0 + \sum_{s=1}^{n} \sum_{i_1<...<i_s}^{n} f_{i_1...i_s}(x_{i_1}, ..., x_{i_s}); \quad 1 \le i_1 < ... < i_s \le n \quad (3)$$

where f_0 is the average of f over I^n given by

$$f_0 = \int_0^1 ... \int_0^1 f(x) dx_1 ... dx_n = \int f(x) dx \quad (4)$$

It is important to note that the terms of eq 2 of the form $f_{i_1...i_s}(x_{i_1},...,x_{i_s})$ represent the combined nonlinear effect of $x_{i_1},...,x_{i_s}$ on $f(x)$. That is, $f_i(x_i)$ is the independent effect of x_i, $f_{ij}(x_i, x_j)$ is the combined effect of x_i and x_j, and so forth.

Furthermore, for the decomposition to be based on ANOVA, we require that

$$\int f_{i_1...i_s}(x_{i_1}, ..., x_{i_s}) dx_{i_1} ... dx_{i_s} = 0 \quad (5)$$

and

$$\int f_{i_1...i_s}(x_{i_1}, ..., x_{i_s}) dx_{i_1} ... dx_{i_s} f_{k_1...k_s}(x_{k_1}, ..., x_{k_s}) dx_{k_1} ... dx_{k_s} = 0 \quad \text{if} \{i_1, ..., i_s\} \ne \{k_1, ..., k_s\} \quad (6)$$

From eqs 2, 5, and 6, it can be shown that

$$\int f(x) \prod_{k \ne i} dx_k = f_0 + f_i(x_i) \quad (7)$$

$$\int f(x) \prod_{k \ne i,j} dx_k = f_0 + f_i(x_i) + f_j(x_j) + f_{ij}(x_i, x_j) \quad (8)$$

Therefore, mathematically

$$f_{ij}(x_i, x_j) = \int f(x) \prod_{k \ne i,j} dx_k + \int f(x) dx - \int f(x) \prod_{k \ne i} dx_k - \int f(x) \prod_{k \ne j} dx_k \quad (9)$$

We can now define total variance, D, of f in I^n as

$$D = \int f^2 dx - f_0^2 = \sum_{s=1}^{n} \sum_{i_1<...<i_s}^{n} \int f_{i_1...i_s}^2 (x_{i_1}, ..., x_{i_s}) dx_{i_1} ... dx_{i_s} = \sum_{s=1}^{n} \sum_{i_1<...<i_s}^{n} D_{i_1...i_s} \quad (10)$$

where D_i or, more generally, $D_{i_1...i_s}$ is the variance of f_i or $f_{i_1...i_s}$ respectively.

The global sensitivity of parameter x_i is given by

$$S_i = \frac{D_i}{D} \quad (11)$$

The global sensitivity of the combination of parameters $x_{i_1},...,x_{i_s}$ is given by

$$S_{i_1...i_s} = \frac{D_{i_1...i_s}}{D} \quad (12)$$

We can further define the total sensitivity of parameter x_i by collecting all terms containing it, that is,

$$S_i^T = \sum_{\langle i \rangle} S_{i_1...i_s} \quad (13)$$

Global sensitivities have several valuable properties. First, by definition, they add up to 1

$$\sum_{s=1}^{n} \sum_{i_1<...<i_s}^{n} S_{i_1...i_s} = 1 \quad (14)$$

Second, if some of the parameters of x, let us say, z is fixed to some nominal value, z_0, while the rest y (i.e., $x \equiv (y, z)$) are allowed to vary, the approximation error of $f(y, z_0)$ over the parameter space, given by

$$\delta(z_0) = \frac{1}{D} \int [f(x) - f(y, z_0)]^2 dx \quad (15)$$

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is of the same order as S_z^T. Indeed, it can be shown that

$$\delta(z_0) \geq S_z^T \tag{16}$$

and

$$P\left\{\delta(z_0) \leq \left(1 + \frac{1}{\varepsilon} S_z^T\right)\right\} \geq 1 - \varepsilon; \qquad 0 \leq \varepsilon \leq 1 \tag{17}$$

where $P\{\alpha\}$ is the probability that α is true.

Third, it can be shown that, for two real values C_l and C_u such that $C_l \leq \left|\frac{df}{dx_i}\right| \leq C_u$ over the parameter space, we have

$$\frac{C_l^2}{12\,D} \leq S_i^T \leq \frac{C_u^2}{12\,D} \tag{18}$$

thereby relating global sensitivity with the partial derivative.

