Detangling hair using feedback-driven robotic brushing

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***Abstract*— The brushing of hair requires a complex understanding of the interaction between soft hair fibers and the soft brushing device. It is also reliant on having both visual and tactile information. Guided by a recently developed model of soft tangled fiber bundles, we develop a method for optimizing hair brushing by robots which seeks to minimize pain and avoid the build up of jammed entanglements. Using an experimental setup with a custom force measuring sensor and a soft brush end effector, we perform closed-loop experiments on hair brushing of different curliness. This utilizes computer vision to assess the curliness of the hair, after which the hair is brushed using a closed loop controller. To demonstrate this approach hair brushing experiments have been performed on a wide variety of wigs with amount of curl. In addition to hair brushing the insight provided by this model driven approach could be applied to brushing of fibers for textiles, or animal fibers.**

I. INTRODUCTION

Soft robotics is rapidly furthering our understanding of the design, control and fabrication of softer structures and systems [1]. It is also extending our understanding of how we interact with complex, soft objects and environments [2] through the provision of improved modeling techniques and controllers. This expands the range of materials and environments within which robots can operate effectively, allowing robots to perform some of the more complex tasks that humans perform with ease. One such application where this is particularly true is in assistance and care robots [3]. With the globally increasing population, longer life expectancy and growing demands on health care systems, the use of robots in personal care and assistance is one area where robots could make a significant humanitarian impact [4].

With current advances in soft robotic technologies, machine learning and modelling, developing robots for care and health care applications is becoming increasing feasible [5]. Within this domain one task which has had limited exploration is hair-brushing. Although a routine tasks for humans, it relies on a complex understanding between the interaction between the deformable brush bristles and the soft hair fibers, and requires both visual and tactile feedback. Hair brushing is typically a self-care task, however for the elderly, young, or those who can not physically perform the task, it has been shown that having assistance in this task benefits both mental and physical health [6].

Within the domain of robots for personal care, there have been a number of notable examples. There has been the development of robotic systems for hair washing [7], shaving

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Fig. 1. Experimental setup for the investigation of robotic hair brushing showing the wig, sensorized brush and robot arm.

and make-up assistant robot [8], [9], and rehabilitation robots [10] . There is also increasing interest in the possibilities for robotic care-givers, or robots that support human care-givers[11]. To allow robots to extend their task solving abilities to more complex tasks such as hairbrushing, we need not only novel safe hardware, but also an understanding of the complex behavior of the soft hair and tangled fibers.

The goal of this work is to develop a platform to explore the complex task of the manipulation and brushing of hair fibers–in particular, to develop a model of soft fiber bundles which considers the complex tangling behavior when undergoing manipulation or brushing interactions. The ability to manipulate soft fiber bundles has a wide range of applications including hair brushing or textile fiber manipulation. In this project we will focus on how the model can be applied to the problem of hair brushing, developing a control approach to allow a robot with a sensorized soft bristle brush to comb hair using an approach which minimizes the 'pain' felt by the user, and the time spend undertaking the procedure.

This task is complex as every head of hair is different, and the interaction between hairs when combing is highly complex, with jamming and tangles forming depending on the brushing strategy used. If the incorrect brushing strategy is used, the process can be very painful and damaging to the hair. Thus, we need to understand the interactions between the soft hair strands to allow the principles behind an efficient hair brushing to be identified. In addition, we need to develop methods of incorporating sensory information (visual and force feedback) to create an effective brushing strategy. Whilst there has been some prior modelling and investigation of hair brushing and combing, the focus has predominantly been on the mechanical, dynamic and visual properties of hair [12]–[14], opposed to the tangling and combing behavior.