We finally note that

$$S_{ij} = \frac{\int f_{ij}^2(x_i, x_j)\mathrm{d}x_i\mathrm{d}x_j}{D} \tag{19}$$

An illustration of S_{ij} using simple nonlinear models is given in the Supporting Information S1.6. To summarize that discussion, S_i captures the influence of parameter i alone; S_{ij} essentially captures the influence of the combination of parameters i and j. In essence, it captures the nonlinearity of the model and the extent to which the derivative with respect to parameter i (or j) is dependent on the value of the other. S_i^T captures the net influence of parameter i, independently or via combination with other parameters.

The important properties of ANOVA-based global sensitivities are summarized here. First, the sensitivities add up to 1. As sensitivities are non-negative by definition (because variances are non-negative), any sensitivity value has to be between 0 and 1 and a larger value implies that the parameter (or a combination of parameters) is more important. Second, the error introduced by fixing a parameter can be rigorously estimated using eqs 16 and 17; therefore, while ranking the parameters, their importance can be quantified. This would allow one to rigorously say, for instance, that the first n' of the important parameters account for x % in the total variance of a function. Third, ANOVA-based global sensitivities are related to local derivatives by eq 18; therefore, for small parameter ranges, $C_l \approx C_u$ and $S_i \sim C_u^2/12\,D$. Fourth, the global sensitivities allow for evaluating the importance of combined influence (or correlated behavior) of two or more parameters arising from the structure of the function f.

Global Sensitivities for Reaction Systems. For an arbitrary reaction system with n elementary steps, we know that the steady-state reaction rate r, or its natural logarithm, $\ln(r)$, are real-valued functions of kinetic and/or equilibrium parameters. Here, for conceptual simplicity, we begin by considering the kinetic parameters $(k_1, ..., k_n)$; however, we note that the method of global sensitivity analysis is agnostic to the specific nature of the parameters one wishes to include. We further define a dimensionless parameter $e_i \in [0, 1]$ because of the following: $e_i = (Ea_i - Ea_{i,l})/(RT\,\Delta)$, where $Ea_i = -RT\ln(k_i/A_i)$ is the activation barrier of step i, $Ea_{i,l}$ is the lower bound of this barrier, $\Delta = (Ea_{i,u} - Ea_{i,l})/RT$ is the chosen parameter range of interest, and $Ea_{i,u}$ is the upper bound of the barrier. Note that one can use the free energies of the transition state instead of the activation barrier as the parameter sets. We can write the ANOVA decomposition of $\ln(r)$ as

$$\ln(r) = r_0 + \sum_i r_i(e_i) + \sum_{i<j} r_{ij}(e_i, e_j) + ... + r_{12...n}(e_1, e_2, ..., e_n) \tag{20}$$

where r_0 is the average over the parameter space, r_i is the unique contribution of parameter e_i, r_{ij} is the combined contribution of e_i and e_j, and so forth.

We note that simplifying approximations can be made as needed to limit the number of terms in eq 20; specifically, we can truncate the equation to limit the level of interactions we permit. This approximates eq 20 as (if only third-order combinations are permitted)

$$\ln(r) \approx r_0 + \sum_i r_i(e_i) + \sum_{i<j} r_{ij}(e_i, e_j) + \sum_{i<j<k} r_{ijk}(e_i, e_j, e_k) \tag{21}$$

We can now expand our parameter set to include the energies of the intermediates as well by defining a dimensionless parameter $e_j \in [0, 1]$ for species j as $e_j = (G_j - G_{j,l})/(RT\,\Delta)$, where G_j is the free energy of the intermediate, $G_{j,l}$ is the lower bound of free energy, $\Delta = (G_{j,u} - G_{j,l})/RT$ is the chosen free energy range of interest, and $G_{j,u}$ is the upper bound of free energy.

At this stage, we note that the parameters are assumed to be independent and uniformly distributed across the space. This represents a generic scenario of an uninformed but reasonable prior. Information about correlations between parameters to reflect physical realities or data-derived statistical distributions can also be treated in the general framework, as briefly discussed in the Supporting Information.

Computing the Global Sensitivities. Traditionally, ANOVA-based global sensitivities are computed through Monte Carlo integration of individual variance integrals. This can be quite cumbersome and slow to converge and is often considered an impediment to the implementation of this method despite its popularity. Recently, however, it has been shown that if a surrogate model of $f(x)$ can be built via multivariate orthogonal polynomial expansions in a data-driven manner,[37] the individual sensitivities can be computed with comparative computational efficiency. The details of this formulation are provided in the Supporting Information. For independent and uniformly distributed variables, the chosen basis set is the Legendre polynomials, while if the parameter distributions are known a priori, other polynomial basis sets can be employed (which, however, are beyond the scope of this work).