Our approach to robot hair brushing utilizes a model of entangled soft fiber bundles as sets of entwined double helices. This provides an improved understanding of hair brushing and entangled hairs and allows us to identify the key characteristics of soft hair which influence brushing strategies. To brush and manipulate the hair we have created a soft bristle end effector which is sensorized to allow forces during brushing to be measured. Using this sensorized soft brushing end effector (Fig. 1), we can explore how this model can be used to optimize the control strategy for hair brushing. Using this setup, we propose a control strategy that uses force-feedback from the sensorized brush. We demonstrate this approach on a number of wigs which represent a wide range of different hair styles and types, and demonstrate how our control approach utilizes our improved understanding of hair combing to minimize both time and pain. As such, we make a number of contributions:

- A robotic setup with a sensorized soft brushing end effector that allows for hair-brushing to be investigated
- A control strategy based on a novel model of the knotting and tangling of soft bundles of fibers
- Demonstration of the approach on a number of different hair types and different measured of tangle

The remainder of the paper is organized as follows. Section II introduces the methods, including the model of tangled fibers used, and how this has been used to inform the development of a control strategy. Section III presents the experimental setup for the hair brushing experiments, and the methods of sensorizing the system and implementing the control strategy. The results from experiments are given in Section IV, which is followed by Section V, a discussion and of the outcomes of the paper and a review of the main conclusions.

II. Methods

A. Model of Untangling Natural Curl

The complexity of modeling hair combing arises from the many-body and extended nature of the entangled hair interactions. To better understand the behavior of untangling, we consider a minimal model for the combing of a double helix [15]. Although combing involves interactions with the many-body bundle of entangled hairs, recent experimental results in [15] suggest that the dominant interactions are of a pairwise, i.e. two-body nature. Thus a model that considers two entwined helices of the same chirality, clamped at the top end and hanging freely at the bottom (Fig. 2), with a single, stiff comb bristle moving through the double-helix is sufficient to explain the mechanics of combing qualitatively.

There are two key phenomena involved in this interaction: link current and over/under winding. Over- and under-winding describe the stretching of the clamped end of the helices (between the fixed upper base and the comb) and the compression of the free end, respectively. When the comb is moved through the helix there is generally an initial rise in the force extension curve before a leveling off, corresponding to an initial over-winding of the helix in front of the tine before a current of link [16], a topological measure of entanglement, leaves through the free end and softens the curve.

Considering these phenomena can also help explain the utility of the minimal model in relation to the complex reality. The likelihood of an interaction more complex than the pairwise one modeled by the entwined helices increases with the curliness (later defined by Eqn 1) of the hairs [15]. However, the closer an entanglement is to the free end, the closer it is to being removed from the hairs. This is captured in the dynamical balance between the jamming-like over-winding of the helices and the detangling loss of link through the free end. Additionally, combing nearer the free ends reduces this many-body complexity concern as pairwise links are can be removed before neighboring hairs can get more involved.

From this model, there are two key results which inform the development of a brushing controller. Firstly, small pitch and more tightly wound helices lead to larger forces required, thus the curlier the hair, the greater the resultant brushing forces. Secondly, starting combing nearer to the free end allows link to propagate out the free end faster and easier. This allows for untangling before the link density in front of the comb becomes too high and the comb/tine begins to jam. Hence we recover the intuitive result that one should start combing ones hair far from the scalp and gradually work upward. Thus, to detangle hair, the hair should be brushed starting from near the free ends and working up, optimally removing a given amount of link entanglements with each brush stroke.

Fig. 2a shows an illustration of this model, and the key parameters which define the double helix model. These are P, the length of one entanglement which corresponds to the length of one period of the double helix, R the radius of the curl, r the radius of this hair and l_0, the length of the hair. From this, a measure of the amount of curl can be given by P/R, and the number of entanglements to remove through brushing is given by P/l. In this work we are going to make the simplifying assumption that r is approximately constant for human hair.

We note that the simulations used in [15] can be extended to model the brushing of a many-body tangle of hair. In such a model, each hair would be treated as a Cosserat elastic rod [17] and each comb tine as a rigid rod, with all tines and hairs interacting via contact forces. A small-scale demonstration of how such a bundle would be initialized is shown in the Supporting Information of [15]. We propose applying this simulation scheme to a large bundle of hairs combed by several tines. In such a simulation, one could track over what length scale of combing two-body interactions accumulate into many-body interactions and produce more complex tangles in the bundle. These results could further