RESULTS—DEMONSTRATION USING THE WATER-GAS SHIFT REACTION ON CU(111)

Illustrative Example. Here, we consider the water-gas shift (WGS) reaction as an example. The elementary set of species and reactions are given in Tables 1 and 2. The adsorption energies and activation barriers were obtained from previously reported density functional theory (DFT) calculations for Cu(111). These parameters were further adjusted by constructing a Bayesian inference problem to learn from the kinetic experimental data of WGS on Cu(111)[38] and the experimental adsorption energy dataset (see Tian and Rangarajan[13] for more details). The optimized kinetic parameters are set as the nominal point for our analysis and the rate and equilibrium constants of each elementary step are shown in Table 2 for a representative temperature (610 K). The microkinetic model was formulated

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Table 1. List of Gaseous Species and Surface Intermediates in the Mechanism of WGS Reaction (Taken from Grabow and Mavrikakis[12])

index	intermediate
I_1	H*
I_2	O*
I_3	OH*
I_4	H_2O*
I_5	CO*
I_6	CO_2*
I_7	HCOO*
I_8	COOH*
G_1	H_2(g)
G_2	CO(g)
G_3	CO_2(g)
G_4	H_2O(g)

Table 2. Elementary Reaction Network and Optimized Kinetic Parameters of WGS Reaction on Cu(111)[a]

index	reaction	k_i (s^{-1})	K_i[b]
R_1	H_2(g) + 2* → 2H*	5.43×10^5	3.97×10^{-5}
R_2	CO(g) + * → CO*	1.18×10^8	1.34×10^{-6}
R_3	CO_2(g) + * → CO_2*	9.42×10^7	1.92×10^{-8}
R_4	H_2O(g) + * → H_2O*	1.47×10^8	1.52×10^{-5}
R_5	CO* + O* → CO_2* + *	5.43×10^5	2.04×10^8
R_6	CO* + OH* → COOH* + *	8.95×10^5	7.21×10^{-4}
R_7	COOH* + * → CO_2* + *	6.06×10^4	3.89×10^5
R_8	COOH* + OH* → CO_2* + H_2O*	5.66×10^{12}	7.01×10^7
R_9	H_2O* + * → OH* + H*	3.46×10^2	5.55×10^{-3}
R_{10}	OH* + * → O* + H*	4.95×10^0	1.38×10^{-6}
R_{11}	2OH* → H_2O* + O*	6.05×10^7	2.48×10^{-4}
R_{12}	CO_2* + H* → HCOO* + *	7.07×10^6	1.49×10^1

[a]Forward rate constant (k_i) and equilibrium constant (K_i) at temperature 610 K are listed. "*" denotes a free site (used in itself) or a surface intermediate (used after a species). [b]The unit of K_i is unitless for the surface reaction; atm^{-1} for the adsorption reaction.

as a dynamic continuous stirred-tank reactor under differential conditions and the steady state rates were obtained by integrating to a large enough time (>10^3 s). The comparison between the microkinetic model prediction at the nominal point and kinetic experiments on Cu(111)[38] is shown in Figure S1.

Global Sensitivities. Figure 1 shows the values of the degree of rate control (kinetic or thermodynamic) of each of the reaction steps and surface intermediates in Tables 1 and 2 for Cu(111). As per the prediction of the optimized microkinetic model, (i) H_2O* dissociation to H* and OH* is the most rate-controlling, (ii) OH* dissociation to H* and O* also has a significant positive degree of rate control, and (iii) step 1, H_2(g) dissociation also has a positive degree of rate control. Furthermore, Figure 2 shows the global sensitivities (S_i and S_i^T) pertaining to these steps for different parameter ranges of deviation of activation barriers and binding energy values from the DFT-calculated values (from [−0.1, 0.1] to [−20, 20] kJ/mol). Figure 3 shows the heat map of S_{ij} (which can be thought of as a 2D correlation plot). S_{ij} quantifies the extent to which the activity depends on the combination of kinetics of parameters i and j. The global sensitivities depend largely on the parameter range specified by the user.

For small deviations from the nominal point, we can see that global sensitivities are qualitatively similar to local sensitivities. It should be noted that the degree of rate control can be negative for some catalytic systems, while global sensitivities are non-negative by definition. A comparison of the degree of rate control with the global sensitivities for only the kinetic parameters is shown in the Supporting Information. We can note in Figure S2 that for small deviations, the global sensitivities are also close to the degree of rate control. Furthermore, for small deviations, we note that $S_i \sim S_i^T$, indicating that the individual contribution of the parameters dominates the total variance. This is also seen in Figure 3, wherein all values are near zero. As the range of allowed deviation is increased, more parameters become important and the sensitivity of R_9, R_{10}, I_3, and I_4 are progressively diminished but still have a level of contribution. In general, global sensitivities reliably capture important parameters; however, there can be situations wherein a parameter is significant in a very narrow region (the system is highly nonlinear) and relatively insignificant everywhere else (the target parameter range is large). In such circumstances, the variance-based global sensitivity method will fail to identify the parameter as important. In addition, we can now start seeing that $S_i < S_i^T$, indicating that the combined contribution of multiple parameters becomes more significant. Figure 3 is consistent with this, as the average magnitude of S_{ij} values progressively increases as the range is expanded from [−0.1, 0.1] to [−20, 20] kJ/mol. For instance, in the largest chosen parameter range ([−20, 20] kJ/mol for each parameter), the largest values are for the combinations: R_1, R_9, R_{10}, I_3, I_4, and I_5. This is not to say that the individual S_{ij} values keep increasing or decreasing monotonously as they are dependent on the change in the total variance. R_9 and R_{10} are correlated because they shared intermediates H* and OH*, and both reactions are kinetically significant. The fact that I_4 and R_{10}, I_3 and R_{10}, and I_5 and R_6 are correlated is meaningful because the intermediates are included in the reactions.

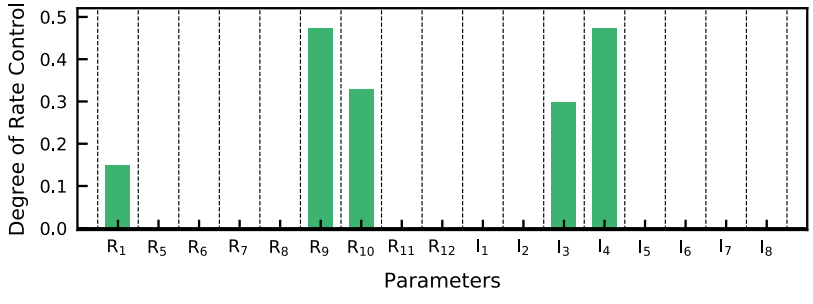

Figure 1. Degree of rate control of each reaction step and intermediate (see Tables 1 and 2) of the WGS reaction on Cu(111) based on k, K in Table 2 at 610 K, P_{H_2O} = 1.3 kPa, and P_{CO} = 3.5 kPa.

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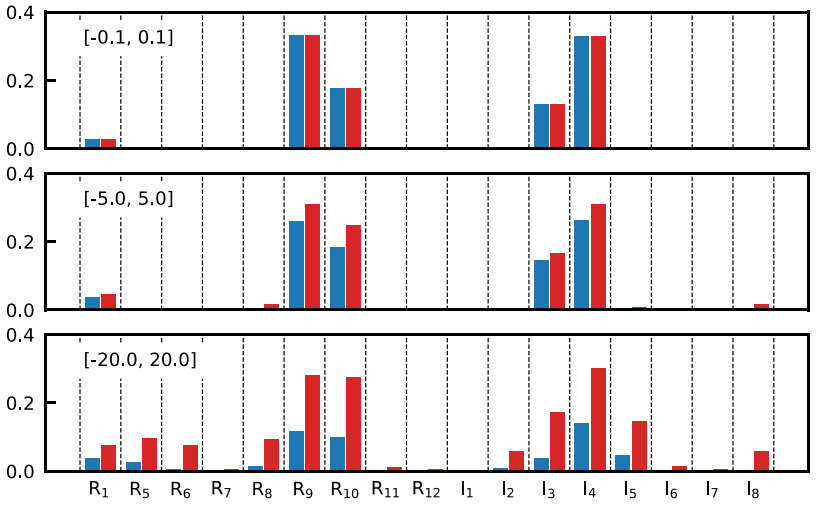

Figure 2. Global sensitivity values S_i and S_i^T for each of the reactions and intermediates of the WGS reaction on Cu(111) at 610 K, P_{H_2O} = 1.3 kPa, and P_{CO} = 3.5 kPa for various ranges ([−0.1, 0.1] to [−20, 20] kJ/mol) of parameter deviations. The ranges are set around the re-optimized energies which are taken as the reference (nominal) point.