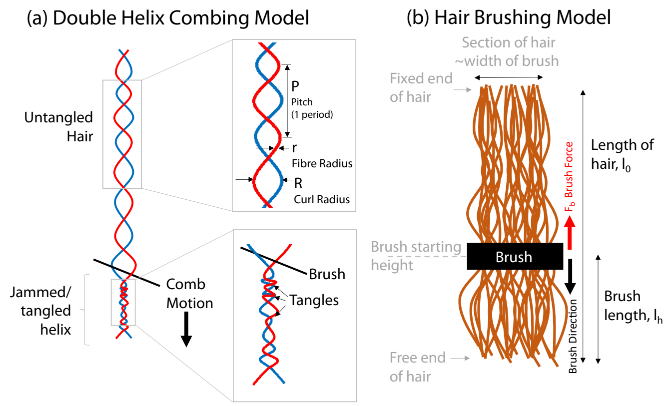

Fig. 2. a) Simplified minimal model of a double helix of hair undergoing combing. b) Model of the hair brushing problem showing the hair brush and the quantities that describe the problem.

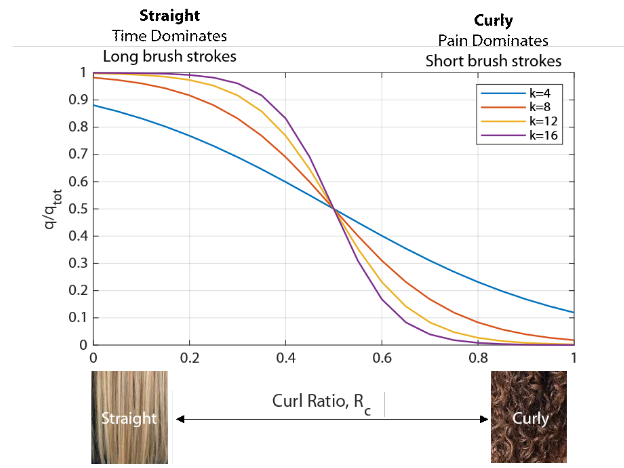

Fig. 3. Illustration of the optimization problem showing the different regions in which different costs (i.e. pain or time) dominate.

inform the control algorithm to design a combing strategy that maximizes the stroke length of each brushing iteration while avoiding the build-up of two-body interactions into many-body tangles.

B. Problem Definition

Given this model, we can define the hair brushing problem. This is the removal of entanglements from hair which is hanging free from the top of the head. The hair can be described by the length, l_0, and we define a curliness ratio as given in (1) as $R_C = R/P$ as:

$$R_C = R/P \tag{1}$$

such that the number of entanglements in the section of the hair can be given by:

$$q_0 = l_0/P \tag{2}$$

A control approach must be developed to optimize for brushing time, whilst ensuring that the maximum force when brushing (F_{max}) does not exceed the pain threshold F_T. This pain threshold has been identified experimentally by brushing hair with a sensorized hairbrush, and identifying the force at which pain starts to be felt. Using the principles identified by the model, the brushing should start from the free end and work upwards to gradually release entanglements. The length and height of the brushing process should be optimized for a given hair type and length. The brush length, b_l, for a specific iteration of brushing is measured from the bottom of the hair to the height at which the brushing starts. The combing device is kept parallel to the and brushed downwards. The force during brushing is measured in the direction of brushing.

C. Optimization

The cost of hair brushing has two key components: pain inflicted and time taken. The brushing length (b_l), and hence number of entanglements removed in each brush must optimized with respect to these two costs. The relative costs of these two metric varies for different amount of curl and entanglements (R_C).

The cost related to pain, can be considered to be proportional to the maximum force experienced, and thus is high for longer brushing lengths on curlier hair. In contrast, the cost relating to time is determined by the number of brushing cycles, so longer brush lengths across any hair will minimize this cost. For straight hair with low values of R_C the time cost will dominate, thus longer brushing value should be selected. In contrast, for curlier hair, the pain cost will dominate so shorter brush lengths should be selected. From the model of hair entangling, we know the relationship between force and curliness is not linear, thus we need a more representative way of formulating this optimization problem.