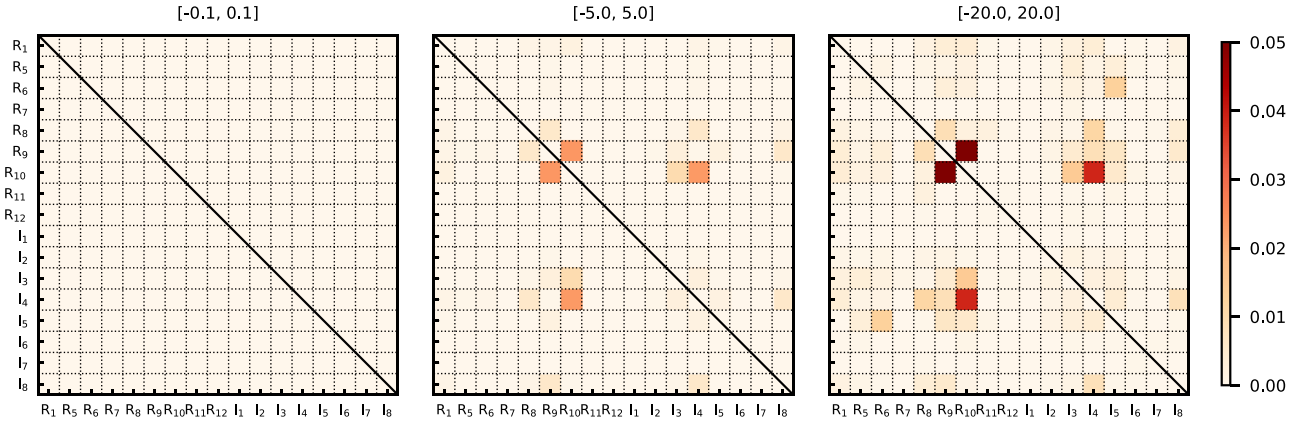

Figure 3. Heat maps of S_{ij} values for each of the reactions and intermediates of the WGS reaction on Cu(111) at 610 K, P_{H_2O} = 1.3 kPa, and P_{CO} = 3.5 kPa for various ranges ([−0.1, 0.1] to [−20, 20] kJ/mol) for various parameter ranges. Note that the map is symmetric, that is, $S_{ij} = S_{ji}$ and S_{ii} is not defined. On an average, more S_{ij} values are non-zero as the parameter range is expanded.

We can also estimate the bounds on the approximation error upon fixing some of the parameters using eqs 16 and 17. Table 3 lists various sets of insensitive parameters, z, the combined total sensitivities of these parameters, S_z^{tot}, and the approximation error $\delta(z_0)$ for the parameter range [−20, 20]. Then, clearly, about 50% of the parameters can be fixed so that the approximation error is still only 50%.

Table 3. Estimation of Approximation Errors upon Fixing Some of the Parameters to Its Nominal Value When the Parameter Range Is [−20, 20]

z	S_z^{tot}	$\delta(z_0)$
R_7, R_{11}, R_{12}, I_1, I_6, I_7	0.04	[0.04, 0.12]
R_7, R_{11}, R_{12}, I_1, I_6, I_7, I_8	0.09	[0.09, 0.27]
R_7, R_{11}, R_{12}, I_1, I_2, I_6, I_7, I_8	0.15	[0.15, 0.45]

[a]The last column provides the 50% probability bounds of approximation error (in fraction of the total original variance, D).

DISCUSSION

From inequality 18 and the definition of e_i for kinetic parameters, we can note that

$$\frac{\left(\frac{\partial \ln r}{\partial \ln k_i}\right)_{min}^2}{12\,D} \le S_i^T \le \frac{\left(\frac{\partial \ln r}{\partial \ln k_i}\right)_{max}^2}{12\,D} \quad (22)$$

That is, the total sensitivity of parameter i is related to the degree of rate control but applicable in a global sense. Therefore, we can define a "global" average degree of rate control of a reaction, $G_{RC,i}$, as

$$G_{RC,i} = \sqrt{12\,DS_i^T} \quad (23)$$

As D is constant for a specific parameter range, we can say that $G_{RC,i} \propto (S_i^T)^{1/2}$; similar to Campbell's degree of rate control, $G_{RC,i}$ is larger for a more important parameter. It also holds several important properties derived from S_i^T: (i) $0 \le S_i^T \le 1$, (ii) $\sum_i S_i^T \ge 1$, and (iii) $S_i^T \le \delta_i$, where δ_i is the approximation error of fixing parameter i, and (iv) $\delta_i \le (3, 5, 6, 11)\ S_i^T$ with greater than 50, 75, 80, and 90% probability, respectively.

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Global sensitivity can be computed for any property of the reaction system with respect to any parameter. For instance, the function $f(x)$ can be the selectivity to a specific product, conversion of a reactant, contribution of a specific reaction or pathway to the overall flux. Similarly, the parameters, x, can be related to kinetics, thermodynamics, or reactor operation (feed flow rate, temperature, pressure, concentration, etc.). We argue that global sensitivities and the global degree of rate control are useful in the modeling and design of catalytic systems. In particular, they can be used in the following.

Uncertainty Quantification. There are intrinsic errors in energies computed using standard functionals.[39] Such errors can either be quantified by comparing with single-crystal thermochemistry data,[13] a higher level of theory, or can be extracted from functionals such as the Bayesian error exchange functional.[40] These uncertainties can be forward-propagated through a microkinetic model by simply sampling the energies from the distribution, computing the associated thermochemistry/kinetics, solving the microkinetic model with those parameters, collecting an ensemble of such model predictions, and providing a statistical distribution.[13,41] This process can be computationally demanding for larger reaction networks;[11] instead, global sensitivities can be computed to rank the order of the important parameters, so that only the errors of these parameters are propagated through the microkinetic model to quantify uncertainties (while keeping other parameters fixed). Furthermore, instead of the microkinetic model, the surrogate orthogonal polynomial expansion (see the Supporting Information Section S1) can be used to compute the turnover frequency at any point. This leads to a substantial reduction in the cost of computation. We note here that the set of important parameters that can be allowed to vary (and therefore the fixed ones) can be rigorously identified based on estimating $\delta(z)$. To validate the global sensitivity in uncertainty quantification, in the Supporting Information S2.4, we have compared different methods for predicting the distribution of the turnover frequency assuming that each kinetic and thermodynamic parameter has an error of ±20 kJ/mol. As shown in Figure S5, the global method is more accurate than the local method when the parameter space is wide.

Parameter Estimation. Given the intrinsic errors in DFT, often the kinetic and thermodynamic parameters are re-evaluated to fit to kinetic experiments. This can be achieved using the maximum likelihood estimator (or obtaining the parameters that minimize the sum-of-squared errors) or via Bayesian inference. In both cases, global sensitivity analysis can be used to identify the most important parameters in a certain parameter space. In the former case, the parameter estimation (via nonlinear optimization) can be restricted to only the most important parameters. This can actually improve the robustness of the estimation process as fewer parameters will have to be reoptimized. For Bayesian inference,[13,42] again the most important parameters along with the surrogate polynomial expansion of the full microkinetic model can be used in the Markov Chain Monte Carlo process of obtaining the posterior parameter distribution.

Catalyst Design. Once the design space of interest (i.e., the material space, e.g., binary alloy catalysts composed of late transition metals) is specified, and if the corresponding parameter space (the bounds) can be estimated, global sensitivity analysis can be used to identify the set of important parameters for a chosen metric (e.g., activity or selectivity). These parameters then provide the direction for searching for a better catalyst in that space. New catalysts can be evaluated by computing only the top few parameters in this list and then using the surrogate model to quickly estimate the overall rate (or selectivity, etc.) as long as all parameters are guaranteed to be within the design space (i.e., within the corresponding parameter space). Alternatively, machine-learned models, such as those reported for binding energies, can be developed only for the top few properties and these models can then be used to screen a large library of catalysts. For example, the important parameters for designing WGS catalysts are R_8−R_{10} and I_2−I_6 when the parameter range is $[-20, 20]$ with R_9 and I_4 being the most important. In effect, then, globally sensitive parameters can be used as descriptors of the design space.

It should be noted that a local sensitivity method such as DRC can also be used, in principle, for each of those three cases. However, this requires making a linear extrapolation which can have considerable error if the parameter space is large (shown in the Supporting Information S2.4). Although global sensitivities are more computationally demanding to compute, they offer the following: (1) a more robust and quantitative (via eqs 16 and 17) way for the identification of important parameters and (2) a good surrogate model (compared to linear extrapolation) that can be used in lieu of explicit evaluation of rates via a microkinetic model.

CONCLUSIONS

We discussed the use of variance-based global sensitivity analysis in heterogeneous catalysis. The global sensitivity values of a kinetic model for its parameters offer a quantitative means of ranking important parameters. Compared to local sensitivities, global sensitivities capture the importance of parameters in a broad parameter space; however, they are still bounded by the extrema in local sensitivity values. As a result, it can be argued that the square root of the variance-based total global sensitivity values can be viewed as a measure of the "global" degree of rate control. Global sensitivities can be efficiently computed using non-intrusive orthogonal polynomial expansions which are also good surrogates for the full kinetic model. Global sensitivities can be used in parameter estimation, uncertainty quantification, and catalyst design. These sensitivities allow one to rigorously identify the set of unessential parameters that can be fixed, while the surrogate model (computed in the process) provides for quick estimation of the reactivity, thereby reducing the computational burden significantly.

ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acscatal.0c03150.

Implementation details of orthogonal polynomial expansion; calculation of global sensitivity indices from the orthogonal polynomial model; additional examples and discussion related to global sensitivity analysis and surrogate modeling; and additional results of WGS reaction examples The code about the global sensitivity analysis of microkinetic modeling in this article are available in the gsMK repository at https://github.com/thj2009/gsMk.git (PDF)

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Notes

The authors declare no competing financial interest.

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REFERENCES

(1) Kozuch, S.; Shaik, S. How to conceptualize catalytic cycles? The energetic span model. *Acc. Chem. Res.* **2011**, *44*, 101−110.