We propose using an adapted Sigmoid function, specifically the Logistic function, to map from curliness (R_C), to the selection of the initial brushing height (b_{l0}). A Sigmoid function offers a monotonically decreasing function in which for low values of curliness we get a large brushing length, whereas for high curliness we get a shorter brushing length. This is illustrated in Fig. 3. Although other non-linear functions are suitable, this function is particular suited as a single parameter can be used to capture the rate of change of the 'S' shaped properties of the function. Using this function, we can describe the optimal brushing height as a function of R_C using:

$$b_{l0} = \frac{q}{q_{tot}} = \frac{1}{1 + e^{(k(R_c+0.5))}} \tag{3}$$

where the value of k should be optimized experimentally by finding the maximum brush length (b_l/l_0) for a hair with a given curl ratio that does not exceed F_T.

D. Control Approach

Using the optimized mapping identified in (3) an estimation of the initial brushing height can be identified. Although, as a base strategy we can increment each brushing iteration by this brush height until the full length is brushed, this

Algorithm 1: Closed-loop brushing control

Take picture, compute R_C eqn (5), b_{l0} eqn (3);
Initialize sensor and arm;
iteration, i = 0;
$b_l[i] = b_{l0}$;
while *finished != true* **do**
    move to brushing height $b_l[i]$;
    start brushing in -z direction ;
    f=0 ;
    while *brush height* $\leq$ *bottom of hair* **do**
        f= [f; read force] ;
        if $f \geq f_T$ **then**
            stop brushing, break;
        end
    end
    stop brushing;
    $f_{max} = max(f)$;
    if $f_{max} \geq f_T$ **then**
        $b_l[i+1] = b_l[i](F_T/F_{max})$;
    else
        $b_l[i+1] = b_l[i] + b_i - b_i(F_{max}/0.5F_T)$;
    end
    $i = i + 1$;
end

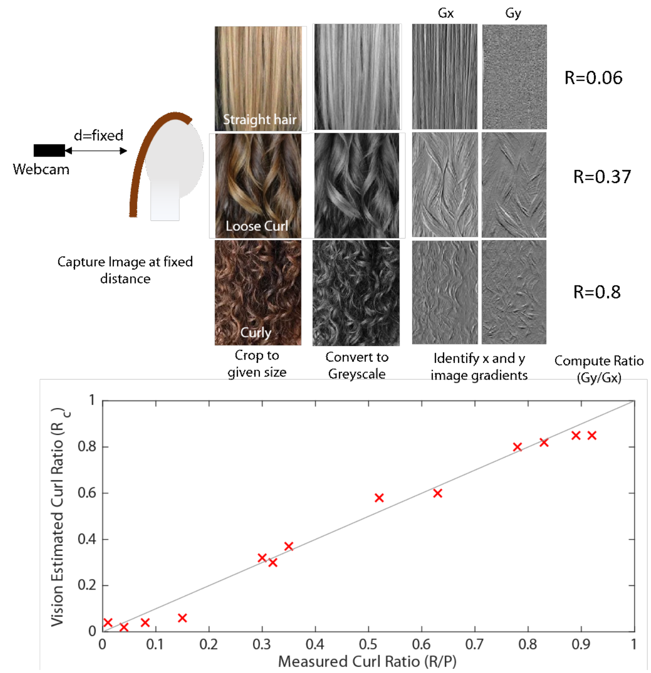

Fig. 4. Top - the vision pipeline on example images: cropping, conversion to greyscale, determining gradients and then determining the ratio. Bottom - figure showing the R_c captured using vision, and the measured (R/P) value.

assumes ideal behavior and that each entanglement is fully removed and the hair behaves as a perfect helix.

Using a closed-loop controller allows for online optimization and customization in response of the specific hair. In particular, we can ensure if there is particularly knotted or entangled areas, we can adapt the brushing height to ensure the maximum pain threshold F_T is not exceeded. We propose the following closed-loop controller for iterative brushes which is based upon the optimized brush length identified in (3). Under this controller, after starting from the height optimized for the specific curl ratio, the following round of brushing is adapted is the force experienced is too high, allowing for repeated brushing at low heights if knots occur, or entanglements are not brushed out in a single iteration. In addition, the incremental length added can be increased is forces are significantly lower than the maximum threshold.

An additional component of this controller, is identifying when the hair section is fully brushed. We define this as when the full length of the hair is being brushed, and there is minimal change in the maximum force between brushing iterations (i). This boolean condition is defined as:

$$finished = \begin{cases} 1, & if|f_{max,\,i} - f_{max,\,i-1}| < 0.4 \ \ \& \ \ b_l \geq l_0 \\ 0 & otherwise \end{cases} \tag{4}$$

where the threshold value was found experimentally.

E. Vision Pipeline

The control approach relies on having an estimation of the metric that defines the curl of the hair, R_C. To estimate this, an image is taken at a fixed distance away from the head. The image is cropped so to include only the hair and is converted to a greyscale image. From the greyscale image the x and y direction image gradients (G_x and G_y) are found using the Sobel gradient operator [18]. Gradients have been shown to be useful in other texture identification tasks [19]. Images of the hair which are straighter have a far higher component of edges in the x direction, where are curlier has a more equal distribution of edges in the x and y plane. This is visually demonstrated in the gradient plots in Fig. 4. Thus, by taking the ratio between the sum of these two gradient fields we can calculate a metric that provides a ratio of the straightnesss', to the 'curl' of the hair:

$$R_c = abs|\frac{\sum G_y}{\sum G_x}| \tag{5}$$

For straight hair,$\sum G_y$ is very small, and thus the ratio R_c is approximately 0, where as for curly hair $\sum G_y$ starts to become closer to $\sum G_x$, resulting in a ratio which is closer to 1. By placing an upper bound on R_c of 1, R_c provides a metric of describing the ratio of P to r.

To demonstrate that these metrics are representative and sufficiently accurate for presenting an reasonable measure of the curliness of hair, for a number of different images of hair, we have computed the ratio, R_c, and have plotted against the measured ratio (R/P), which forms the ground truth for this experiment. This results are shown in shown in Fig. 4.

III. EXPERIMENTAL SETUP

To explore the control strategies for hair brushing we have developed an instrumented soft brushing end effector which

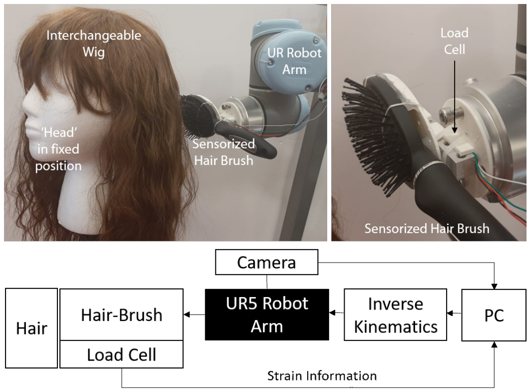

Fig. 5. Top) The experimental setup and the sensorized hair brush. Bottom) Block diagram of the hair brushing system.

is mounted on the end of a robot arm. The setup is shown in Fig.5. The robot arm allows for a wide variety of trajectories to be performed and provide speed control. The end effector is constructed from a hairbrush which has soft bristles. The brush is mounted on a pain which connects to a load cell which is mounted on the end effector of the robot arm. This allows the force normal to the load cell to be measured. The signal from the load cell is amplified using an instrumental amplifyer and measured with an microcontroller. The load cell has been calibrated, and the readings are converted into Newtons and send over serial to the control PC at 10Hz. The control policy is implemented, and the hairbrush controlled by sending position control commands to the the robot arm to which the hair brush is attached. A system diagram of the system is shown in also shown in Fig.5.

To test the system and controller, we use a head mount on which various wigs can be attached using hair pins. The head is mounted on a variable height mount such that the bottom of the hair is kept in in a fixed location relative to the robot and camera setup. Each wig is set in to a 'pre-brushed' state by turning upside down and shaking for 10-15 seconds. This returns the wigs to an entangled state, allows experiments to repeated with a similar state of entagledness.

The wig is placed such that the free end of the hair is at a fixed point relative to the arm. A webcam is also mounted in a fixed position relative to this position, and is used to capture the image to determine the curl ratio R_C. The brushing height can then be determined, and the brush moved to this height before the brushing regime starts. The brush is moved from the brush height in a fixed plane downwards to below the end of the hair. This is repeated until the robot determines the processes is finished. The depth of brushing (horizontal distance between the hair and the brush) is kept constant for all experiments and was chosen experimentally such that brush bristles interact with both straight hair which has less volume, and also curly hair.