(2) Cortright, R.; Dumesic, J. Kinetics of heterogeneous catalytic reactions: Analysis of reaction schemes. *Advances in Catalysis*; Academic Press, 2001; Vol. *46*; pp 161−264.

(3) Dumesic, J. A. Analyses of reaction schemes using De Donder relations. *J. Catal.* **1999**, *185*, 496−505.

(4) Motagamwala, A. H.; Dumesic, J. A. Analysis of reaction schemes using maximum rates of constituent steps. *Proc. Natl. Acad. Sci. U.S.A.* **2016**, *113*, E2879−E2888.

(5) Kang, M.; DeWilde, J. F.; Bhan, A. Kinetics and mechanism of alcohol dehydration on γ-Al_2O_3: effects of carbon chain length and substitution. *ACS Catal.* **2015**, *5*, 602−612.

(6) Cheung, P.; Bhan, A.; Sunley, G.; Law, D.; Iglesia, E. Site requirements and elementary steps in dimethyl ether carbonylation catalyzed by acidic zeolites. *J. Catal.* **2007**, *245*, 110−123.

(7) Bhan, A.; Hsu, S.; Blau, G.; Caruthers, J.; Venkatasubramanian, V.; Delgass, W. Microkinetic modeling of propane aromatization over HZSM-5. *J. Catal.* **2005**, *235*, 35−51.

(8) Bhandari, S.; Rangarajan, S.; Maravelias, C. T.; Dumesic, J. A.; Mavrikakis, M. Reaction mechanism of vapor-phase formic acid decomposition over platinum catalysts: DFT, reaction kinetics experiments, and microkinetic modeling. *ACS Catal.* **2020**, *10*, 4112−4126.

(9) Rangarajan, S.; Maravelias, C. T.; Mavrikakis, M. Sequential-optimization-based framework for robust modeling and design of heterogeneous catalytic systems. *J. Phys. Chem. C* **2017**, *121*, 25847−25863.

(10) Medford, A. J.; Shi, C.; Hoffmann, M. J.; Lausche, A. C.; Fitzgibbon, S. R.; Bligaard, T.; Nørskov, J. K. CatMAP: a software package for descriptor-based microkinetic mapping of catalytic trends. *Catal. Lett.* **2015**, *145*, 794−807.

(11) Matera, S.; Schneider, W. F.; Heyden, A.; Savara, A. Progress in accurate chemical kinetic modeling, simulations, and parameter estimation for heterogeneous catalysis. *ACS Catal.* **2019**, *9*, 6624−6647.

(12) Grabow, L. C.; Mavrikakis, M. Mechanism of methanol synthesis on Cu through CO_2 and CO hydrogenation. *ACS Catal.* **2011**, *1*, 365−384.

(13) Tian, H.; Rangarajan, S. Leveraging thermochemistry data to build accurate microkinetic models. *J. Phys. Chem. C* **2020**, *124*, 5740−5748.

(14) Bhandari, S.; Rangarajan, S.; Mavrikakis, M. Combining computational modeling with reaction kinetics experiments for elucidating the in situ nature of the active site in catalysis. *Acc. Chem. Res.* **2020**, *53*, 1893−1904.

(15) Stegelmann, C.; Schiødt, N. C.; Campbell, C. T.; Stoltze, P. Microkinetic modeling of ethylene oxidation over silver. *J. Catal.* **2004**, *221*, 630−649.

(16) Salciccioli, M.; Stamatakis, M.; Caratzoulas, S.; Vlachos, D. G. A review of multiscale modeling of metal-catalyzed reactions: Mechanism development for complexity and emergent behavior. *Chem. Eng. Sci.* **2011**, *66*, 4319−4355.

(17) Stegelmann, C.; Andreasen, A.; Campbell, C. T. Degree of rate control: How much the energies of intermediates and transition states control rates. *J. Am. Chem. Soc.* **2009**, *131*, 8077−8082.

(18) Campbell, C. T. The degree of rate control: A powerful tool for catalysis research. *ACS Catal.* **2017**, *7*, 2770−2779.

(19) Avanesian, T.; Gusmão, G. S.; Christopher, P. Mechanism of CO_2 reduction by H_2 on Ru(0001) and general selectivity descriptors for late-transition metal catalysts. *J. Catal.* **2016**, *343*, 86−96.

(20) Foley, B. L.; Bhan, A. Degree of rate control and De Donder relations − An interpretation based on transition state theory. *J. Catal.* **2020**, *384*, 231−251.

(21) Mao, Z.; Campbell, C. T. Apparent activation energies in complex reaction mechanisms: A simple relationship via degrees of rate control. *ACS Catal.* **2019**, *9*, 9465−9473.

(22) Jørgensen, M.; Grönbeck, H. Connection between macroscopic kinetic measurables and the degree of rate control. *Catal. Sci. Technol.* **2017**, *7*, 4034−4040.