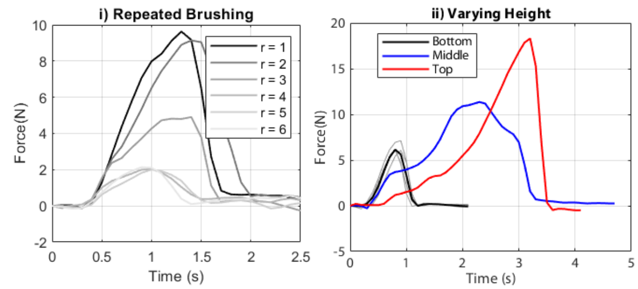

Fig. 6. Exemplar timeseries from the sensorized hairbrush when brushing i) the same hair at a repeated high for 6 iterations, ii) varying the height of brushing.

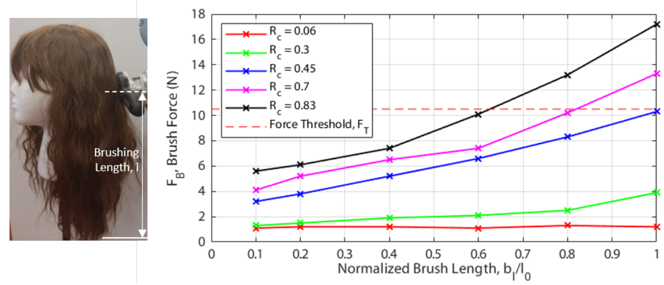

Fig. 7. Variation in maximum force with normalized brushing length for hair with different curliness.

IV. EXPERIMENTAL RESULTS

A. Exploration of hair brushing

To demonstrate and validate the model, we have performed a set of brushing experiments for which we show the progression of the force with brushing time. First, we show repeated brushing of a single wig with the brushing height fixed. We see the expected rise in force as the hair fibers start to jam, after which the force applied to the brush overcomes this jamming force and brushes free. The brush force is the highest for the first brush, however, reduces with brushing iterations as with each brush a number of entanglements are removed. After three brush cycles the maximum forces measured on each brush cycle starts to plateau, demonstrating that entanglements and jamming has been removed. This validates the method of identifying when brushing has finished as given in (4).

We next perform a similar experiment but where the brushing height is varied. Each height experiment was repeated 5 times, with the hair 'reset' between experiments. The average of each set of experiments is shown in Fig. 6b. When brushing the entire length of the hair fibers we see the build up forces, with the rate increasing as the fibers jam, until the brush pulls through these entanglements. When the brush height is reduced, the maximum force that is reached reduces significantly, we also see more reliability in the repeated experiments.

The model suggests that the velocity of brushing should have a minimal effect on the experienced force. To test this we use the hair brushing platform to explore the variation in force measured for different speeds of brushing. Fig. 8

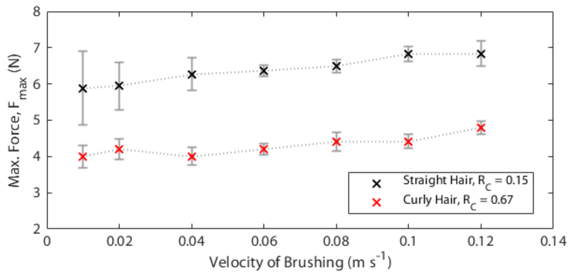

Fig. 8. Average maximum force for different velocities and orientations of brushing for a fixed brush length, and on the same hair ($R_C = 0.4$).

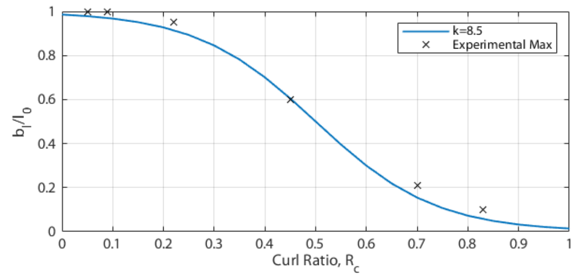

Fig. 9. Optimized sigmoid function (3) to allow identification of the optimal starting brush height for hair with different amounts of curl.

shows the variation in maximum brushing force for different brushing speeds. This is performed for the two wigs, one straight, one curl, with the brushing height fixed. The wig reset between each tests.

As expected we see that the velocity has minimal effect on the brushing force, in particular for straight hair. For curly hair, whilst the maximum force is approximately constant, there is far greater uncertainty for slower motion. This suggests that although velocity does not affect the maximum force, a slightly faster speed reduces variability in the forces experienced. The maximum speed of this experiments was limited by the capabilities of the robot arm, thus remains relatively low; further work could explore the effects of faster more jerky hair brushing motions.

B. Optimization

To optimize the selection of brushing height, we must fit the sigmoid function proposed in (3) to experimental data. For a number of wigs with different hair types, we identify the maximum normalized brushing length that can be performed without exceeding the brushing force threshold, F_T. We can then fit a sigmoid function to these data points to identify the optimal value of k to ensure efficient brushing without exceeding the maximum force threshold.

The results of this process shown in Fig. 9, where the optimal value of k is found to be 8.5. It can be seen that the sigmoid function has been fitted to the points such that the curve is lower than any of the experimental results to ensure that the maximum force threshold is not exceed at any points. There appears to be a close fitting between the experimental results and the sigmoid function, validiating this choice of function.

C. Brushing Demonstration

To demonstrate the hair brushing approach, we test the controller on a number of wigs which have been selected to show a variety of different lengths, hair types and curl ratios. We have selected four examples with varying hair type to demonstrate the process. In each of these, we show the iterative brush height that is selected, and the measured maximum force for each of these iterations. This is shown in Fig. 10 alongside before and after photos of the hair. The visual results of the hair before and after are also shown. In can be see than in the case of straight hair, the approach is simplistic, and the estimated ratio perform works well, and minimal iterations are required. As the amount of curl increases, more brush strokes are required, and the forces increase. Finally, when we move to the curliest hair we see the ratio predicted using vision is initially too high, and thus the brush height must be reduced to remove some of the entanglements that form. In all cases we can visually see an improvement of the hair after brushing.

To benchmark and contrast the performance of the control approach developed, we compare the performance in comparison to a 'naive' brushing approach and human brushing. In the 'naive' approach we brush at a fixed interval which increases by thirds each time (e.g. $B_l = 0.33, 0.67, 1$). We also compare it a human baseline where the human was asked to brush the section of hair using the sensorized hairbrush. Whilst this human baseline considers only a single person and therefore their specific approach, it does provide an approximate order of magnitude of their performance. All control strategies were repeated on wigs with a variety of curl ratio with the total time to brush, and also the maximum force recorded. This comparison between approaches is shown in Fig. 11.

When considering the maximum force that is achieved the optimized robot solutions show similar response to that of human brushing, although the human performs marginally better at the extremes. The naive approach leads to significantly higher forces for the curly hair, significantly exceeding the pain threshold ($F_T \;\; 10N$), however, this is a faster than the optimized solution. Throughout the experiments the human performs faster, however the robots movement is limited by the capabilities of the robot arm, and thus there is considerable room to optimize this process and close the performance gap in comparison to humans.

V. DISCUSSION & CONCLUSION

In this work we introduce an experimental model for the combing of entangled chiral hairs using a visuo-motor feedback loop that builds on recent work to comb a double helix [15]. Our model provides significant insight in to the behaviors of the combing of hair with respect to the number of entanglements, and how these can be efficiently and effectively brushed out by choosing appropriate brushing lengths. We propose using a sigmoid function to provide selection of