(23) Mao, Z.; Campbell, C. T. Kinetic isotope effects: Interpretation and prediction using degrees of rate control. *ACS Catal.* **2020**, *10*, 4181−4192.

(24) Norskov, J. K.; Bligaard, T.; Kleis, J. Rate control and reaction engineering. *Science* **2009**, *324*, 1655−1656.

(25) Avanesian, T.; Christopher, P. Scaled degree of rate control: Identifying elementary steps that control differences in performance of transition-metal catalysts. *ACS Catal.* **2016**, *6*, 5268−5272.

(26) Foley, B. L.; Bhan, A. Degrees of rate control at non(pseudo)-steady-state conditions. *ACS Catal.* **2020**, *10*, 2556−2564.

(27) Wolcott, C. A.; Medford, A. J.; Studt, F.; Campbell, C. T. Degree of rate control approach to computational catalyst screening. *J. Catal.* **2015**, *330*, 197−207.

(28) Iooss, B.; Lemaître, P. *Uncertainty Management in Simulation-Optimization of Complex Systems*; Springer, 2015; pp 101−122.

(29) Sobol, I. M.; Kucherenko, S. Derivative based global sensitivity measures and their link with global sensitivity indices. *Math. Comput. Simulat.* **2009**, *79*, 3009−3017.

(30) Sobol, I. M. Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates. *Math. Comput. Simulat.* **2001**, *55*, 271−280.

(31) Li, G.; Wang, S.-W.; Rabitz, H.; Wang, S.; Jaffé, P. Global uncertainty assessments by high dimensional model representations (HDMR). *Chem. Eng. Sci.* **2002**, *57*, 4445−4460.

(32) Li, G.; Rosenthal, C.; Rabitz, H. High dimensional model representations. *J. Phys. Chem. A* **2001**, *105*, 7765−7777.

(33) Kramer, Z. C.; Gu, X.-K.; Zhou, D. D. Y.; Li, W.-X.; Skodje, R. T. Following molecules through reactive networks: surface catalyzed decomposition of methanol on Pd(111), Pt(111), and Ni(111). *J. Phys. Chem. C* **2014**, *118*, 12364−12383.

(34) Zádor, J.; Zsély, I. G.; Turányi, T. Local and global uncertainty analysis of complex chemical kinetic systems. *Reliab. Eng. Syst. Saf.* **2006**, *91*, 1232−1240.

(35) Döpking, S.; Plaisance, C. P.; Strobusch, D.; Reuter, K.; Scheurer, C.; Matera, S. Addressing global uncertainty and sensitivity in first-

principles based microkinetic models by an adaptive sparse grid approach. *J. Chem. Phys.* **2018**, *148*, 034102.
(36) Sutton, J. E.; Guo, W.; Katsoulakis, M. A.; Vlachos, D. G. Effects of correlated parameters and uncertainty in electronic-structure-based chemical kinetic modelling. *Nat. Chem.* **2016**, *8*, 331−337.
(37) Sudret, B. Global sensitivity analysis using polynomial chaos expansions. *Reliab. Eng. Syst. Saf.* **2008**, *93*, 964−979.
(38) Campbell, C.; Daube, K. A surface science investigation of the water-gas shift reaction on Cu(111). *J. Catal.* **1987**, *104*, 109−119.
(39) Wellendorff, J.; Silbaugh, T. L.; Garcia-Pintos, D.; Nørskov, J. K.; Bligaard, T.; Studt, F.; Campbell, C. T. A benchmark database for adsorption bond energies to transition metal surfaces and comparison to selected DFT functionals. *Surf. Sci.* **2015**, *640*, 36−44.
(40) Wellendorff, J.; Lundgaard, K. T.; Møgelhøj, A.; Petzold, V.; Landis, D. D.; Nørskov, J. K.; Bligaard, T.; Jacobsen, K. W. Density functionals for surface science: Exchange-correlation model development with Bayesian error estimation. *Phys. Rev. B: Condens. Matter Mater. Phys.* **2012**, *85*, 235149.
(41) Medford, A. J.; Wellendorff, J.; Vojvodic, A.; Studt, F.; Abild-Pedersen, F.; Jacobsen, K. W.; Bligaard, T.; Norskov, J. K. Assessing the reliability of calculated catalytic ammonia synthesis rates. *Science* **2014**, *345*, 197−200.
(42) Savara, A.; Walker, E. A. CheKiPEUQ Intro 1: Bayesian parameter estimation considering uncertainty or error from both experiments and theory. *ChemCatChem* **2020**, DOI: 10.1002/cctc.202000953.

https://dx.doi.org/10.1021/acscatal.0c03150
ACS Catal. 2020, 10, 13535−13542