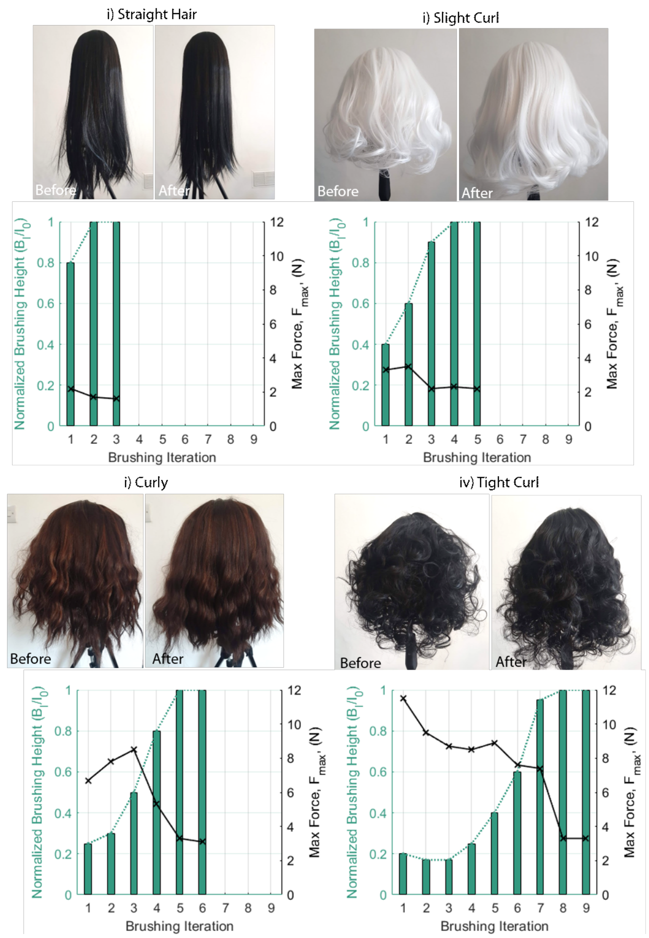

Fig. 10. Demonstration of hair with increasing curliness using the control approach showing the maximum force and normalized brushing length for each iteration.

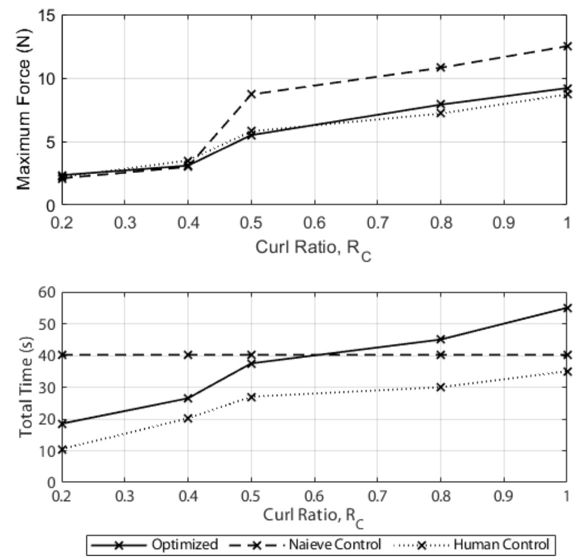

Fig. 11. Comparison between developed controller, human brushing and also a naive controller for the consideration of maximum force (top) and brushing time (bottom)..

brushing height for different hair, where computer vision is used to identify the curliness of hair.

Using this model, we have developed an control approach for brushing hair for various hair types. By developing a sensorized soft brush end effector, we have validated this approach experimentally and benchmarked it in comparison to a human lead approach and a naive approach. Whilst this work has demonstrated effective hair-brushing, there is significant further work for improvement, including investigating more complex trajectories and increasing the complexity of our model through many-hair combing simulations. Performing such simulations would allow us to increase the complexity of our algorithm to capture nuances in the differences in the mechanics of combing across different hair types and hair styles.

Another area for future work is performing more realistic experiments on humans, to gain more subjective feedback from the experimental subjects. Pain is a highly complex phenomena, and to truly understand the performance of the robot with respect to this metric, human experiments are required.

One advantage of this approach presented is the adaptability of the approach, due to the use of a sigmoid function to optimize the starting brush height. This allows the model to be easily adapted when the tasks is varied, for example of a different brushing device, or a different tasks - such as brushing wider/complex complex textile based fibers. By fitting the sigmoid function to a set of minimal experimental data the approach can be readily adapted.

In addition to further exploration of hair brushing, the model of entwined soft hairs could have significant further applications. For example, it could be used to assist with the development of robotic approaches to dealing with ropes, and fibrous systems, or even robots that efficiently manipulate spaghetti.

ACKNOWLEDGMENT

We would like to thank the NSF for their support under Grant EFRI-1830901.

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